HEAVY SEMILEPTONIC DECAYS WITH A FULLY RELATIVISTIC MIXED ACTION

J. Frison, A. Bussone, G. Herdoíza, C. Pena, J.Á. Romero, J. Ugarrio

Instituto de Física Teórica UAM-CSIC, Madrid, Spain

Abstract

The first phase of a heavy quark program based on twisted mass valence quarks has been presented at last years's lattice conference. The CLS $N_f = 2+1$ ensembles were used for their fine lattice spacing, while twisting the masses is expected to reduce discretisation errors even further and allow for a fully relativistic calculation. In this poster, we present our strategy and first very preliminary results on three point functions, corresponding to $D \to K$ and $D \to \pi$ semileptonic decays.

Introduction

Flavour physics has always been an attractive place to look for new physics, in particular because it is the origin of most of the free parameters of the Standard Model, for which one naturally hopes to find a more fundamental explanation. In the last years this turned very concretely into some **promising experimental tensions in $b \to s$ and $b \to c$** transitions, which will keep accumulating statistics in the near future from **LHCb and Belle II**. In the meantime, the more discrete experiments **BESIII and CLEO-c** are going to improve our knowledge of D decays, which have long been overlooked because of many technical difficulties. On top of the hopes about new physics, one should not forget that it is still a way to get more and more precise values of the aforementioned parameters.
As those new experimental results come in, our theoretical computations have to improve too. Lattice field theory has become a very powerful method for this objective, since ensembles **fine enough to fit dynamical heavy quark** can now be generated. In the last years, this front as well has seen a large increase of the influx of new results [1], but much remains to be done in order to have fully reliable estimates in all the observables of interest. While most of the previous results are based on some effective heavy quark action, it has been shown that some quantities including even B mesons can be accessed directly with a fully relativistic action. This is the path we will follow.

General idea

In this project we will take advantage of the **automatic $O(a)$ improvement** [6] of twisted mass fermions at maximal twist, which precisely gets rid of the dangerous $O(am_c)$ terms. Unlike the strategy of ETMC [8] however, this action will only be used in the valence, while the $N_f = 2+1$ Wilson action in the sea comes from the very fine lattices obtained by CLS with the help of open boundary conditions. The control of the order of discretisation errors and the size of the lattice spacing are of course two crucial features when it comes to heavy quark physics. Here those two actions only differ by the choice of mass parameters, so the **renormalisation factors (in a massless scheme) are kept unchanged and no matching is required** except the tuning of the valence masses. This matching has already been described last year in [3], as well as first results for leptonic decays in [4].
With this setup, our next step consists in focusing on **semileptonic $D \to \pi$** (and $D \to K$) decays to obtain V_{cd} (V_{cs}). While charm physics studies are relevant by themselves for phenomenology and a better understanding of QCD, they also allow to establish the expected benefits regarding cutoff effects in view of B-sector computations. This paves the way for the later study of processes such as $B \to \pi$ (V_{ub}), $B \to D^{(*)}$ ($V_{cb}, R(D^{(*)})$), $B \to K^{(*)}$ ($R(K^{(*)})$, as well as potentially $D \to \rho, a_0, f_0$ (for which BESIII published some new experimental results ([9]).

Ensembles and correlators

The *CLS* ensembles considered in the current scope of this study are described below:

Id	β	a[fm]	N_s	N_t	m_π[MeV]	m_K[MeV]	$m_\pi L$
H101	3.40	0.087	32	96	420	420	5.8
H102	3.40	0.087	32	96	350	440	4.9
H105	3.40	0.087	32	96	280	460	3.9
H400	3.46	0.077	32	96	420	420	5.2
H401	3.46	0.077	32	96	550	550	7.3
H402	3.46	0.077	32	96	450	450	5.7
H200	3.55	0.065	32	96	420	420	4.3
N202	3.55	0.065	48	128	420	420	6.5
N203	3.55	0.065	48	128	340	440	5.4
N200	3.55	0.065	48	128	280	460	4.4
D200	3.55	0.065	64	128	200	480	4.2
N300	3.70	0.050	48	128	420	420	5.1
J303	3.70	0.050	64	192	260	260	4.1

Therefore for the finest ensembles here, $am_c^{\overline{\rm MS},2\,{\rm GeV}} \sim 0.32$ while $am_b^{\overline{\rm MS},2\,{\rm GeV}} \sim 1.06$. And as a first step we are going to focus on the $SU(3)$ symmetric ensembles H101 and N300, which almost only differ by their lattice spacing.

