A strategy for the calculation of disconnected contributions to QED and strong isospin-breaking effects

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Outline

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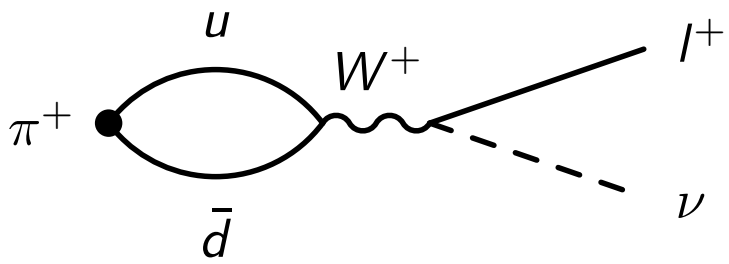
- Leptonic decays
- All-to-all approach & Meson fields
- Isospin-breaking corrections
- Disconnected isospin-breaking corrections
- Summary

[github.com/paboyle/Grid]

Hadrons

[github.com/paboyle/Grid/tree/develop/Hadrons]

Leptonic decays



- The leading order (α^0) decay rate is:

$$\Gamma(\pi^+ \to l^+\nu) = \frac{m_\pi}{8\pi} G_F^2 |f_\pi|^2 |V_{ud}|^2 m_l^2 \left[1 - \frac{m_l^2}{m_\pi^2}\right]^2$$

- The isospin breaking correction to the rate:

$$\Gamma(\pi^+ \to l^\pm \nu_l) = \Gamma_\pi^{\text{tree}} \cdot \left[1 + \delta R_\pi\right]$$

[1502.00257] [1711.06537]

- QED isospin breaking (IB):

$$\langle O \rangle = \langle O \rangle_0 + \frac{e^2}{2} \frac{\partial^2}{\partial e^2} \langle O \rangle \Big|_{e=0}$$

[1303.4896]

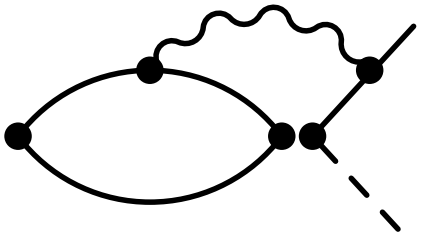
- Strong isospin breaking:

$$\langle O \rangle = \langle O \rangle_0 + \frac{(m_d - m_u)}{2} \langle O S \rangle$$

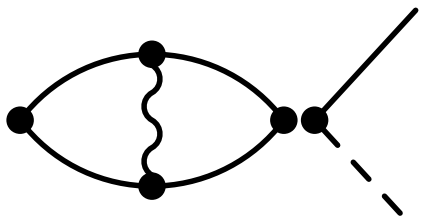
[1110.6294]

IB corrections Leptonic decay rates

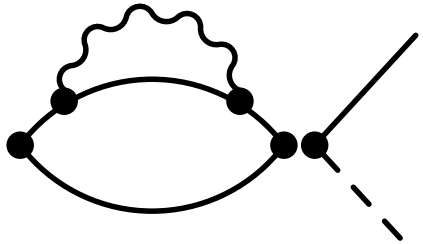
- Connected QED isospin breaking corrections to the decay rate: (A.Portelli's talk)



Quark-lepton Diagram

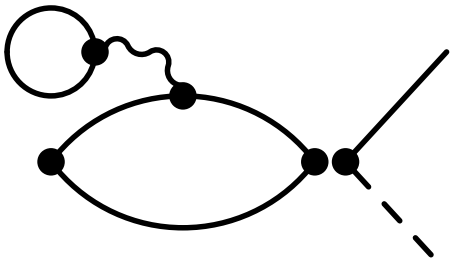


Exchange diagram

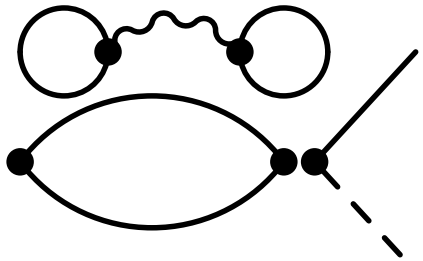


Self-energy diagram

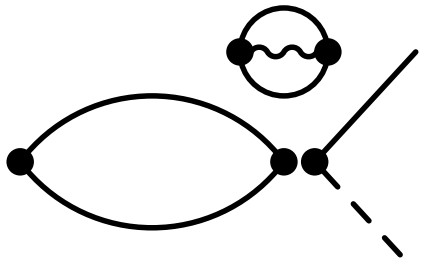
- Disconnected qq QED isospin breaking corrections to the decay rate: [1502.00257]



Tadpole diagram

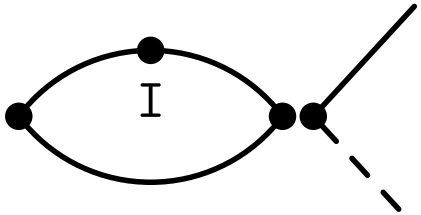


Specs diagram

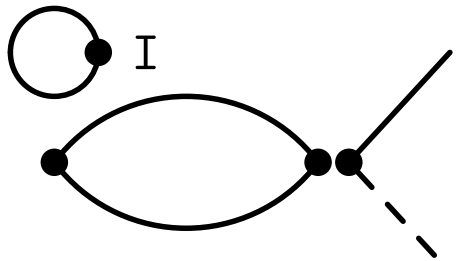


Burger diagram

- Strong isospin breaking corrections:



Strong diagram



Strong disconnected diagram

All-to-all propagators

- All-to-all propagator:

$$D^{-1}_{A2A}(x,y) = \sum_{i=1}^{N_l+N_h} v_i(x) w_i^\dagger(y) = \sum_{l=1}^{N_l} v_l(x) w_l^\dagger(y) + \sum_{h=N_l+1}^{N_l+N_h} v_h(x) w_h^\dagger(y)$$

