$B \rightarrow D^*$ form factors, $R(D^*)$, and $|V_{cb}|$

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Intro & Motivation

- Semileptonic $B \rightarrow D^{(*)}l\nu$ decays: used to extract $|V_{cb}|$.
- Long-standing tension $\sim 3\sigma$ in inclusive/exclusive determinations.
- Observables $R(D^{(*)})$ have persistent $2 - 3\sigma$ tensions with SM.

What is the role of lattice QCD in resolving or understanding these tensions?

More generally, what is the outlook for $b \rightarrow c$ semileptonic transitions.
Outline

1. Intro & Motivation.

2. Theoretical framework.

3. Expt’l prospects @ Belle-II & LHCb

4. Status of $B(s) \rightarrow D^{(*)}(s) \ell \nu$ lattice calculations.

5. Conclusions & Future outlook.
$B \rightarrow D^{(*)} l \nu$

\[
\frac{d\Gamma}{dw}(B \rightarrow D) = (\text{known}) |V_{cb}|^2 (w^2 - 1)^{3/2} |\mathcal{G}(w)|^2
\]

\[
\frac{d\Gamma}{dw}(B \rightarrow D^*) = (\text{known}) |V_{cb}|^2 (w^2 - 1)^{1/2} \chi(w) |\mathcal{F}(w)|^2
\]

\[
w = v_B \cdot v_{D^{(*)}} = \frac{M_B^2 + M_{D^{(*)}}^2 - q^2}{2M_B M_{D^{(*)}}}
\]

At zero recoil $w = 1$, and $\mathcal{F}(1) = h_{A_1}(1)$. 

$\Gamma_{\text{tree}} = d\Gamma_{\text{tree}} = 1104.5484$
The form factors $\mathcal{F}$ and $\mathcal{G}$ can be determined from QCD matrix elements computed on the lattice.

\[
\frac{\langle D|V^{\mu}|B\rangle}{\sqrt{m_B m_D}} = (v_B + v_D)^{\mu} h_+(w) + (v_B - v_D)^{\mu} h_-(w)
\]

\[
\frac{\langle D^*|V^{\mu}|B\rangle}{\sqrt{m_B m_{D^*}}} = \varepsilon^{\mu\nu\rho\sigma} v_B^{\nu} v_D^{\rho}_{\alpha}^* \epsilon^*_{\sigma} h_V(w)
\]

\[
\frac{\langle D^*|A^{\mu}|B\rangle}{\sqrt{m_B m_{D^*}}} = i\epsilon^{*\nu}_{\alpha} \left[ h_{A_1}(w)(1 + w)g^{\mu\nu} - (h_{A_2}(w)v_B^{\mu} + h_{A_3}(w)v_{D^*}^{\mu})v_B^{\nu} \right]
\]

At zero recoil $w = 1$, and $\mathcal{F}(1) = h_{A_1}(1)$. 
There is a long-standing tension in extractions of $|V_{cb}|$ from inclusive and exclusive semileptonic $B$ decays, hovering at $\approx 3\sigma$.

The most precise exclusive channel is $B \to D^*l\nu$.

Lattice calculations are more precise near zero-recoil, whereas experimental data is suppressed here.

To connect theory and experiment, the experimental results are extrapolated to zero recoil using HQET (CLN parameterization) and this is compared with the form factor computed at zero recoil to extract $|V_{cb}|$. 
Form factor extrapolation

Experimental data traditionally analyzed in HQET framework: Fit form factors as function of $w$ to extract $\mathcal{F}(1)\eta_{\text{EW}}|V_{cb}|$, compare with $h_{A_1}(1)$ from lattice to extract $|V_{cb}|$.

CLN parameterization:

$$h_{A_1}(z) = h_{A_1}(1)(1 - 8\rho^2 z + (53\rho^2 - 15)z^2 - (231\rho^2 - 91)z^3)$$
$$R_1(w) = R_1(1) - 0.12(w - 1) + 0.05(w - 1)^2$$
$$R_2(w) = R_2(1) - 0.11(w - 1) - 0.06(w - 1)^2$$

BGL parameterization uses only weak unitarity constraints: In terms of $z = (\sqrt{w + 1} - \sqrt{2})/(\sqrt{w + 1} + \sqrt{2})$, $z \in [0, 0.056]$:

$$f_i(z) = \frac{1}{P_i(z)\phi_i(z)} \sum_{k=0}^{n_i} a_k^i z^k$$
Recent analyses indicating CLN may be too restrictive. 1703.06124, 1703.08170, 1708.07134

- 1702.01521 Belle tagged data set, strong model dependence observed CLN/BGL.

- 1903.10002 Babar tagged analysis w/ BGL, model dependence but discrepancy persists.

