

K_{l3} form factors in $N_f = 2 + 1$ QCD
at physical point on large volume

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Wuhan, China

- Introduction
- Construction of semileptonic form factors
- Set up

- Results
 - interpolation to zero momentum transfer(NLO SU(3) ChPT + NNLO analytic)
 - extrapolation to infinity time separation
 - Z_V dependence of form factors
 - CKM matrix element
- Conclusion

Introduction

CKM matrix unitarity check (up quark row)

$$\Delta_u \equiv |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 - 1 \quad \text{If CKM matrix is unitary,}$$
$$\Delta_u = 0$$

In recent result of CKM matrix element (PDG 2018)

M. Tanabashi *et al.* (Particle Data Group), Phys. Rev. D **98**, 030001 (2018).

$$|V_{ud}| = 0.97420(21) \quad \begin{array}{l} \text{from superallowed nuclear } \beta \text{ decay} \\ \text{Hardy and Towner PoS CKM2016, 028 (2016).} \end{array}$$

$$|V_{us}| = 0.2231(8) \quad \begin{array}{l} \text{from combination experiment and semileptonic} \\ \text{decay form factor by } N_f=2+1 \text{ lattice calculations} \end{array}$$

$$|V_{us}| = 0.2253(7) \quad \begin{array}{l} \text{from combination experiment and the ratio of} \\ \text{decay constants } f_K/f_\pi \text{ by } N_f=2+1 \text{ lattice calculations} \end{array}$$

$$|V_{ub}| = 3.94(36) \times 10^{-3} \quad \begin{array}{l} \text{from combination of inclusive} \\ \text{and exclusive decay} \end{array}$$

Introduction

Current status of CKM matrix element of up quark parts

$$|V_{us}| = 0.2231(8) \quad \text{from combination experiment and semileptonic decay form factor by } N_f=2+1 \text{ lattice calculations}$$

decay rate of semileptonic decays

$$\Gamma(K \rightarrow \pi l \nu) = \frac{G_F^2 M_K^5}{192\pi^3} C S_{EW} (1 + \delta_{EM} + \delta_{SU(2)})^2 I |V_{us}|^2 f_+(0)^2$$

I : phase space integral

$$C = 1, (K^0) \quad C = \frac{1}{2}, (K^\pm)$$

S_{EW} : short distance electroweak correction

δ_{EM} : long distance electromagnetic correction

$\delta_{SU(2)}$: isospin-breaking correction

Introduction

Current status of CKM matrix element of up quark parts

$$|V_{us}| = 0.2231(8)$$

from combination experiment and semileptonic decay form factor by $N_f=2+1$ lattice calculations

decay rate of semileptonic decays

$$\Gamma(K \rightarrow \pi l \nu)$$

I : phase space integral

Determination
by experiment

$$S_{EW}$$

$$\delta_{EM}$$

$$\delta_{SU(2)}$$

Determination by phenomenology

Introduction

Current status of CKM matrix element of up quark parts

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from combination experiment and semileptonic decay form factor by $N_f=2+1$ lattice calculations

decay rate of semileptonic decays

$$\Gamma(K \rightarrow \pi l \nu) = \frac{G_F^2 M_K^5}{192\pi^3} C S_{EW} (1 + \delta_{EM} + \delta_{SU(2)})^2 I |V_{us}|^2 f_+(0)^2$$

The average of five decay modes yields $|V_{us}| f_{\{+,0\}}(0) = 0.21654(41)$

$$K_L^0 \rightarrow \pi e \nu \quad K_L^0 \rightarrow \pi \mu \nu \quad K_S^0 \rightarrow \pi e \nu$$

$$K^\pm \rightarrow \pi^0 \mu \nu \quad K^\pm \rightarrow \pi^0 e^\pm \nu$$

the form factor is determined by Lattice QCD calculation

Introduction

the result of “unitarity”
 is assumed $\Delta_u = 0$
 and ignoring V_{ub}

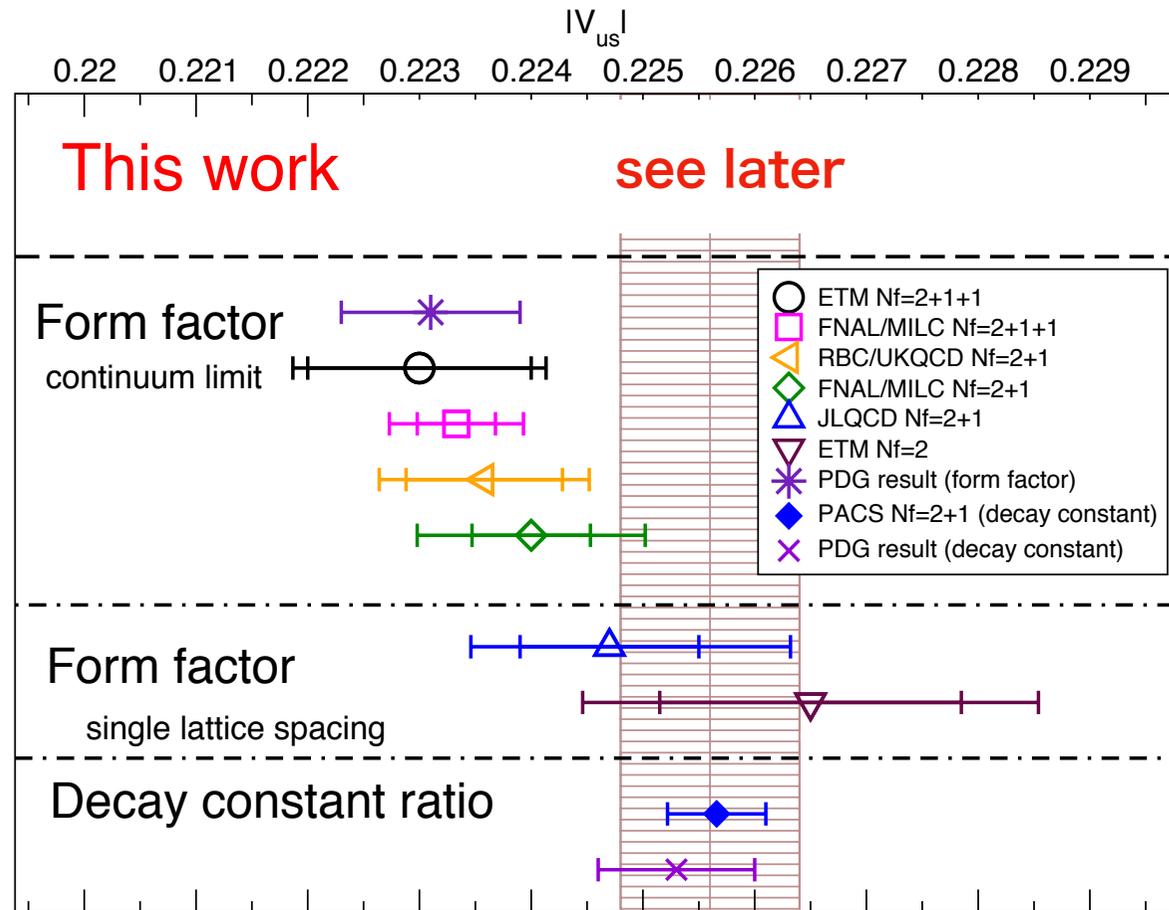
right panel : lattice calculations of V_{us}
 from semileptonic form factor and
 result assuming unitarity (brown band)

