Electromagnetic Corrections to Decay Amplitudes: Real Emissions in Leptonic Decays

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DIPARTIMENTO DI FISICA

SAPIENZA UNIVERSITÀ DI ROMA

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PLAN OF THE TALK

Leptonic decays e.g. $\pi+ \rightarrow \mu+ \nu_{\mu} (\gamma)$ *& heavier*

Real Emissions (RM123 method)

Conclusion & Outlook

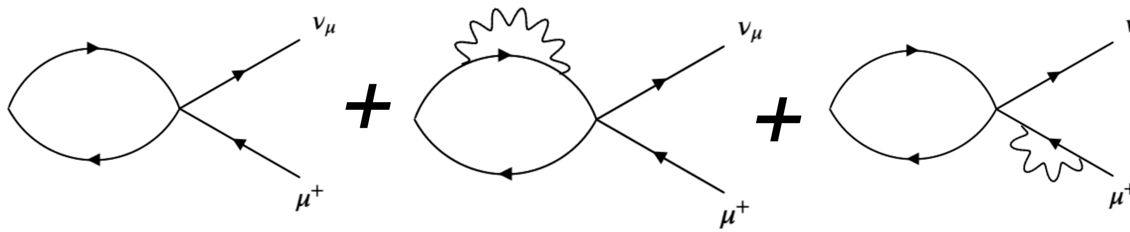
for infrared divergences, finite volume, continuation from Minkowsky to Euclidean, renormalisation of the relevant operators see also talk by C. Sachrajda on Wednesday 19th at 11:30

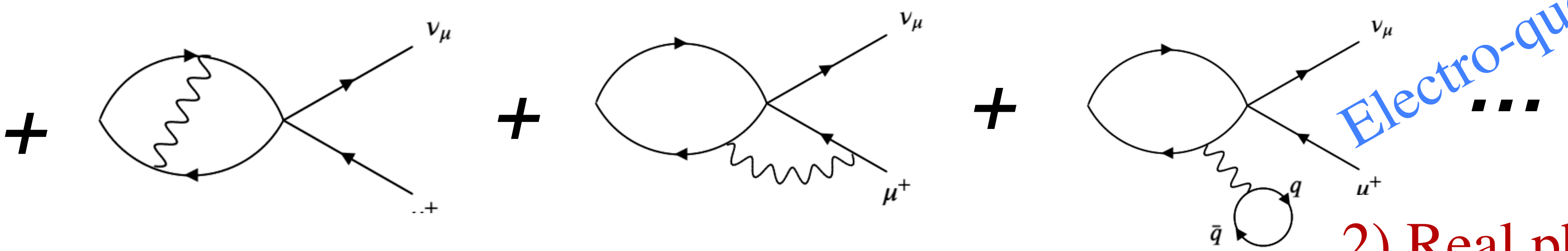
Rate at O(α)

$$\Gamma(\Delta E) = \int_0^{\Delta E} dE_\gamma \frac{d\Gamma}{dE_\gamma} = \Gamma_0 + \Gamma_1(\Delta E)$$

1) Virtual photons

$$\Gamma(\Delta E) = \int d\Omega_2 \;\Big|$$



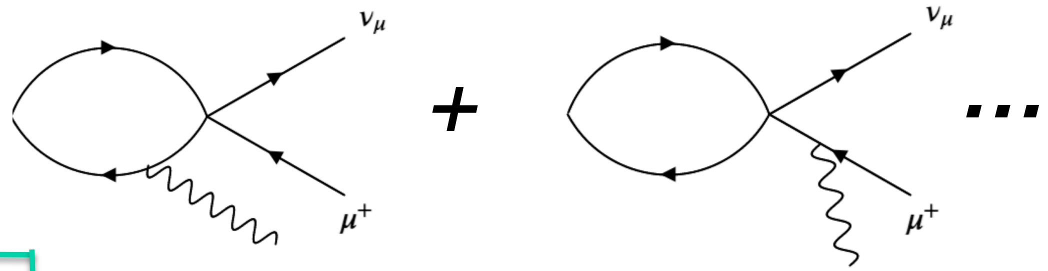


Electro-quenched

$$\Big|^2$$

2) Real photons

$$+ \int_{E_\gamma \leq \Delta E} d\Omega_3 \;\Big|$$



$$\Big|^2$$

$d\Omega_{2,3} = 2-3$ body phase $-$ space

The Infrared Problem

$$\Gamma(\Delta E) = \Gamma_0 + \Gamma_1(\Delta E)$$

QED Corrections to Hadronic Processes in Lattice QCD,
N.Carrasco, V.Lubicz, G.Martinelli, C.T.Sachrajda, N.Tantalo, C.Tarantino and M.Testa,
Phys. Rev. D **91** (2015) no.7, 07450 [arXiv:1502.00257 [hep-lat]].

Finite-Volume QED Corrections to Decay Amplitudes in Lattice QCD,
V.Lubicz, G.Martinelli, C.T.Sachrajda, F.Sanfilippo, S.Simula and N.Tantalo,
Phys. Rev. D **95** (2017) no.3, 034504 [arXiv:1611.08497 [hep-lat]].

First Lattice Calculation of the QED Corrections to Leptonic Decay Rates,
D.Giusti, V.Lubicz, G.Martinelli, C.T.Sachrajda, F.Sanfilippo, S.Simula, N.Tantalo and C.Tarantino,
Phys. Rev. Lett. **120** (2018) 072001 [arXiv:1711:06537]

Light-meson leptonic decay rates in lattice QCD+QED
M.Di Carlo, D.Giusti, V.Lubicz, G.Martinelli, C.T.Sachrajda, F.Sanfilippo, S.Simula and N.Tantalo,
arXiv:1904.08731.