[0505023]

All-to-all propagators

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[0505023]

- Low modes (from eigenvectors):

$$v_l(x) = \phi_l(x)$$
$$w_l(y) = \phi_l(y)/\lambda_l$$

- High modes (from stochastic solves):

$$v_h(x) = D^{-1}_{\text{defl}} \eta_h(x)$$
$$w_h(y) = \eta_h(y)$$

All-to-all propagators

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- All-to-all propagator:

$$D^{-1}_{A2A}(x,y) = \sum_{i=1}^{N_l+N_h} v_i(x) w_i^\dagger(y) = \sum_{l=1}^{N_l} v_l(x) w_l^\dagger(y) + \sum_{h=N_l+1}^{N_l+N_h} v_h(x) w_h^\dagger(y)$$

[0505023]

- Low modes (from eigenvectors):

$$v_l(x) = \phi_l(x)$$
$$w_l(y) = \phi_l(y)/\lambda_l$$

- High modes (from stochastic solves):

$$v_h(x) = D^{-1}_{\text{defl}} \eta_h(x)$$
$$w_h(y) = \eta_h(y)$$

- Ensembles:

vol.	a^{-1}	m_π	N	N_l
$48^3 \times 96$	1.73GeV	140MeV	19	2000
$24^3 \times 64$	1.78GeV	340MeV	25	600

[1411.7017]

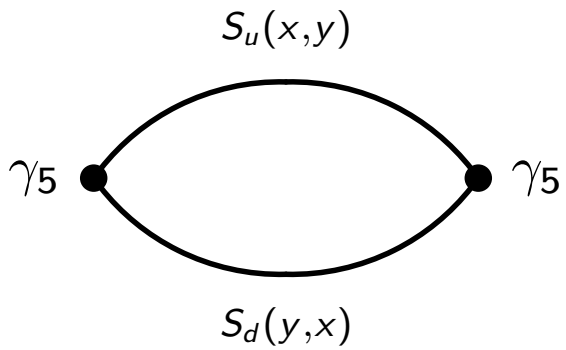
[0804.0473]

- QED:
  - Local vector currents
  - Feynman gauge
  - QEDL [0804.2044]
  - stochastic photons
    $\Delta_{\mu\nu}(x-y) = \langle A_\mu(x) A_\nu(y) \rangle$

[1704.06561] [1706.05293]

Meson fields

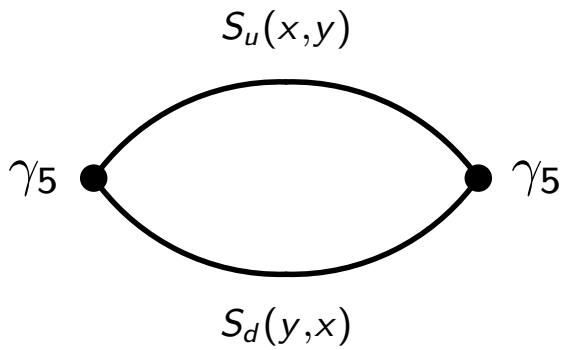
- Two point function:

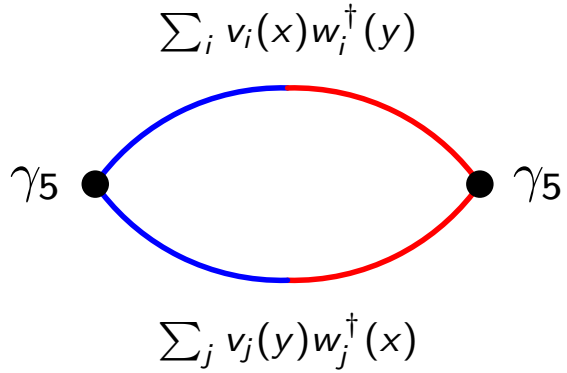


$$\sum_{\vec{x},\vec{y}} tr\left[\gamma_5 S_u(x,y)\gamma_5 S_d(y,x)\right]$$

Meson fields

- Two point function:



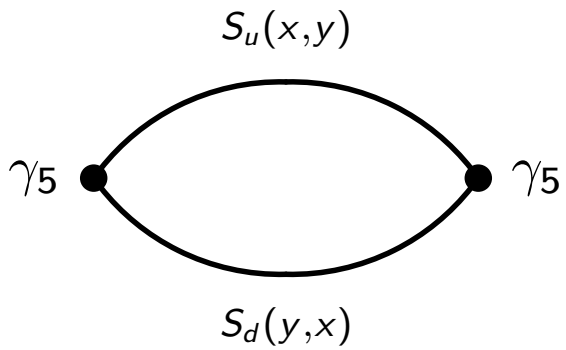


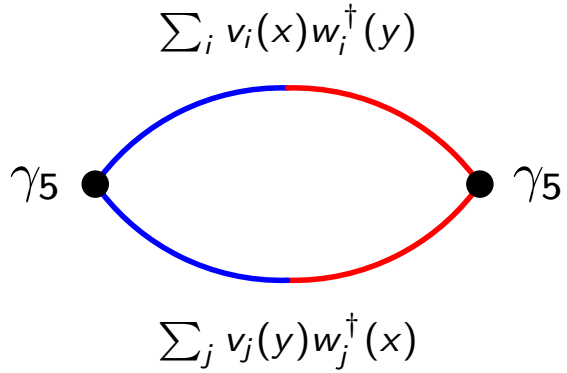
$$\sum_{\vec{x},\vec{y}} tr\left[\gamma_5 S_u(x,y)\gamma_5 S_d(y,x)\right]$$

$$\sum_{\vec{x},\vec{y}} tr\left[\gamma_5 \sum_i v_i(x) w_i^\dagger(y)\gamma_5 \sum_j v_j(y) w_j^\dagger(x)\right]$$