FLAG, arXiv:1902.08191 [hep-lat]

JLQCD, arXiv:1705.00884v1 [hep-lat]

FNAL/MILC, arXiv:1809.02827v1 [hep-lat]

inner error : stat. only
 outer error : total error
 (stat.+syst.+exp.)



uncertainty from lattice calculation

chiral extrapolation, finite size effect, continuum limit, etc...

suppressing the uncertainties, we could check the significance

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Construction of the form factors

Semileptonic form factor

vector form factor $\langle \pi(\vec{p}_\pi) | V_\mu | K(\vec{p}_K) \rangle = (p_K + p_\pi)_\mu f_+(q^2) + (p_K - p_\pi)_\mu f_-(q^2)$

scalar form factor $f_0(q^2) = f_+(q^2) + \frac{-q^2}{(m_K^2 - m_\pi^2)} f_-(q^2) = f_+(q^2) \left(1 + \frac{-q^2}{(m_K^2 - m_\pi^2)} \xi(q^2) \right)$

kinematic constraint $f_+(0) = f_0(0)$ $\left(\xi(q^2) = \frac{f_-(q^2)}{f_+(q^2)} \right)$

to construct form factors, we calculate

vector current 3point functions and 2point functions

$$C_\mu^{XY}(\vec{p}', \vec{p}, t) = \langle 0 | O_Y(\vec{p}', t_f) V_\mu(\vec{q}, t) O_X^\dagger(\vec{p}, t_i) | 0 \rangle$$
$$\rightarrow \frac{Z_Y(\vec{p}') Z_X(\vec{p})}{4E_Y(\vec{p}') E_X(\vec{p})} \frac{1}{Z_V} \langle Y(\vec{p}') | V_\mu | X(\vec{p}) \rangle e^{-E_Y(\vec{p}')(t_f-t)} e^{-E_X(\vec{p})(t-t_i)}$$

$t_i \ll t \ll t_f \quad X, Y = \pi, K$

$$C^X(\vec{p}, t, t_i) = \langle 0 | O_X(\vec{p}, t) O_X^\dagger(\vec{p}, t_i) | 0 \rangle$$

and construct ratios by 3(or 2)point function

the ratios (combination of 2point and 3point functions)

JLQCD, arXiv:1705.00884v1 [hep-lat]

$$d_1 = \frac{C_4^{\pi K}(\vec{0}, \vec{0}) C_4^{K\pi}(\vec{0}, \vec{0})}{C_4^{KK}(\vec{0}, \vec{0}) C_4^{\pi\pi}(\vec{0}, \vec{0})} \rightarrow \frac{(m_K + m_\pi)^2}{4m_K m_\pi} \boxed{(f_0(q_{min}^2))^2}$$

$$q_{min}^2 = -(m_K - m_\pi)^2$$

$$d_2 = \frac{C_4^{\pi K}(\vec{0}, \vec{p}) C^\pi(\vec{0})}{C_4^{\pi K}(\vec{0}, \vec{0}) C^\pi(\vec{p})} \rightarrow \left(\frac{E_\pi(\vec{p}) + m_K}{m_\pi + m_K} + \frac{E_\pi(\vec{p}) - m_K}{m_\pi + m_K} \xi(q^2) \right) \frac{\boxed{f_+(q^2)}}{\boxed{f_0(q_{min}^2)}}$$

$$d_3 = \frac{C_i^{\pi K}(\vec{0}, \vec{p}) C_4^{KK}(\vec{0}, \vec{p})}{C_i^{KK}(\vec{0}, \vec{p}) C_4^{\pi K}(\vec{0}, \vec{p})} \rightarrow \left(\frac{E_\pi(\vec{p}) + m_K}{p_i} \frac{p_i - \boxed{p_i \xi(q^2)}}{E_\pi(p) + m_K + \boxed{(m_K - E_\pi(p)) \xi(q^2)}} \right)$$

$$(0 \ll t \ll t_f, i = 1 - 3)$$

construct the form factors from the ratios

$$\boxed{d_1 \rightarrow f_0(q_{min}^2)} \rightarrow \boxed{d_2 \rightarrow f_+(q^2)} \rightarrow f_0(q^2) = f_+(q^2) \left(1 + \frac{-q^2}{(m_K^2 - m_\pi^2)} \xi(q^2) \right)$$

$$\boxed{d_3 \rightarrow \xi(q^2) = \frac{f_-(q^2)}{f_+(q^2)}}$$

Simulation details

configuration parameter

PACS10 configuration (HPCI System Research project)

