# $B \to D^*$ form factors, $R(D^*)$, and $|V_{cb}|$

Andrew Lytle
INFN Roma Tor Vergata

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# Intro & Motivation

- Semileptonic $B \to D^{(*)} l\nu$ decays: used to extract $|V_{cb}|$.
- Long-standing tension $\sim 3\sigma$ in inclusive/exclusive determinations.
- Observables $R(D^{(*)})$ have persistent $2-3\sigma$ tensions with SM.

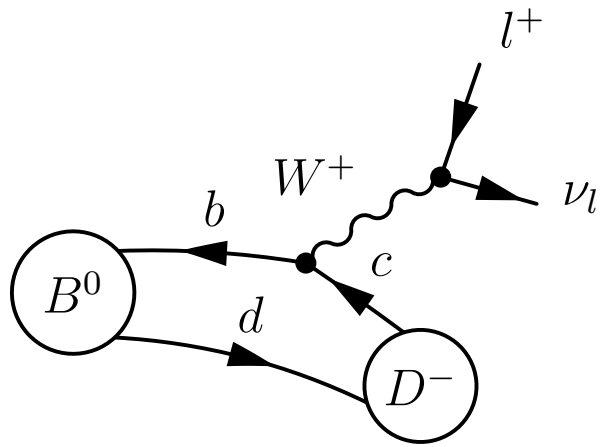
What is the role of lattice QCD in resolving or understanding these tensions?

More generally, what is the outlook for $b \to c$ semileptonic transitions.

# Outline

1. Intro & Motivation.
2. Theoretical framework.
3. Expt'l prospects @ Belle-II & LHCb
4. Status of $B_{(s)} \to D^{(*)}_{(s)}\, l\nu$ lattice calculations.
5. Conclusions & Future outlook.

# $B \to D^{(*)} l\nu$




1104.5484

$$\frac{d\Gamma}{dw}(B \to D) = (\text{known})|V_{cb}|^2(w^2-1)^{3/2}|\mathcal{G}(w)|^2$$
$$\frac{d\Gamma}{dw}(B \to D^*) = (\text{known})|V_{cb}|^2(w^2-1)^{1/2}\chi(w)|\mathcal{F}(w)|^2$$

$$w = v_B \cdot v_{D^{(*)}} = \frac{M_B^2 + M_{D^{(*)}}^2 - q^2}{2M_B M_{D^{(*)}}}$$

At zero recoil $w = 1$, and $\mathcal{F}(1) = h_{A_1}(1)$.

# $B \to D^{(*)} l\nu$ matrix elements

The form factors $\mathcal{F}$ and $\mathcal{G}$ can be determined from QCD matrix elements computed on the lattice.

$$\frac{\langle D|V^\mu|B\rangle}{\sqrt{m_B\, m_D}} = (v_B + v_D)^\mu h_+(w) + (v_B - v_D)^\mu h_-(w)$$

$$\frac{\langle D^*_\alpha|V^\mu|B\rangle}{\sqrt{m_B\, m_{D^*}}} = \varepsilon^{\mu\nu\rho\sigma} v_B^\nu v_{D^*}^\rho \epsilon_\alpha^{*\sigma} h_V(w)$$

$$\frac{\langle D^*_\alpha|A^\mu|B\rangle}{\sqrt{m_B\, m_{D^*}}} = i\epsilon_\alpha^{*\nu} \left[h_{A_1}(w)(1+w)g^{\mu\nu} - (h_{A_2}(w)v_B^\mu + h_{A_3}(w)v_{D^*}^\mu)v_B^\nu\right]$$

At zero recoil $w = 1$, and $\mathcal{F}(1) = h_{A_1}(1)$.

# $|V_{cb}|$ and the $V_{cb}$ puzzle

- There is a long-standing tension in extractions of $|V_{cb}|$ from inclusive and exclusive semileptonic $B$ decays, hovering at $\approx 3\sigma$.
- The most precise exclusive channel is $B \to D^* l\nu$.
- Lattice calculations are more precise near zero-recoil, whereas experimental data is suppressed here.
- To connect theory and experiment, the experimental results are extrapolated to zero recoil using HQET (CLN parameterization) and this is compared with the form factor computed at zero recoil to extract $|V_{cb}|$.

# Form factor extrapolation

Experimental data traditionally analyzed in HQET framework:
Fit form factors as function of $w$ to extract $\mathcal{F}(1)\eta_{\rm EW}|V_{cb}|$,
compare with $h_{A_1}(1)$ from lattice to extract $|V_{cb}|$.
CLN parameterization:

$$h_{A_1}(z) = h_{A_1}(1)(1 - 8\rho^2 z + (53\rho^2 - 15)z^2 - (231\rho^2 - 91)z^3)$$
$$R_1(w) = R_1(1) - 0.12(w-1) + 0.05(w-1)^2$$
$$R_2(w) = R_2(1) - 0.11(w-1) - 0.06(w-1)^2$$

BGL parameterization uses only weak unitarity constraints: In terms of $z = (\sqrt{w+1} - \sqrt{2})/(\sqrt{w+1} + \sqrt{2})$, $z \in [0, 0.056]$:

$$f_i(z) = \frac{1}{P_i(z)\phi_i(z)} \sum_{k=0}^{n_i} a_k^i z^k$$

# Model dependence in $|V_{cb}|_{\text{excl}}$

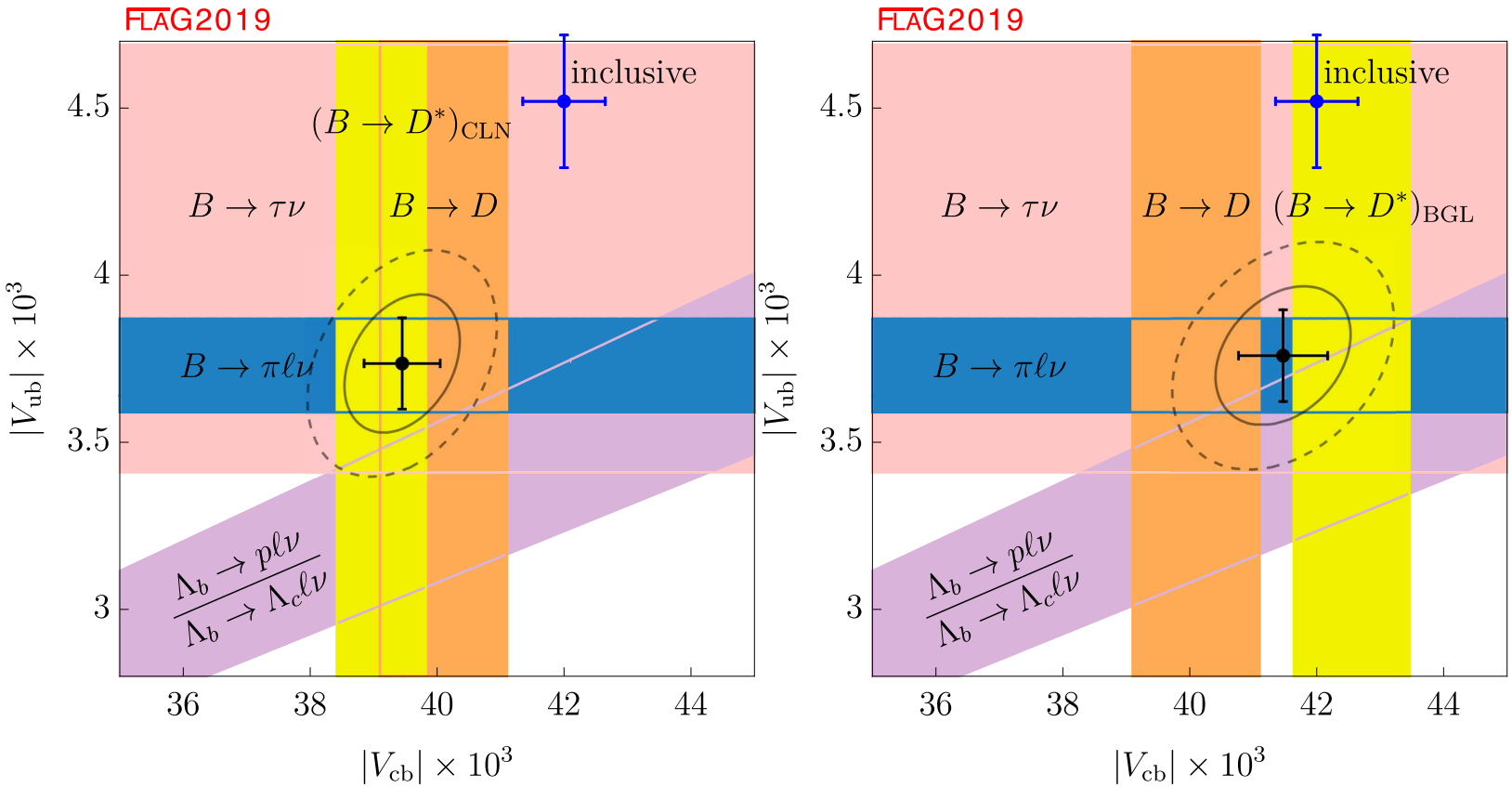
Recent analyses indicating CLN may be too restrictive.
1703.06124, 1703.08170, 1708.07134

- 1702.01521 Belle tagged data set, strong model dependence observed CLN/BGL.
- 1903.10002 Babar tagged analysis w/ BGL, model dependence but discrepancy persists.
- 1809.03290, 1905.08209 Belle untagged data set, no strong model dependence, $|V_{cb}|$ discrepancy persists.

