An exploratory study of heavy-light semileptonics using distillation

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17th June 2019

THE UNIVERSITY of EDINBURGH

Lattice 19
Wuhan·China | June 16-22

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Outline

1. Introduction
   - Motivation
   - heavy-light semileptonic decays
2. Distillation (in Grid)
3. Analysis
   - First glance at data
   - Comparison to Z_2 noise with sequential solves
4. Conclusions & Outlook

Motivation

- Semi-leptonic decays give access to CKM matrix elements, e.g. $|V_{cd}|$ from $D \to \pi\ell\nu$.
- current tension in lepton flavour universality detected in $R(D^{(*)}) = \frac{\mathcal{B}(B\to D^{(*)}\tau\nu_\tau)}{\mathcal{B}(B\to D^{(*)}\ell\nu_\ell)}$
  - $\Rightarrow$ clear first-principles determination needed
- Interesting processes suffer from bad signal-to-noise ratios
  - $\Rightarrow$ advanced methods needed
- Testing ground for recently implememted distillation code in Grid (& Hadrons)

GRID

[github.com/paboyle/Grid]

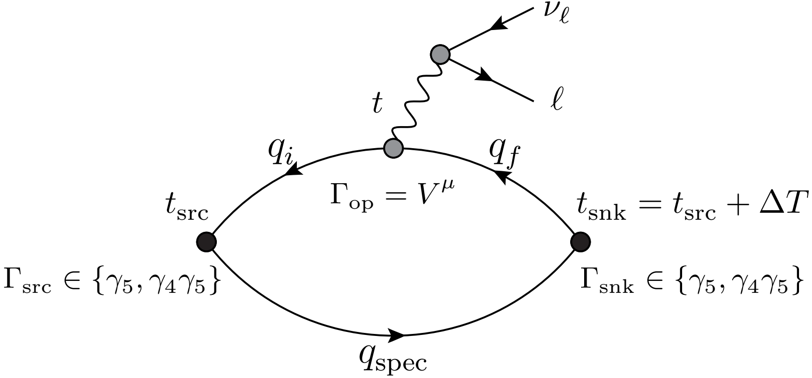
Hadrons

[github.com/paboyle/Grid/tree/develop/Hadrons]

Related RBC/UKQCD charm-to-bottom programme talks:

- Semileptonic B decays with RHQ b quarks **[Mon 16:50, Ryan Hill]**
- Neutral meson mixing and related observables in the $D_{(s)}$ and $B_{(s)}$ meson systems **[Tue 15:40, Tobias Tsang]**
- Semileptonic form factors for exclusive $B_s \to K\ell\nu$ and $B_s \to D_s\ell\nu$ decays **[Tue, Poster, Oliver Witzel]**

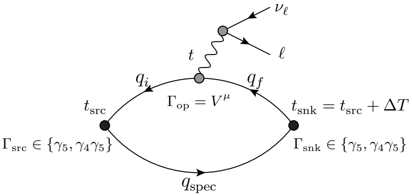
heavy-light semileptonic decays



for $D(B) \to \pi$: $q_{\mathrm{spec}} = l$, $q_{\mathrm{i}} = h_1, h_2$, $q_{\mathrm{f}} = l$

heavy-light semileptonic decays

Pointlike weak operator
Three-point functions C_3^{pp} have the form:



$$\tilde{C}_3^{pp}(t, \Delta T) \propto \left(e^{E_i t} - e^{E_f(\Delta T - t)}\right).$$

we want to map out a large momentum transfer, so $D(\mathrm{p_i} = 0) \to \pi(\mathrm{p_f})$ are best suited.
We study different values of source-sink separations ΔT and different momenta $\mathbf{p}_f$.