arXiv:1807.06237v1 [hep-lat]

$$L^3 \times T = 128^3 \times 128 \quad m_\pi L \approx 7.5 \quad (\kappa_{ud}, \kappa_s) = (0.126117, 0.124902)$$

$$\beta = 1.82, n_{stout} = 6, \rho = 0.1, c_{sw} = 1.11$$

$$a^{-1} = 2.333(\text{GeV}) \rightarrow a = 0.084(\text{fm})$$

$N_f = 2+1$ Iwasaki gauge + stout smeared link Wilson clover action

Simulation details

measurement parameter

$ t_f - t_i $	36	42	48
$ t_f - t_i $ (fm)	3.05	3.55	4.05
# of config.	20	20	20
# of time slice	8	8	8
# of temporal dir.	4	4	4
# of rand. source	1	2	2

directions of
source momenta $\vec{p} = \frac{2\pi}{L} \vec{n}$

$ \vec{n}^2 = n$	# dir. of \vec{n}	perm. of $\vec{n} =$
0	1	(0, 0, 0)
1	6	(1, 0, 0)
2	12	(1, 1, 0)
3	8	(1, 1, 1)
4	6	(2, 0, 0)
5	9	(2, 1, 0)
6	9	(2, 1, 1)

sink momentum : 0

periodic boundary condition in temporal direction

without twisted boundary condition in spatial directions

jackknife analysis bin size:10 traj.

resources : Oakforest-PACS (JCAHPC)

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interpolation to zero momentum transfer (NLO SU(3) ChPT + NNLO analytic)

NLO SU(3) ChPT

JHEP 9805 (1998) 014
Nucl.Phys.B669:341-362,2003

$$\mu = m_\rho = 0.77\text{GeV}$$

$$f_0 = 0.074\text{GeV}$$

\bar{A} , \bar{B} , \bar{B}_{ij}

loop integral of ChPT

**one fit parameter is
in each form factor**

$$f_+(q^2) = 1 - 2\frac{q^2}{f_0^2}L_9(\mu) + \frac{3}{8f_0^2}(\bar{A}(m_\eta^2, \mu) + \bar{A}(m_\pi^2, \mu) + 2\bar{A}(m_K^2, \mu)) \\ + \frac{3}{2f_0^2}(\bar{B}_{22}(m_\pi^2, m_K^2, q^2, \mu) + \bar{B}_{22}(m_K^2, m_\eta^2, q^2, \mu))$$

$$f_0(q^2) = 1 - 4\frac{q^2}{f_0^2}L_5(\mu) + \frac{3}{8f_0^2}(\bar{A}(m_\eta^2, \mu) + \bar{A}(m_\pi^2, \mu) + 2\bar{A}(m_K^2, \mu)) \\ + \frac{3}{2f_0^2}(\bar{B}_{22}(m_\pi^2, m_K^2, q^2, \mu) + \bar{B}_{22}(m_K^2, m_\eta^2, q^2, \mu))$$

$$+ \frac{1}{f_0^2}(1/2\bar{A}(m_\eta^2) - 5/12\bar{A}(m_\pi^2) + 7/12\bar{A}(m_K^2)) \\ + \bar{B}(m_\pi^2, m_K^2, q^2, \mu)(-m_\pi^2/12 - 5m_K^2/12 - 5q^2/12) \\ + \bar{B}(m_K^2, m_\eta^2, q^2, \mu)(m_\pi^2/12 - 7m_K^2/12 - q^2/4) \\ + \bar{B}_1(m_\pi^2, m_K^2, q^2, \mu)(-7m_\pi^2/12 - 19m_K^2/12 - 5q^2/12) \\ + \bar{B}_1(m_K^2, m_\eta^2, q^2, \mu)(-1m_\pi^2/12 + 23m_K^2/12 + q^2/4) \\ + \bar{B}_{21}(m_\pi^2, m_K^2, q^2, \mu)(3m_\pi^2/2 - 3m_K^2/2 + 5q^2/6) \\ + \bar{B}_{21}(m_K^2, m_\eta^2, q^2, \mu)(3m_\pi^2/2 - 3m_K^2/2 + q^2/2) \\ + \bar{B}_{22}(m_\pi^2, m_K^2, q^2, \mu)(-5/6) \\ + \bar{B}_{22}(m_K^2, m_\eta^2, q^2, \mu)(-1/2)$$

eta meson mass is composed
of pion's one and kaon's one
(GMOR)

$$3m_\eta^2 = 4m_K^2 - m_\pi^2$$

interpolation to zero momentum transfer
(NLO SU(3) ChPT + NNLO analytic)

NLO SU(3) ChPT

$$f_+(q^2) = 1 - 2\frac{q^2}{f_0^2}L_9(\mu) + \frac{3}{8f_0^2}(\bar{A}(m_\eta^2, \mu) + \bar{A}(m_\pi^2, \mu) + 2\bar{A}(m_K^2, \mu)) \\ + \frac{3}{2f_0^2}(\bar{B}_{22}(m_\pi^2, m_K^2, q^2, \mu) + \bar{B}_{22}(m_K^2, m_\eta^2, q^2, \mu))$$

