$B \to \pi$ form factors with NRQCD/HISQ on the $n_f = 2 + 1$ asqtad ensembles

Chris Bouchard, University of Glasgow

with P. Lepage, C. Monahan, and J. Shigemitsu (HPQCD)

HPQCD

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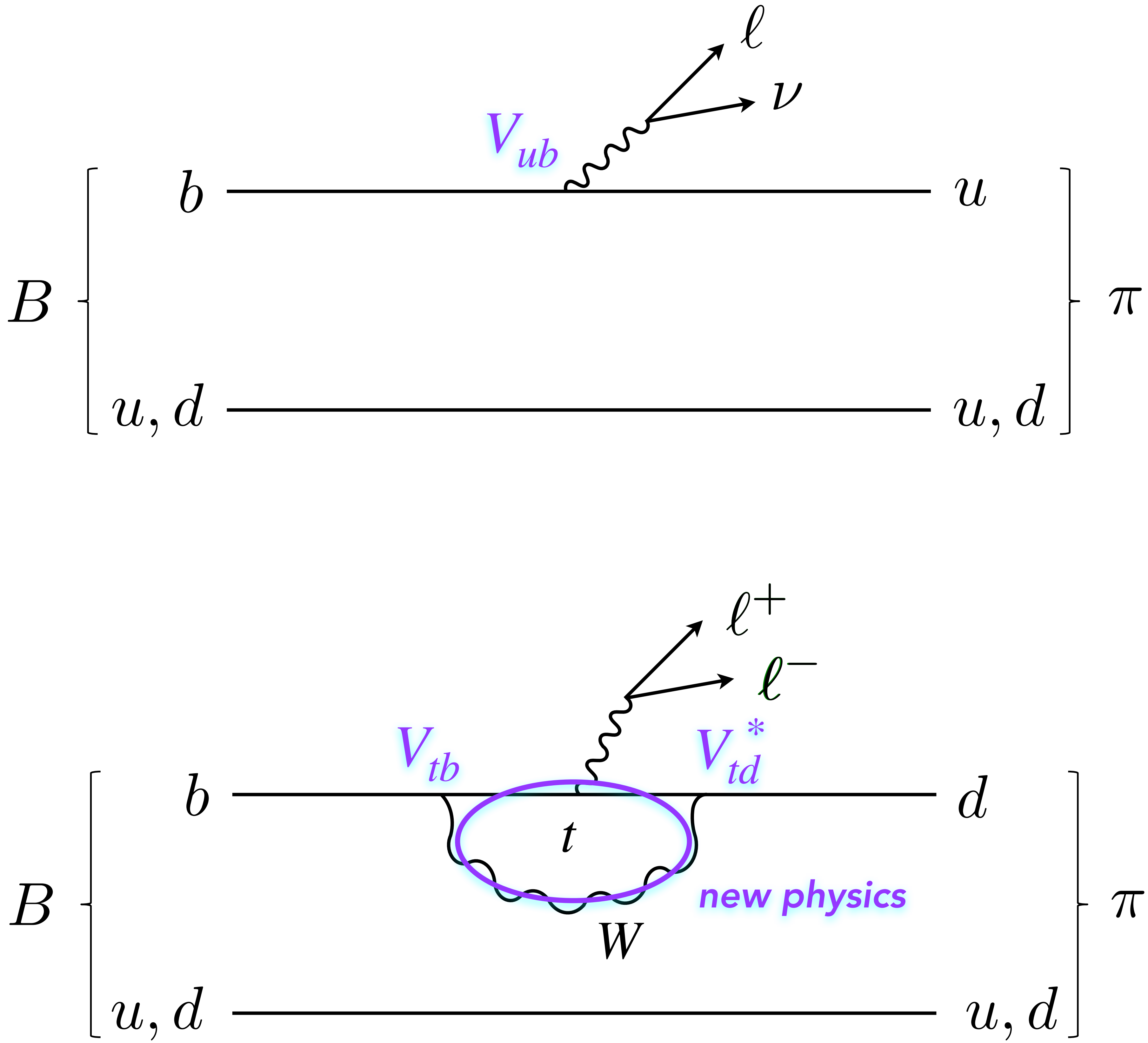
LATTICE 2019, Wuhan, China