Distillation

- small ΔT: cannot isolate ground state
- large ΔT: bad signal-to-noise ratio

$\Rightarrow$ need good smearing technique and advanced numerical method

- Distillation with (stochastic) LapH might help with both **[arxiv:0905.2160]** **[arxiv:1104.3870]**

Distillation

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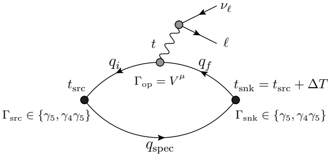
idea: construct a smearing matrix from N_{vec} low modes of the 3D lattice Laplacian (tuneable parameter):

$$S_{xy}(t) = \sum_{k=1}^{N_{\mathrm{vec}}} V_k(x,t) V_k^{\dagger}(y,t)\,,$$

- expensive only once, assembly of correlation functions (momenta, Γ-structure) is the last step of the computation
- Smeared propagators can be projected into smaller subspace
  - $\Rightarrow$ Can be re-used in other projects

Distillation

We want to compute:



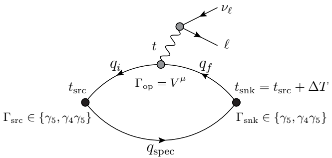
$$C_3 = \langle \Gamma_{\rm snk} D^{-1}(t_{\rm snk}, t)\Gamma_{\rm op} D^{-1}(t, t_{\rm src})\Gamma_{\rm src} D^{-1}(t_{\rm src}, t_{\rm snk})\rangle \,.$$

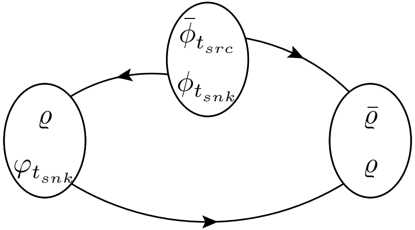
The central semileptonic insertion must be unsmeared and we want to smear with $S(t) = V(t)V^\dagger(t)$ (spatial sums implicit) only at the $\Gamma_{\rm src}, \Gamma_{\rm snk}$ insertions:

$$\begin{aligned} C_3 = \langle &\Gamma_{\rm snk} V(t_{\rm snk})V^\dagger(t_{\rm snk}) D^{-1}(t_{\rm snk}, t)\Gamma_{\rm op} D^{-1}(t, t_{\rm src}) V(t_{\rm src})V^\dagger(t_{\rm src}) \\ &\Gamma_{\rm src} V(t_{\rm src})V^\dagger(t_{\rm src}) D^{-1}(t_{\rm src}, t_{\rm snk}) V(t_{\rm snk})V^\dagger(t_{\rm snk})\rangle \,. \end{aligned}$$

Distillation

using γ_5 hermiticity we can invert some of the quark lines and can write, using meson fields





- $\phi_{t'}(x,t) = D^{-1}_{x,t,x',t'} V_x(t')$ are the unsmeared sinks, i.e. a solve on a source with support on timeslice t'. [arxiv:1403.5575]
- Constructed from sources on two timeslices, $t_{\rm src}$ and $t_{\rm snk}$.
- ϕ cannot be projected into a smaller subspace (i.e. into a perambulator object).

Lattice setup

- RBC-UKQCD's 2+1 flavour domain wall fermions
- feasibility study on $L^3 \cdot T = 24^3 \cdot 64$ lattice, $m_\pi \approx 340\mathrm{MeV}$
- one light quark ($am_l = 0.005, M_5 = 1.8, L_s = 16$)
- two different heavy-quark masses with $am_h = 0.58$ and $am_h = 0.64$ using a stout-smeared action ($\rho = 0.1, N = 3$) with $M_5 = 1.0, L_s = 12$ and Moebius-scale $= 2$ **[arxiv:1812.08791]**