Meson fields

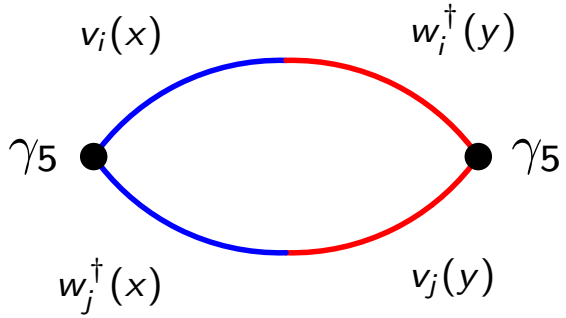
- Two point function:





$$\sum_{\vec{x},\vec{y}} tr\left[\gamma_5 S_u(x,y)\gamma_5 S_d(y,x)\right]$$

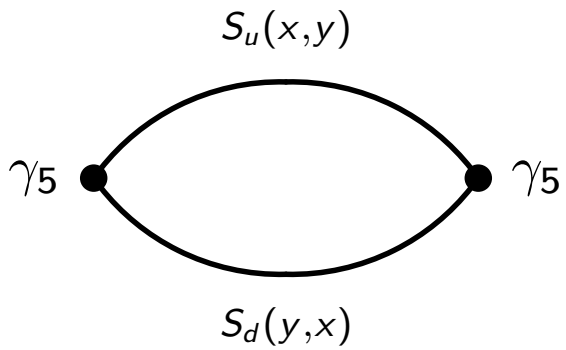
$$\sum_{\vec{x},\vec{y}} tr\left[\gamma_5 \sum_i v_i(x) w_i^\dagger(y)\gamma_5 \sum_j v_j(y) w_j^\dagger(x)\right]$$

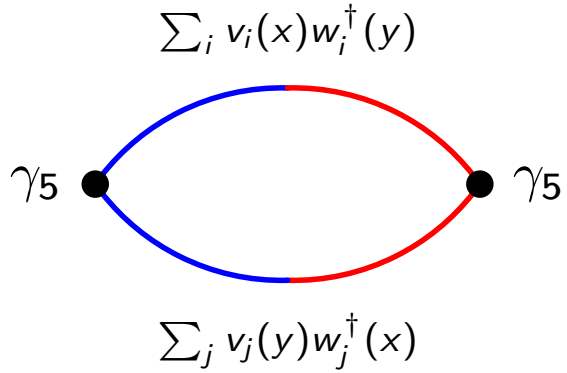


$$\sum_{i,j} tr\left[\sum_{\vec{y}} w_i^\dagger(y)\gamma_5 v_j(y) \sum_{\vec{x}} w_j^\dagger(x)\gamma_5 v_i(x)\right]$$

Meson fields

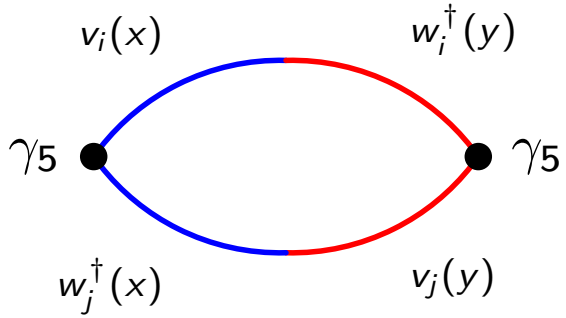
- Two point function:





$$\sum_{\vec{x},\vec{y}} tr\left[\gamma_5 S_u(x,y)\gamma_5 S_d(y,x)\right]$$

$$\sum_{\vec{x},\vec{y}} tr\left[\gamma_5 \sum_i v_i(x) w_i^\dagger(y)\gamma_5 \sum_j v_j(y) w_j^\dagger(x)\right]$$



- Meson Field:

$$\Pi_{ji}(t_x;\gamma_5) = \sum_{\vec{x}} w_j^\dagger(x)\gamma_5 v_i(x)$$

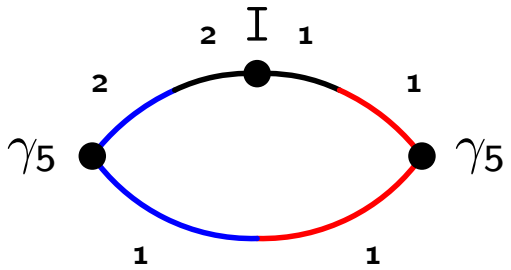
$$\sum_{i,j} tr\left[\sum_{\vec{y}} w_i^\dagger(y)\gamma_5 v_j(y) \sum_{\vec{x}} w_j^\dagger(x)\gamma_5 v_i(x)\right]$$

Strong isospin breaking using Meson fields

- Strong isospin breaking:

$$\langle \mathcal{O}\bar{q}(z)\mathtt{I}q(z)\rangle = \langle \sum_{\vec{x},\vec{y},z} tr(\gamma_5 S_l(x,z)\mathtt{I}S_l(z,y)\gamma_5 S_l(y,x))\rangle$$

$$= \langle \sum_{i,j,k}\sum_{t_z} \Pi_{ij}(t_x,\gamma_5)\Pi_{jk}(t_z,\mathtt{I})\Pi_{ki}(t_y,\gamma_5)\rangle$$