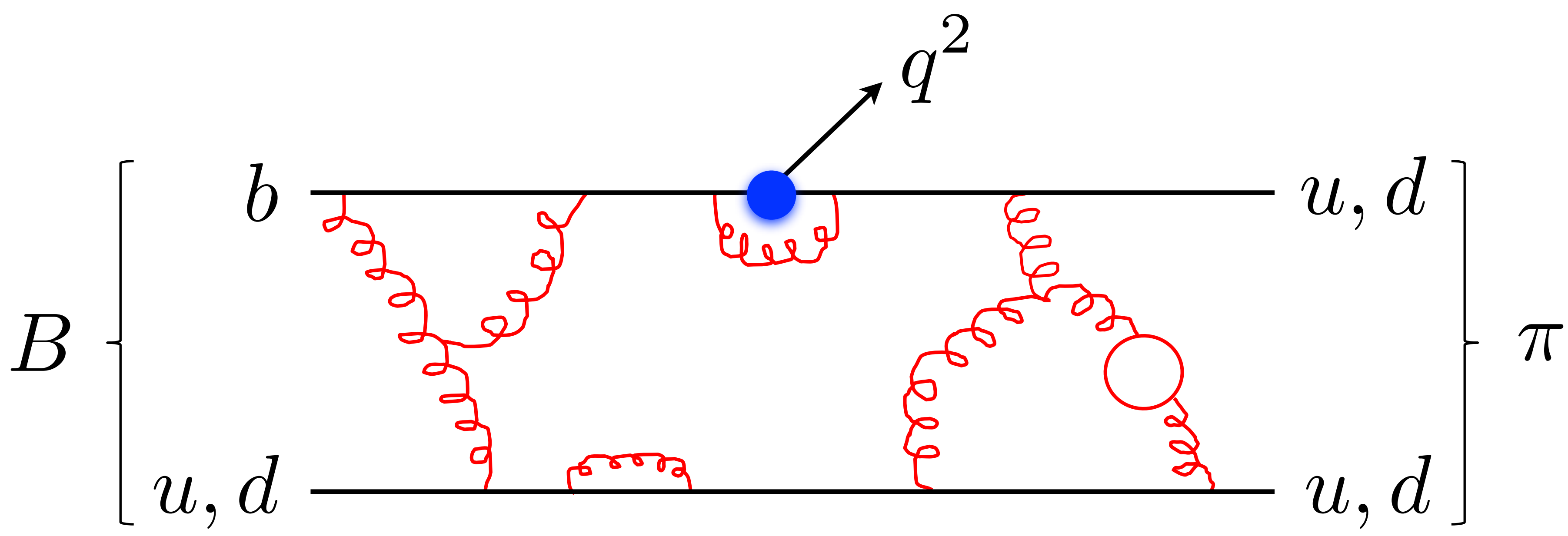
- motivation & role of LQCD
- simulation & correlator construction
- correlator fitting
- chiral, continuum, & kinematic extrapolation
- phenomenology
- summary/outlook

Motivation ...



ℓ
ν
V_{ub}
b
u
B
π
u, d
u, d
ℓ^+
ℓ^-
V_{tb}
V_{td}^*
b
d
t
new physics
B
π
W
u, d
u, d

Role of LQCD ...



Physics at disparate scales factorizes

$$\frac{d\Gamma}{dq^2} = \left(\sum_i C_i(V_{\mathrm{CKM}}) \, \langle \pi | J_i | B \rangle \right)^2$$

- Wilson coefficients: short distance, *perturbative*
- hadronic matrix elements: long distance, *nonperturbative*

form factors: f_0, f_+, & f_T

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MILC $n_f = 2+1$ asqtad ensembles

ensemble	a / fm	M_π / MeV	$N_s^3 \times N_t$	N_{srcs}	N_{cfg}
C1 □	0.12	267	$24^3 \times 64$	4	2096
C2 ○	0.12	348	$20^3 \times 64$	2	2242
C3 △	0.12	489	$20^3 \times 64$	2	1200
F1 ○	0.09	313	$32^3 \times 96$	4	1896
F2 △	0.09	438	$32^3 \times 96$	4	1200