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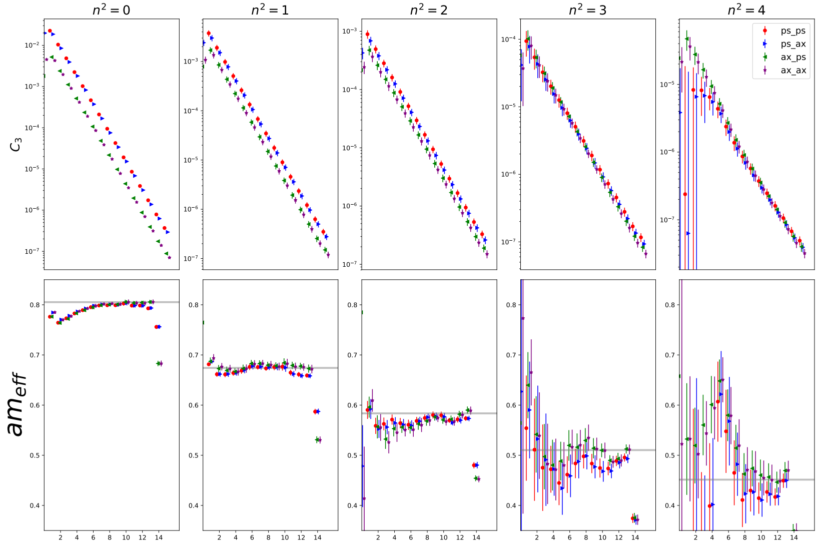
current level of statistics:

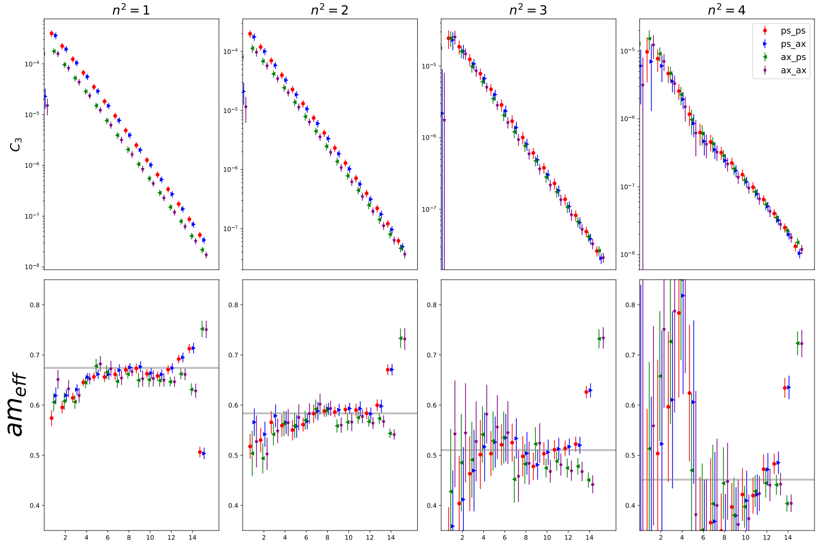
- 2 configurations
- 16 solves on each config
- 4 different $\Delta T = 12, 16, 20, 24$
- all lattice momenta up to $n^2 = 4$

V^0, $\Delta T = 16$, $m_h = 0.58$



grey bands: free energies $E_D - E_\pi(p_f)$

V^i, $\Delta T = 16$, $m_h = 0.58$



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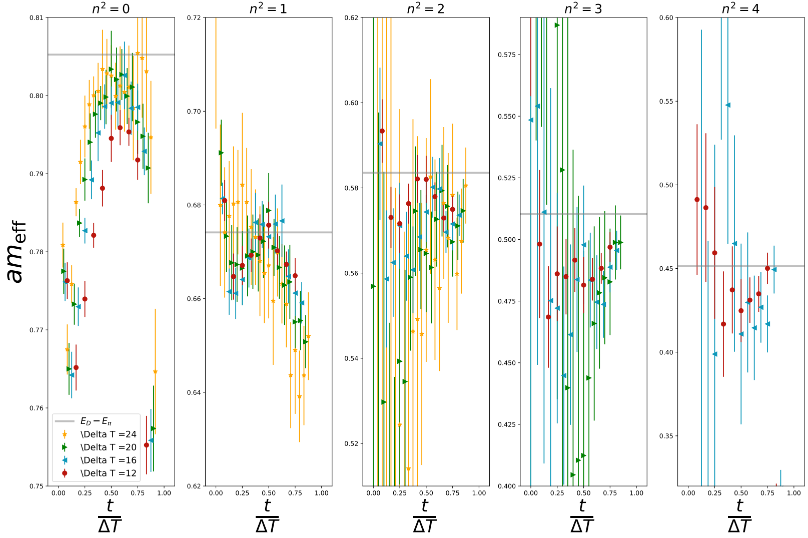
V^0, comparison of ΔT (pp channel), $m_h = 0.58$