Lattice data for $w > 1$ is extremely useful to help resolve this issue! 1708.07134, 1905.08209

# $|V_{cb}|$ and the $V_{cb}$ puzzle

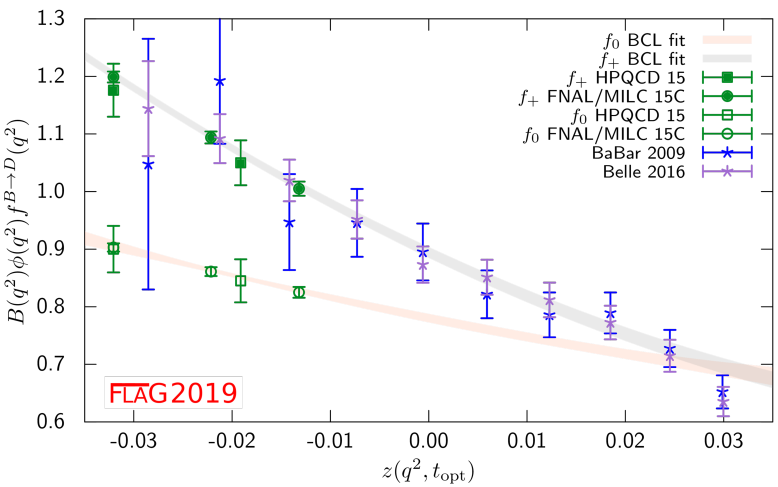
1902.08191

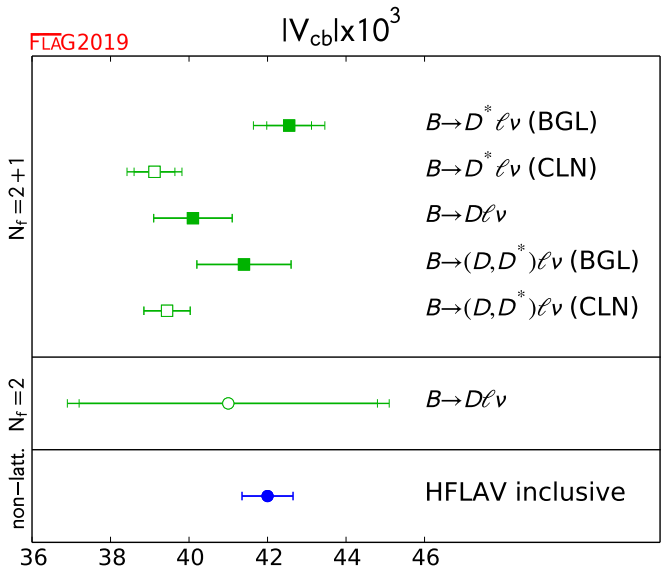


# $|V_{cb}|$ and the $V_{cb}$ puzzle

1503.07237 [MILC], 1505.03925 [HPQCD]
1510.03675 [Belle], 0904.4063[BaBar]

Combining lattice and expt'l data in $B \to Dl\nu$




compatible with $B \to D^*$ extraction and inclusive:
$|V_{cb}| = 40.49(97)10^{-3}$ 1606.08030

# $R$-ratios

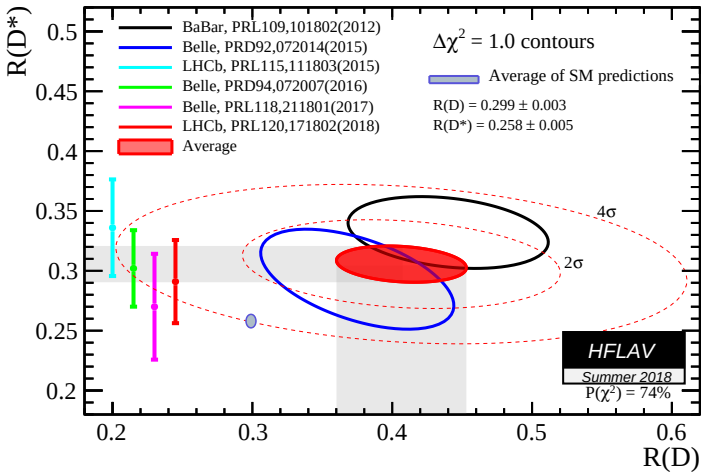
$R$-ratio for $B \to D^{(*)}$ semileptonic decay defined as

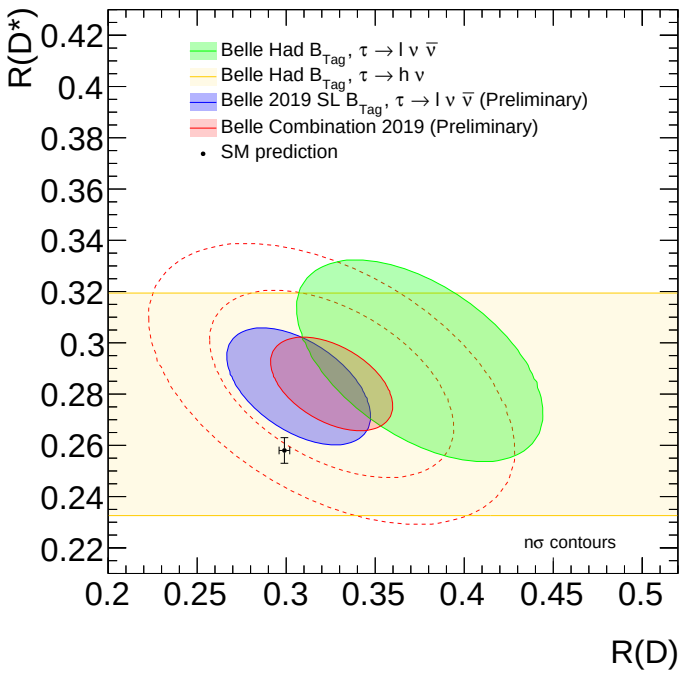
$$R(B \to D^{(*)}) = \frac{\mathcal{B}(B \to D^{(*)}\,\tau\nu)}{\mathcal{B}(B \to D^{(*)}\,l\nu)}\,, \quad l = \mu,\, e$$

- Test lepton flavour universality.
- Experimentally and theoretically attractive.
  - Msm't systematics cancel in ratio.
  - Form factor dependence largely cancels in ratio.
- There are persistent 2-3$\sigma$ anomalies in the ratios $R(B \to D^*)$ and $R(B \to D)$, as well as recent msm't $R(B_c \to J/\psi)$ 1711.05623 ($\lesssim 2\sigma$).

# $R(D^{(*)})$

hflav18, 1904.08794




- $R(D)_{\mathrm{SM}}$ from lattice calculations.
  1503.07237 [FNAL/MILC]
  1505.03925 [HPQCD]

- Belle combination moves $R(D^{(*)})$ closer to SM.

- Would be good to have $R(D^*)_{\mathrm{SM}}$ prediction from lattice. $\rightarrow$ all ffs over full kinematic range.

# $R(B_c \to J/\psi)$

- LHCb 1711.05623
  $R(J/\psi) = 0.71(17)_{\text{stat}}(18)_{\text{syst}}$
- w/in $2\sigma$ of theory range [0.25, 0.28]
- Preliminary lattice result 0.290(7)

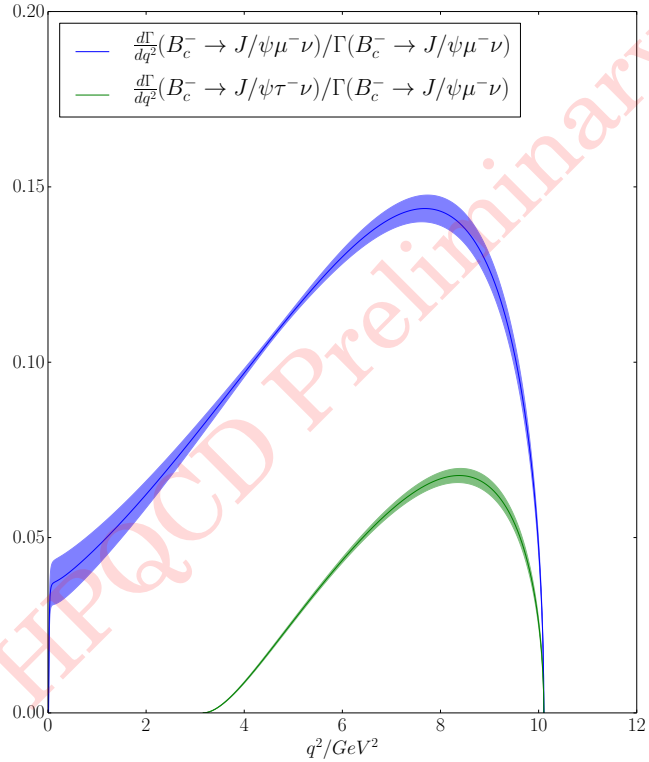



Fig. courtesy J. Harrison

# Projections - Belle II 1808.10567

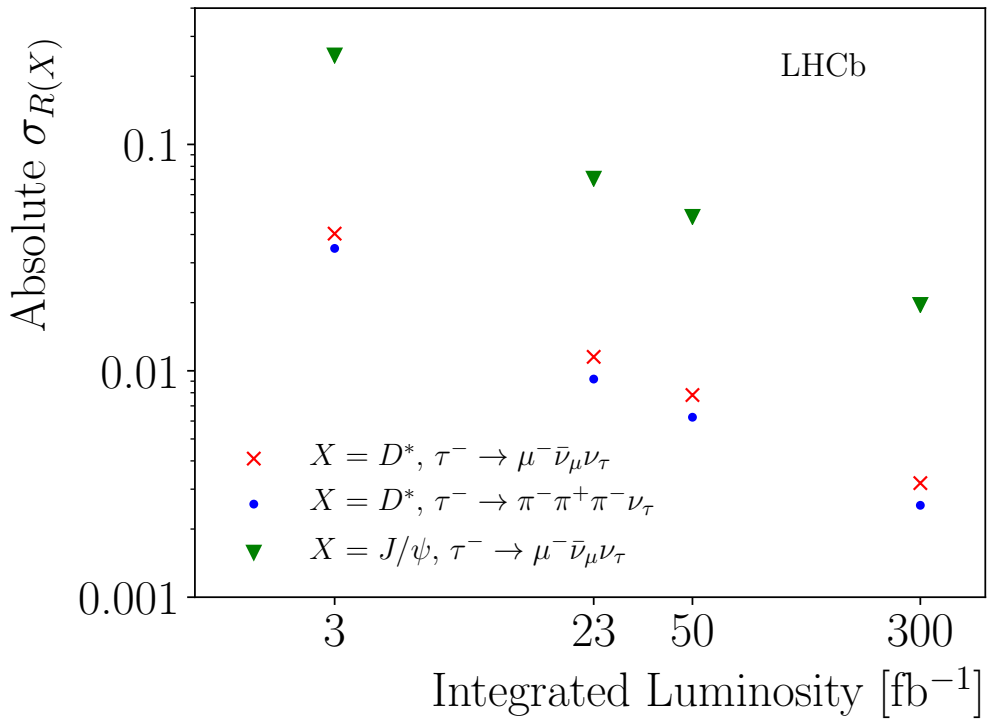
Expect increasingly precise results from large Belle II datasets.
Integrated luminosity:

- 5 $\text{ab}^{-1} \sim 2021$
- 50 $\text{ab}^{-1} \sim 2025$

| | Belle | Belle II | |
|---|---|---|---|
| | (2017) | 5 $\text{ab}^{-1}$ | 50 $\text{ab}^{-1}$ |
| $\lvert V_{cb}\rvert$ excl. | 3.3 % | 1.8% | 1.4% |
| $\lvert V_{cb}\rvert$ incl. | 1.8% | 1.2% | - |
| $R(D)$ (Had. tag) | 16.5% | 6% | 3 % |
| $R(D^*)$ (Had. tag) | 7.4% | 3% | 2 % |

# Projections - LHCb 1808.08865

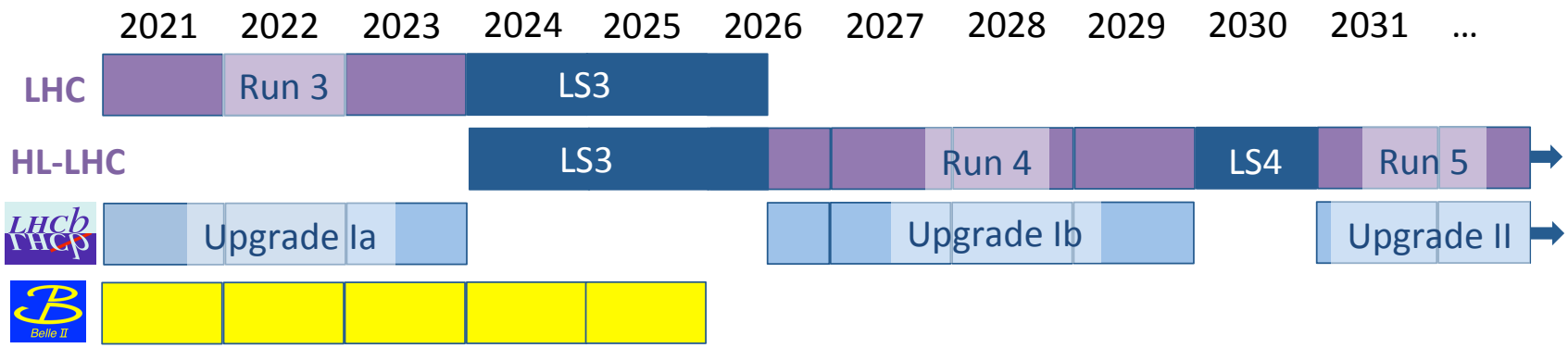
Increasingly precise $B \to D^{(*)}$, $B_c \to J/\psi$.