On those ensembles we will have to compute three point and two point functions with momenta. Those are obtained by inversion on a **stochastic source** and the use of a **sequential propagator**:

$$Q_{f,i}(x' \mid p) = e^{-ipx'} D_f^{-1}(x',x)\xi_i(x)e^{ipx}\delta_{x_0-t_{src}}, \quad \text{where } \sum_i \xi_i(x)\xi_i^*(y) \to \delta(x-y) \quad (1)$$

$$W_{f'f,i}(x' \mid p', p) = e^{-ip'x'} D_{f'}^{-1}(x',x)e^{ip'x}\delta_{x_0-t_{snk}} Q_{f,i}(x \mid p), \quad (2)$$

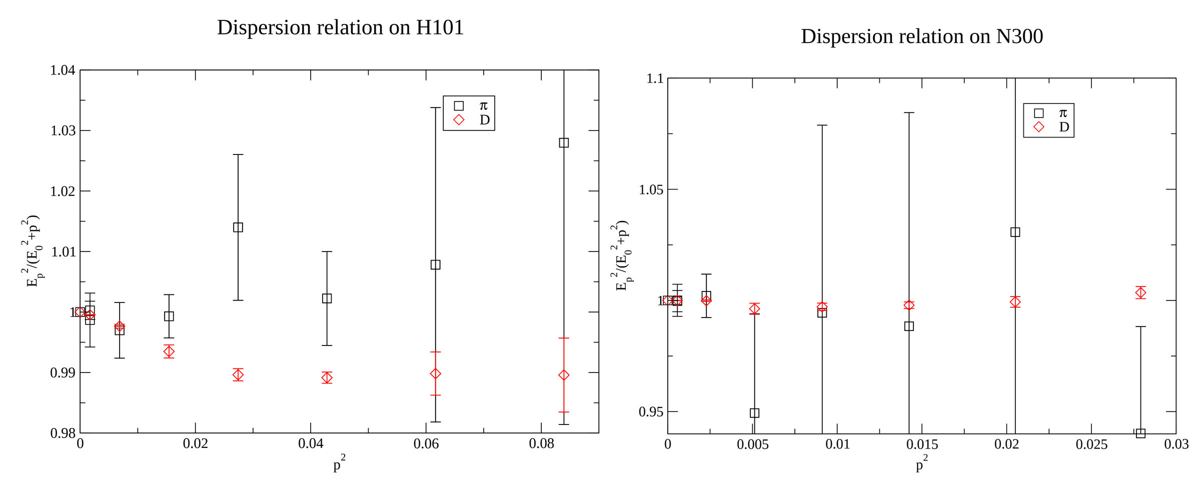
As a result the correlators are obtained from

$$C^{2pt}_{ff'} = \mathrm{Tr}\left[Q_f Q^\dagger_{f'}\right] \quad (3)$$

$$C^{3pt,\Gamma}_{ff'f''} = \mathrm{Tr}\left[Q_f W^\dagger_{f'f''}\gamma_5\Gamma\gamma_5\right] \quad (4)$$

This way we obtain easily the matrix elements for any Γ (in particular the scalar and vector form factors of the Standard Model $D \to \pi$) and can vary as we want the time at which this operator is inserted, while the external mesons are restricted to be pseudoscalar for now and are each on a fixed time slice.
The flavour indices shown here are chosen so that $D \to D$, $D \to P$, $D \to P$ and $P \to P$ transitions are all available, which allows to consider various methods to extract the matrix elements and also allows some sanity checks for this preparatory phase. The twisted mass assignment is such that all mesons involved are associated to conserved currents, thus circumventing issues with $O(a^2)$ flavour breaking effects [7].
In the currently running jobs, the momentum of the spectator quark is kept to zero while each of the two other quarks has a momentum which can take 15 values, each one being imposed by a different twisted boundary condition for the quark inversion. Those values are positive and negative (7 of each, going up to 700 MeV) because a parity average is necessary to keep the $O(a)$ improvement. As a result we have 15 light inversions and 15 heavy inversions, but this gives us $15^2 = 225$ kinematics on the correlator, covering all the range from $q^2 = q^2_{max}$ to a slightly negative q^2. Finally, all these momenta are on a fixed spacial direction, but this direction is non-democratic for a reason we will explain later.
All being considered, this leads to 14430 correlators per configuration for a single noise hit, a single source-sink separation, degenerate light quarks (u,d,s) and a single choice of charm mass (tuned to its physical mass, later we will want to cover all values on $[m_c, m_b]$). Despite this impressive number, we have checked that the cost of contractions remains negligible.