$$f_0(q^2) = 1 - 4\frac{q^2}{f_0^2}L_5(\mu) + \frac{3}{8f_0^2}(\bar{A}(m_\eta^2, \mu) + \bar{A}(m_\pi^2, \mu) + 2\bar{A}(m_K^2, \mu)) \\ + \frac{3}{2f_0^2}(\bar{B}_{22}(m_\pi^2, m_K^2, q^2, \mu) + \bar{B}_{22}(m_K^2, m_\eta^2, q^2, \mu)) \\ + \frac{1}{f_0^2}(1/2\bar{A}(m_\eta^2) - 5/12\bar{A}(m_\pi^2) + 7/12\bar{A}(m_K^2)) \\ + \bar{B}(m_\pi^2, m_K^2, q^2, \mu)(-m_\pi^2/12 - 5m_K^2/12 - 5q^2/12) \\ + \bar{B}(m_K^2, m_\eta^2, q^2, \mu)(m_\pi^2/12 - 7m_K^2/12 - q^2/4) \\ + \bar{B}_1(m_\pi^2, m_K^2, q^2, \mu)(-7m_\pi^2/12 - 19m_K^2/12 - 5q^2/12) \\ + \bar{B}_1(m_K^2, m_\eta^2, q^2, \mu)(-1m_\pi^2/12 + 23m_K^2/12 + q^2/4) \\ + \bar{B}_{21}(m_\pi^2, m_K^2, q^2, \mu)(3m_\pi^2/2 - 3m_K^2/2 + 5q^2/6) \\ + \bar{B}_{21}(m_K^2, m_\eta^2, q^2, \mu)(3m_\pi^2/2 - 3m_K^2/2 + q^2/2) \\ + \bar{B}_{22}(m_\pi^2, m_K^2, q^2, \mu)(-5/6) \\ + \bar{B}_{22}(m_K^2, m_\eta^2, q^2, \mu)(-1/2))$$

However
there is no analytic
term in NLO of $f_{\{+,0\}}(0)$

interpolation to zero momentum transfer
(NLO SU(3) ChPT + NNLO analytic)

NLO SU(3) ChPT

+ analytic terms

$$f_+(q^2) = 1 - 2\frac{q^2}{f_0^2}L_9(\mu) + \frac{3}{8f_0^2}(\bar{A}(m_\eta^2, \mu) + \bar{A}(m_\pi^2, \mu) + 2\bar{A}(m_K^2, \mu)) \\ + \frac{3}{2f_0^2}(\bar{B}_{22}(m_\pi^2, m_K^2, q^2, \mu) + \bar{B}_{22}(m_K^2, m_\eta^2, q^2, \mu))$$

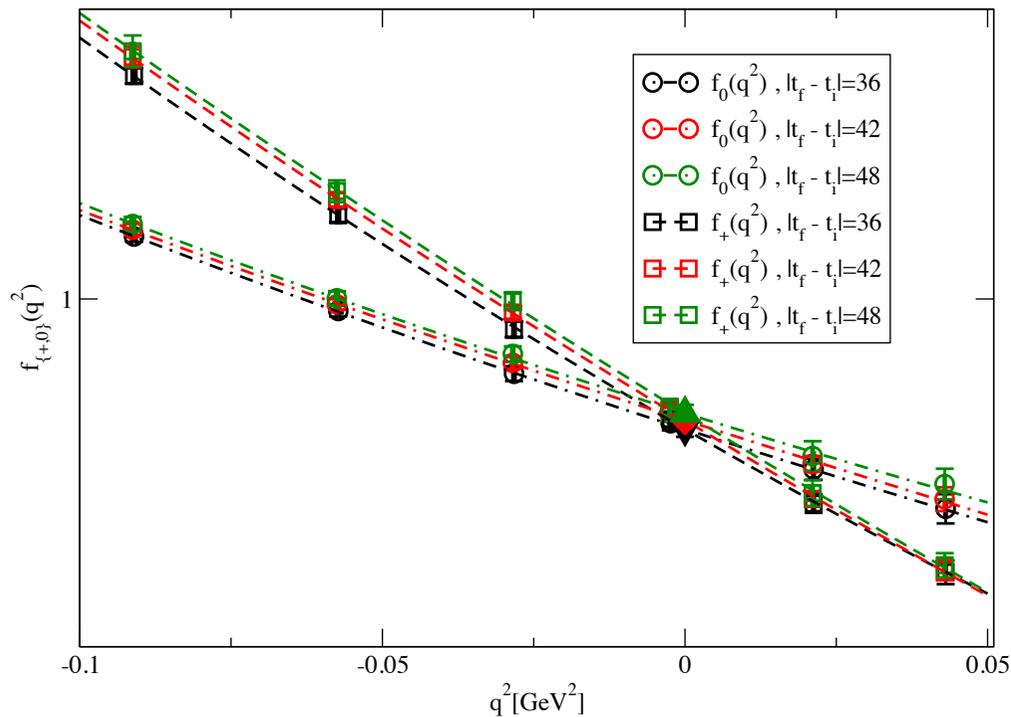
$$f_0(q^2) = 1 - 4\frac{q^2}{f_0^2}L_5(\mu) + \frac{3}{8f_0^2}(\bar{A}(m_\eta^2, \mu) + \bar{A}(m_\pi^2, \mu) + 2\bar{A}(m_K^2, \mu)) \\ + \frac{3}{2f_0^2}(\bar{B}_{22}(m_\pi^2, m_K^2, q^2, \mu) + \bar{B}_{22}(m_K^2, m_\eta^2, q^2, \mu)) \\ + \frac{1}{f_0^2}(1/2\bar{A}(m_\eta^2) - 5/12\bar{A}(m_\pi^2) + 7/12\bar{A}(m_K^2)) \\ + \bar{B}(m_\pi^2, m_K^2, q^2, \mu)(-m_\pi^2/12 - 5m_K^2/12 - 5q^2/12) \\ + \bar{B}(m_K^2, m_\eta^2, q^2, \mu)(m_\pi^2/12 - 7m_K^2/12 - q^2/4) \\ + \bar{B}_1(m_\pi^2, m_K^2, q^2, \mu)(-7m_\pi^2/12 - 19m_K^2/12 - 5q^2/12) \\ + \bar{B}_1(m_K^2, m_\eta^2, q^2, \mu)(-1m_\pi^2/12 + 23m_K^2/12 + q^2/4) \\ + \bar{B}_{21}(m_\pi^2, m_K^2, q^2, \mu)(3m_\pi^2/2 - 3m_K^2/2 + 5q^2/6) \\ + \bar{B}_{21}(m_K^2, m_\eta^2, q^2, \mu)(3m_\pi^2/2 - 3m_K^2/2 + q^2/2) \\ + \bar{B}_{22}(m_\pi^2, m_K^2, q^2, \mu)(-5/6) \\ + \bar{B}_{22}(m_K^2, m_\eta^2, q^2, \mu)(-1/2)$$