MASTER FORMULA for the rate at O(α)

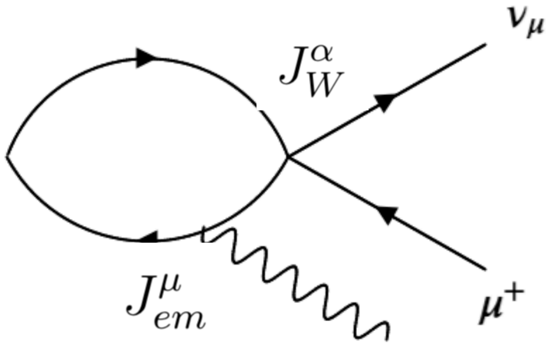
$$\Gamma(\Delta E) = \lim_{V\to\infty}(\Gamma_0 - \Gamma_0^{\rm pt}) + \lim_{V\to\infty}(\Gamma_0^{\rm pt} + \Gamma_1(\Delta E))$$

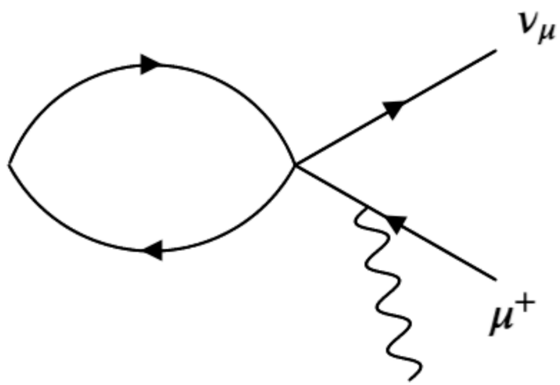
pt = point-like & perturbative

- the infrared divergences in Γ_0 and Γ_0^{pt} are universal and cancel in the difference
- $\Gamma(\Delta E) = \Gamma_0^{pt} + \Gamma_1(\Delta E)$ is infrared finite since is a physical, well defined quantity *F. Bloch, A. Nordsieck Phys.Rev. 52 (1937) T.D. Lee, M. Nauenberg Phys.Rev. 133 (1964)*
- the infrared divergences in $\Delta\Gamma_0(L) = \Gamma_0 - \Gamma_0^{pt}$ and
- $\Gamma(\Delta E) = \Gamma_0^{pt} + \Gamma_1(\Delta E)$ cancel separately hence they can be regulated with different infrared cutoff
- Γ_0 and Γ_0^{pt} are also ultraviolet finite

We now discuss the non-perturbative determination of $\Gamma_1(\Delta E)$

Real photons





- the leptonic part factorizes;
- the photon is not really there, we just give a pinch that carries away some momentum at the vertex where the (conserved) electromagnetic current is inserted;
- therefore finite volume effects are exponentially suppressed.

- can be computed in perturbation theory;
- it has no relation with the non perturbative structure of the hadron.

The relevant hadronic amplitude (T-product)

$$H_W^{\alpha\, r}(k,p) = \varepsilon_\mu^r(k)\, H_W^{\alpha\, \mu}(k,p) = \varepsilon_\mu^r(k) \int d^4y\, e^{ik\cdot y}\, \langle 0|T\{j_W^\alpha(0) j_{em}^\mu(y)\}|P(p)\rangle$$

- $j_{em}^\mu(y)$ electromagnetic current
- $j_W^\alpha(0)$ weak current
- $P(p)$ pseudoscalar meson with momentum p
- $\epsilon_\mu^r(k)$ polarisation vector of the photon with momentum *k*

Decomposition of the amplitude in form-factors

$$
\begin{aligned}
H_W^{\alpha\mu}(k,p) \;=\;& H_1\left[k^2 g^{\alpha\mu} - k^\alpha k^\mu\right] + H_2\left[\left(p\cdot k - k^2\right)k^\mu - k^2\,(p-k)^\mu\right]\,(p-k)^\alpha \\
& - i\frac{F_V}{m_P}\epsilon^{\mu\alpha\gamma\beta}k_\gamma p_\beta + \frac{F_A}{m_P}\left[\left(p\cdot k - k^2\right)g^{\alpha\mu} - k^\alpha\,(p-k)^\mu\right] \\
& + \; f_P\left[g^{\alpha\mu} + \frac{(2p-k)^\mu\,(p-k)^\alpha}{2p\cdot k - k^2}\right]
\end{aligned}
$$

The last term is dictated from the Ward id

$$
k_\mu H_W^{\alpha\mu}(k,p) = i\langle 0|j_W^\alpha|P(p)\rangle = f_P\,p^\alpha
$$

it is the same of a point-like scalar particle

it is related to the i.r. divergence

- 4 independent form-factors;
- the form factors only depend on k^2 e p.k;
- we will only discuss the real photon case, $k^2 = 0$ $\varepsilon^r.k = 0$;
- in the future it may be interesting to consider $\Gamma^{exp}(K \to \ell\nu_\ell\, l^+ l^-)$

Amplitude: decomposition in scalar form-factors

$$
\begin{aligned}
H_W^{\alpha\,\mu}(k,p) \;&= H_1\left[k^2 g^{\alpha\,\mu} - k^\alpha k^\mu\right] + H_2\left[\left(p\cdot k - k^2\right) k^\mu - k^2\left(p-k\right)^\mu\right]\left(p-k\right)^\alpha \\
&- i\frac{F_V}{m_P}\epsilon^{\mu\alpha\gamma\beta} k_\gamma p_\beta + \frac{F_A}{m_P}\left[\left(p\cdot k - k^2\right) g^{\alpha\,\mu} - k^\alpha \left(p-k\right)^\mu\right] \\
&+ \; f_P\left[g^{\alpha\,\mu} + \frac{\left(2p-k\right)^\mu \left(p-k\right)^\alpha}{2p\cdot k - k^2}\right]
\end{aligned}
$$

Real Photon Emission

$$
H_W^{\alpha\,\mu}(k,p) \;=\; -i\frac{F_V}{m_P}\epsilon^{\mu\alpha\gamma\beta} k_\gamma p_\beta + \left(\frac{F_A}{m_P} + \frac{f_P}{p\cdot k}\right)\left(p\cdot k\, g^{\alpha\,\mu} - k^\alpha p^\mu\right) + \frac{f_P}{p\cdot k}\, p^\alpha p^\mu
$$

1. different tensors can be separated by using suitable projectors, we then have to disentangle F_A from f_P ;
2. the point-like term is related to the infrared divergence in 1/p.k that has to cancel the corresponding divergence of the virtual correction in the rate;

It is F_A (together with F_V) the relevant structure dependent quantity to be determined.