- 1809.03290, 1905.08209 Belle untagged data set, no strong model dependence, $|V_{cb}|$ discrepancy persists.

Lattice data for $w > 1$ is extremely useful to help resolve this issue! 1708.07134, 1905.08209
$|V_{cb}|$ and the $V_{cb}$ puzzle

FLAG2019

$B \to D^*$

$B \to \tau \nu$

$B \to D$

$B \to \pi \ell \nu$

$\Lambda_b \to p \ell \nu$

$\Lambda_b \to \Lambda_c \ell \nu$

$|V_{cb}| \times 10^3$

3.0

3.5

4.0

4.5

36 38 40 42 44

1902.08191
$|V_{cb}|$ and the $V_{cb}$ puzzle

1503.07237 [MILC], 1505.03925 [HPQCD]
1510.03675 [Belle], 0904.4063 [BaBar]

Combining lattice and expt’l data in $B \to D\ell\nu$

compatible with $B \to D^*\ell\nu$ extraction and inclusive:

$|V_{cb}| = 40.49(97)10^{-3}$ 1606.08030
$R$-ratios

$R$-ratio for $B \to D^{(*)}$ semileptonic decay defined as

$$R(B \to D^{(*)}) = \frac{B(B \to D^{(*)} \tau \nu)}{B(B \to D^{(*)} l \nu)}, \quad l = \mu, e$$

- Test lepton flavour universality.
- Experimentally and theoretically attractive.
  - Msm’t systematics cancel in ratio.
  - Form factor dependence largely cancels in ratio.
- There are persistent 2-3$\sigma$ anomalies in the ratios $R(B \to D^*)$ and $R(B \to D)$, as well as recent msm’t $R(B_c \to J/\psi)$ 1711.05623 ($\lesssim 2\sigma$).
\( R(D^{(*)}) \)

- \( R(D)_{\text{SM}} \) from lattice calculations.
  
  1503.07237 [FNAL/MILC]
  
  1505.03925 [HPQCD]

- Belle combination moves \( R(D^{(*)}) \) closer to SM.

- Would be good to have \( R(D^{*})_{\text{SM}} \) prediction from lattice. \( \rightarrow \) all ffs over full kinematic range.
$R(B_c \rightarrow J/\psi)$

- LHCb 1711.05623
  
  $R(J/\psi) = 0.71(17)_{\text{stat}}(18)_{\text{syst}}$

- w/in 2\(\sigma\) of theory range [0.25, 0.28]

- Preliminary lattice result 0.290(7)

Fig. courtesy J. Harrison
Expect increasingly precise results from large Belle II datasets. Integrated luminosity:

- $5 \text{ ab}^{-1} \sim 2021$
- $50 \text{ ab}^{-1} \sim 2025$

<table>
<thead>
<tr>
<th></th>
<th>Belle (2017)</th>
<th>Belle II 5 ab$^{-1}$</th>
<th>Belle II 50 ab$^{-1}$</th>
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<tr>
<td>$</td>
<td>V_{cb}</td>
<td>$ excl.</td>
<td>3.3 %</td>
</tr>
<tr>
<td>$</td>
<td>V_{cb}</td>
<td>$ incl.</td>
<td>1.8%</td>
</tr>
<tr>
<td>$R(D)$ (Had. tag)</td>
<td>16.5%</td>
<td>6%</td>
<td>3 %</td>
</tr>
<tr>
<td>$R(D^*)$ (Had. tag)</td>
<td>7.4%</td>
<td>3%</td>
<td>2 %</td>
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</table>
Increasingly precise $B \to D^{(*)}$, $B_c \to J/\psi$.

- 23 fb$^{-1}$ Run 3 (end 2023)
- 50 fb$^{-1}$ Run 4 (end 2029)
- 300 fb$^{-1}$ Upgrade II (2030s)
• Increasingly precise $B \rightarrow D^{(*)}$, $B_c \rightarrow J/\psi$.

• $B \rightarrow D^{(*)}$ differential measurements. (Upgrade II)

• Measurements of new modes $B_s \rightarrow D_s^{(*)}$

• $\sigma_{R(D_s^{(*)})} \sim 6\%$ Run 3, 2.5\% Upgrade II.
Lattice QCD results
$B \to D^*$ at zero recoil from LQCD

\[ a \approx 0.15 \text{ fm} \]
\[ a \approx 0.12 \text{ fm} \]
\[ a \approx 0.09 \text{ fm} \]
\[ a \approx 0.06 \text{ fm} \]
\[ a \approx 0.045 \text{ fm} \]

extrapolated value

\[ h_{A_1}(1) = 0.906(4)(12) \]

\[ h_{sA_1}(1) = 0.883(12)(28) \]

**FNAL/MILC 1403.0635**
- $n_f = 2 + 1$ MILC asqtad ensembles
- Clover $b$ with Fermilab interpretation
- $h_{A_1}(1) = 0.906(4)(12)$