- 23 fb$^{-1}$ Run 3 (end 2023)
- 50 fb$^{-1}$ Run 4 (end 2029)
- 300 fb$^{-1}$ Upgrade II (2030s)

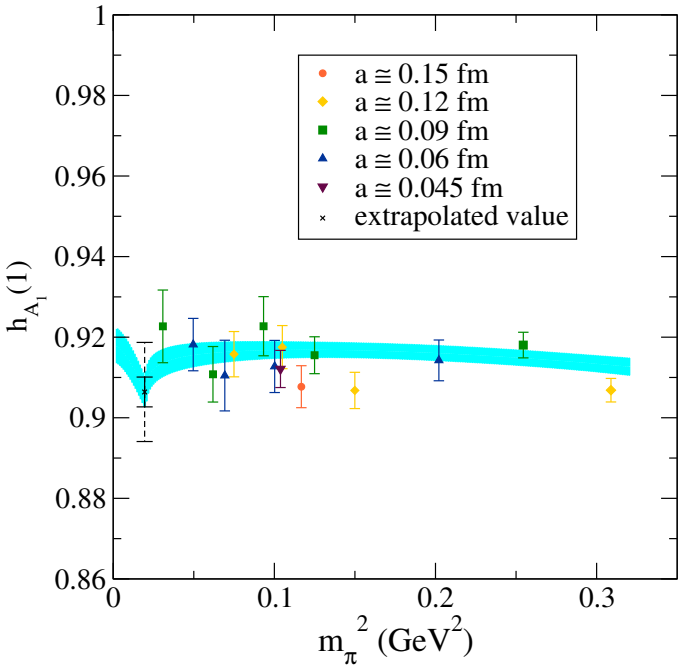
# Projections - LHCb 1808.08865

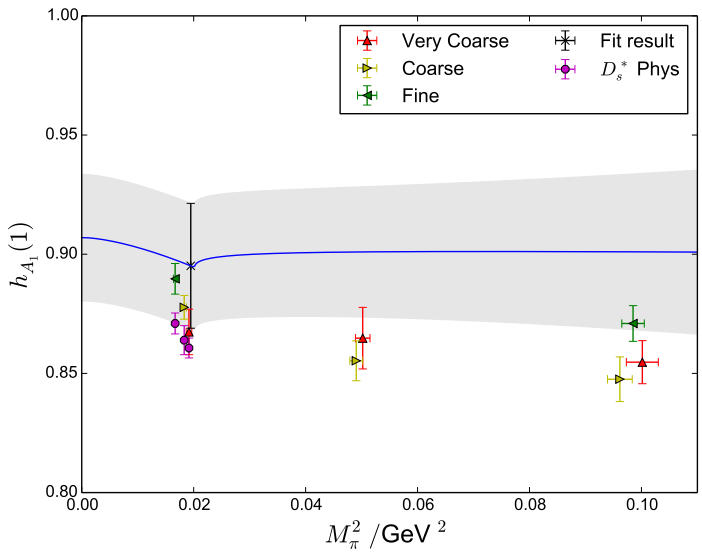
- Increasingly precise $B \to D^{(*)}$, $B_c \to J/\psi$.
- $B \to D^{(*)}$ differential measurements. (Upgrade II)
- Measurements of new modes $B_s \to D_s^{(*)}$
- $\sigma_{R(D_s^{(*)})} \sim 6\%$ Run 3, 2.5% Upgrade II.



# Lattice QCD results

# $B \to D^*$ at zero recoil from LQCD




FNAL/MILC 1403.0635

- $n_f = 2 + 1$ MILC asqtad ensembles
- Clover $b$ with Fermilab interpretation
- $h_{A_1}(1) = 0.906(4)(12)$

HPQCD 1711.11013

- $n_f = 2 + 1 + 1$ MILC HISQ ensembles
- NRQCD $b$ quark
- $h_{A_1}(1) = 0.895(10)(24)$
- $h^s_{A_1}(1) = 0.883(12)(28)$

# Treatment of heavy quarks

Treatment of $c$ and especially $b$ quarks challenging in lattice simulaiton due to lattice artifacts which grow as $(am_h)^n$

- Generally one uses an effective theory framework to handle the $b$ quark.
  - Fermilab interpretation, RHQ, OK, NRQCD
  - Pros: Solves problem w/ $am_h$ artifacts.
  - Cons: Requires matching, can still have $ap$ artifacts.
- Also possible to use fully relativistic action provided $a$ is sufficiently small $am_c \ll 1$, $am_b < 1$.
  - Use improved actions e.g. $\mathcal{O}(a^2) \to \mathcal{O}(\alpha_s a^2)$
  - Pros: Absolutely normalised current, straightforward continuum extrap.
  - Cons: Numerically expensive, extrapolate $m_h \to m_b$.

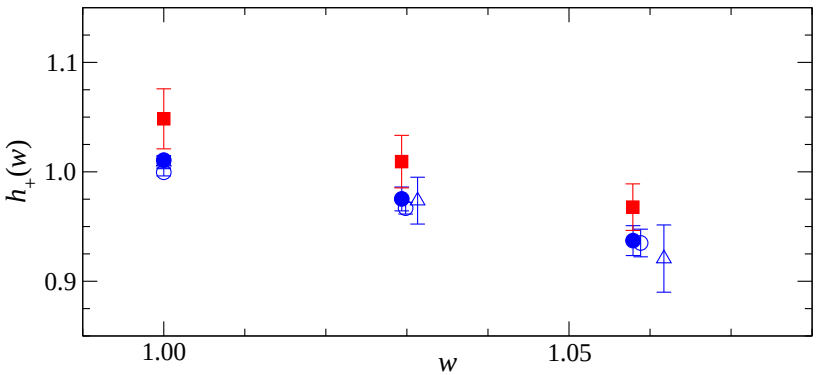
# JLQCD $B \to D^{(*)}$ 1811.00794

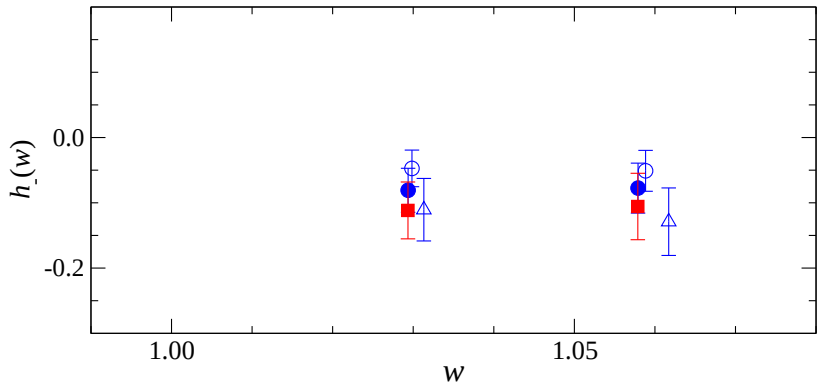
- 'Relativistic-$b$' approach on Möbius $n_f = 2 + 1$ DWF ensembles.
- $a^{-1} = 2.5$, 3.6, (4.5) GeV, $M_\pi = 310$, (230) MeV.
- $am_h < 0.8$ ($m_h$ up to $2.4m_c$)
- Work in range $w \in [1.0, 1.07]$

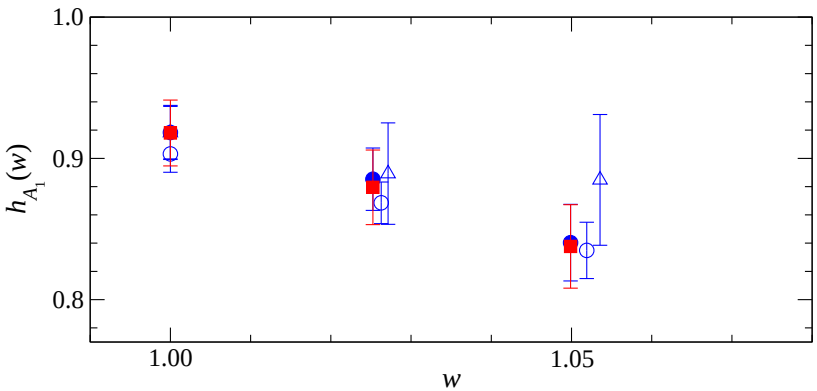
See talk by Takashi Kaneko, Mon. 15:00 [WD&ME].