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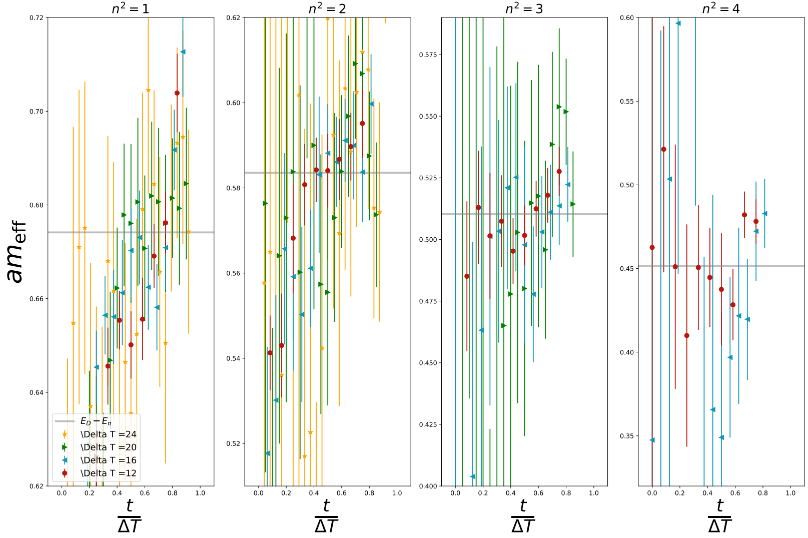
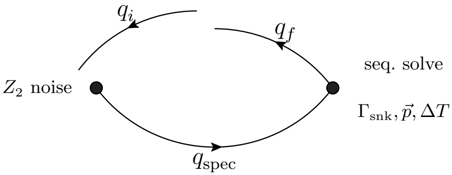


grey bands: free energies $E_D - E_\pi(p_f)$

Z_2 noise with sequential solves

- Z_2 noise at source
- exploit γ_5 hermiticity for q_i quark
- compute q_spec quark line
- sequential solve on q_spec quark line

$\Rightarrow$ 1 inversion for each $\vec{p}$, Γ_snk, ΔT



Z_2 noise with sequential solves

- Z_2 noise at source
- exploit γ_5 hermiticity for $q_{\rm i}$ quark
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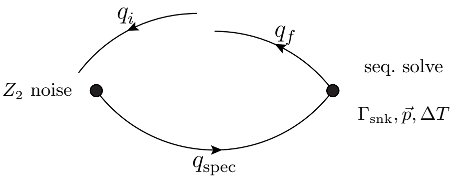
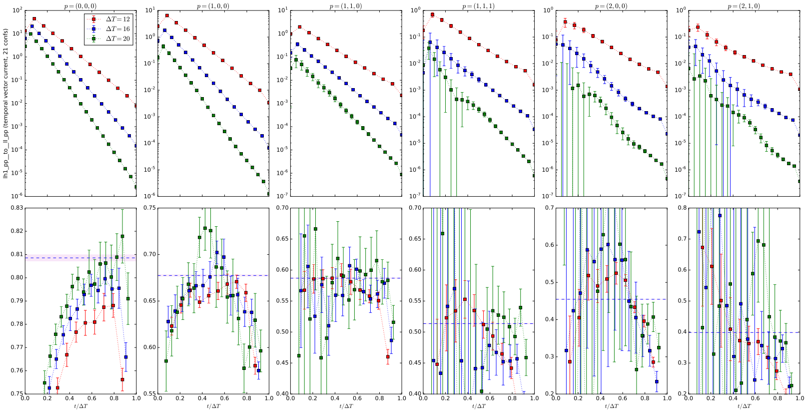


current level of statistics:

- 21 configurations
- 2 solves on each config
- 3 different $\Delta T = 12, 16, 20$
- one lattice momentum each up to $n^2 = 5$

V^0, comparison of ΔT (pp channel)



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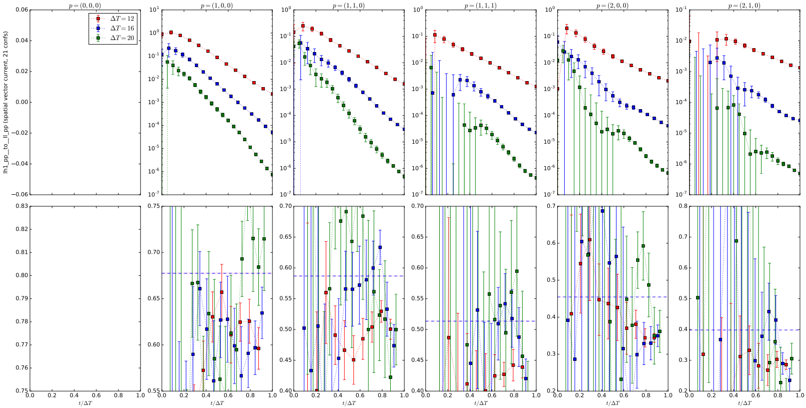


cost of production per configuration

Presented here: $q_{\rm spec} = l$, $q_f = l$, $q_i = h_{1,2,\ldots}$

	Distillation	Z_2 seq.
#Inv / conf / $t_{\rm src}$	$N_{vec} \times 4$	$N_{\Delta T} \times N_{\vec{p}} \times N_{\Gamma_{\rm snk}}$
total #Inv	7680	1008

$\Rightarrow$ $\approx$ factor 8 in inversion cost for Distillation, plus non-negligible cost for meson fields (another factor 2-3)

- $\approx$ factor 2-6 error reduction in Distillation, depending on **p** and $\Gamma_{\rm op}$
  - $\Rightarrow$ no clear winner

Possible setup we are interested in:
$D \to \pi$, $D \to K$, $D_s \to K$, $D_{(s)} \to D^{'}_{(s)}$:
$q_{\rm spec} = l, s$, $q_f = l, s, h$, $q_i = h_{1,2,\ldots}$

	Distillation	Z_2 seq.
#Inv / conf / $t_{\rm src}$	$N_{vec} \times 4$	$N_{\Delta T} \times N_{\vec{p}} \times N_{\Gamma_{\rm snk}} \times \#\{q_{\rm spec}, q_f\}$
total #Inv	unchanged	increased by factor ≈ 5

$\Rightarrow$ Cost of Distillation might pay off

Conclusions & Outlook

Conclusions:

- computed 2pt-functions and 3pt-functions to study heavy-light semileptonics using the newly implented distillaton code in Grid and Hadrons
- Distillation is expensive, but has several advantages:
  - different momenta and Γ_{snk} are free
  - smearing automatically implemented
  - perambulators (inversions) can be re-used for other projects

Outlook:

- we want to study $D \to \pi$, $D \to K$, $D_s \to K$, $D_{(s)} \to D'_{(s)}$ with larger statistics on the 24^3 ensemble
- Some interesting RBC-UKQCD ensembles in production at the moment ($2.8\mathrm{GeV}$ at the physical point) **[Thu, 14:00, Robert Mawhinney]**

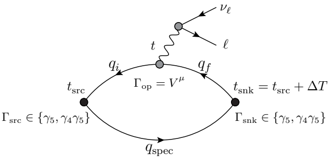
Longer-term goal:

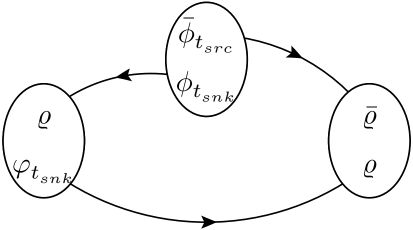
- use ensembles with smaller lattice spacing to extrapolate to physical B

Thank you!