**HPQCD 1711.11013**
- $n_f = 2 + 1 + 1$ MILC HISQ ensembles
- NRQCD $b$ quark
- $h_{A_1}(1) = 0.895(10)(24)$
- $h_{sA_1}(1) = 0.883(12)(28)$
Treatment of heavy quarks

Treatment of $c$ and especially $b$ quarks challenging in lattice simulation due to lattice artifacts which grow as $(am_h)^n$

- Generally one uses an effective theory framework to handle the $b$ quark.
  - Fermilab interpretation, RHQ, OK, NRQCD
  - Pros: Solves problem w/ $am_h$ artifacts.
  - Cons: Requires matching, can still have $ap$ artifacts.

- Also possible to use fully relativistic action provided $a$ is sufficiently small $am_c \ll 1$, $am_b < 1$.
  - Use improved actions e.g. $O(a^2) \rightarrow O(\alpha_s a^2)$
  - Pros: Absolutely normalised current, straightforward continuum extrap.
  - Cons: Numerically expensive, extrapolate $m_h \rightarrow m_b$. 
• ‘Relativistic-b’ approach on Möbius $n_f = 2 + 1$ DWF ensembles.

• $a^{-1} = 2.5, 3.6, (4.5) \text{ GeV}, M_\pi = 310, (230) \text{ MeV}.$

• $am_h < 0.8$ ($m_h$ up to $2.4m_c$)

• Work in range $w \in [1.0, 1.07]$
**JLQCD** $B \rightarrow D^{(*)}$

\[ a^{-1} \sim 2.5 \text{ GeV}, \ m_b = 1.25^2 m_c, \ M_\pi \sim 500 \text{ MeV} \]
\[ a^{-1} \sim 2.5 \text{ GeV}, \ m_b = 1.25^2 m_c, \ M_\pi \sim 300 \text{ MeV} \]
\[ a^{-1} \sim 3.6 \text{ GeV}, \ m_b = 1.25^2 m_c, \ M_\pi \sim 500 \text{ MeV} \]
\[ a^{-1} \sim 3.6 \text{ GeV}, \ m_b = 1.25^4 m_c, \ M_\pi \sim 500 \text{ MeV} \]
JLQCD $B \rightarrow D^*$

Figs. courtesy Takashi Kaneko.
Indications of significant corrections to NLO HQET.
On the other hand $R_1(w)$ favors CLN. PRD96 091503
JLQCD $B \rightarrow D^{(*)}$

Figs. courtesy Takashi Kaneko.
• $n_f = 2 + 1$ asqtad ensembles

• $a^{-1} = 0.15–0.045$ fm

• Asqtad light quarks

• Clover heavy w/ Fermilab interpretation

• Range $w \in [1.0, 1.15]$

See talk by Alex Vaquero, Mon. 14:40 [WD&ME].
Figs. courtesy A. Vaquero
FNAL/MILC $B \to D^*$

Figs. courtesy A. Vaquero
• $n_f = 2 + 1 + 1$ HISQ MILC lattices.

• Oktay-Kronfeld valence for charm and bottom, reduce discretisation error as compared to Fermilab action.

• Preliminary results at zero recoil. ($\rho_{A_1}$ blind) 1812.07675

See poster by Seungyeob Jwa this evening.
• \( n_f = 2 + 1 \) DWF ensembles

• Optimized Möbius DWF to simulate charm, \( am_q < 0.4 \).

• \( a^{-1} = 1.79, 2.38, 2.77 \) GeV

• \( b \) quark RHQ action

\[ B_s \rightarrow D_s \]

See poster by Oliver Witzel.
See talk by Felix Erben, Mon. 15:40 [WD&ME].
See talk by Ryan Hill, Mon. 16:50 [WD&ME].
**HPQCD** $B_{(s)} \rightarrow D_{(s)}^{(*)}$

- $n_f = 2 + 1 + 1$ HISQ MILC ensembles, $a^{-1} = 2.19, 3.33, 4.48$ GeV.
- Fully relativistic approach: $am_h < 0.8$ ($m_h$ up to $\sim 0.9 m_b$).
- Work directly over the full kinematic range.