# JLQCD $B \to D^{(*)}$

1811.00794

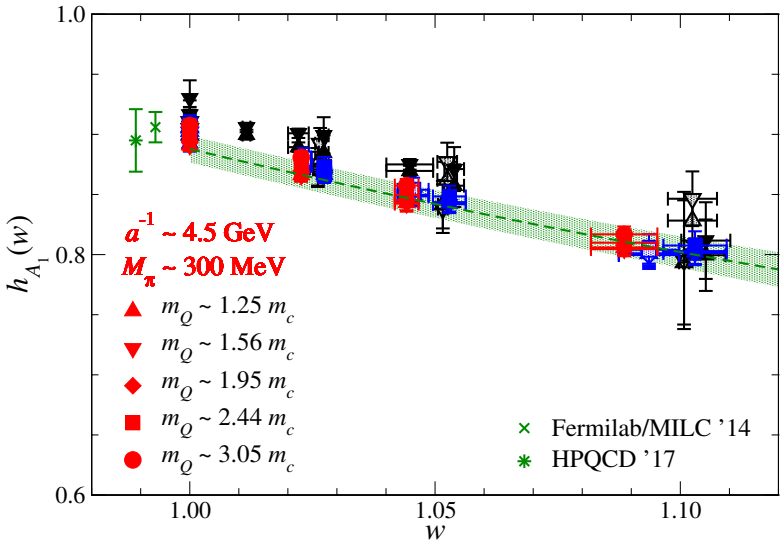


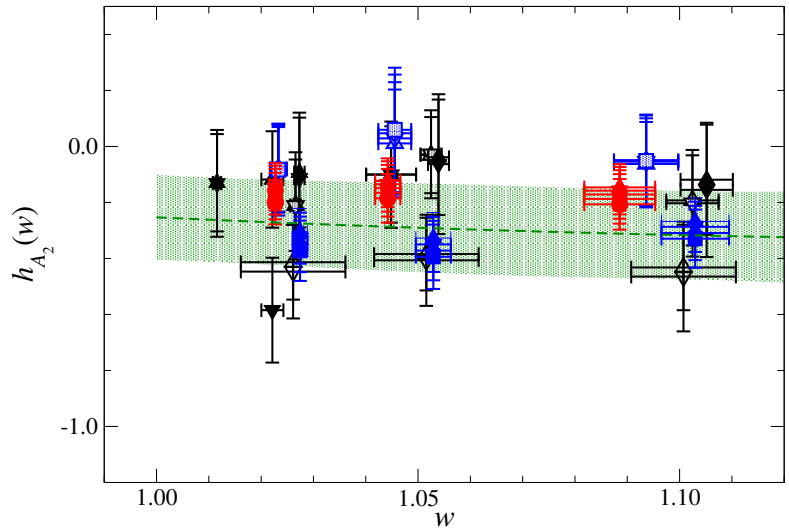



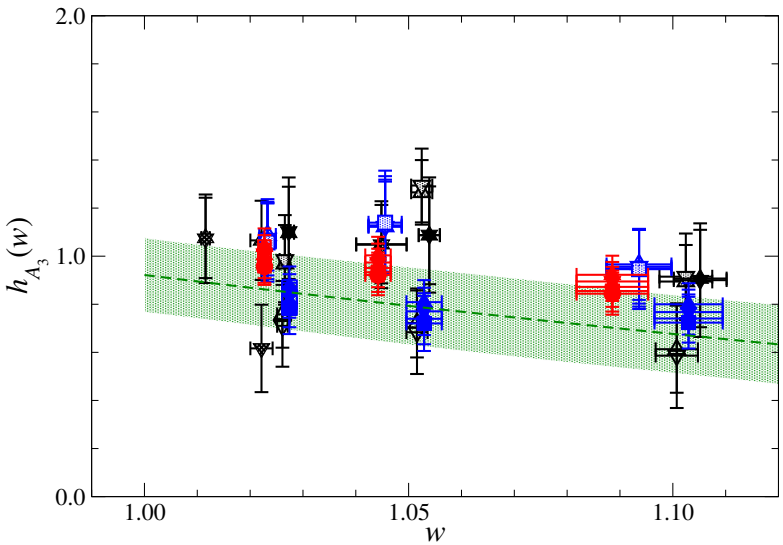



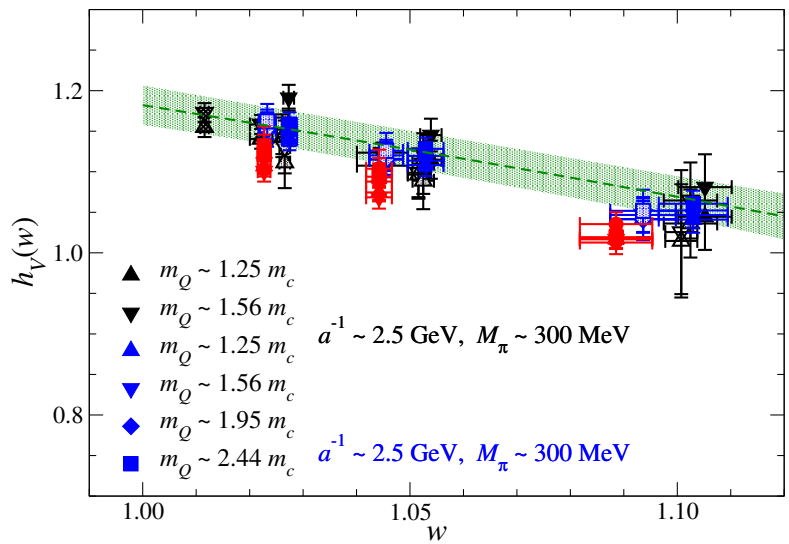

- $a^{-1} \sim 2.5$ GeV, $m_b = 1.25^2 m_c$, $M_\pi \sim 500$ MeV
- $a^{-1} \sim 2.5$ GeV, $m_b = 1.25^2 m_c$, $M_\pi \sim 300$ MeV
- $a^{-1} \sim 3.6$ GeV, $m_b = 1.25^2 m_c$, $M_\pi \sim 500$ MeV
- $a^{-1} \sim 3.6$ GeV, $m_b = 1.25^4 m_c$, $M_\pi \sim 500$ MeV

# JLQCD $B \to D^*$



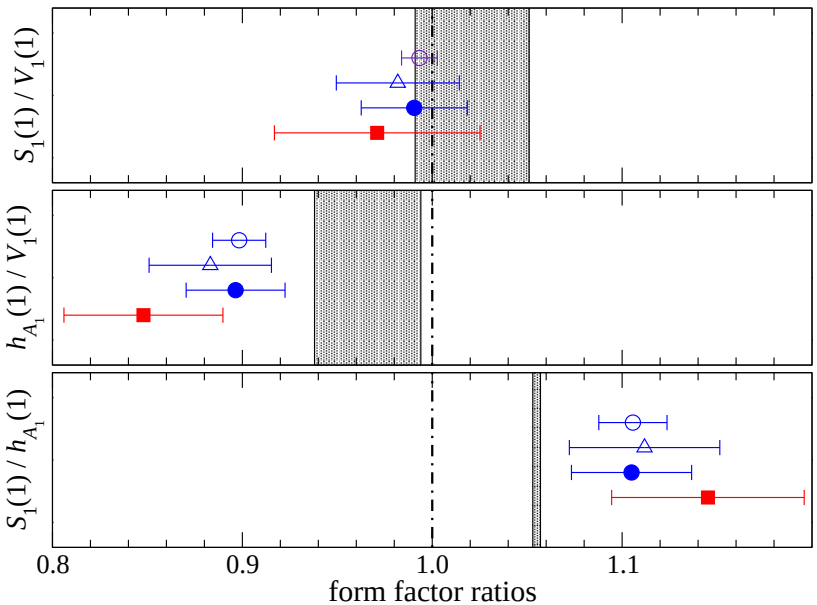



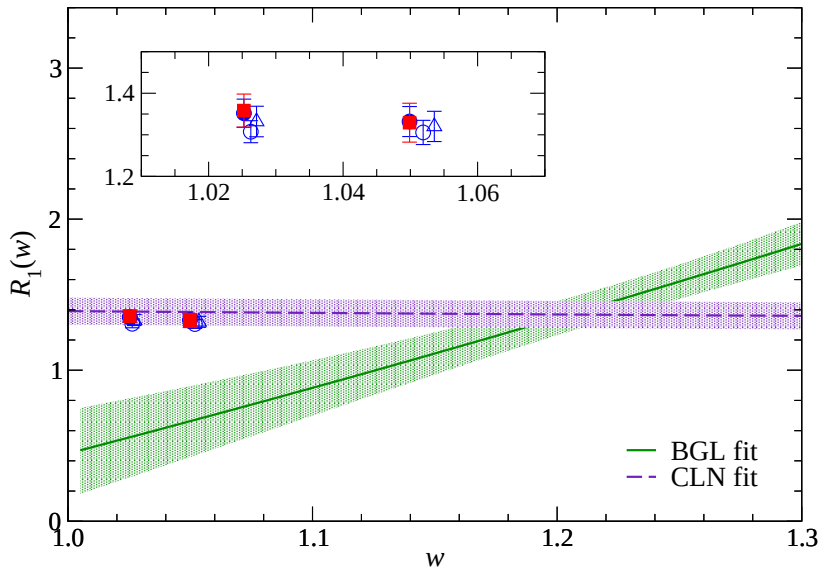




Figs. courtesy Takashi Kaneko.

# JLQCD $B \to D^{(*)}$

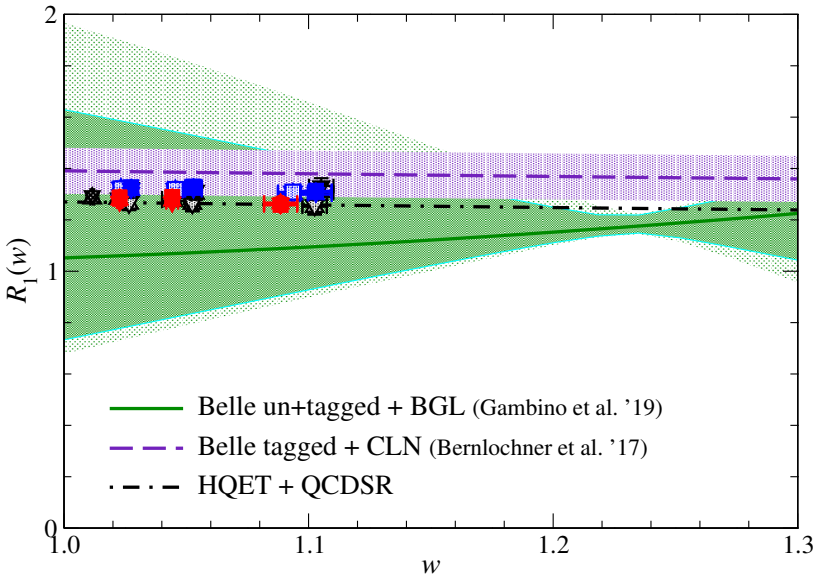
1811.00794

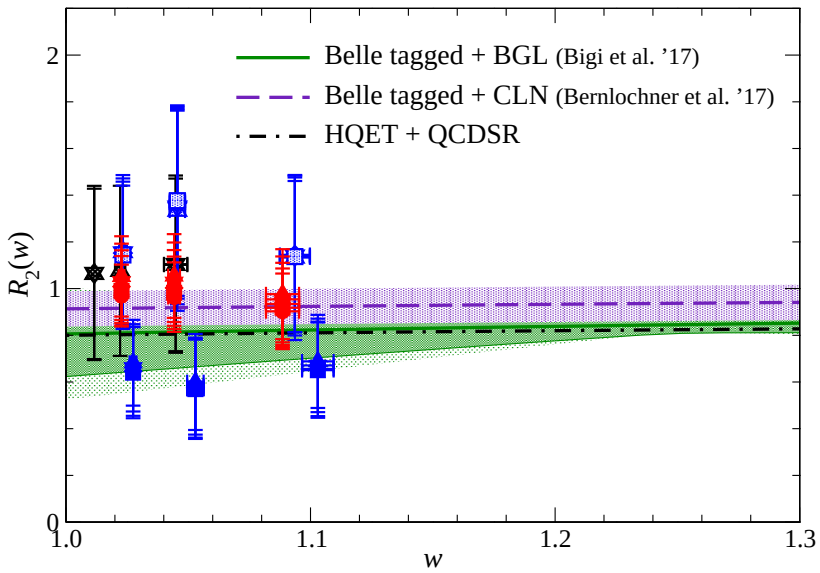



- Indications of significant corrections to NLO HQET.
- On the other hand $R_1(w)$ favors CLN. PRD96 091503

