An exploratory study of heavy-light semileptonics using distillation

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1 Introduction
   • Motivation
   • heavy-light semileptonic decays

2 Distillation (in Grid)

3 Analysis
   • First glance at data
   • Comparison to $Z_2$ noise with sequential solves

4 Conclusions & Outlook
Motivation

• Semi-leptonic decays give access to CKM matrix elements, e.g. $|V_{cd}|$ from $D \to \pi \ell \nu$.
• Current tension in lepton flavour universality detected in $R(D^{(*)}) = \frac{B(B \to D^{(*)} \tau \nu \tau)}{B(B \to D^{(*)} \ell \nu \ell)}$ ⇒ clear first-principles determination needed
• Interesting processes suffer from bad signal-to-noise ratios ⇒ advanced methods needed
• Testing ground for recently implemented distillation code in Grid (& Hadrons)

Related RBC/UKQCD charm-to-bottom programme talks:

• Semileptonic $B$ decays with RHQ $b$ quarks [Mon 16:50, Ryan Hill]
• Neutral meson mixing and related observables in the $D_s$ and $B_s$ meson systems [Tue 15:40, Tobias Tsang]
• Semileptonic form factors for exclusive $B_s \to K \ell \nu$ and $B_s \to D_s \ell \nu$ decays [Tue, Poster, Oliver Witzel]
heavy-light semileptonic decays

\[ t_{\text{snk}} = t_{\text{src}} + \Delta T \]

\[ t_{\text{src}} \in \{ \gamma_5, \gamma_4 \gamma_5 \} \]

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\[ \Gamma_{\text{op}} = V^\mu \]

\[ \Gamma_{\text{snk}} \in \{ \gamma_5, \gamma_4 \gamma_5 \} \]

\[ q_{\text{spec}} \]

\[ q_i = h_1, h_2, q_f = l \]

for \( D(B) \rightarrow \pi \): \( q_{\text{spec}} = l, q_i = h_1, h_2, q_f = l \)
Pointlike weak operator

Three-point functions $C_{3}^{pp}$ have the form:

$$
\tilde{C}_{3}^{pp}(t, \Delta T) \propto \left( e^{E_{i}t} - e^{E_{f}(\Delta T - t)} \right).
$$

we want to map out a large momentum transfer, so $D(p_{i} = 0) \rightarrow \pi(p_{f})$ are best suited.

We study different values of source-sink separations $\Delta T$ and different momenta $p_{f}$. 
Distillation

- small $\Delta T$: cannot isolate ground state
- large $\Delta T$: bad signal-to-noise ratio

$\Rightarrow$ need good smearing technique and advanced numerical method

- Distillation with (stochastic) LapH might help with both [arxiv:0905.2160]
  [arxiv:1104.3870]

- idea: construct a smearing matrix from $N$ vec low modes of the 3D lattice Laplacian (tuneable parameter):

$$S_{xy}(t) = N \text{vec} \sum_{k=1}^{V} V_k(x,t) V_k^*(y,t),$$

- expensive only once, assembly of correlation functions (momenta, $\Gamma$-structure) is the last step of the computation
- Smeared propagators can be projected into smaller subspace $\Rightarrow$ Can be re-used in other projects
• small $\Delta T$: cannot isolate ground state
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idea: construct a smearing matrix from $N_{\text{vec}}$ low modes of the 3D lattice Laplacian (tuneable parameter):

$$S_{xy}(t) = \sum_{k=1}^{N_{\text{vec}}} V_k(x, t) V_k^\dagger(y, t),$$

• expensive only once, assembly of correlation functions (momenta, $\Gamma$-structure) is the last step of the computation
• Smeeared propagators can be projected into smaller subspace
  $\Rightarrow$ Can be re-used in other projects
We want to compute:

\[ C_3 = \left\langle \Gamma_{\text{snk}} D^{-1}(t_{\text{snk}}, t) \Gamma_{\text{op}} D^{-1}(t, t_{\text{src}}) \Gamma_{\text{src}} D^{-1}(t_{\text{src}}, t_{\text{snk}}) \right\rangle. \]

The central semileptonic insertion must be unsmeared and we want to smear with \( S(t) = V(t) V^\dagger(t) \) (spatial sums implicit) only at the \( \Gamma_{\text{src}}, \Gamma_{\text{snk}} \) insertions:

\[ C_3 = \left\langle \Gamma_{\text{snk}} V(t_{\text{snk}}) V^\dagger(t_{\text{snk}}) D^{-1}(t_{\text{snk}}, t) \Gamma_{\text{op}} D^{-1}(t, t_{\text{src}}) V(t_{\text{src}}) V^\dagger(t_{\text{src}}) \Gamma_{\text{src}} V(t_{\text{src}}) V^\dagger(t_{\text{src}}) D^{-1}(t_{\text{src}}, t_{\text{snk}}) V(t_{\text{snk}}) V^\dagger(t_{\text{snk}}) \right\rangle. \]
using $\gamma_5$ hermiticity we can invert some of the quark lines and can write, using meson fields

- $\phi_{t'}(x, t) = D^{-1}_{x,t,x',t'} V_x(t')$ are the unsmeared sinks, i.e. a solve on a source with support on timeslice $t'$. [arxiv:1403.5575]

- Constructed from sources on two timeslices, $t_{\text{src}}$ and $t_{\text{snk}}$.