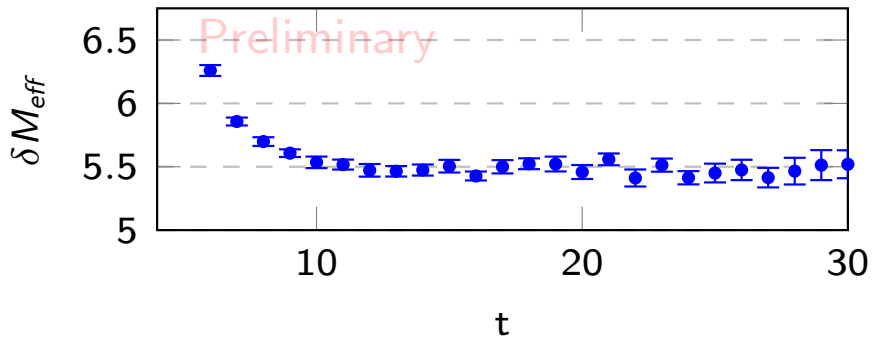


Strong diagram

- Mass correction:

$$\delta C_\pi(t) = C_\pi(t)\left[\frac{\delta \mathcal{A}_\pi}{\mathcal{A}_\pi} - \delta M_\pi t\right] \qquad \delta M_{eff} = \frac{\delta C_\pi(t)}{C_\pi(t)} - \frac{\delta C_\pi(t+1)}{C_\pi(t+1)}$$

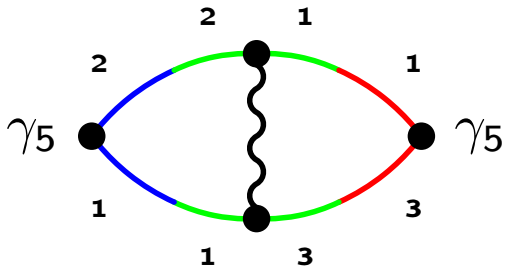
- 24^3 Strong isospin breaking corrections:



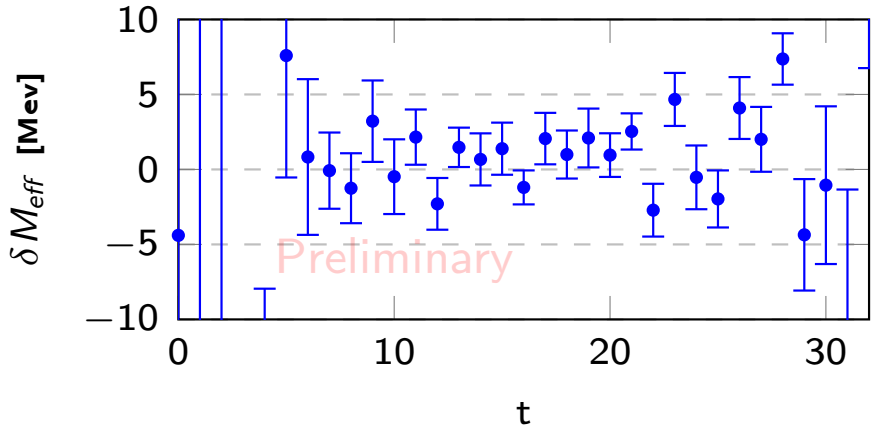
QED isospin breaking using Meson fields

- A-slash field:
- Exchange:

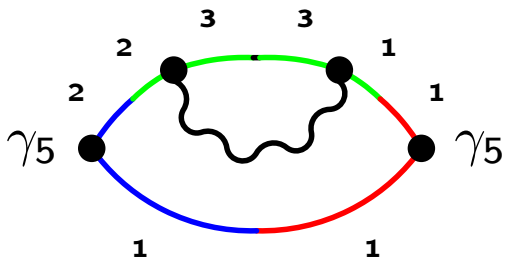
$$\Pi_{ij}(t_x; \not{A}(x)) = \sum_{\vec{x}} w_i^\dagger(x) \not{A}(x) v_j(x)$$



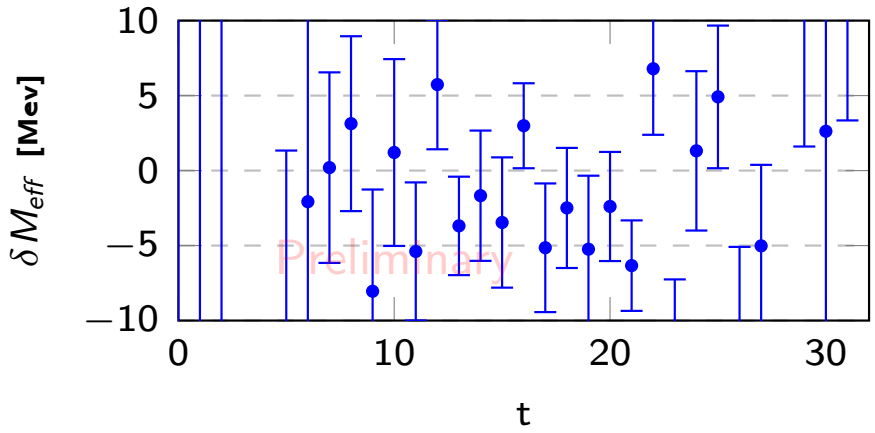
Exchange diagram



- Self Energy:



Self Energy diagram

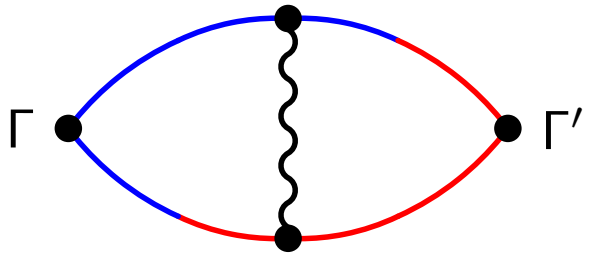


Sequential Meson fields

- Instead sequentially insert the photons.