# JLQCD $B \to D^{(*)}$



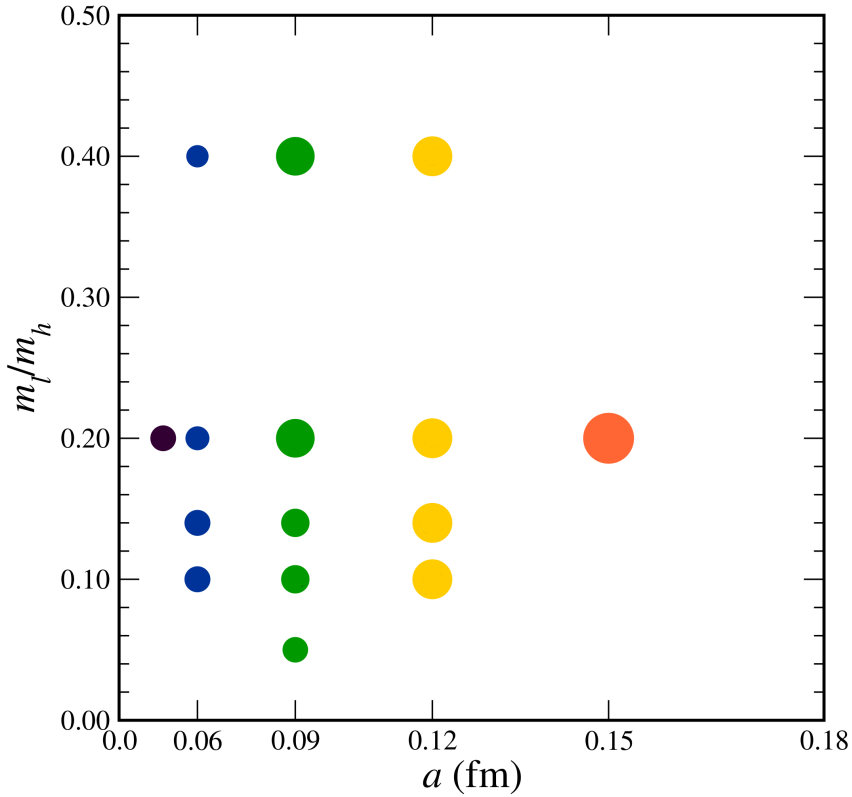




Figs. courtesy Takashi Kaneko.

# FNAL/MILC $B \to D^*$

1710.09817

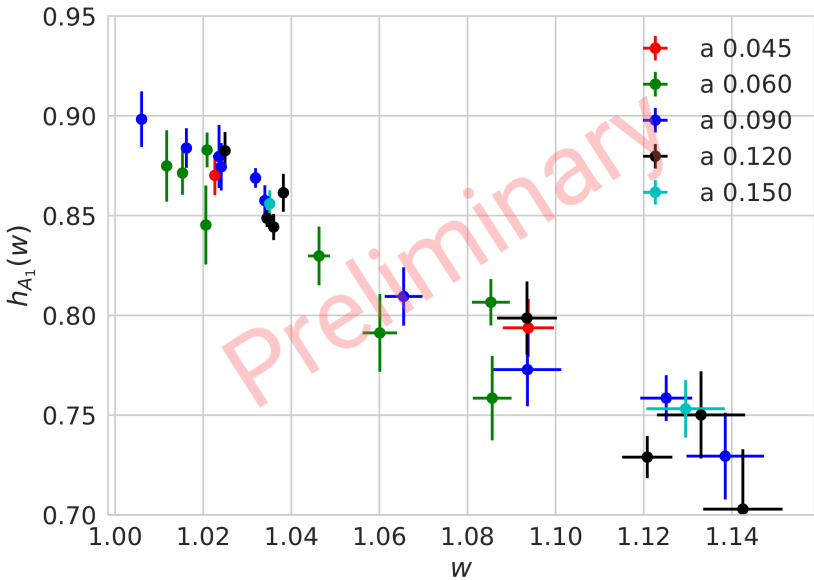
- $n_f = 2 + 1$ asqtad ensembles
- $a^{-1} = 0.15$–$0.045$ fm
- Asqtad light quarks
- Clover heavy w/ Fermilab interpretation
- Range $w \in [1.0, 1.15]$

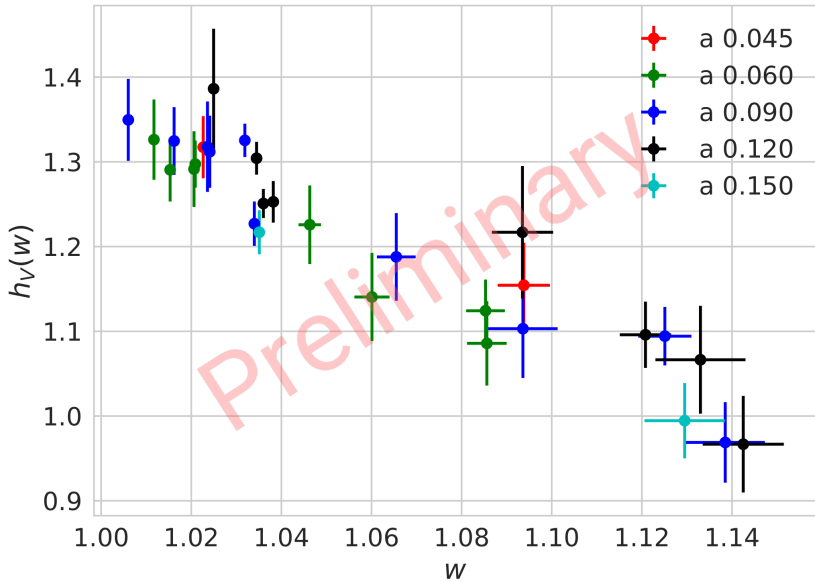


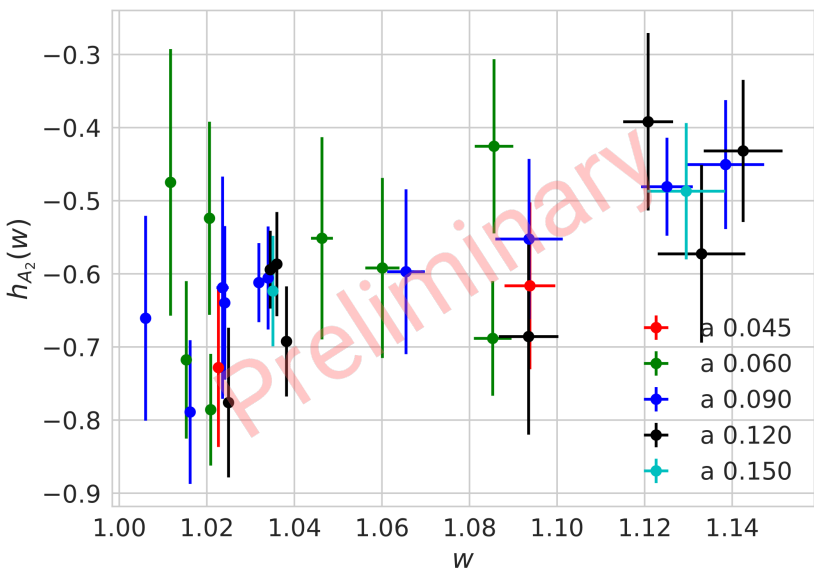

See talk by Alex Vaquero, Mon. 14:40 [WD&ME].