- $\phi$ cannot be projected into a smaller subspace (i.e. into a perambulator object).
Lattice setup

- RBC-UKQCD’s 2+1 flavour domain wall fermions
- feasibility study on $L^3 \cdot T = 24^3 \cdot 64$ lattice, $m_\pi \approx 340\text{MeV}$
- one light quark ($a m_l = 0.005, M_5 = 1.8, L_s = 16$)
- two different heavy-quark masses with $a m_h = 0.58$ and $a m_h = 0.64$
  using a stout-smeared action ($\rho = 0.1, N = 3$) with $M_5 = 1.0, L_s = 12$ and Moebius-scale = 2 [arxiv:1812.08791]
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  $M_5 = 1.0, L_s = 12$ and Moebius-scale = 2 [arxiv:1812.08791]

Current level of statistics:

- 2 configurations
- 16 solves on each config
- 4 different $\Delta T = 12, 16, 20, 24$
- all lattice momenta up to $n^2 = 4$
$V^0$, $\Delta T = 16$, $m_h = 0.58$

grey bands: free energies $E_D - E_\pi(p_f)$
$V^i, \Delta T = 16, m_h = 0.58$

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$V^0$, comparison of $\Delta T$ (pp channel), $m_h = 0.58$

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$E_D - E_{\pi}$

grey bands: free energies $E_D - E_{\pi}(p_f)$
$Z_2$ noise with sequential solves

- $Z_2$ noise at source
- exploit $\gamma_5$ hermiticity for $q_i$ quark
- compute $q_{\text{spec}}$ quark line
- sequential solve on $q_{\text{spec}}$ quark line

$\Rightarrow$ 1 inversion for each $\vec{p}$, $\Gamma_{\text{snk}}$, $\Delta T$
Z\(_2\) noise with sequential solves

- Z\(_2\) noise at source
- exploit \(\gamma_5\) hermiticity for \(q_i\) quark
- compute \(q_{\text{spec}}\) quark line
- sequential solve on \(q_{\text{spec}}\) quark line

\[\Rightarrow 1\text{ inversion for each } \vec{p}, \Gamma_{\text{snk}}, \Delta T\]

current level of statistics:

- 21 configurations
- 2 solves on each config
- 3 different \(\Delta T = 12, 16, 20\)
- one lattice momentum each up to \(n^2 = 5\)
$V^0$, comparison of $\Delta T$ (pp channel)

lh1_pp → ll_pp (temporal vector current, 21 confs)

$p = (0, 0, 0)$

$\Delta T = 12$

$p = (1, 0, 0)$

$\Delta T = 16$

$p = (1, 1, 0)$

$\Delta T = 20$

$p = (2, 0, 0)$

$p = (2, 1, 0)$

$p = (2, 2, 0)$
$V^i$, comparison of $\Delta T$ (pp channel)
Presented here: $q_{\text{spec}} = l, q_{f} = l, q_{i} = h_{1,2,\ldots}$

<table>
<thead>
<tr>
<th>#Inv / conf / $t_{\text{src}}$</th>
<th>Distillation</th>
<th>$Z_{2}$ seq.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_{\text{vec}} \times 4$</td>
<td>$N_{\Delta T} \times N_{\vec{p}} \times N_{\Gamma_{\text{snk}}}$</td>
<td></td>
</tr>
<tr>
<td>total #Inv</td>
<td>7680</td>
<td>1008</td>
</tr>
</tbody>
</table>

$\Rightarrow \approx$ factor 8 in inversion cost for Distillation, plus non-negligible cost for meson fields (another factor 2-3)

- $\approx$ factor 2-6 error reduction in Distillation, depending on $p$ and $\Gamma_{\text{op}}$

$\Rightarrow$ no clear winner

Possible setup we are interested in:
$D \rightarrow \pi, D \rightarrow K, D_{s} \rightarrow K, D_{(s)} \rightarrow D'_{(s)}$: $q_{\text{spec}} = l, s, q_{f} = l, s, h, q_{i} = h_{1,2,\ldots}$

<table>
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<td>$N_{\text{vec}} \times 4$</td>
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<td></td>
</tr>
<tr>
<td>total #Inv</td>
<td>unchanged</td>
<td>increased by factor $\approx 5$</td>
</tr>
</tbody>
</table>