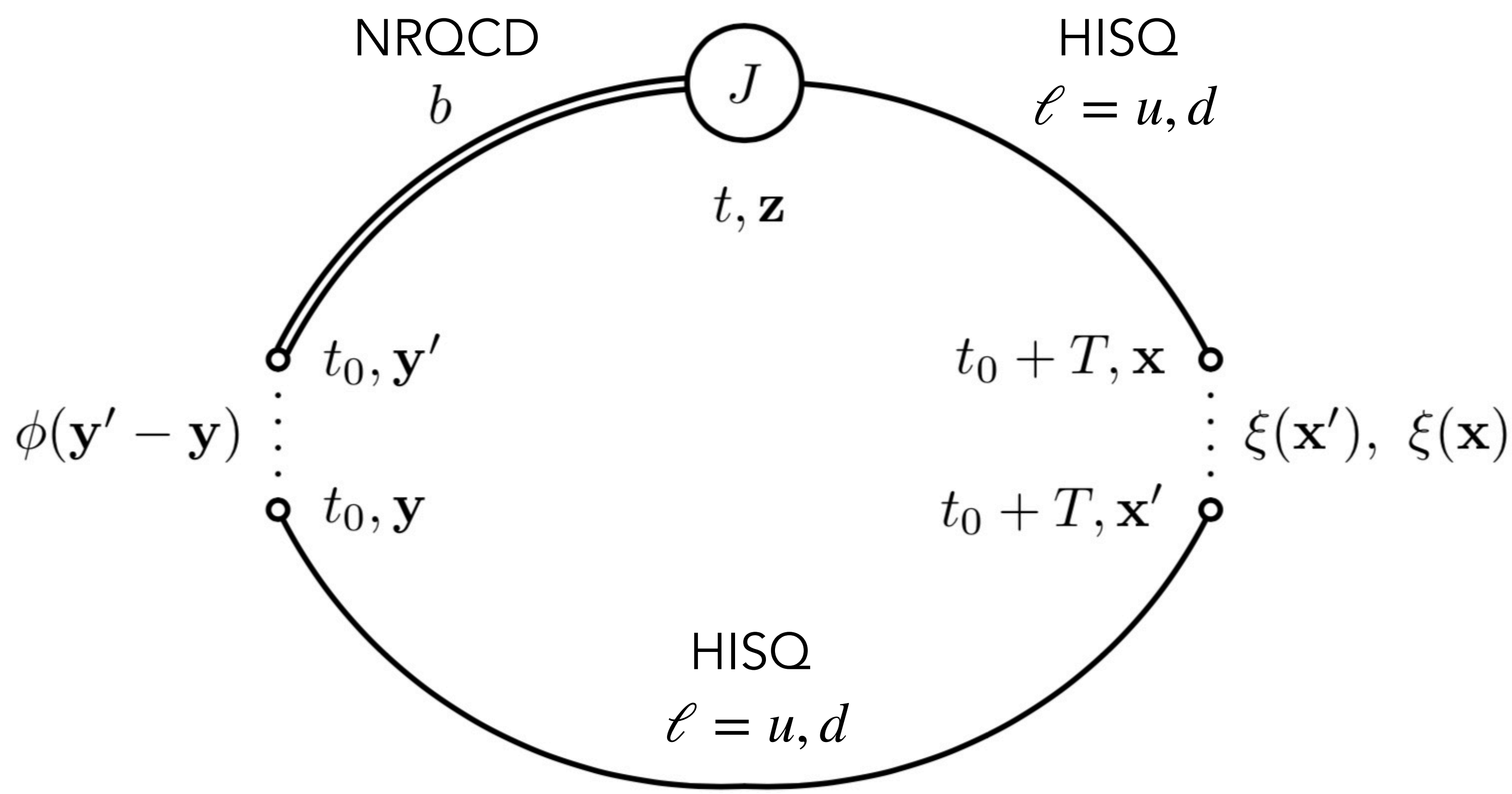
Bazavov et al, RMP 82, 1349 (2010)

Simulated momenta reaches $q^2 = 0.4\ \mathrm{GeV}^2$

ensemble	a / fm	M_π / MeV	$\boldsymbol{p}_\pi L / (2\pi)$ q^2 / GeV^2							
C1 □	0.12	267	(0,0,0),	(1,0,0),	(1,1,0),	(1,1,1),	(2,0,0)			
			25.5,	22.9,	21.2,	19.9,	18.7			
C2 ○	0.12	348	(0,0,0),	(1,0,0),	(1,1,0),	(1,1,1)				
			24.8,	21.8,	19.8,	18.2				
C3 △	0.12	489	(0,0,0),	(1,0,0),	(1,1,0),	(1,1,1),	(2,0,0),	(3,0,0),	(4,0,0),	(5,0,0)
			23.7,	21.2,	19.4,	17.9,	16.5,	11.3,	5.9,	0.4
F1 ○	0.09	313	(0,0,0),	(1,0,0),	(1,1,0),	(1,1,1),			(4,0,0)	
			24.9,	21.8,	19.7,	18.1,			5.8	
F2 △	0.09	438	(0,0,0),	(1,0,0),	(1,1,0),	(1,1,1),	(2,0,0),	(3,0,0)*		
			23.9,	21.3,	19.4,	17.8,	16.5,	11.2		

* Generated 300 cfgs with $\boldsymbol{p}_\pi L / (2\pi) = (2,2,1)$ to search for Lorentz-violating effects relative to (3,0,0). None found.

3pt correlator construction ...