Lattice Calculation

Relevant Correlator

$E_\gamma = |\mathbf{k}|$

$$C_W^{\alpha r}(t,\mathbf{p},\mathbf{k}) = \epsilon_\mu^r(\mathbf{k}) \int d^4y\; d^3x\; e^{t_y E_\gamma - i\mathbf{k}\cdot\mathbf{y} + i\mathbf{p}\cdot\mathbf{x}}\; \mathrm{T}\langle 0| j_W^\alpha(t) j_{em}^\mu(y) P(0,\mathbf{x})|0\rangle$$

suitable projector on the different form factors

Euclidean extraction of the photon momentum k

source of the pseudoscalar meson with momentum p

the convergence of the integral over t_y is ensured by the safe analytic continuation from Minkowsky to Euclidean, namely by the absence of intermediate states lighter than the pseudoscalar meson. The physical form factors can be extracted directly from the Euclidean correlation functions

$$\sum_n \int_{-\infty}^0 dt\; e^{tE_\gamma} \langle 0|j_W^\alpha(0)|n\rangle\langle n|e^{tH} j_{em}^\mu(0,\mathbf{k}) e^{-tH}|P\rangle \;\to e^{t(E_\gamma + E_n - E_P)}$$

$$E_\gamma + E_n > E_P$$

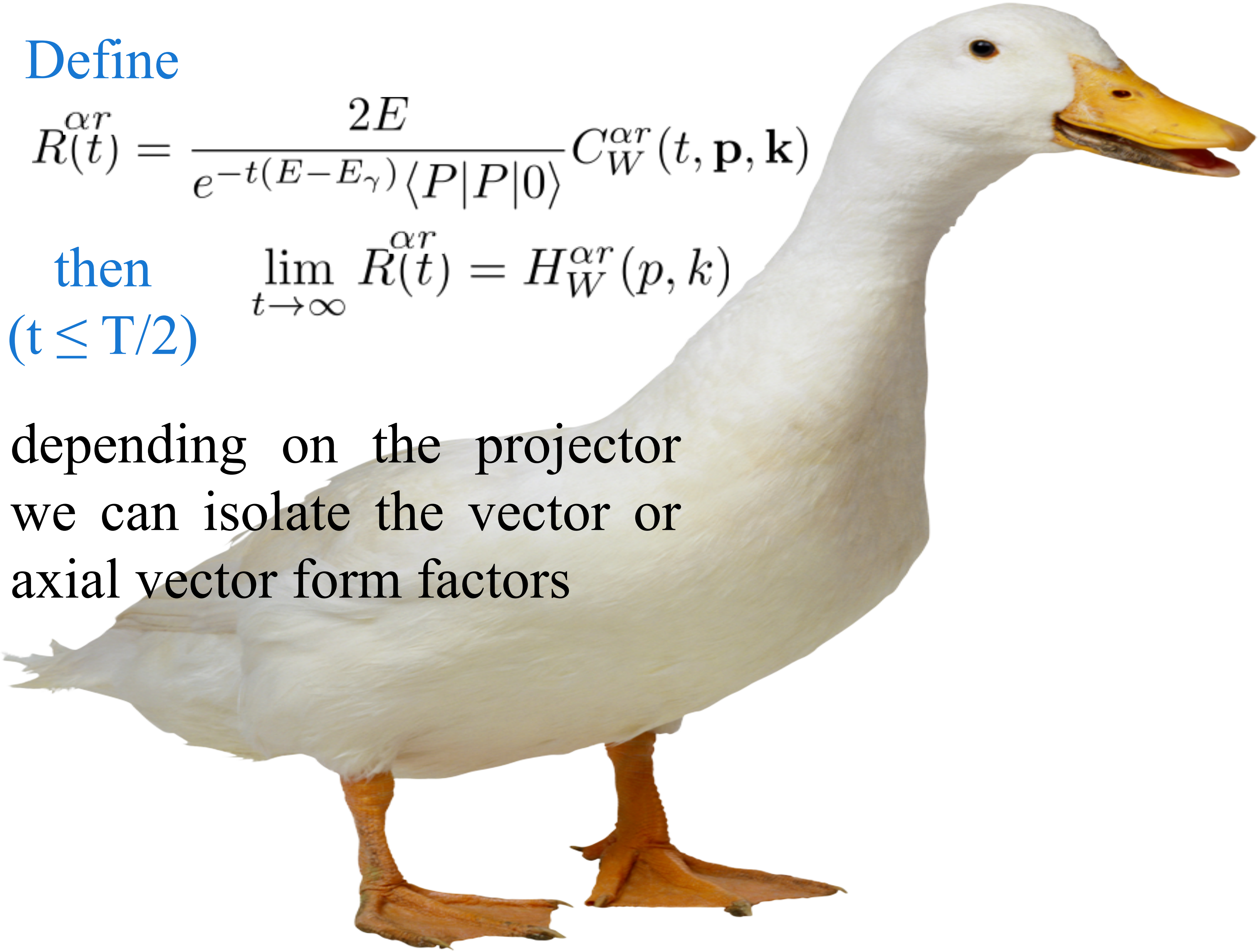
Define

$$R^{\alpha r}(t) = \frac{2E}{e^{-t(E-E_\gamma)}\langle P|P|0\rangle} C_W^{\alpha r}(t, \mathbf{p}, \mathbf{k})$$

then
(t ≤ T/2)

$$\lim_{t \to \infty} R^{\alpha r}(t) = H_W^{\alpha r}(p, k)$$

depending on the projector we can isolate the vector or axial vector form factors