Distillation

using γ_5 hermiticity we can invert some of the quark lines and can write, using meson fields





$$C = M_{\Gamma_{op}}(\bar{\phi}_{t_{src}}, \phi_{t_{snk}}, t) M_{\Gamma_{src}}(\varphi_{t_{snk}}, \varrho, t_{src}) M_{\Gamma_{snk}}(\bar{\varrho}, \varrho, t_{snk}) \,,$$

Distillation

$$C = M_{\Gamma_{op}}(\bar{\phi}_{t_{src}}, \phi_{t_{snk}}, t) M_{\Gamma_{src}}(\varphi_{t_{snk}}, \varrho, t_{src}) M_{\Gamma_{snk}}(\bar{\varrho}, \varrho, t_{snk}) \,,$$

where

$$\varrho^{[n,d]}_{a\alpha}(\vec{x}, t) = \sum_{k,l,t',\beta} v_{ka}(\vec{x}; t) P^{[d]}_{k\alpha,l\beta}(t, t') \rho^{[n]}_{l\beta}(t') \,,$$

$$\phi^{[n,d]}_{t,a\alpha}(\vec{x}', t') = \sum_{a,b,\beta,t,\vec{x}} D^{-1}_{a\alpha,b\beta}(\vec{x}', t'; \vec{x}, t) \varrho^{[n,d]}_{b\beta}(\vec{x}, t) \,,$$

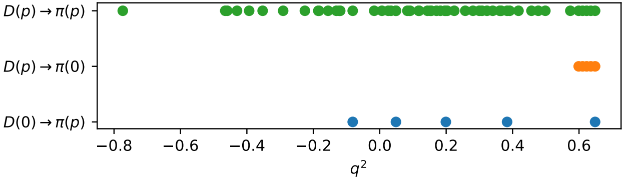
$$\tau^{[n,d]}_{t,k\alpha}(t') = \sum_{a,b,\beta,t,\vec{x}',\vec{x}} v_{ka}(\vec{x}'; t')^{\dagger} \phi^{[n,d]}_{t,a\alpha}(\vec{x}', t') \,,$$

$$\varphi^{[n,d]}_{t,a\alpha}(\vec{x}, t) = \sum_{k} v_{ka}(\vec{x}; t) \tau^{[n,d]}_{k\alpha}(t) \,.$$

Distillation

$$
\begin{aligned}
M_{\Gamma}^{[n_1,d_1;n_2,d_2]}&(\varphi_q, \varrho_{q'}, t, \vec{p}) \\
&= \sum_{\vec{x},a,\alpha,\beta} e^{-i\vec{p}\cdot\vec{x}} (\varphi_q)_{a\alpha}^{[n_1,d_1]}(\vec{x},t) \Gamma_{\alpha\beta} (\varrho_{q'})_{a\beta}^{[n_2,d_2]}(\vec{x},t) \,.
\end{aligned}
$$

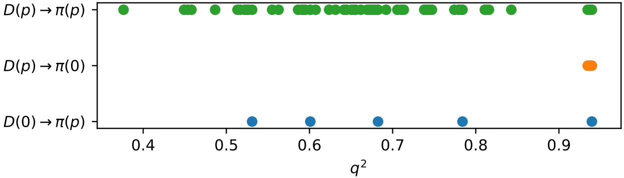
q^2 range, 24^3 ensemble



$$q^2 = (E_D - E_\pi)^2 - (\mathbf{p}_D - \mathbf{p}_\pi)^2$$
$$= m_D^2 + \mathbf{p}_D^2 + m_\pi^2 + \mathbf{p}_\pi^2 - 2\sqrt{m_D^2 + \mathbf{p}_D^2}\sqrt{m_\pi^2 + \mathbf{p}_\pi^2} - (\mathbf{p}_D - \mathbf{p}_\pi)^2$$