$B_s \rightarrow D_s$ 1906.00701
HPQCD $B_S \to D_S$

$\frac{1}{|V_{cb}|^2} \frac{d}{dq^2} \times 10^{12}$ [GeV$^{-1}$] $B_s \to D_s \mu \bar{\mu}$

$B_s \to D_s \tau \nu$
HPQCD \( B(s) \rightarrow D^*_s \)

\[ B_s \rightarrow D^*_s \]

\[ B \rightarrow D^* \]

Figs. courtesy Judd Harrison
Conclusions - I

- Presently several observables in tension with SM: $|V_{cb}|$, incl./excl., $R(D^{(*)})$, $R(J/\psi)$.
- Until now in $B \to D^*$, only lattice calculations for $h_{A_1}(1)$.
- Reliability of $|V_{cb}|$ using CLN has been called into question – need non-zero recoil form factors to resolve the issue.
- Incl./excl. tension $|V_{cb}|$ from $B \to D$ reduced when lattice results available away from zero recoil. (Also makes competitive with $B \to D^*$)
• Fortunately, many new calculations are on the horizon. Different strategies, actions, etc.
  ▶ Effective treatment for $b$: Fermilab, NRQCD...
  ▶ ‘Direct’ approach on fine lattices.

• Expect new data of increasing precision from Belle II and LHCb.

• Expect results in new modes: $B_s \rightarrow D_s^{(*)}$, $B_c \rightarrow J/\psi$ both from theory and experiment.
Thank you!
\[
\chi^2 / \text{d.o.f.} = 0.73, \text{ p-value} = 0.78
\]

<table>
<thead>
<tr>
<th>Uncertainty</th>
<th>( h_{A_1}(1) )</th>
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<tbody>
<tr>
<td>Statistics</td>
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<tr>
<td>Scale ((r_1)) error</td>
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<tr>
<td>( \chiPT ) fits</td>
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<tr>
<td>( gD^*D\pi )</td>
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</tr>
<tr>
<td>Isospin</td>
<td>0.1%</td>
</tr>
<tr>
<td>Total</td>
<td>1.4%</td>
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</tbody>
</table>
HPQCD $B_s \rightarrow D^*_s$

![Graph showing $h^{(s)}(q^2_{\text{max}})\text{[fine]}$ vs. $M_{\eta h} \text{[GeV]}$]

Fig. courtesy Euan McLean
\[ aH_{\text{NRQCD}} = aH_0 + a\delta H \]

\[ aH_0 = -\frac{\Delta^{(2)}}{2am_b} \]

\[ a\delta H = -c_1\frac{(\Delta^{(2)})^2}{8(am_b)^3} + c_2\frac{i}{8(am_b)^2} (\nabla \cdot \mathbf{E} - \mathbf{E} \cdot \nabla) \]

\[ - c_3\frac{1}{8(am_b)^2} \sigma \cdot (\nabla \times \mathbf{E} - \mathbf{E} \times \nabla) \]

\[ - c_4\frac{1}{2am_b} \sigma \cdot \mathbf{B} + c_5\frac{\Delta^{(4)}}{24am_b} \]

\[ - c_6\frac{(\Delta^{(2)})^2}{16n(am_b)^2} \]
$f_0$ and $f_+$ are determined in the NRQCD formalism from matrix elements of the vector current $\langle V_\mu^{\text{nrqcd}} \rangle$, where

$$V_0^{\text{nrqcd}} = (1 + \alpha_s z_0^{(0)}) \left[V_0^{(0)} + (1 + \alpha_s z_0^{(1)})V_0^{(1)} + \alpha_s z_0^{(2)}V_0^{(2)}\right]$$

and

$$V_k^{\text{nrqcd}} = (1 + \alpha_s z_k^{(0)}) \left[V_k^{(0)} + (1 + \alpha_s z_k^{(1)})V_k^{(1)} + \alpha_s z_k^{(2)}V_k^{(2)} + \alpha_s z_k^{(3)}V_k^{(3)} + \alpha_s z_k^{(4)}V_k^{(4)}\right].$$
Experimental data traditionally analyzed in HQET framework: Fit form factors as function of $w$ to extract $\mathcal{F}(1)\eta_{EW}|V_{cb}|$, compare with $h_{A_1}(1)$ from lattice to extract $|V_{cb}|$.

Caprini-Lellouch-Neubert (CLN) parameterization:

$$h_{A_1}(z) = h_{A_1}(1)(1 - 8\rho^2 z + (53\rho^2 - 15) z^2 - (231\rho^2 - 91) z^3)$$

$$R_1(w) = \frac{h_V(w)}{h_{A_1}(w)} = R_1(1) - 0.12(w - 1) + 0.05(w - 1)^2$$

$$R_2(w) = \frac{r h_{A_2}(w) + h_{A_3}(w)}{h_{A_1}(w)} = R_2(1) - 0.11(w - 1) - 0.06(w - 1)^2$$

(Boyd-Grinstein-Lebed) BGL parameterization uses only weak unitarity constraints: In terms of $$z = (\sqrt{w + 1} - \sqrt{2})/(\sqrt{w + 1} + \sqrt{2}), z \in [0, 0.056]$$:

$$f_i(z) = \frac{1}{P_i(z)\phi_i(z)} \sum_{k=0}^{n_i} a_k^i z^k$$