J: current insertion, with momentum

$\phi(\mathbf{y}' - \mathbf{y})$: local and Gaussian smeared NRQCD b quark

$\xi(\mathbf{x}')$, $\xi(\mathbf{x})$: U(1) phases for random wall HISQ sources, with momentum

T : separations of $12, 13, 14, 15$ $(21, 22, 23, 24)$ used on 0.12 fm (0.09 fm) ensembles

Matching NRQCD …

- Generate 3pt correlator data for the following currents,

$$\mathscr{V}^{(0)}_\mu = \bar{q}\gamma_\mu b\ , \qquad \mathscr{V}^{(1)}_\mu = \frac{-1}{2am_b}\,\bar{q}\gamma_\mu\gamma\cdot\nabla b\ , \qquad \mathscr{V}^{(2)}_\mu = \frac{-1}{2am_b}\,\bar{q}\gamma\cdot\overleftarrow{\nabla}\gamma_0\gamma_\mu b\ ,$$

$$\mathscr{V}^{(3)}_k = \frac{-1}{2am_b}\,\bar{q}\,\nabla_k b\ , \qquad \mathscr{V}^{(4)}_k = \frac{-1}{2am_b}\,\bar{q}\,\overleftarrow{\nabla}_k b\ .$$

- Match through $\mathcal{O}\left(\alpha_s\frac{\Lambda_{\rm QCD}}{m_b}, \frac{\alpha_s}{am_b}, \alpha_s a\Lambda_{\rm QCD}\right)$:

$$\langle V_0\rangle = (1+\alpha_s\rho^{(0)}_0)\langle\mathscr{V}^{(0)}_0\rangle \ + \ (1+\alpha_s\rho^{(1)}_0)\langle\mathscr{V}^{(1),{\rm sub}}_0\rangle \ + \ \alpha_s\rho^{(2)}_0\langle\mathscr{V}^{(2)}_0\rangle$$

$$\langle V_k\rangle = (1+\alpha_s\rho^{(0)}_k)\langle\mathscr{V}^{(0)}_k\rangle \ + \ (1+\alpha_s\rho^{(1)}_k)\langle\mathscr{V}^{(1),{\rm sub}}_k\rangle \ + \ \alpha_s\sum_{i=2}^{4}\rho^{(i)}_k\langle\mathscr{V}^{(i)}_k\rangle$$

$$\langle T_{k0}\rangle = (1+\alpha_s\rho^{(0)}_T)\langle\mathscr{T}^{(0)}_{k0}\rangle \ + \ \langle\mathscr{T}^{(1),{\rm sub}}_{0k}\rangle$$

with power-law subtraction $\langle J^{(1),{\rm sub}}\rangle = \langle J^{(1)}\rangle \ - \ \alpha_s\xi_J\langle J^{(0)}\rangle$.

- Correlator data are combined with priors for matching coefficients.

Monahan et al, PRD 87, 034017 (2013) Gulez et al, PRD 73, 074502 (2006); D75, 119906(E) (2007)

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Correlator fitting ...