Numerical Results

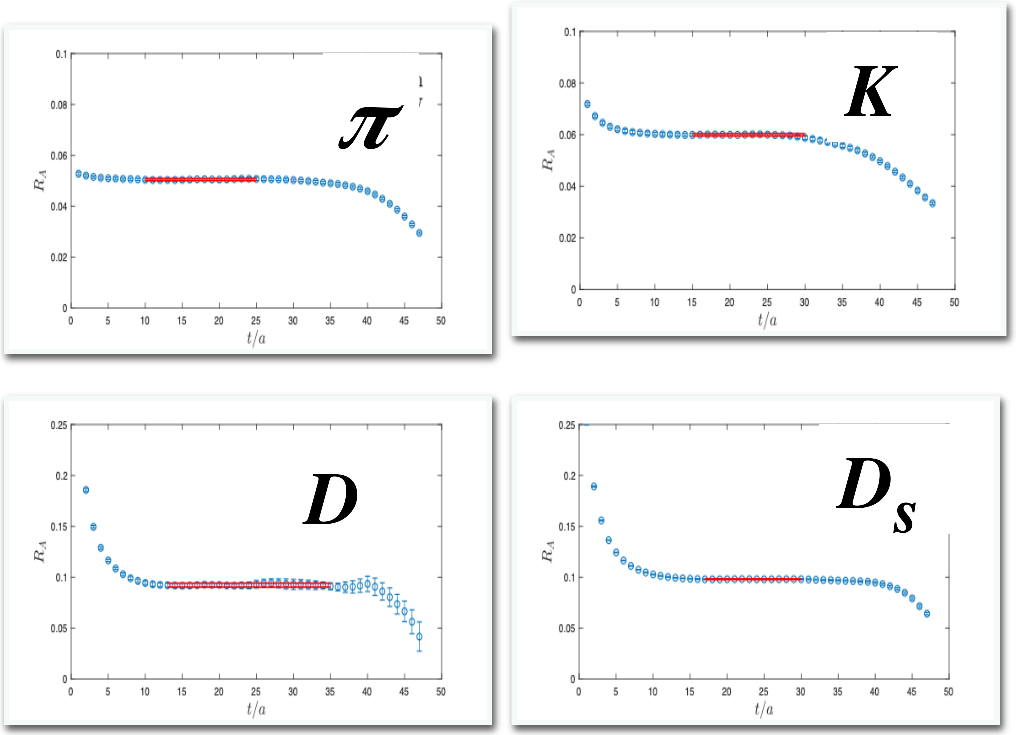
(all the results in the following are preliminary)

- *2+1+1 twisted mass fermions*
- *Three different lattice spacings*
- *3-4 different pion masses for each lattice spacing*
- *Pion masses down to ~230 MeV*
- *100 different momentum configurations*

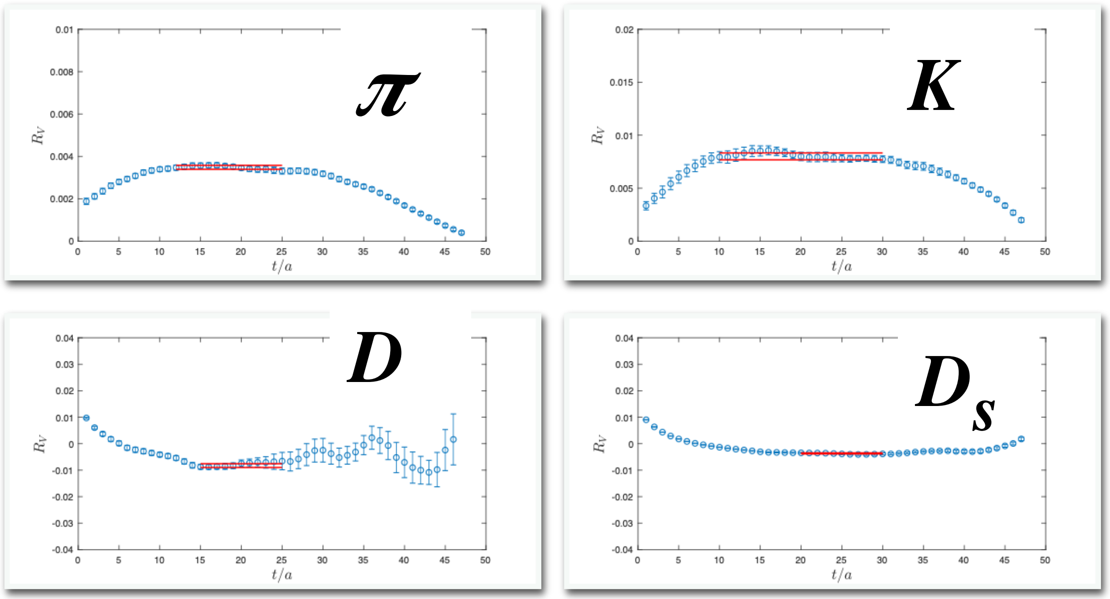
for the plateaus we find rather good signals