Dispersion relation



In this preliminary data with limited statistics and non-finalised fits, we see that:

- The pion is much more sensitive to the introduction of momentum than the D in terms of statistical (relative) error bars. But it does not show any obvious systematic deviation, which should be linear in $(ap)^2$ at moderate p.
- The D meson could have an order 0.5% deviation on its energy at large momentum on the coarse ensemble (left), starting as the expected $O((ap)^2)$ and then getting flatter, but it could be due to the bad quality of the fit rather than to a discretisation effect. And on the fine ensemble (right) this deviation does not go beyond 0.02%.

Parametrisation

The scalar and vector form factors, following arguments of Lorentz symmetry, have the structure:

$$\langle S \rangle = \frac{M_D^2 - M_P^2}{\mu_c - \mu_q} f_0(q^2) + O(a^2) \quad (5)$$

$$\langle \hat{V}_\mu \rangle = P_\mu f_+(q^2) + q_\mu \frac{M_D^2 - M_P^2}{q^2}\left[f_0(q^2) - f_+(q^2)\right] + O(a^2) \quad (6)$$

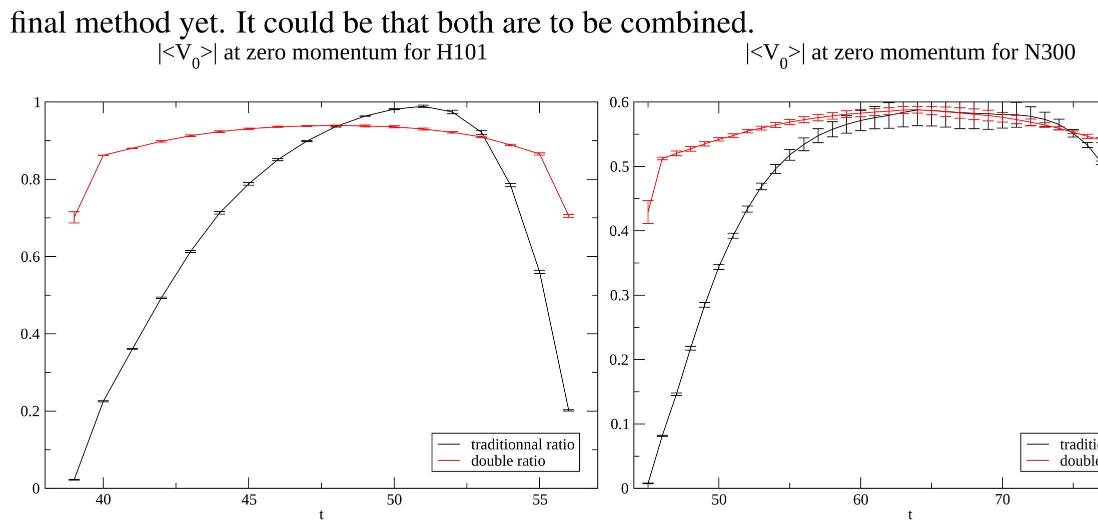
where $P = p_D + p_P$, $q = p_D - p_P$, S is unrenormalised and $\hat{V} = Z_V V$ is renormalised.
But first we need to extract the matrix elements from the correlators. We are considering two methods:
The first one is the simple ratio

$$\langle \Gamma \rangle = \frac{C^{3pt,\Gamma}_{DP}(t,t')}{\sqrt{C^{2pt}_D(t) C^{2pt}_P(t'-t)}} \sqrt{\frac{4E_D E_P}{e^{iM_D t} e^{iM_P(t'-t)}}} \quad (7)$$

whose inconvenient is to depend on the result of a fit and to have poor plateaus, and which gives an unrenormalised quantity.
The second one is the double ratio

$$|\langle \hat{\Gamma} \rangle|^2 = \left\{ \begin{array}{c} 4E_D E_P \\ \text{or} \\ 4p_{D\mu}p_{P\mu} \end{array} \right\} \frac{C^{3pt,\Gamma}_{DP}(t,t') C^{3pt,\Gamma}_{PD}(t,t')}{C^{3pt,\Gamma}_{DD}(t,t') C^{3pt,\Gamma}_{PP}(t,t')} \quad (8)$$

in which the renormalisation cancel out and our observation is that the plateaus are better, at the expense of statistical precision on some components.
The analysis and checks are still underway on N300 and H101 and we have not cho final method yet. It could be that both are to be combined.