- Bayesian, using `lsqfit` Lepage, github.com/gplepage/lsqfit

- simultaneous fit to (2pt + 3pt)$_p$

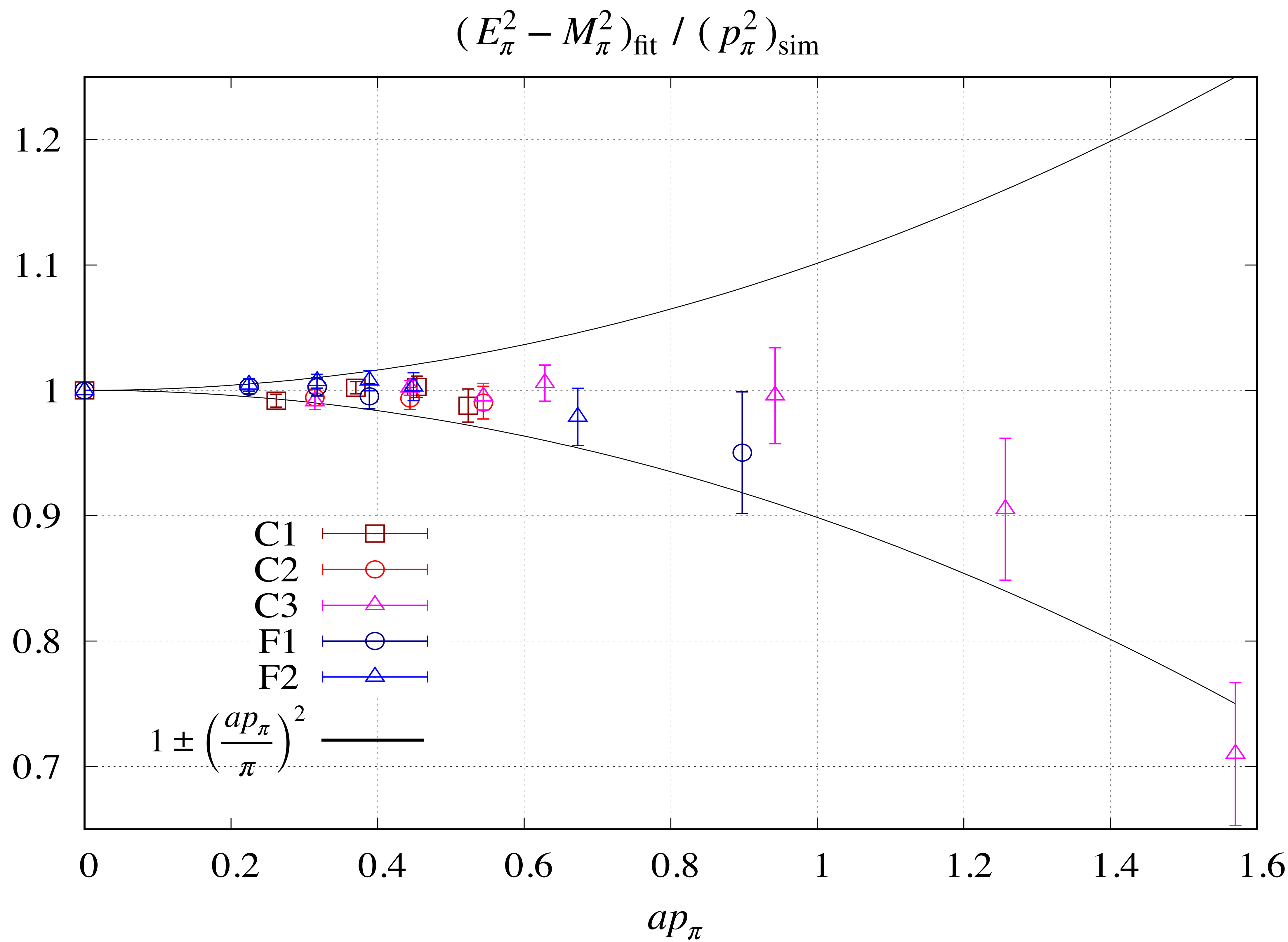
$$C^{\pi}_{2pt}(p;t) = \sum_n |Z^{\pi}_n(p)|^2 \, (-1)^{nt} \left(e^{-E^{\pi}_n(p)t} + e^{-E^{\pi}_n(p)(N_t-t)} \right)$$

$$C^{B\pi}_{3pt}(p;t) = \sum_{n,m} Z^{\pi}_n(p)\, A_{nm}(p)\, Z^B \, (-1)^{mt+n(T-t)} \; e^{-E^b_m t} \; e^{-E^{\pi}_n(p)(T-t)}$$

- posteriors from (2pt + 3pt)$_{(0,0,0)}$ are priors in (2pt + 3pt)$_{(1,0,0)}$

- posteriors from (2pt + 3pt)$_{(1,0,0)}$ are priors in (2pt + 3pt)$_{(1,1,0)}$ ...

Dispersion relation ...



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Matching NRQCD (account for missing higher order effects) ...

- Matrix elements written in terms of intermediate form factors

$$\langle V_0\rangle = \sqrt{2M_B}\, f_\parallel \,, \qquad \langle V_k\rangle = \sqrt{2M_B}\, p_\pi^k\, f_\perp \,, \qquad \langle T_{k0}\rangle = \frac{2M_B p_\pi^k}{M_B + M_\pi}\, f_T \,.$$

- Uncertainty from omitted higher order matching effects are accounted for after correlator fitting:

$$f_{\parallel,\perp,T} \to f_{\parallel,\perp,T}\left(1 + \mu_{\parallel,\perp,T} + \tilde{\mu}_{\parallel,\perp,T}\frac{p_\pi}{p_\pi^{\max}}\right)$$