$$\lim_{t\to\infty} R^{\alpha r}(t) = H_W^{\alpha r}(p,k)$$

Axial vectors



Vectors

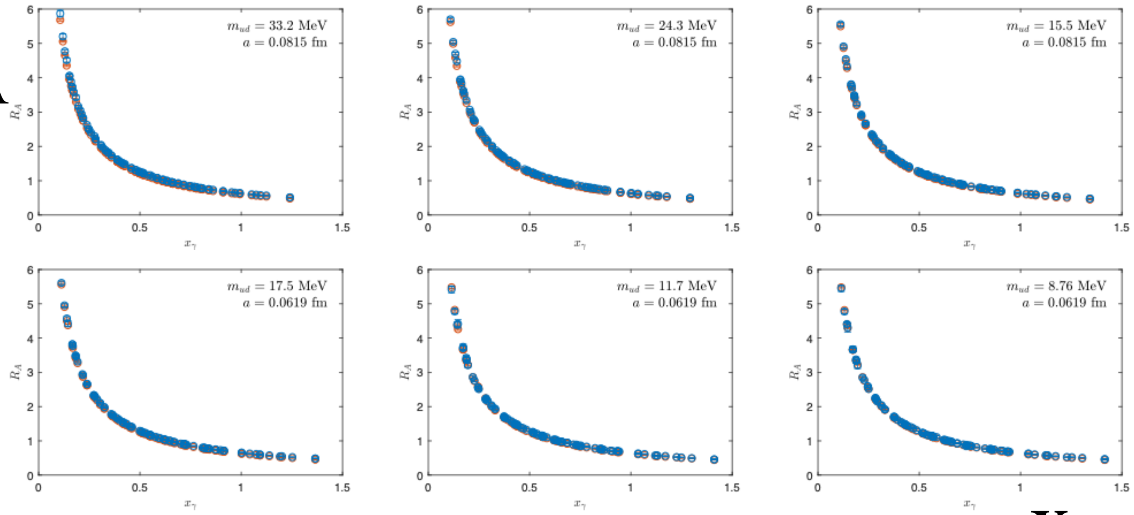


Axial-vector channel for the Kaon

$$R_A = \frac{F_A}{m_P} + \frac{f_P}{p \cdot k}$$

m_{ud} = 33.2 MeV -> decreasing m_{ud} @ a=0.0815 fm m_{ud} = 11.1 MeV

R_A



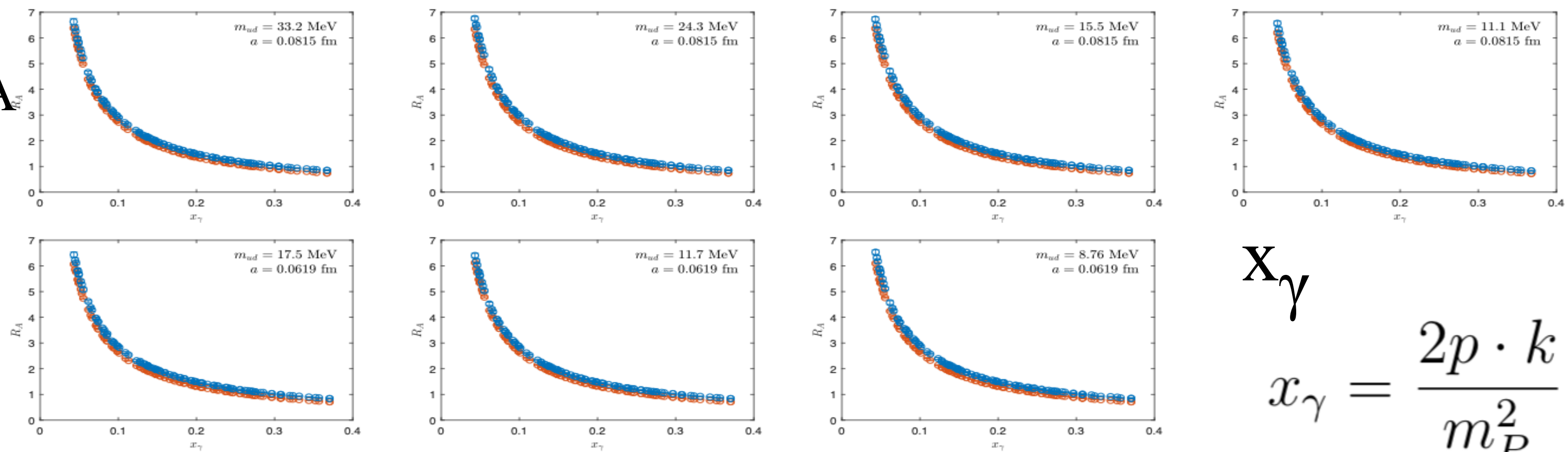
x_γ

a=0.0619 fm

- dominated by the point-like vertex
- no important discretization errors

Axial-vector channel for the D_s

R_A

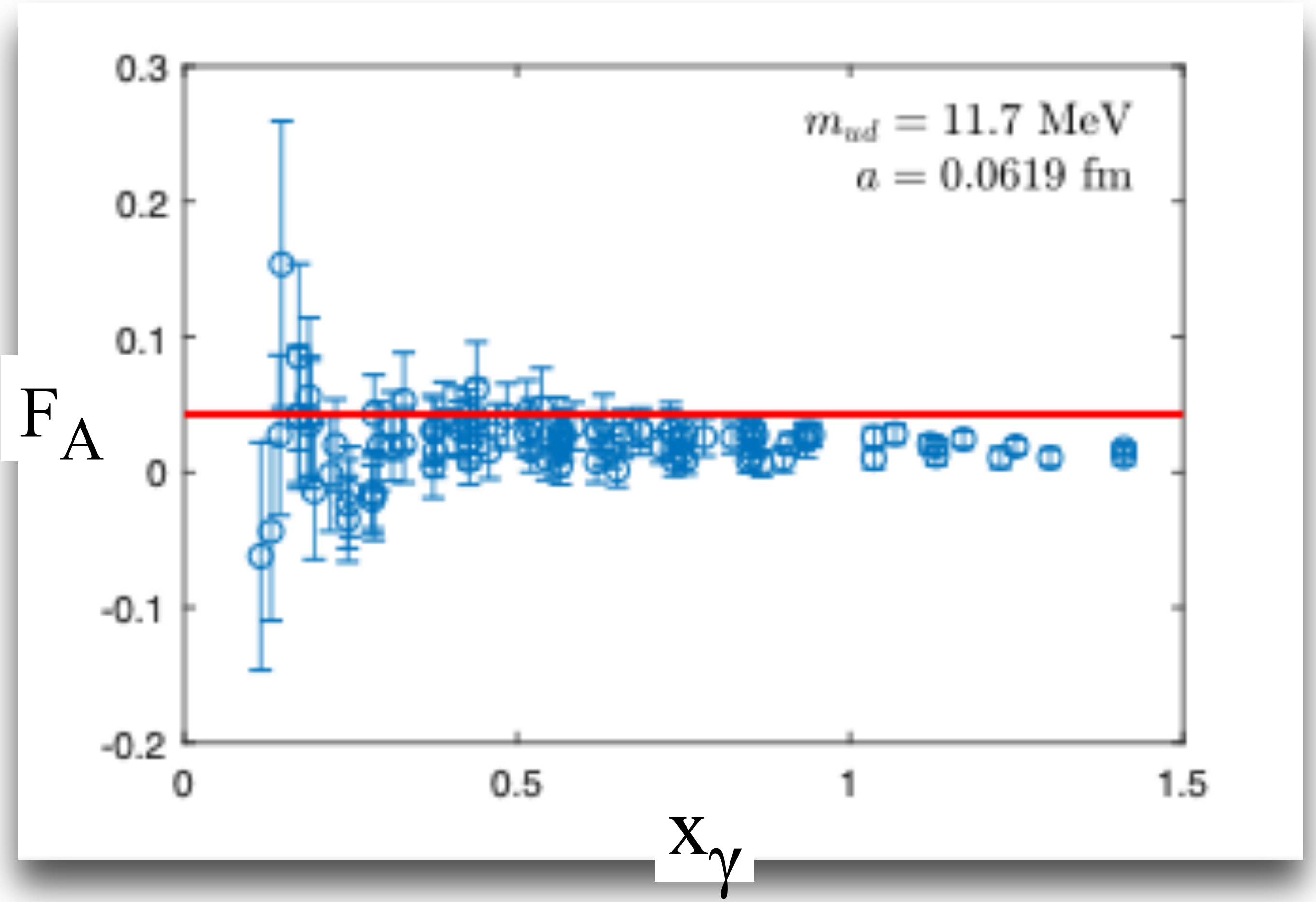


x_γ