Later on, once we have accumulated enough statistics, our objective is to study the h cubic symmetry breaking effects observed by ETMC and described by

$$\langle S \rangle_{hyp} = \frac{a^2}{\mu_c - \mu_q}\left[q^{[4]}\tilde{H}_1 + q^{[3]}P^{[1]}\tilde{H}_2 + q^{[2]}P^{[2]}\tilde{H}_3 + q^{[1]}P^{[3]}\tilde{H}_4 + P^{[4]}\tilde{H}_5\right]$$

$$\langle \hat{V}_\mu \rangle_{hyp} = a^2\left[(q_\mu)^3 H_1 + (q_\mu)^2 P_\mu H_2 + q_\mu (P_\mu)^2 H_3 + (P_\mu)^3 H_4\right].$$

In their observation, the use of democratic momenta was offering very little handle to trol those effects, and their description heavily depends on a global fit procedure. Th moreover no reason to believe that democratic momenta lead to much smaller discreti errors here, because these **hypercubic breaking effects come from the fact that tim space are treated very differently**, not from the spatial subgroup. By using differen mentum components it is easy to show that one can have enough data to solve for m those additional form factor through simple linear algebra at a single q^2, without hav provide a model for their q^2 dependence. For instance in the Breit frame ($P_i = 0$) we

$$\frac{\langle \hat{V}_i \rangle}{q_i} - \frac{\langle \hat{V}_j \rangle}{q_j} = \frac{\langle \hat{V}_i \rangle_{hyp}}{q_i} - \frac{\langle \hat{V}_j \rangle_{hyp}}{q_j} = a^2\left[q_i^2 - q_j^2\right] H_1(P,q)$$

Conclusion

We have presented a strategy to obtain form factor of semileptonic D decays on the with a mixed action using twisted mass fermions in the sea. The source code for sions, contraction and most of the analysis have been written, and we have started lo at the first data. After some exploratory phase and some sanity checks which are w progress, the production phase should start in the coming months.
Our relation of dispersion shows that the introduction of momenta, which had not tested in last year's work on leptonic decays, does not make discretisation errors go control (up to 700 MeV).
We obtain decent plateaus on V_0, but V_i and S are still too noisy to discuss form factor their discretisation errors.

Acknowledgement

We thank the CLS effort for the generation of the gauge configuration ensembles in this study. We acknowledge PRACE for awarding us access to MareNostrum Barcelona Supercomputing Center (BSC), Spain. We thank CESGA for granting acc FinisTerrae II. We thankfully acknowledge support through the Spanish MINECO p FPA2015-68541-P, the Centro de Excelencia Severo Ochoa Programme SEV-2016 and the Ramn y Cajal Programme RYC-2012-10819

References

[1] S. Aoki *et al.* [Flavour Lattice Averaging Group], arXiv:1902.08191 [hep-lat].

[2] A. Bussone, S. Chaves, G. Herdoíza, C. Pena, D. Preti, J. Á. Romero and J. Ug PoS LATTICE **2018** (2019) 270 doi:10.22323/1.334.0270 [arXiv:1812.01474 lat]].

[3] A. Bussone, G. Herdoíza, C. Pena, D. Preti, J. Á. Romero and J. Ugarrio, PoS TICE **2018** (2019) 318 doi:10.22323/1.334.0318 [arXiv:1903.00286 [hep-lat]].

[4] A. Bussone, G. Herdoíza, C. Pena, D. Preti, J. Á. Romero and J. Ug arXiv:1812.05458 [hep-lat].

[5] G. Herdoíza, C. Pena, D. Preti, J. Á. Romero and J. Ugarrio, EPJ Web Conf (2018) 13018 doi:10.1051/epjconf/201817513018 [arXiv:1711.06017 [hep-lat]

[6] R. Frezzotti and G. C. Rossi, Nucl. Phys. Proc. Suppl. **128** (2004) doi:10.1016/S0920-5632(03)02477-0 [hep-lat/0311008].

[7] K. Jansen *et al.* [XLF Collaboration], Phys. Lett. B **624** (2005) doi:10.1016/j.physletb.2005.08.029 [hep-lat/0507032].

[8] V. Lubicz *et al.* [ETM Collaboration], Phys. Rev. D **96** (2017) no.5, 05451 ratum: [Phys. Rev. D **99** (2019) no.9, 099902] doi:10.1103/PhysRevD.96.05 10.1103/PhysRevD.99.099902 [arXiv:1706.03017 [hep-lat]].

[9] M. Ablikim *et al.* [BESIII Collaboration], Phys. Rev. Lett. **122** (2019) no.6, 06 doi:10.1103/PhysRevLett.122.062001 [arXiv:1809.06496 [hep-ex]].