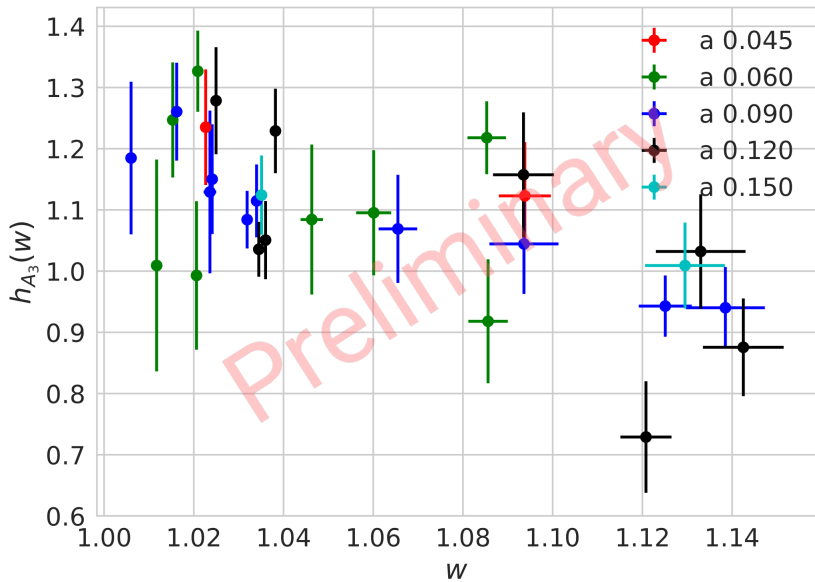
# FNAL/MILC $B \to D^*$

1710.09817

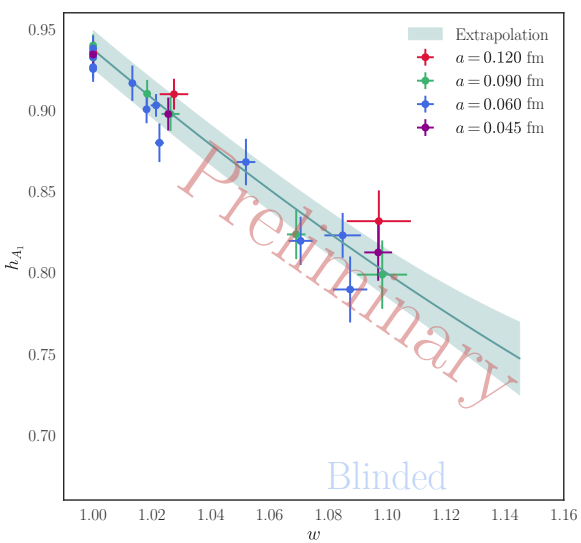


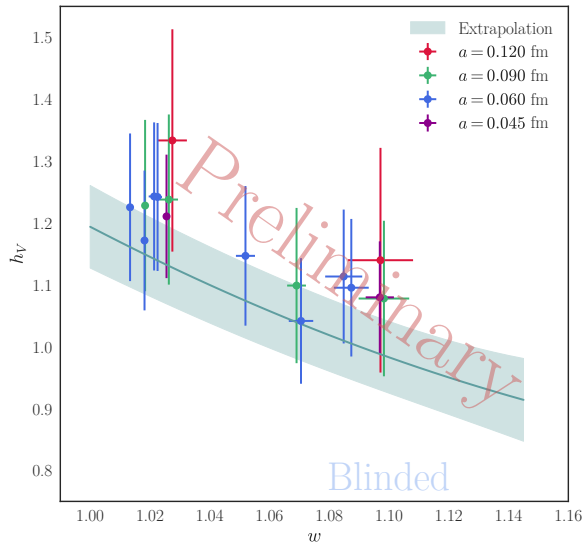



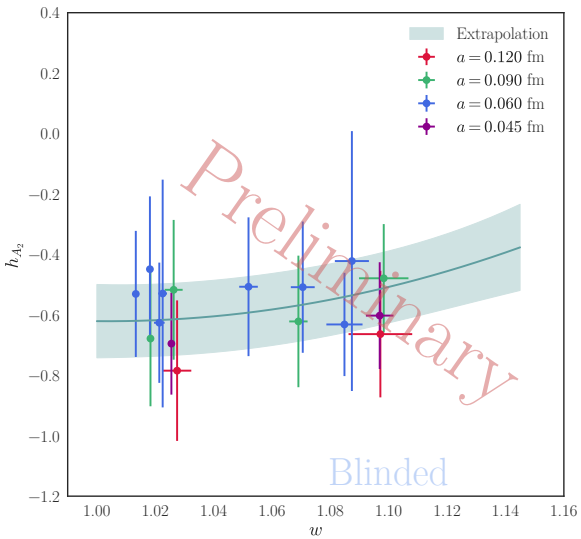



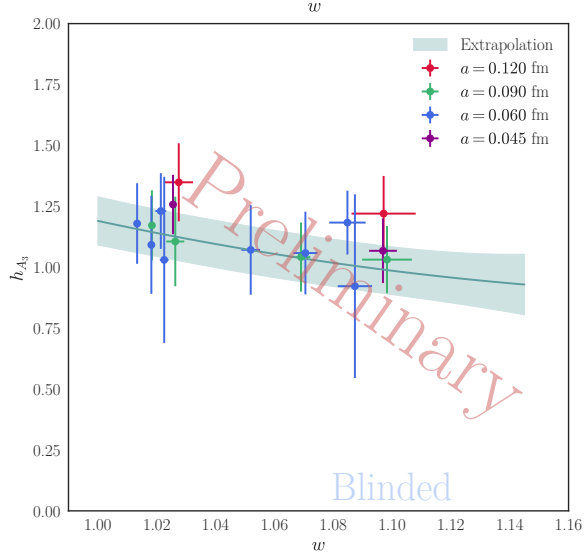



# FNAL/MILC $B \to D^*$



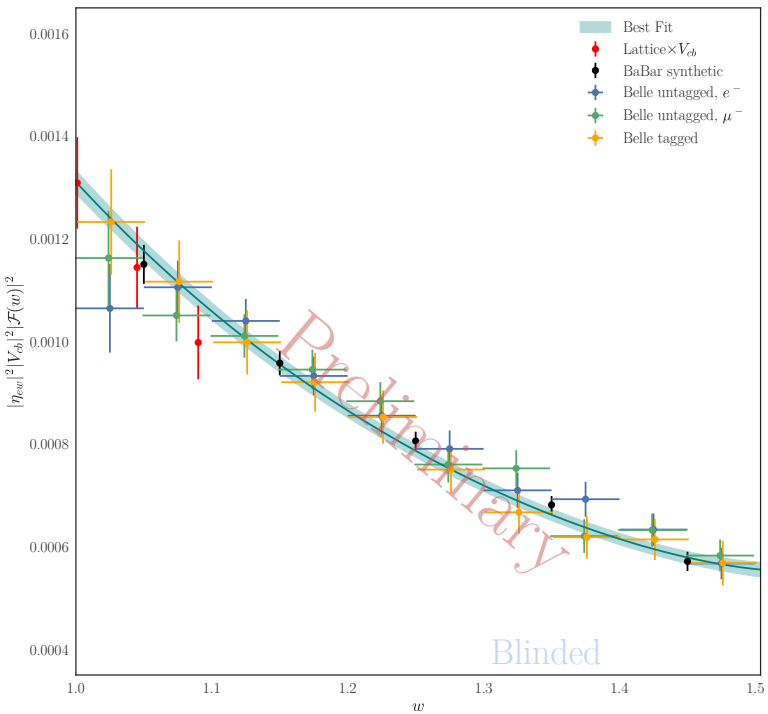



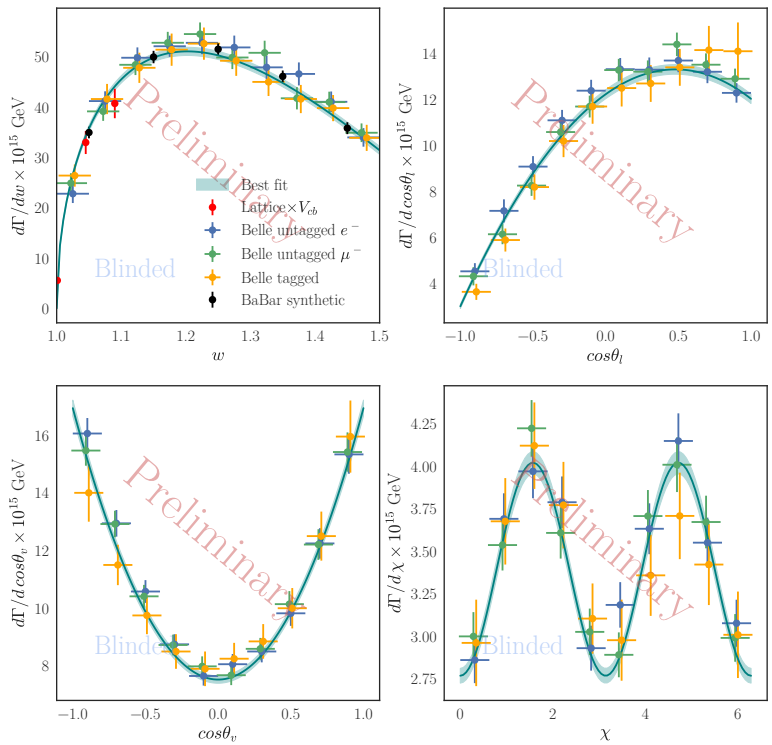




Figs. courtesy A. Vaquero

# FNAL/MILC $B \to D^*$

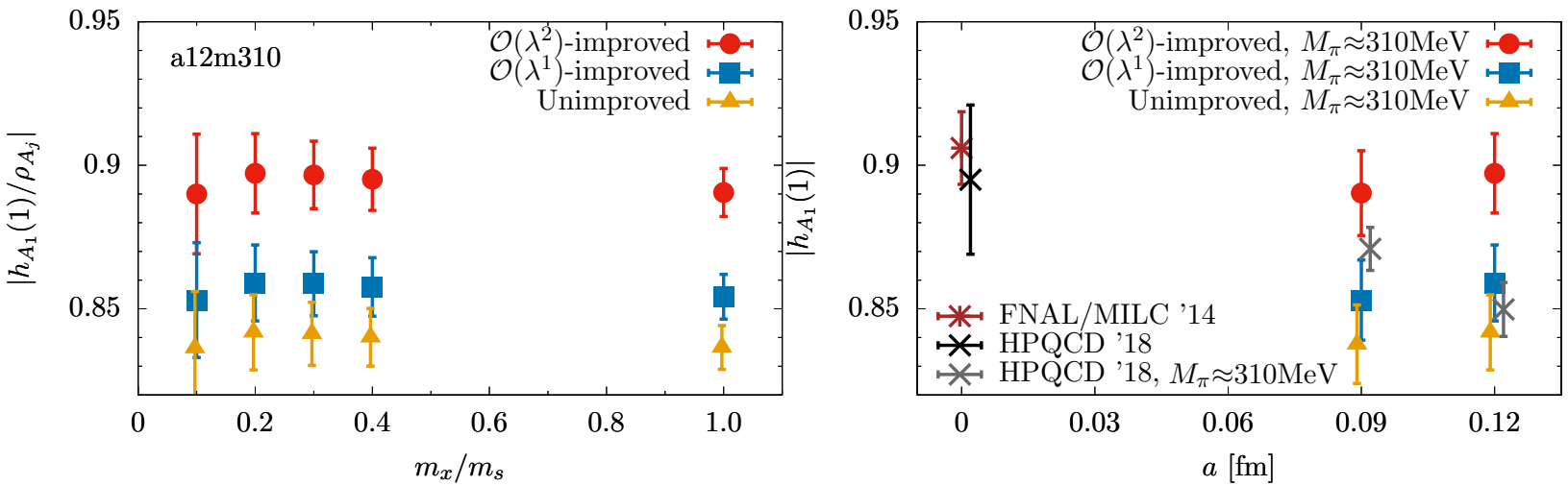



Figs. courtesy A. Vaquero

# LANL-SWME $B \to D^*$ 1711.01786, 1812.07675

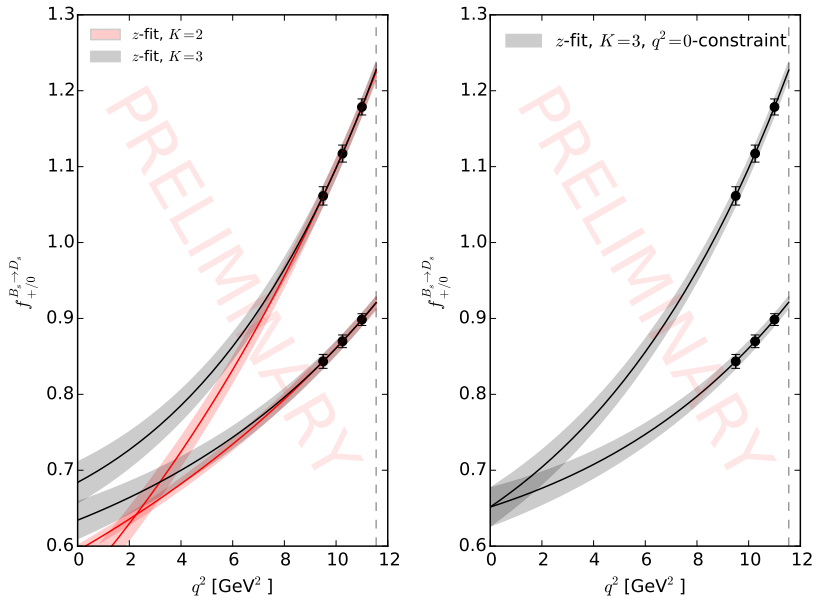
- $n_f = 2 + 1 + 1$ HISQ MILC lattices.
- Oktay-Kronfeld valence for charm and bottom, reduce discretisation error as compared to Fermilab action.
- Preliminary results at zero recoil. ($\rho_{A_1}$ blind) 1812.07675




See poster by Seungyeob Jwa this evening.