$$\mathbf{p}_{\max}^2 = 4$$

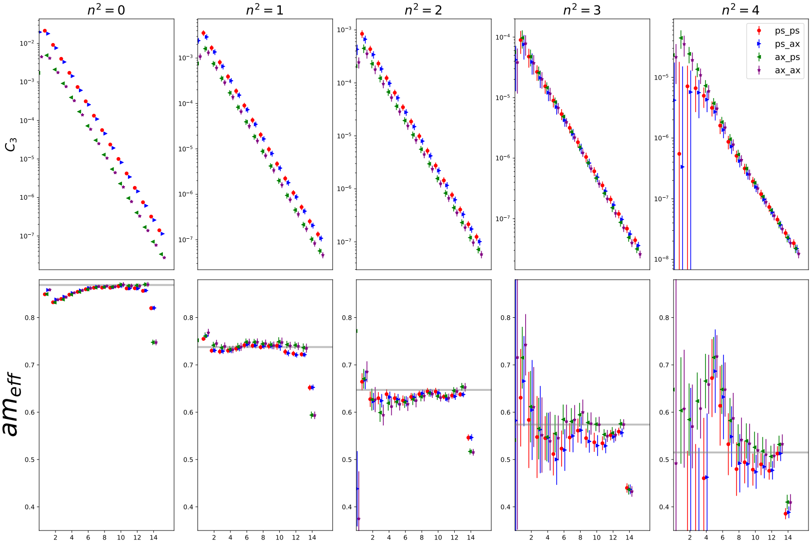
q^2 range, physical meson masses



$$q^2 = (E_D - E_\pi)^2 - (\mathbf{p}_D - \mathbf{p}_\pi)^2$$
$$= m_D^2 + \mathbf{p}_D^2 + m_\pi^2 + \mathbf{p}_\pi^2 - 2\sqrt{m_D^2 + \mathbf{p}_D^2}\sqrt{m_\pi^2 + \mathbf{p}_\pi^2} - (\mathbf{p}_D - \mathbf{p}_\pi)^2$$

$$\mathbf{p}^2_{\mathrm{max}} = 4$$

V^0, $\Delta T = 16$, $m_h = 0.64$



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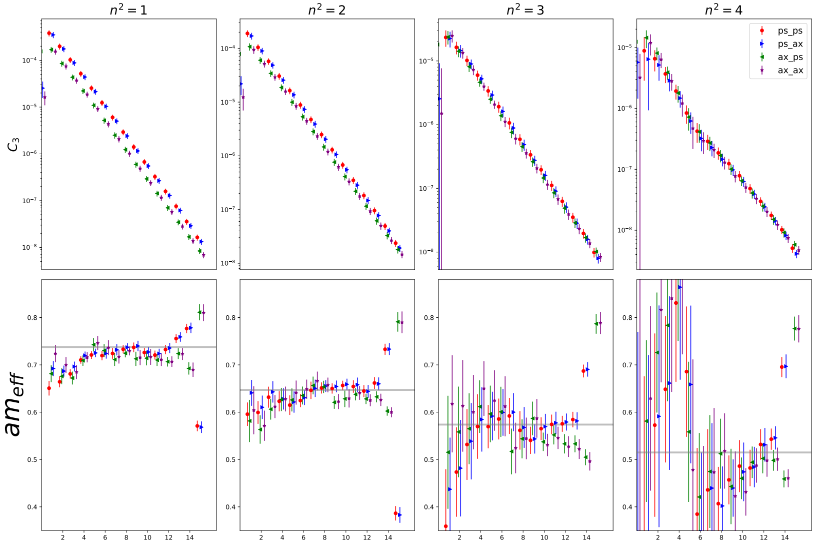