$$v_i'(x) = \sum_z S(x,z) i\not{A}(z) v_i(z)$$

$$\Pi_{ij}'(t_x;\Gamma) = \sum_{\vec{x}} w_i^\dagger(x)\Gamma v_j'(x)$$



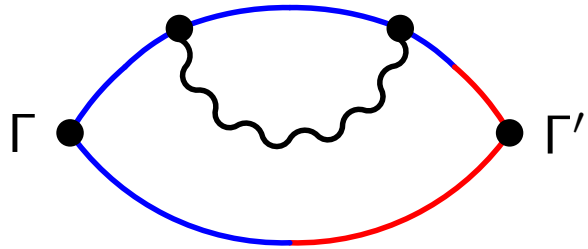
Exchange diagram

- Double *sequential meson fields*:

$$v_i''(x) = \sum_y S(x,y) i\not{A}(y) v_i'(y)$$

$$= \sum_y S(x,y) i\not{A}(y) \sum_z S(y,z) i\not{A}(z) v_i(z)$$

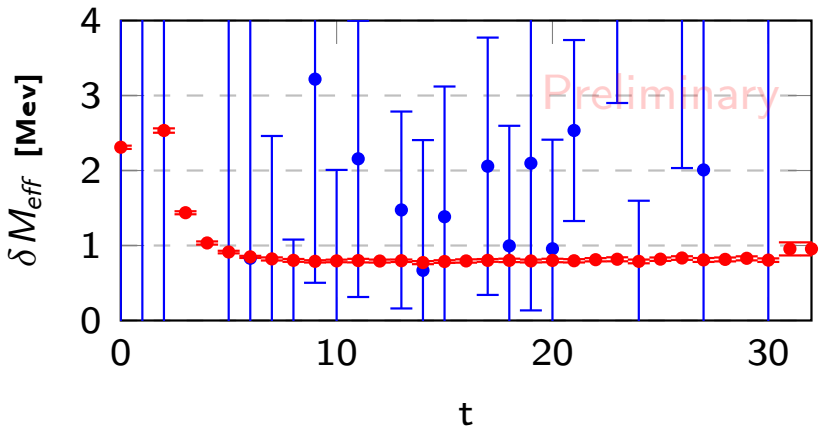
$$\Pi_{ij}''(t_x;\Gamma) = \sum_{\vec{x}} w_i^\dagger(x)\Gamma v_j''(x)$$



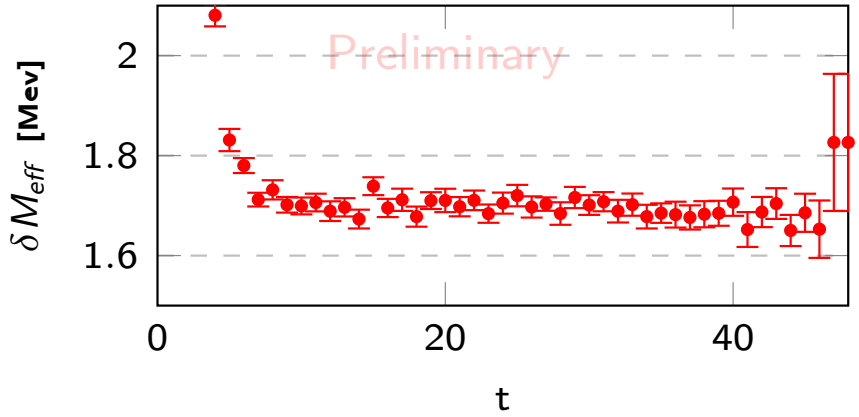
Self Energy diagram

Sequential all-to-all: Comparison and results

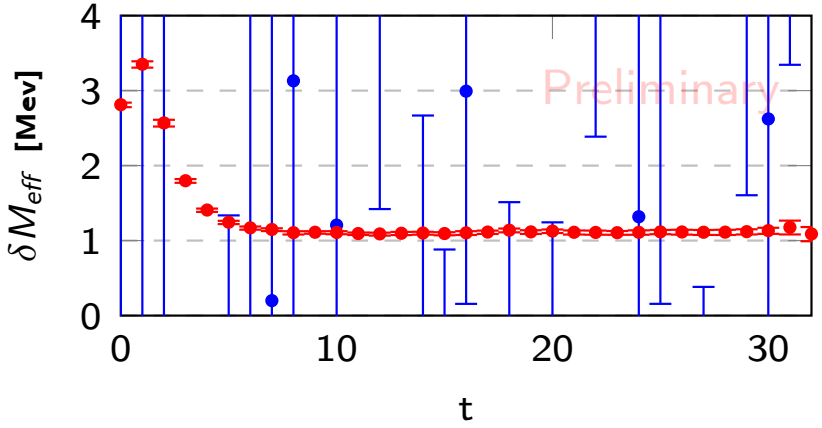
- 24^3 Pion Exchange comparison



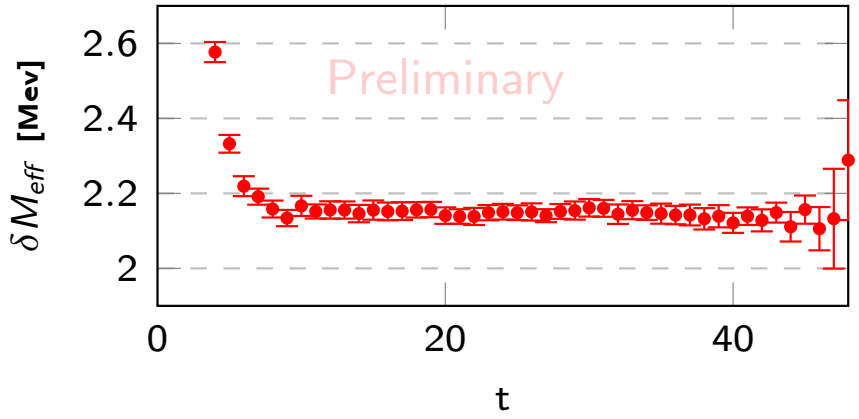
- 48^3 Pion Exchange



- 24^3 Pion Self Energy comparison



- 48^3 Pion Self Energy:



Disconnected Strong isospin breaking corrections

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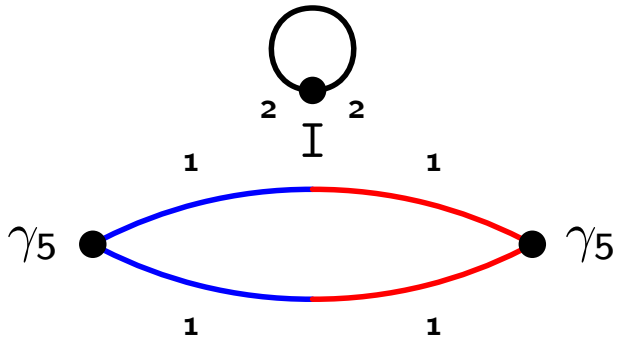
$$\langle \mathcal{O}\bar{q}(z)\mathtt{I}q(z)\rangle - \langle\mathcal{O}\rangle\langle\bar{q}(z)\mathtt{I}q(z)\rangle$$

$$= \sum_{\vec{x},\vec{y},z} \langle tr\left[\gamma_5 S_l(x,y)\gamma_5 S_l(y,x)\right] tr\left[\mathtt{I} S_l(z,z)\right]\rangle$$

$$- \sum_{\vec{x},\vec{y},z} \langle tr\left[\gamma_5 S_l(x,y)\gamma_5 S_l(y,x)\right]\rangle \langle tr\left[\mathtt{I} S_l(z,z)\right]\rangle$$

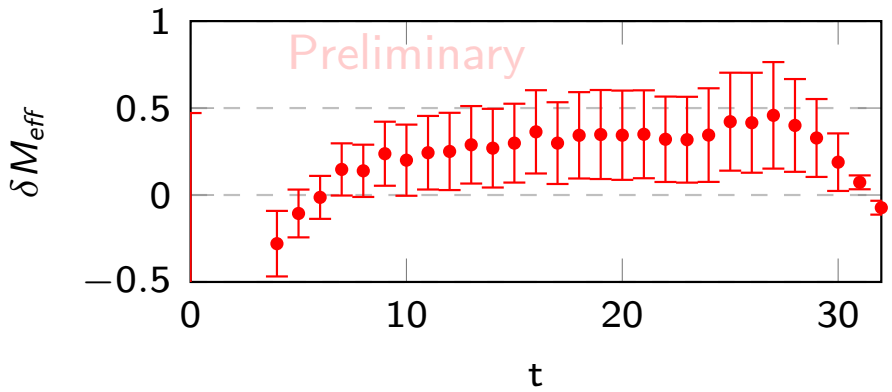
$$= \sum_{i,j,k} \langle \left[\Pi_{ij}(t_x,\gamma_5)\Pi_{ji}(t_y,\gamma_5)\right]\left[\sum_{t_z}\Pi_{kk}(t_z,\mathtt{I})\right]\rangle$$

$$- \sum_{i,j,k} \langle \left[\Pi_{ij}(t_x,\gamma_5)\Pi_{ji}(t_y,\gamma_5)\right]\rangle \langle \left[\sum_{t_z}\Pi_{kk}(t_z,\mathtt{I})\right]\rangle$$



Strong disconnected diagram

- 24^3 effective mass:



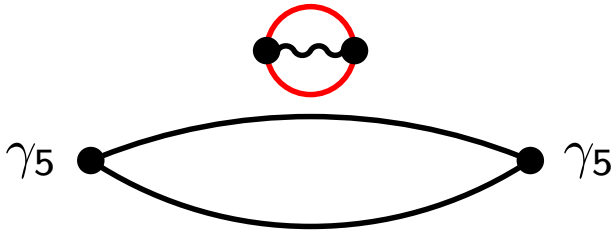
- Simple case without QED.
- Suppressed compa6red to leading order strong IB contribution.
- u and d contributions cancel.

Disconnected QED isospin breaking corrections

- Sequential A-slash field:

$$\Pi'_{ij}(t_x; \not{A}(x)) = \sum_{\vec{x}} w_i^\dagger(x) \not{A}(x) v'_j(x) = \sum_{\vec{x}} w_i^\dagger(x) \not{A}(x) \sum_z S(x,z) i \not{A}(z) v_j(z)$$

- Quark disconnected diagrams:



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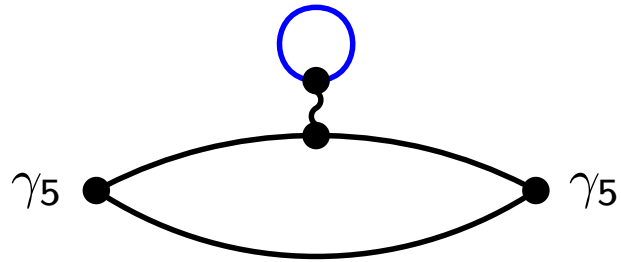
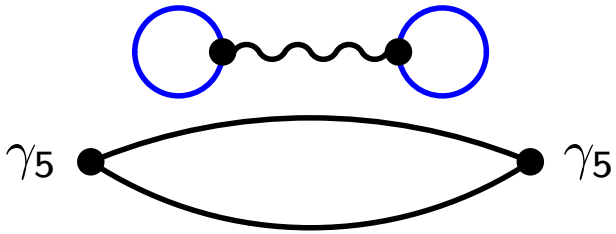
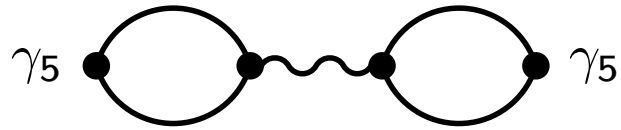