$$x_\gamma = \frac{2p \cdot k}{m_P^2}$$

we may compare with χPT that at $O(p^4)$ gives without any momentum dependence

$$F_A = \frac{8m_K}{f_\pi}\left(L_9^r + L_{10}^r\right)$$

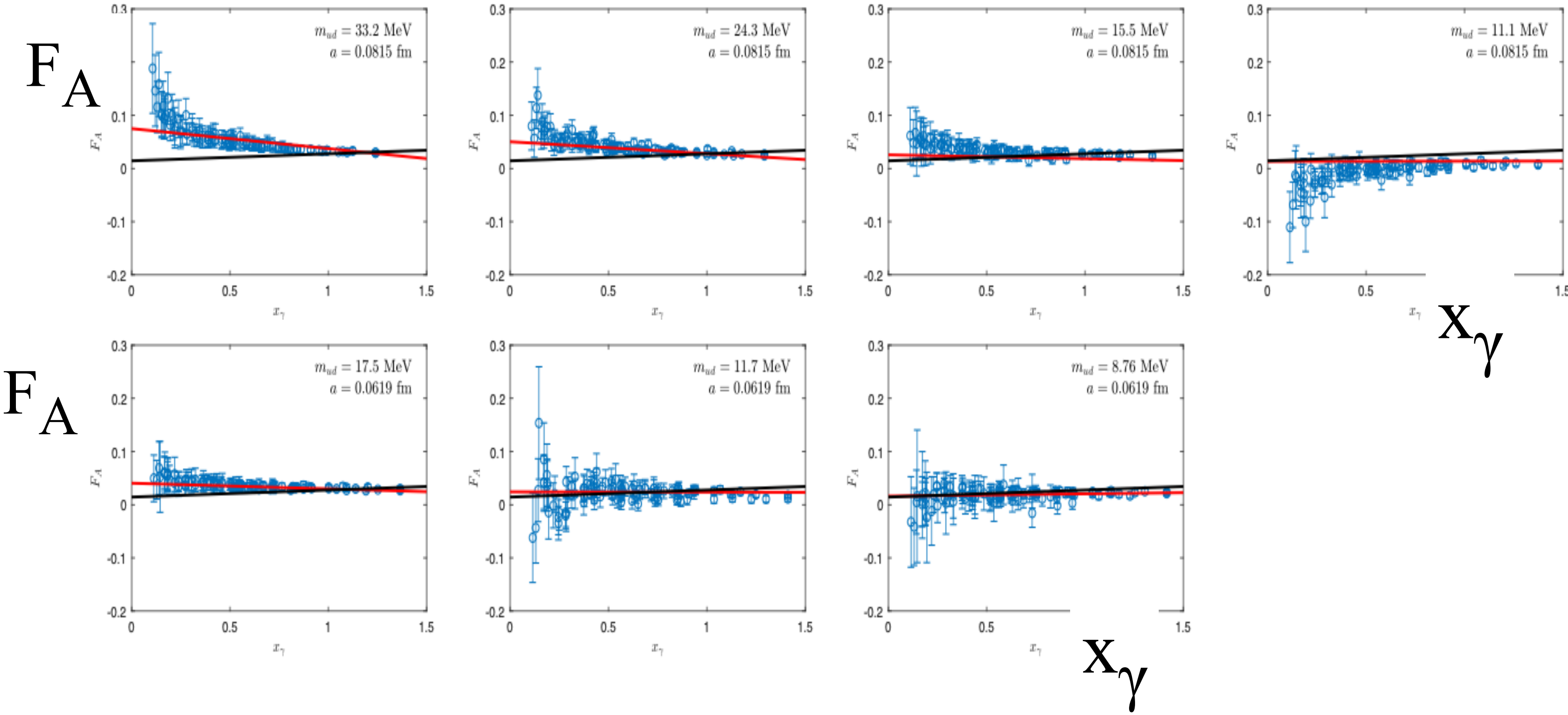


Kaon
F_A form factor

with an improved analysis we will be able to extract the momentum dependence too

Extrapolation to the physical point F_A for the Kaon

after the subtraction of the point-like axial vertex we obtain the relevant structure dependent form factor and its momentum dependence.