# RBC/UKQCD $B_{(s)} \to D_{(s)}^{(*)}$ 1612.05112

- $n_f = 2 + 1$ DWF ensembles
- Optimized Möbius DWF to simulate charm, $am_q < 0.4$.
- $a^{-1} = 1.79, 2.38, 2.77$ GeV
- $b$ quark RHQ action



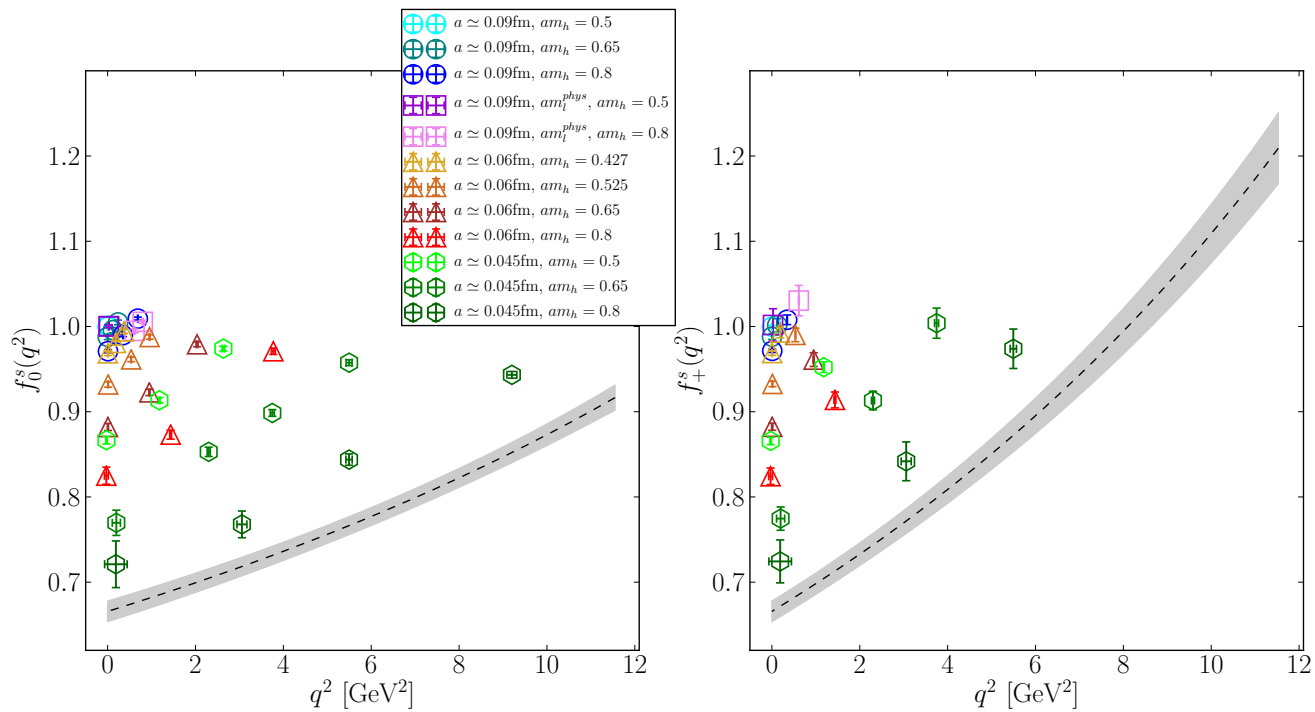

$B_s \to D_s$ 1903.02100

See poster by Oliver Witzel.
See talk by Felix Erben, Mon. 15:40 [WD&ME].
See talk by Ryan Hill, Mon. 16:50 [WD&ME].

# HPQCD $B_{(s)} \to D_{(s)}^{(*)}$

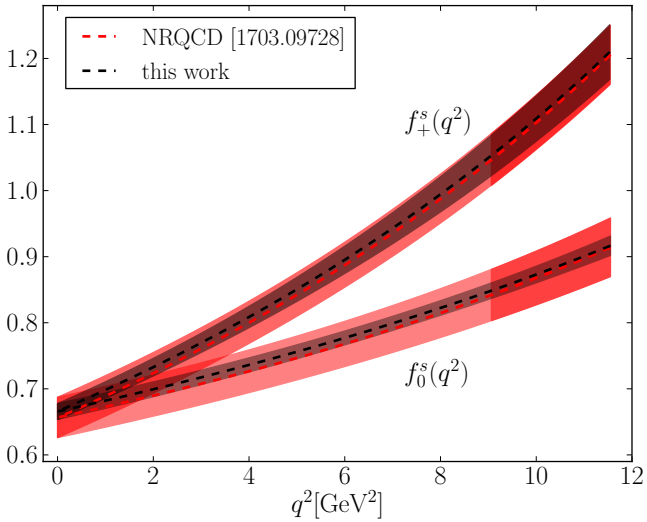
- $n_f = 2+1+1$ HISQ MILC ensembles, $a^{-1} = 2.19$, 3.33, 4.48 GeV.
- Fully relativistic approach: $am_h < 0.8$ ($m_h$ up to $\sim 0.9\, m_b$).
- Work directly over the full kinematic range.