+ $c_0 + (-q^2)^2 c_2$

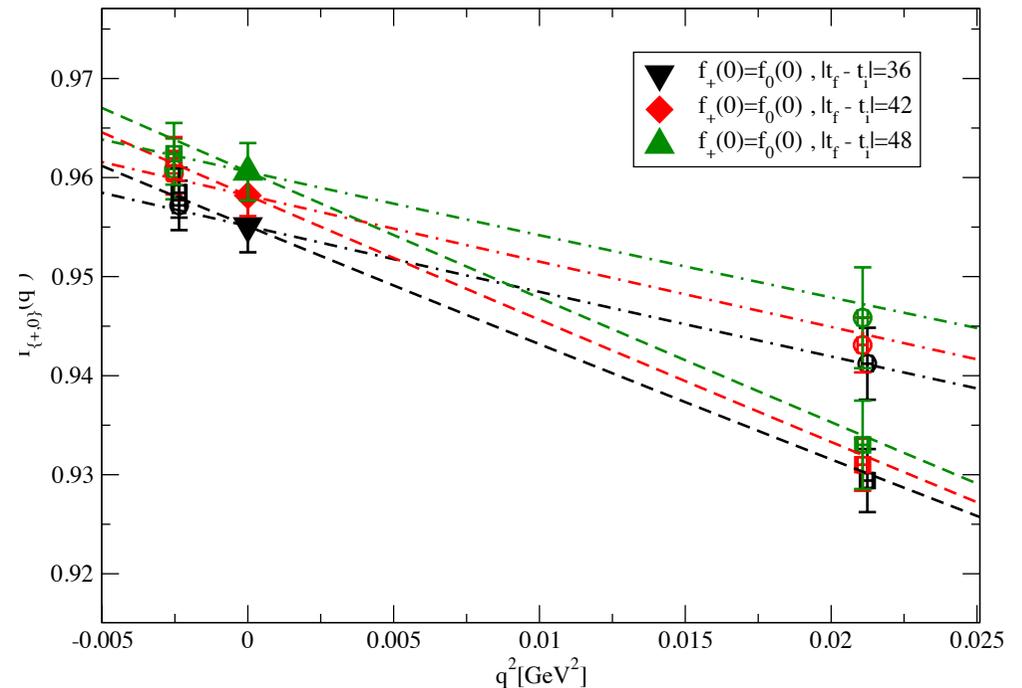
Result : interpolation to zero momentum transfer (NLO SU(3) ChPT + analytic terms)

uncorrelated fit in $n=1-6$

5 free parameters



the fit with the momenta
in $n=1-6$



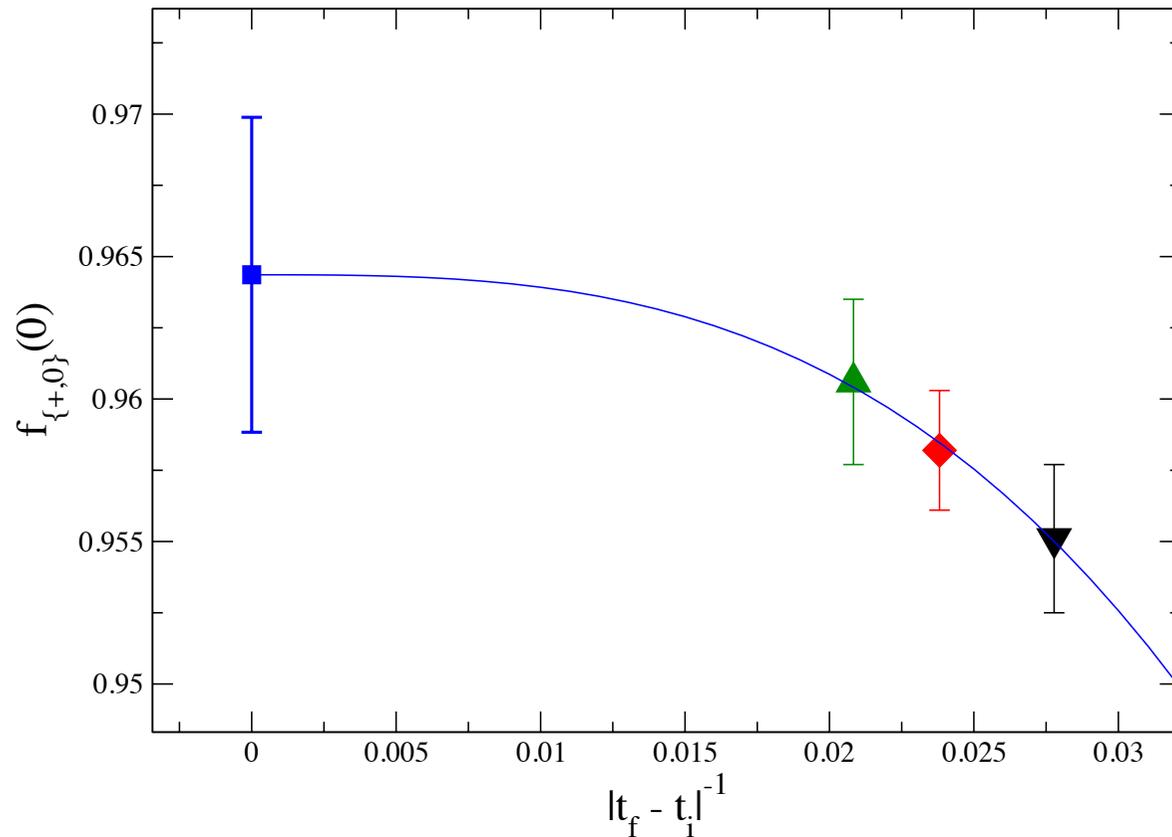
near $q=0$ region,
between $n=4$ and $n=5$ mom.