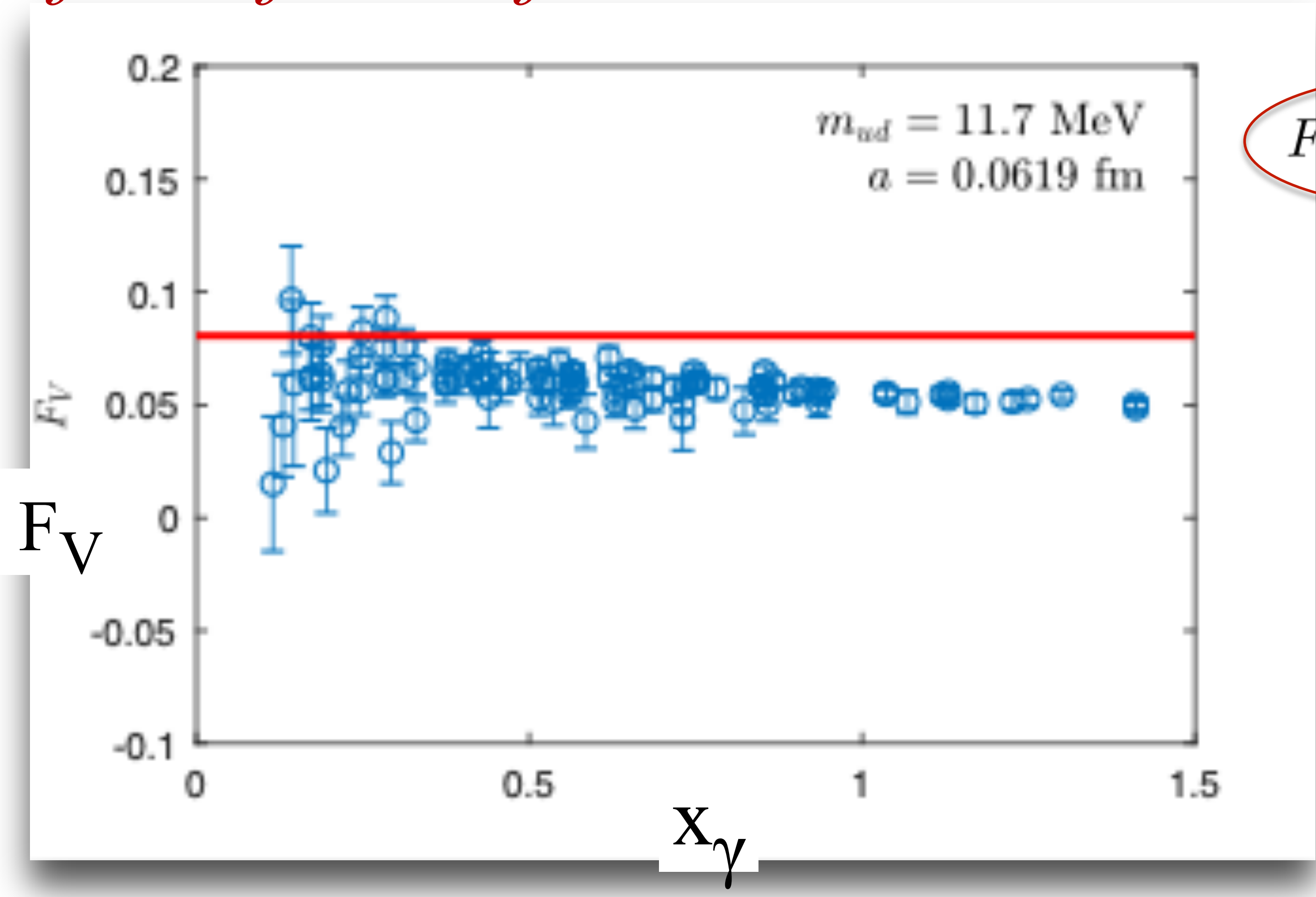


$$F_{(A,V)}(x_\gamma, m_{ud}, a^2, L) = c_0 + c_1\, x_\gamma + d_0\, m_{ud} + d_1 m_{ud} x_\gamma + e_0\, a^2 + e_1\, a^2 x_\gamma + \dots$$