where the coefficients

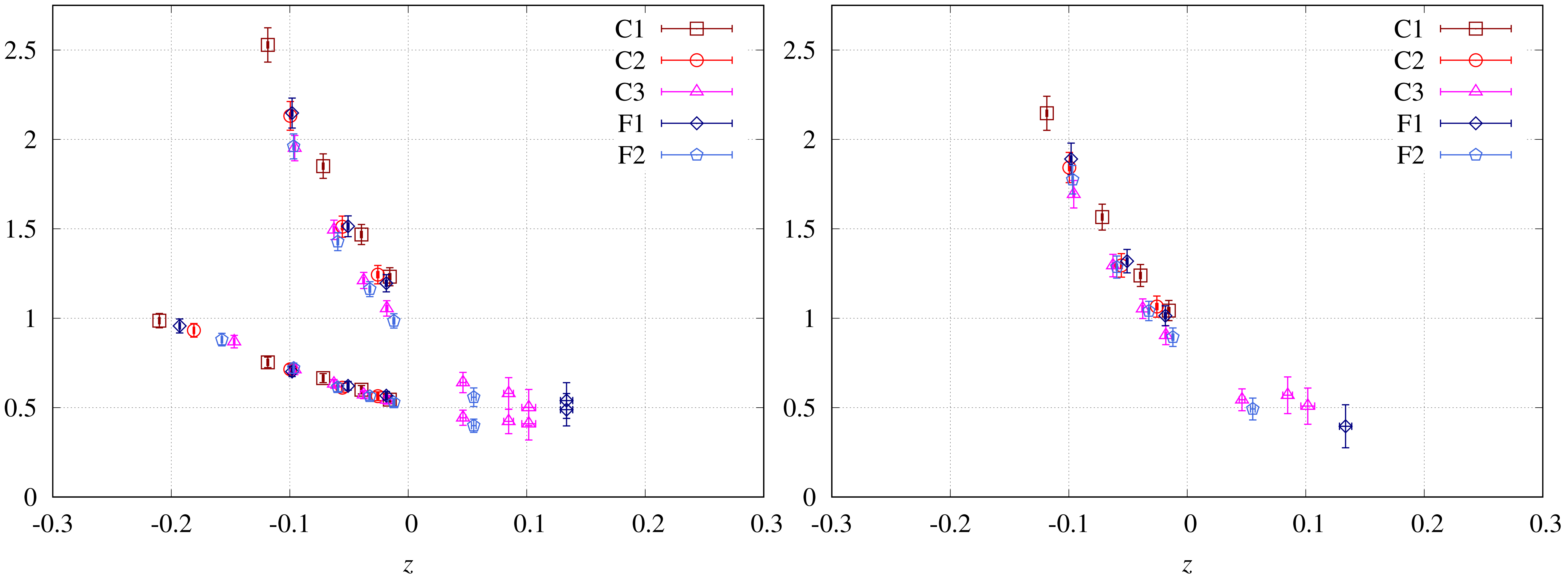
$\mu_{\parallel,\perp,T}$: generic p_π-independent effects

$\tilde{\mu}_{\parallel,\perp,T}$: generic p_π-dependent effects

$$\mu_{\parallel,\perp,T} = 0 \pm 0.04$$
$$\tilde{\mu}_\parallel = 0 \pm 0.02$$
$$\tilde{\mu}_\perp = 0 \pm 0.03$$
$$\tilde{\mu}_T = 0 \pm 0.05$$

are priors in subsequent fit, with values set by observed size of $J^{(i)} / J^{(0)}$.

Chiral, continuum, and kinematic extrapolation...



- HISQ permits large momenta simulations
- want to fit all this data

Chiral, continuum, and kinematic extrapolation...

- Use hard pion ChPT to factorize chiral logs from kinematics

Bijnens and Jemos, NPB 840, 54 (2010); B 844, 182(E) (2011)
Bijnens and Jemos, NPB 846, 145 (2011)

- kinematics: trade E_π for z Bouchard, Lepage, Monahan, Na, and Shigemitsu, PRD 90, 054506 (2014)

$$f(E_\pi) = (1 + [\mathrm{logs}])\,\mathcal{K}(E_\pi) \;\longrightarrow\; f(z) = (1 + [\mathrm{logs}])\,\mathcal{K}(z)$$

- where the hard pion ChPT log is

$$[\mathrm{logs}]_{\mathrm{SU}(2)} = -\frac{3}{8}(1 + 3g^2_{BB^*\pi})\,x_\pi\left[\log\left(\frac{M_\pi^2}{\Lambda_\chi^2}\right) + \delta_{\mathrm{FV}}\right]$$

Chiral, continuum, and kinematic extrapolation...

- write kinematic term $K(z)$ as polynomial in z

$$P_i(q^2) f_i(q^2) = \big(1 + [\text{logs}]\big) \sum_{k=0}^{K} a_k^{(i)} D_k^{(i)} z(q^2)^k$$

- with expansion parameters

$$x_\pi = \left(\frac{M_\pi}{4\pi f_\pi}\right)^2 \qquad \delta_X = \frac{M^2_{X,\,\text{asqtad}} - M_X^2}{(4\pi f_\pi)^2} \qquad x_b = \frac{am_b - \text{avg}(am_b)}{\text{avg}(am_b)}$$