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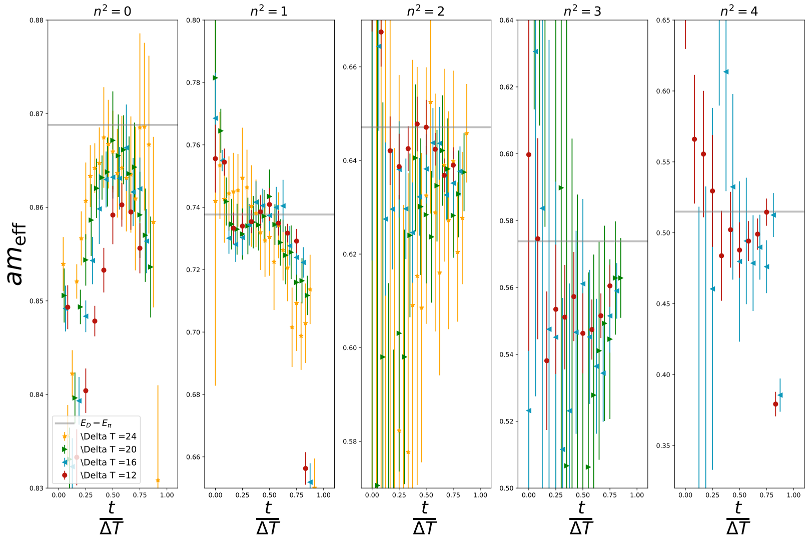
V^0, comparison of ΔT (pp channel), $m_h = 0.64$



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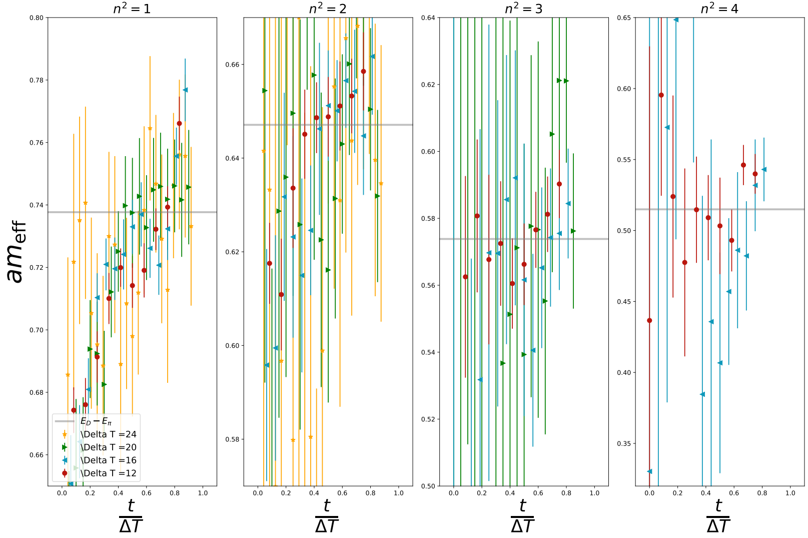

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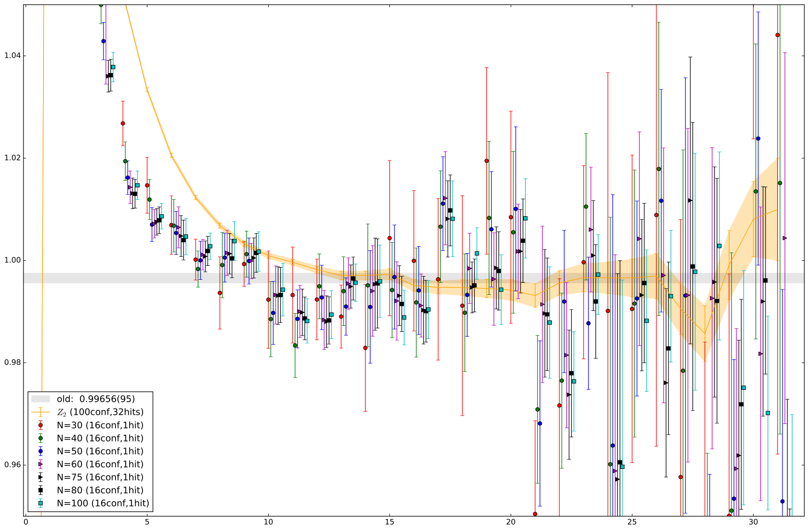
N_{vec} comparison, pp



N_{vec} comparison, ii