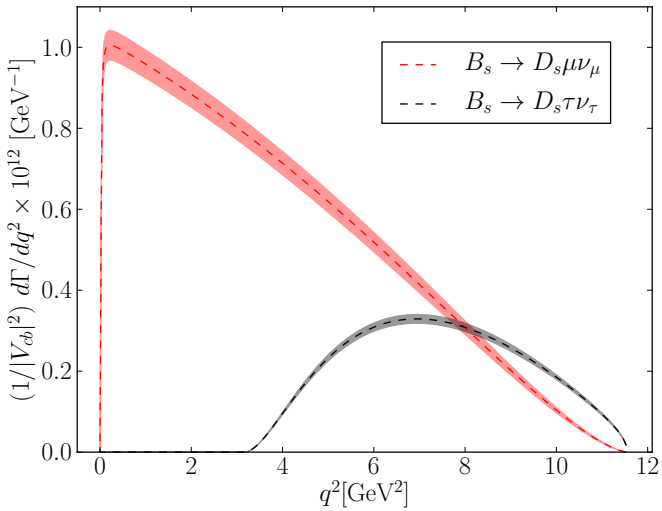
$B_s \to D_s$ 1906.00701

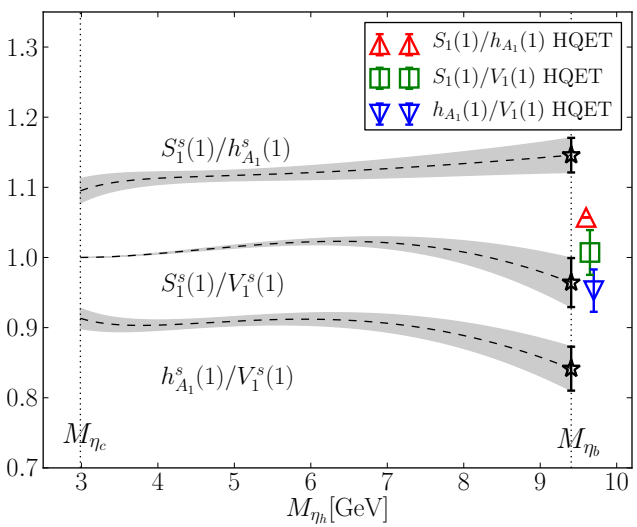


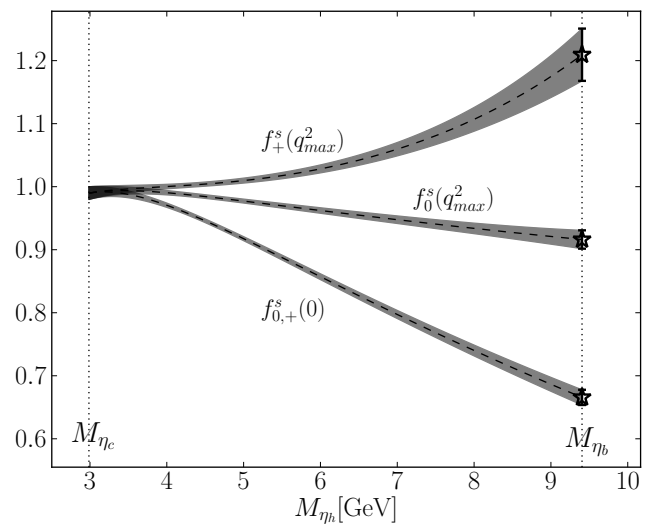
# HPQCD $B_s \to D_s$

1906.00701



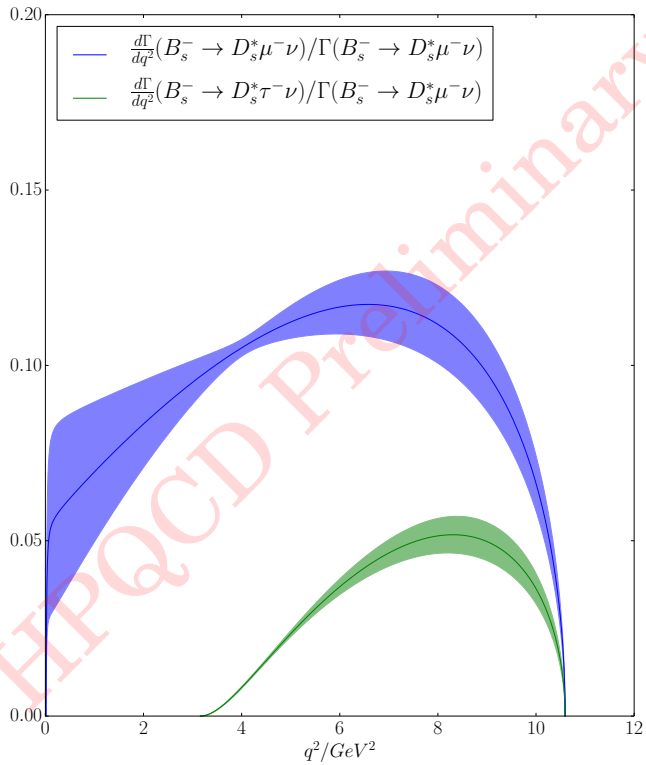



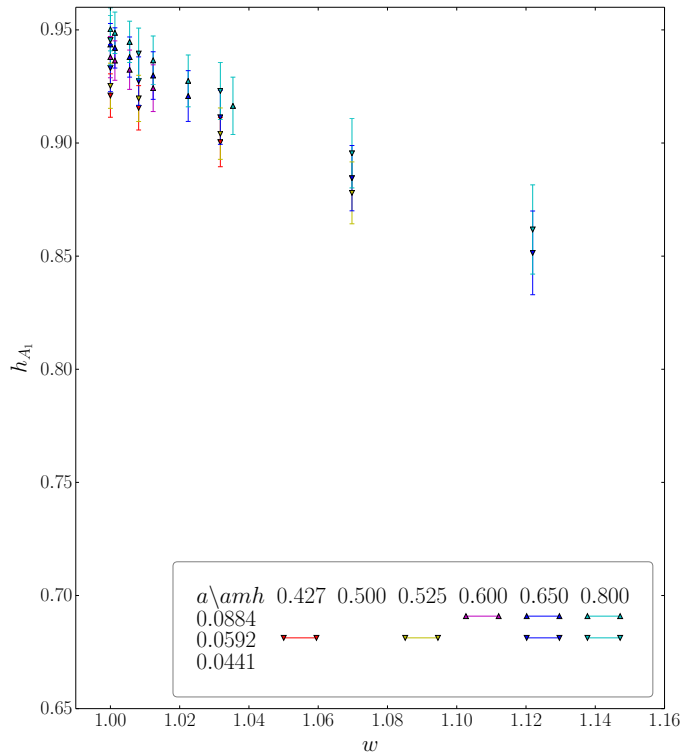



# HPQCD $B_{(s)} \to D^*_{(s)}$

$B_s \to D_s^*$




$B \to D^*$




HPQCD Preliminary

Figs. courtesy Judd Harrison

# Conclusions - I

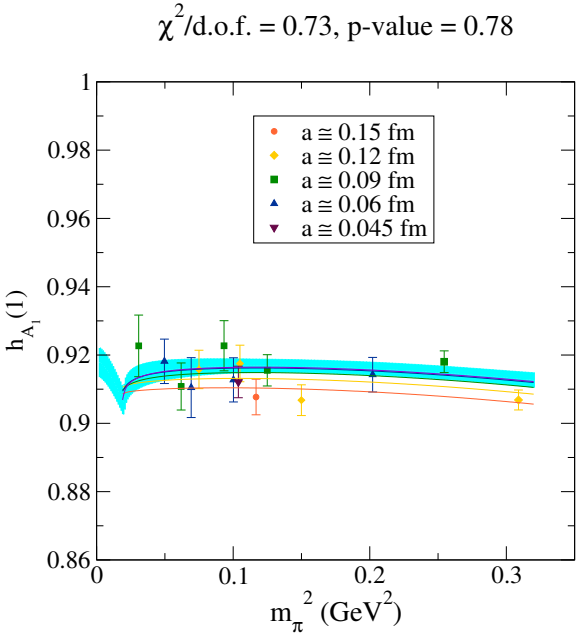
- Presently several observables in tension with SM: $|V_{cb}|$ incl./excl., $R(D^{(*)})$, $R(J/\psi)$.
- Until now in $B \to D^*$, only lattice calculations for $h_{A_1}(1)$.
- Reliability of $|V_{cb}|$ using CLN has been called into question – need non-zero recoil form factors to resolve the issue.
- Incl./excl. tension $|V_{cb}|$ from $B \to D$ reduced when lattice results available away from zero recoil. (Also makes competitive with $B \to D^*$)

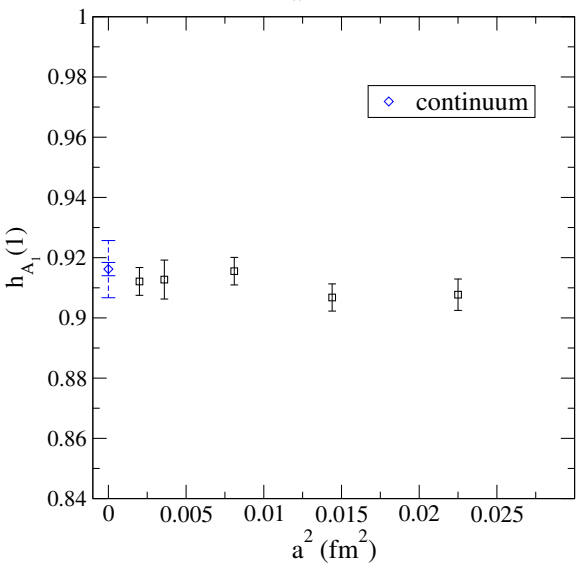
# Conclusions - II

- Fortunately, many new calculations are on the horizon. Different strategies, actions, etc.
  - Effective treatment for $b$: Fermilab, NRQCD...
  - 'Direct' approach on fine lattices.
- Expect new data of increasing precision from Belle II and LHCb.
- Expect results in new modes: $B_s \to D_s^{(*)}$, $B_c \to J/\psi$ both from theory and experiment.

Thank you!

# MILC $B \to D^*$ at zero recoil 1403.0635




| Uncertainty | $h_{A_1}(1)$ |
|---|---|
| Statistics | 0.4% |
| Scale ($r_1$) error | 0.1% |
| $\chi$PT fits | 0.5% |
| $g_{D^*D\pi}$ | 0.3% |
| Discretization errors | 1.0% |
| Perturbation theory | 0.4% |
| Isospin | 0.1% |
| Total | 1.4% |

# HPQCD $B_{(s)} \to D^*_{(s)}$

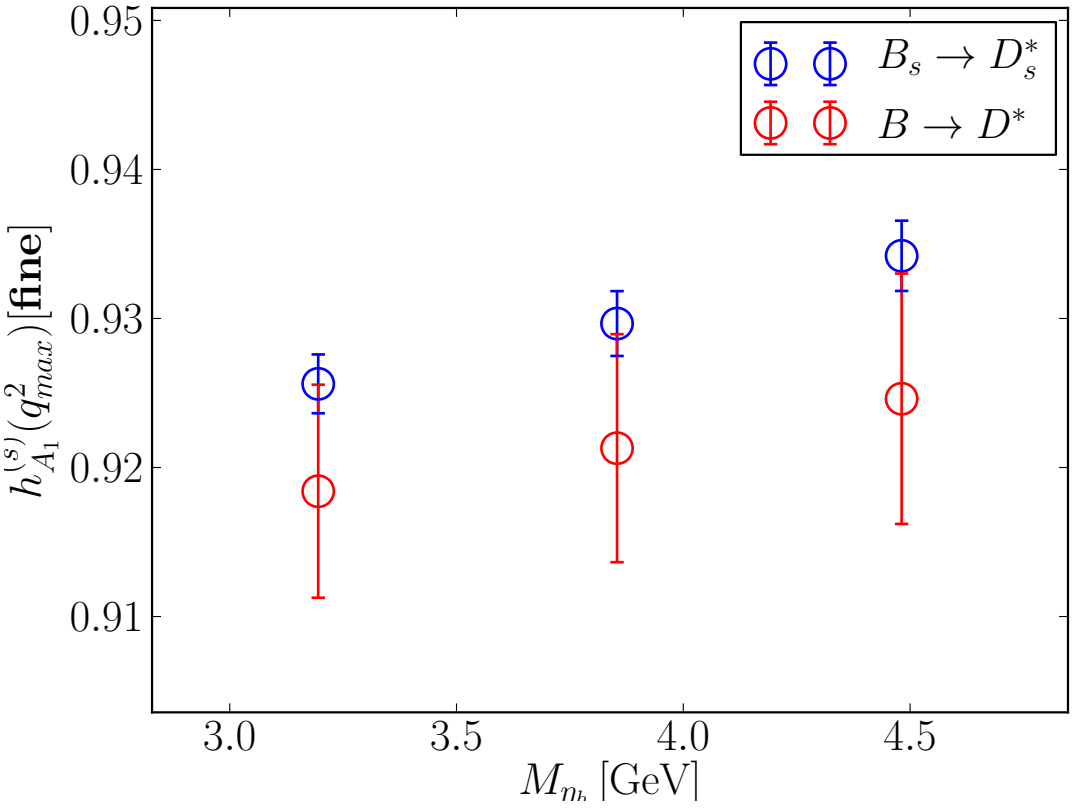



Fig. courtesy Euan McLean

# NRQCD

$$
\begin{aligned}
aH_{\mathrm{NRQCD}} &= aH_0 + a\delta H \\
aH_0 &= -\frac{\Delta^{(2)}}{2am_b} \\
a\delta H &= -c_1 \frac{(\Delta^{(2)})^2}{8(am_b)^3} + c_2 \frac{i}{8(am_b)^2}\left(\nabla\cdot\mathbf{E} - \mathbf{E}\cdot\nabla\right) \\
&\quad - c_3 \frac{1}{8(am_b)^2}\sigma\cdot\left(\nabla\times\mathbf{E} - \mathbf{E}\times\nabla\right) \\
&\quad - c_4 \frac{1}{2am_b}\sigma\cdot\mathbf{B} + c_5\frac{\Delta^{(4)}}{24am_b} \\
&\quad - c_6 \frac{(\Delta^{(2)})^2}{16n(am_b)^2}
\end{aligned}
$$

# NRQCD currents

$f_0$ and $f_+$ are determined in the NRQCD formalism from matrix elements of the vector current $\langle V_\mu^{\rm nrqcd} \rangle$, where

$$V_0^{\rm nrqcd} = (1 + \alpha_s z_0^{(0)}) \left[ V_0^{(0)} + (1 + \alpha_s z_0^{(1)}) V_0^{(1)} + \alpha_s z_0^{(2)} V_0^{(2)} \right]$$

$$V_k^{\rm nrqcd} = (1 + \alpha_s z_k^{(0)}) \Big[ V_k^{(0)} + (1 + \alpha_s z_k^{(1)}) V_k^{(1)} + \alpha_s z_k^{(2)} V_k^{(2)} + \alpha_s z_k^{(3)} V_k^{(3)} + \alpha_s z_k^{(4)} V_k^{(4)} \Big] .$$

# Form factor extrapolation

Experimental data traditionally analyzed in HQET framework:
Fit form factors as function of $w$ to extract $\mathcal{F}(1)\eta_{\rm EW}|V_{cb}|$,
compare with $h_{A_1}(1)$ from lattice to extract $|V_{cb}|$.
Caprini-Lellouch-Neubert (CLN) parameterization:

$$h_{A_1}(z) = h_{A_1}(1)(1 - 8\rho^2 z + (53\rho^2 - 15)z^2 - (231\rho^2 - 91)z^3)$$

$$R_1(w) = \frac{h_V(w)}{h_{A_1}(w)} = R_1(1) - 0.12(w-1) + 0.05(w-1)^2$$

$$R_2(w) = \frac{r h_{A_2}(w) + h_{A_3}(w)}{h_{A_1}(w)} = R_2(1) - 0.11(w-1) - 0.06(w-1)^2$$

(Boyd-Grinstein-Lebed) BGL parameterization uses only weak unitarity constraints: In terms of
$z = (\sqrt{w+1} - \sqrt{2})/(\sqrt{w+1} + \sqrt{2})$, $z \in [0, 0.056]$:

$$f_i(z) = \frac{1}{P_i(z)\phi_i(z)} \sum_{k=0}^{n_i} a_k^i z^k$$