the parameters' errors from F.S.E. $O(e^{-m_\pi L}) = 0.06\%$

→form factor's error from F.S.E. $O(10^{-6})$

Result : extrapolation to infinity time separation

	$ t_f - t_i = 36$	$ t_f - t_i = 42$	$ t_f - t_i = 48$
$f_{+,0}(0)$	0.9551 (26)	0.9582(21)	0.9606 (29)
$\chi^2/d.o.f$	0.05	0.09	0.19



Fit form

$$f_{\{+,0\}}(0) + \frac{C}{|t_f - t_i|^3}$$

Central value :

$$f_{\{+,0\}}(0) = 0.9606(26)$$

$$(|t_f - t_i| = 48)$$

The difference between central value and the extrap. : 0.40%

Result : Z_V dependence of form factors

$$d_1 = \frac{C_4^{\pi K}(\vec{0}, \vec{0}) C_4^{K\pi}(\vec{0}, \vec{0})}{C_4^{KK}(\vec{0}, \vec{0}) C_4^{\pi\pi}(\vec{0}, \vec{0})} \rightarrow \frac{(m_K + m_\pi)^2}{4m_K m_\pi} (f_0(q_{min}^2))^2$$

$$\rightarrow \frac{(m_K + m_\pi)^2}{4m_K m_\pi} (Z_V f_0^{bare}(q_{min}^2))^2$$

Z_V is related to weak current

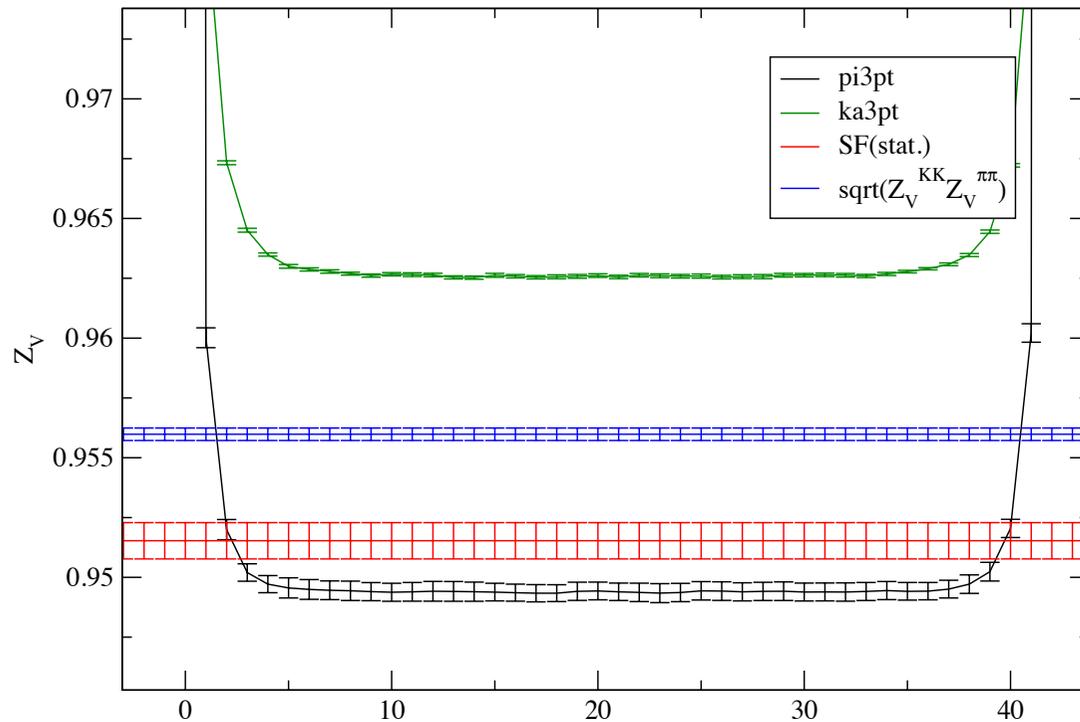
Z_V does not appear explicitly in the other ratios (d2,d3)

$$d_2 = \frac{C_4^{\pi K}(\vec{0}, \vec{p}) C^\pi(\vec{0})}{C_4^{\pi K}(\vec{0}, \vec{0}) C^\pi(\vec{p})}$$

$$d_3 = \frac{C_i^{\pi K}(\vec{0}, \vec{p}) C_4^{KK}(\vec{0}, \vec{p})}{C_i^{KK}(\vec{0}, \vec{p}) C_4^{\pi K}(\vec{0}, \vec{p})}$$