$\Rightarrow$ Cost of Distillation might pay off
Conclusions:

- computed 2pt-functions and 3pt-functions to study heavy-light semileptonics using the newly implemented distillaton code in Grid and Hadrons
- Distillation is expensive, but has several advantages:
  - different momenta and $\Gamma_{snk}$ are free
  - smearing automatically implemented
  - perambulators (inversions) can be re-used for other projects

Outlook:

- we want to study $D \rightarrow \pi$, $D \rightarrow K$, $D_s \rightarrow K$, $D_{(s)} \rightarrow D'_{(s)}$ with larger statistics on the $24^3$ ensemble
- Some interesting RBC-UKQCD ensembles in production at the moment (2.8GeV at the physical point) [Thu, 14:00, Robert Mawhinney]

Longer-term goal:

- use ensembles with smaller lattice spacing to extrapolate to physical $B$
Thank you!
using $\gamma_5$ hermiticity we can invert some of the quark lines and can write, using meson fields

$$C = M_{\Gamma_{op}}(\overline{\phi}_{t_{src}}, \phi_{t_{snk}}, t) M_{\Gamma_{src}}(\varphi_{t_{snk}}, \varphi, t_{src}) M_{\Gamma_{snk}}(\overline{\varphi}, \varphi, t_{snk}),$$
Distillation

\[ C = M_{\Gamma_{\text{op}}} (\bar{\phi}_{t_{\text{src}}} , \phi_{t_{\text{snk}}} , t) M_{\Gamma_{\text{src}}} (\phi_{t_{\text{snk}}} , \rho , t_{\text{src}}) M_{\Gamma_{\text{snk}}} (\bar{\rho} , \rho , t_{\text{snk}}) , \]

where

\[ \rho_{\alpha}^{[n,d]}(\vec{x}, t) = \sum_{k,l,t',\beta} v_{ka}(\vec{x}; t) P_{k\alpha,l\beta}(t, t') \rho^{[n]}_{l\beta}(t') , \]

\[ \phi_{\alpha}^{[n,d]}(\vec{x}', t') = \sum_{a,b,\beta,t,\vec{x}} D_{a\alpha,b\beta}^{-1}(\vec{x}', t'; \vec{x}, t) \rho_{\beta}^{[n,d]}(\vec{x}, t) , \]

\[ \tau_{\alpha}^{[n,d]}(t') = \sum_{a,b,\beta,t,\vec{x}',\vec{x}} v_{ka}(\vec{x}'; t') \phi_{\alpha}^{[n,d]}(\vec{x}', t') , \]

\[ \phi_{\alpha}^{[n,d]}(\vec{x}, t) = \sum_{k} v_{ka}(\vec{x}; t) \tau_{k\alpha}^{[n,d]}(t) . \]
\[ M_{\Gamma}^{[n_1,d_1;n_2,d_2]}(\varphi_q, \varrho_{q'}, t, \vec{p}) = \sum_{\vec{x}, a, \alpha, \beta} e^{-i\vec{p} \cdot \vec{x}} (\varphi_q)_{a \alpha}^{[n_1,d_1]}(\vec{x}, t) \Gamma_{\alpha \beta} (\varrho_{q'})_{a \beta}^{[n_2,d_2]}(\vec{x}, t). \]
\[ q^2 = (E_D - E_\pi)^2 - (p_D - p_\pi)^2 \]

\[ = m_D^2 + p_D^2 + m_\pi^2 + p_\pi^2 - 2\sqrt{m_D^2 + p_D^2}\sqrt{m_\pi^2 + p_\pi^2} - (p_D - p_\pi)^2 \]

\[ p_{\text{max}}^2 = 4 \]
$q^2$ range, physical meson masses

\[
q^2 = (E_D - E_\pi)^2 - (p_D - p_\pi)^2
\]

\[
= m_D^2 + p_D^2 + m_\pi^2 + p_\pi^2 - 2\sqrt{m_D^2 + p_D^2}\sqrt{m_\pi^2 + p_\pi^2} - (p_D - p_\pi)^2
\]

\[
p^2_{\text{max}} = 4
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\(V^0, \Delta T = 16, m_h = 0.64\)

Grey bands: free energies \(E_D - E_\pi(p_f)\)
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$N_{vec}$ comparison, pp

old: 0.99656(95)

$Z_2$ (100 conff, 32 hits)

$N=30$ (16 conff, 1 hit)

$N=40$ (16 conff, 1 hit)

$N=50$ (16 conff, 1 hit)

$N=60$ (16 conff, 1 hit)

$N=75$ (16 conff, 1 hit)

$N=80$ (16 conff, 1 hit)

$N=100$ (16 conff, 1 hit)
$N_{\text{vec}}$ comparison, ii

\begin{itemize}
  \item $Z_2$ (100 conf, 32 hits)
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\end{itemize}