F_V *form factor for the Kaon*

χPT

$$F_V = \frac{m_K}{4\pi^2 f_\pi}$$

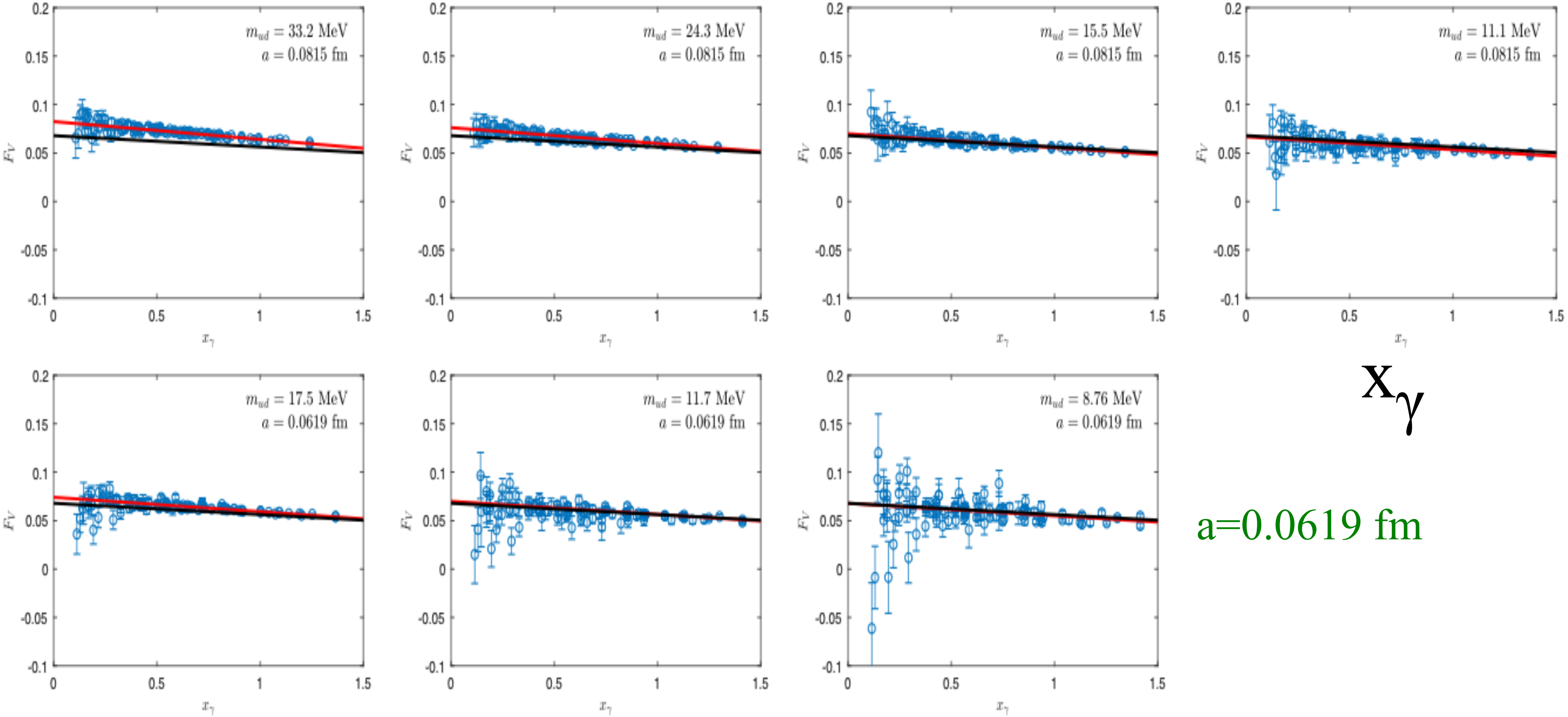


with an improved analysis we will be able to extract the momentum dependence too

Similar results were found for the other mesons π, D and D_s

F_V *form factor for the Kaon*

m_{ud} = 33.2 MeV -> decreasing m_{ud} @ a=0.0815 fm m_{ud} = 11.1 MeV



x_γ

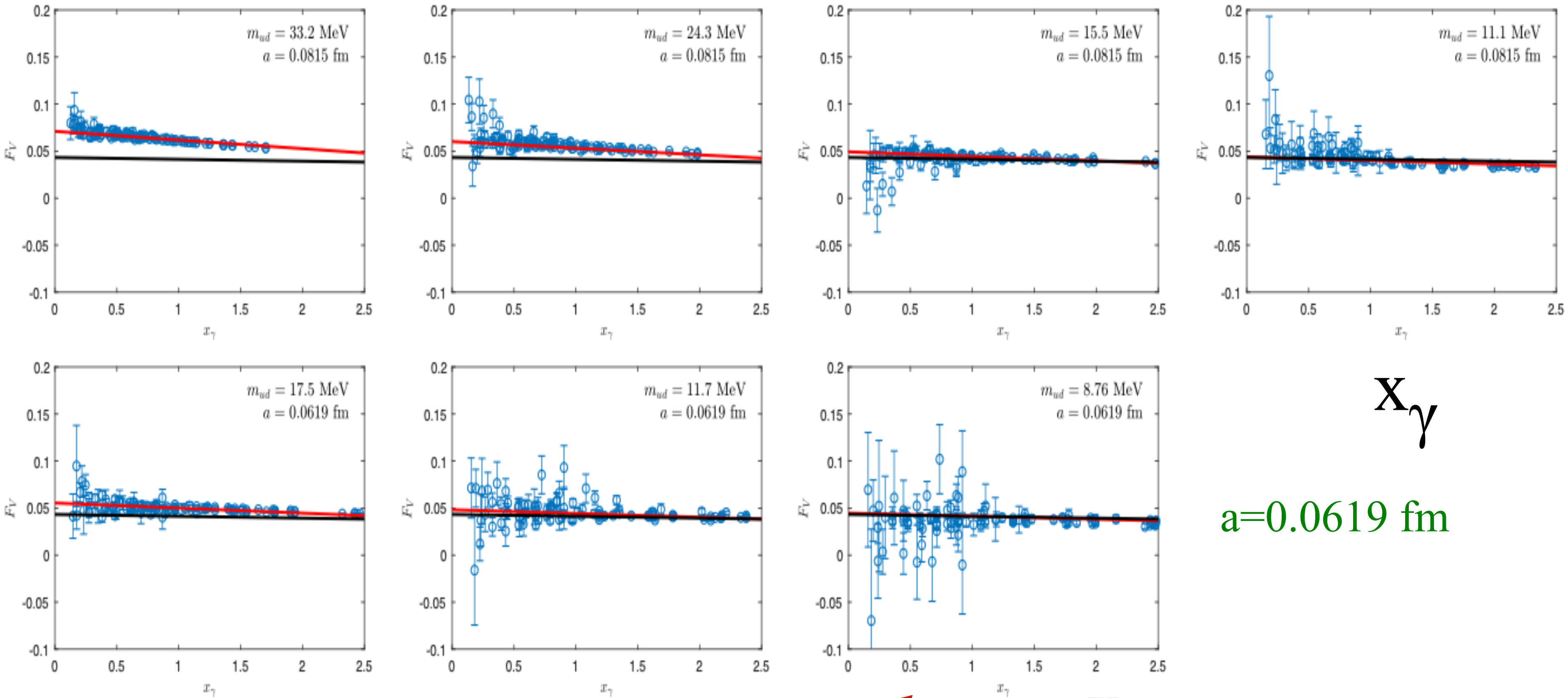
a=0.0619 fm

x_γ

F_V *form factor for the* π

m_{ud} = 33.2 MeV -> decreasing m_{ud} a=0.0815 fm

m_{ud} = 11.1 MeV



x_γ

a=0.0619 fm

x_γ

low mass & small lattice spacing

CONCLUSION and OUTLOOK

Present:

Full lattice calculations of radiative corrections to leptonic decays are possible

The form factors for real emissions are accessible from Euclidean correlators

Our preliminary results are very encouraging, still more work is needed and the analysis is ongoing

Future:

B-mesons are also very interesting, we expect a dynamical enhancement, but they are more difficult on current lattices

THANKS FOR YOUR ATTENTION