Investigation of systematic error from choice of Z_V

Result : Z_V dependence of form factors



pattern 1 : obtain from EM current
3-pt functions and correlators

$$Z_V = \sqrt{Z_V^{\pi\pi} Z_V^{KK}} = 0.95598(26)$$

fit range $t=7-34$

pattern 2 : SF scheme

PoS LATTICE2015 (2016) 271

$$Z_V = Z_V^{SF} = 0.95153(76)(1487)$$

Systematic error by choice of Z_V : 0.47%

$$f_{\{+,0\}}(0) = 0.9606(26) \binom{+38}{-0}(45)$$

Note that our result is single lattice spacing calculation

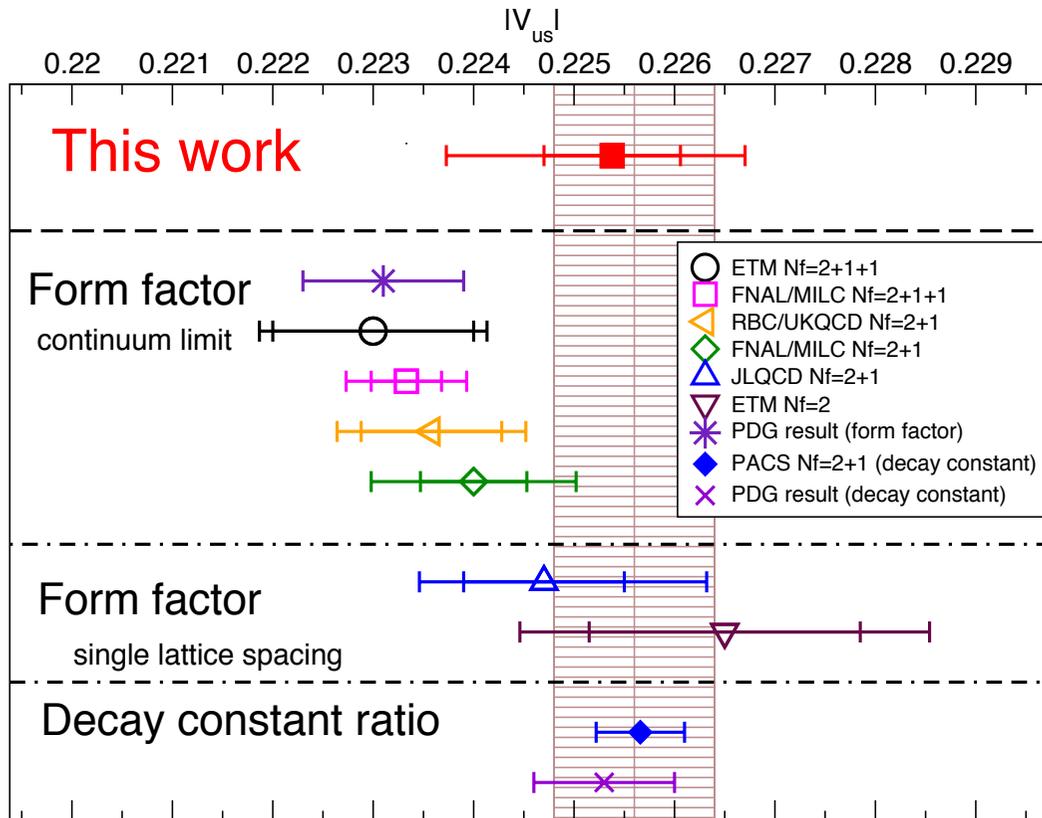
Result : CKM matrix element

In the experimental result

$$|V_{us}| f_{\{+,0\}}(0) = 0.21654(41)$$

our results of CKM

$$|V_{us}| = 0.22538(68) \left(\begin{matrix} +0 \\ -99 \end{matrix} \right) (105)(43)$$



1st error : stat.

2nd error : time sep.

3rd error : Z_V dep.

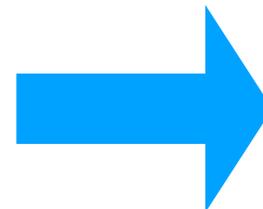
4th error : exp.

inner error : stat. only

outer error : total error

(stat.+syst.+exp.)

our result covers SM
and also previous continuum
limit results



important to reduce
systematic errors

Summary

We calculated vector meson form factors and semileptonic form factor in $N_f=2+1$ Lattice QCD

our result of semileptonic form factors

$$f_{\{+,0\}}(0) = 0.9606(26) \left(\begin{smallmatrix} +38 \\ -0 \end{smallmatrix} \right) (45)$$

(NLO SU(3) ChPT fit + analytic terms)

our result covers the SM and also previous continuum limit results

$$|V_{us}| = 0.22538(68) \left(\begin{smallmatrix} +0 \\ -99 \end{smallmatrix} \right) (105)(43)$$

It is important to reduce systematic errors

Next step

- continuum limit (finer lattice spacing)
- (partially) conserved current analysis

back up

Calculation 3-point function

- connected 3-point function is $C_{\mu}^{XY}(\vec{P}', \vec{P}) = Z_V \langle 0 | O_Y(t_f, \vec{P}') V_{\mu}(t, \vec{q}) O_X^{\dagger}(t_i, \vec{P}) | 0 \rangle$
consist of 3 quark propagators

- 1 random wall source

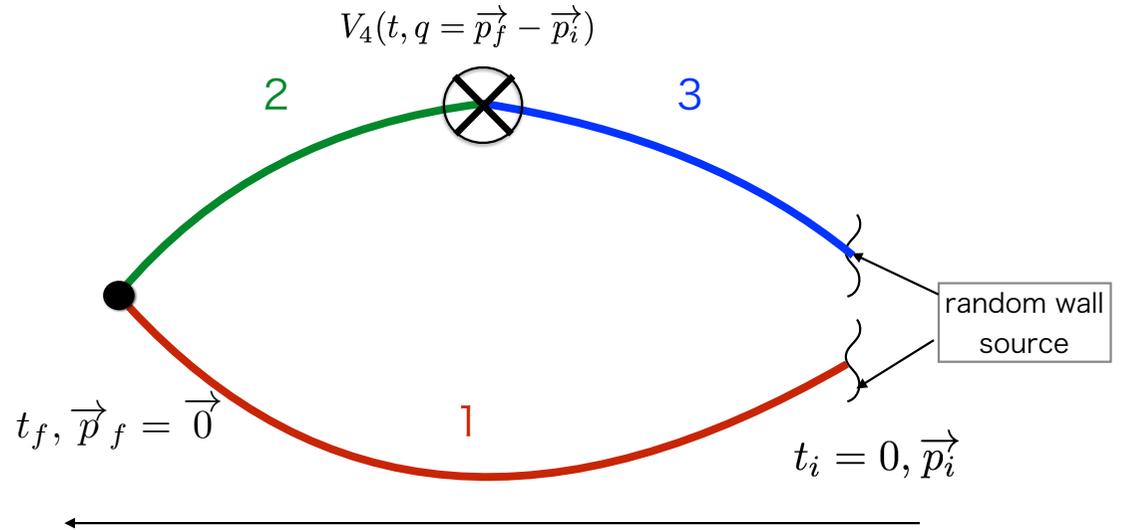
$$\vec{p}_i = \vec{0}$$

- 2 sequential source

$$\vec{p}_f = \vec{0}$$

- 3 random wall source

$$\vec{p}_i \neq \vec{0}$$



random wall source(A,B : color&spinor index)