Tadpole diagram



Specs diagram



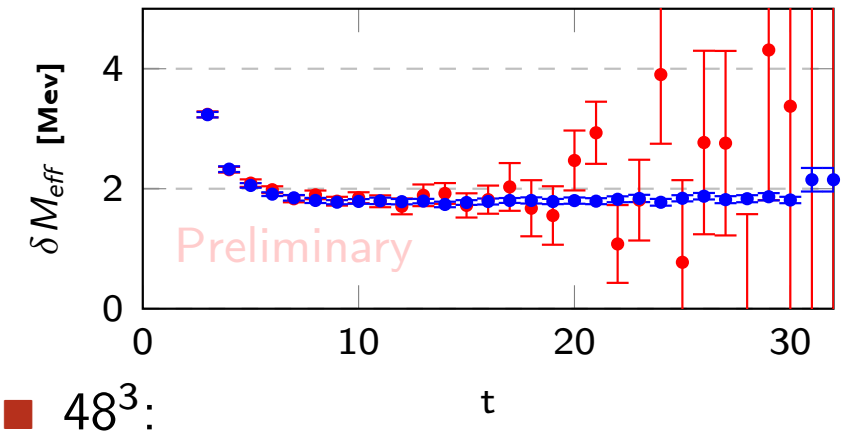
Neutral pion exchange diagram

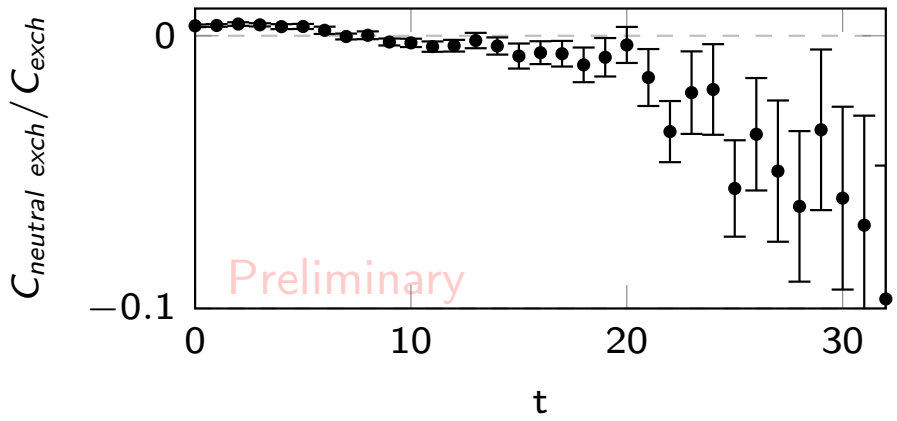
Pion mass splitting

$$M_{\pi^+} - M_{\pi^0} = \frac{(Q_u - Q_d)^2}{2} e^2 \partial_t \left[\frac{C^{\pi}_{exch} - C^{\pi}_{\text{neutral exch}}}{C^{\pi}_0} \right]$$

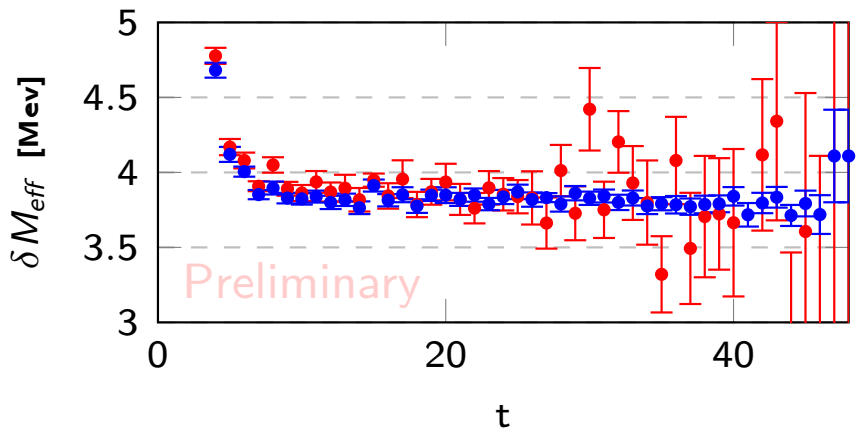
- 24^3:

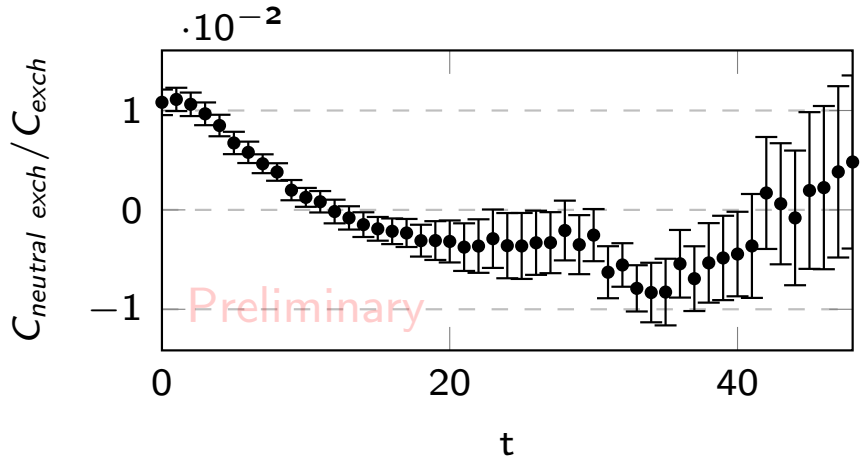
[1303.4896]





- 48^3:





Pion QED isospin breaking sea quark effects

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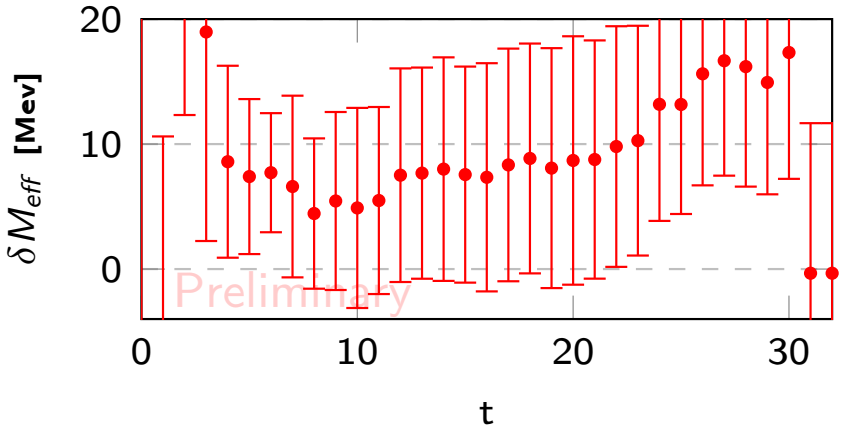
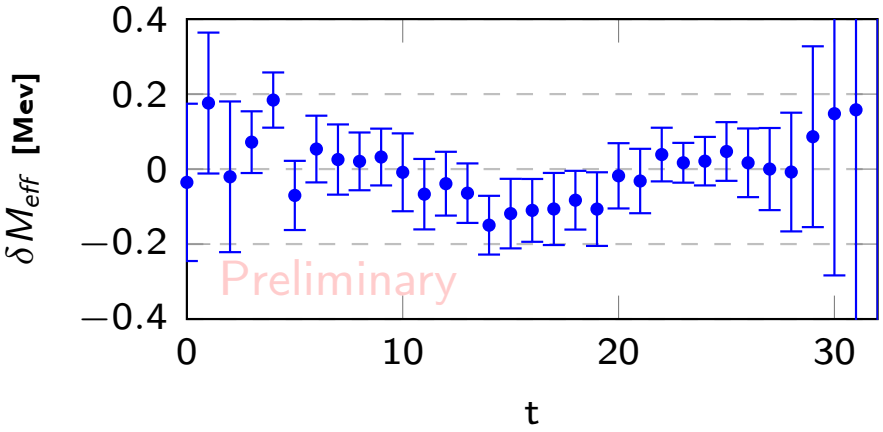
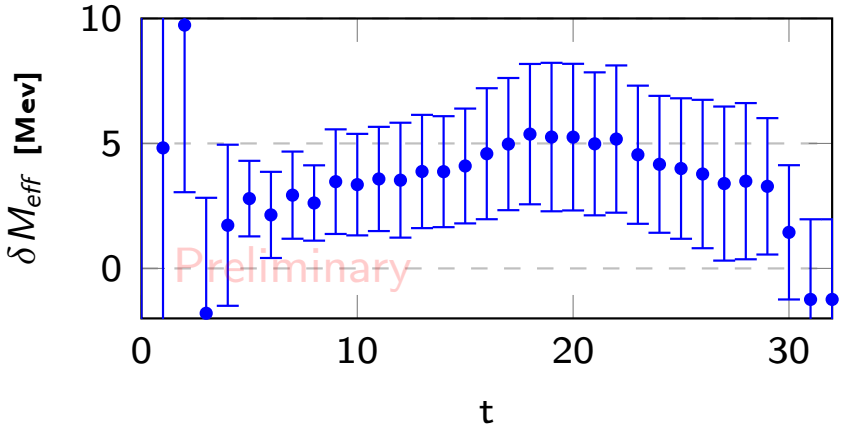
- Burger contribution 24^3:



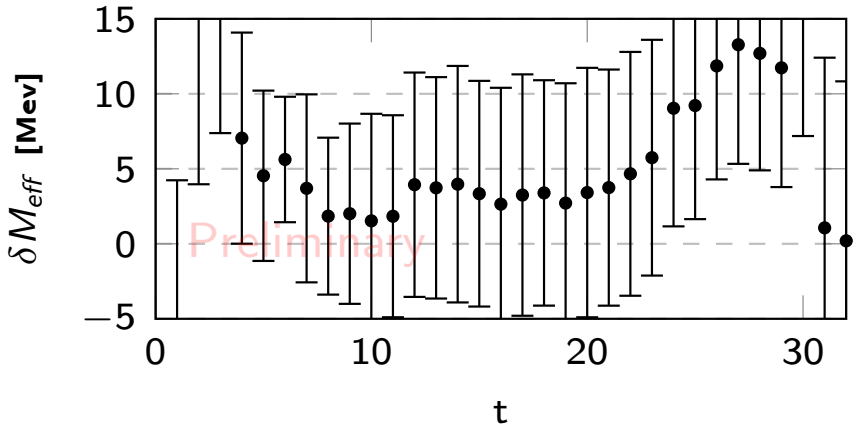
- Tadpole contribution 24^3:



- Specs contribution 24^3:



- Total contribution 24^3:



Summary

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- We use an all-to-all approach to form meson fields that can be used to calculate a variety of physical quantities, including isospin breaking effects.
- We see a significant improvement in signal when using sequential meson fields to form the correlators required to study QED isospin breaking effects.
- We have an initial set of meson fields on our 24^3 and 48^3 ensembles allowing us to investigate a variety of quantities including IB corrections to masses, leptonic decay rates and work on Rare Kaon's (See Fionn's talk on Friday)
- The all-to-all method allows us to calculate disconnected contributions to isospin breaking effects.
- For more on our collaborations work on isospin breaking corrections to leptonic decay rates, Antonin Portelli is speaking next.