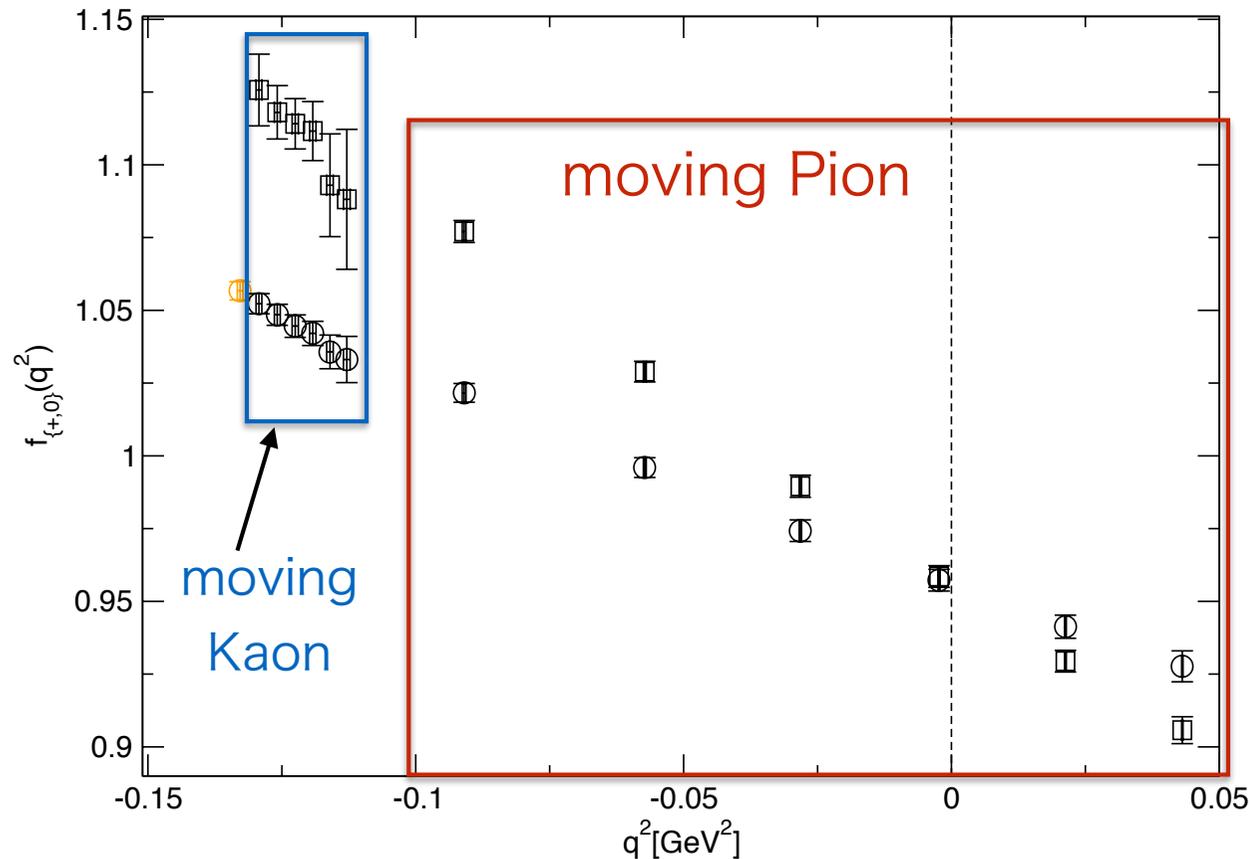
$$\eta_B(\vec{y}, t_i) = \left\{ \frac{1+i}{\sqrt{2}}, \frac{1-i}{\sqrt{2}}, \frac{-1+i}{\sqrt{2}}, \frac{-1-i}{\sqrt{2}} \right\} \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{j=0}^N \eta_A^j(\vec{x}, t_i) \eta_B^{\dagger j}(\vec{y}, t_i) = \delta(\vec{x} - \vec{y}) \delta_{AB}$$

$\in \mathbb{Z}(2) \otimes \mathbb{Z}(2)$

calculation cost is reduced by random wall source

RBC&UKQCD:JHEP(0807 (2008)112)

Preliminary result : momentum transfer dependance of K13 form factor



constant fit
from all the ratios
in $t = 15 - 21$

there is target
momentum ($q=0$)
between mom. of
 $n=4$ and $n=5$

$$n = 0 \rightarrow q_{min}^2 = -(m_K - m_\pi)^2 \approx -0.132(\text{GeV}^2)$$

square symbols: $f_+(q^2)$
circle symbols: $f_0(q^2)$

orange circle: $f_0(q^2_{min})$

the form factors of **Kaon with mom.** are noisier than **Pion's with mom.** and too far to extrapolate to the target
the data of Kaon with mom. is NOT included in the next
analysis

Result : slope of form factor

$$\tilde{f}_{(+,0)}(q^2) = 1 + \lambda'_{+,0} \frac{-q^2}{m_\pi^2} + \lambda''_{+,0} \left(\frac{-q^2}{m_\pi^2}\right)^2 + \dots$$

is measured in Kl3 decay experiment

slope of form factor can be used for the
consistency check with experiment

experiment M. Moulson, PoS CKM2016, 033 [arXiv:1704.04104 [hep-ex]].
JLQCD collaboration, arXiv:1705.00884v1 [hep-lat]

$$\lambda'_{(+)} = 2.58(7) \times 10^{-2} \quad \lambda'_{(0)} = 1.36(7) \times 10^{-2}$$

our result

$$\lambda'_{(+)} = 2.44(6) \times 10^{-2} \quad \lambda'_{(0)} = 1.36(7) \times 10^{-2}$$

our results are consistent with experiment