- analytic chiral and continuum terms are

$$\begin{aligned} D_k^{(i)} &= 1 + c_{1,k}^{(i)} x_\pi + c_{2,k}^{(i)} x_\pi^2 \\ &\quad + d_{0,k}^{(i)} \left(\frac{a}{r_1}\right)^2 \big(1 + l_{0,k}^{(i)} x_\pi + l_{1,k}^{(i)} x_\pi^2\big)\big(1 + h_{0,k}^{(i)} x_b + h_{1,k}^{(i)} x_b^2\big) \\ &\quad + d_{1,k}^{(i)} \left(\frac{a}{r_1}\right)^4 \big(1 + l_{2,k}^{(i)} x_\pi + l_{3,k}^{(i)} x_\pi^2\big)\big(1 + h_{2,k}^{(i)} x_b + h_{3,k}^{(i)} x_b^2\big) \\ &\quad + e_{0,k}^{(i)} \left(\frac{a\mathbf{p}_\pi}{\pi}\right)^2 + e_{1,k}^{(i)} \left(\frac{a\mathbf{p}_\pi}{\pi}\right)^4 \\ &\quad + c_{3,k}^{(i)} \left(\frac{\delta_\pi}{2} + \delta_K\right) + c_{4,k}^{(i)} \frac{M_{\eta_s}^2 - (M_{\eta_s}^{\text{phys.}})^2}{(4\pi f_\pi)^2} \end{aligned}$$

Chiral, continuum, and kinematic extrapolation...

- write kinematic term $K(z)$ as polynomial in z

$$P_i(q^2) f_i(q^2) = \big(1 + [\mathrm{logs}]\big) \sum_{k=0}^{K} a_k^{(i)} D_k^{(i)} z(q^2)^k$$

- fit stable from $K = 3$

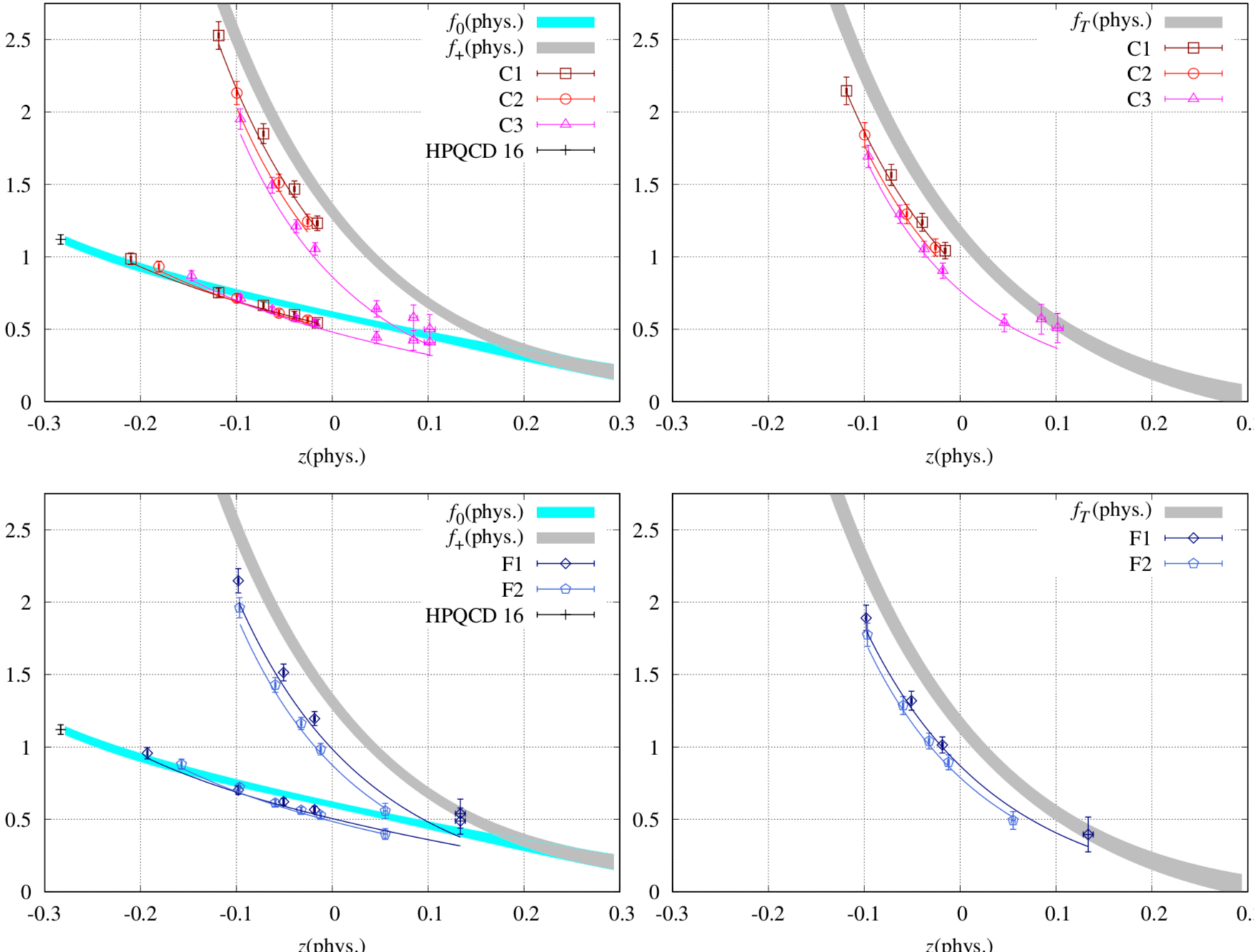
- impose constraints in continuum:

  - kinematic $\quad f_0(q^2 = 0) = f_+(q^2 = 0)$

  - large q^2 scaling of f_+ and f_T (i.e. BCL type z-expansion)

- in physical limit, the BCL z-expansion coefficients are then

$$\lim_{\substack{m \to m_{\mathrm{physical}} \\ a \to 0}} (1 + [\mathrm{logs}])\, a_k D_k$$



$$O(z^3),\ \chi^2/\mathrm{dof} = 60.8/76$$

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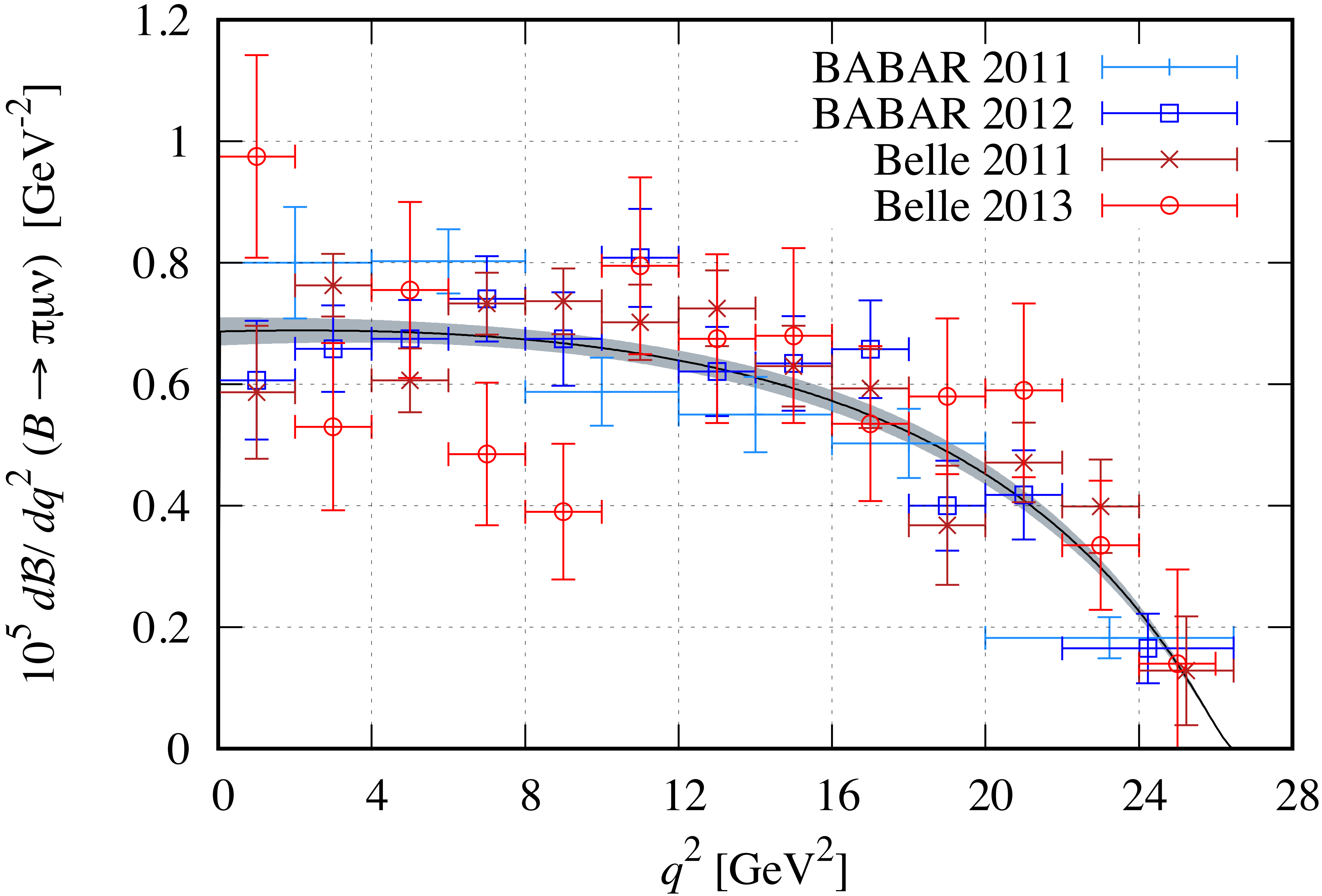
Incorporate Experimental Results...

- redo mod-z fit, including BaBar and Belle results for the differnital branching fraction

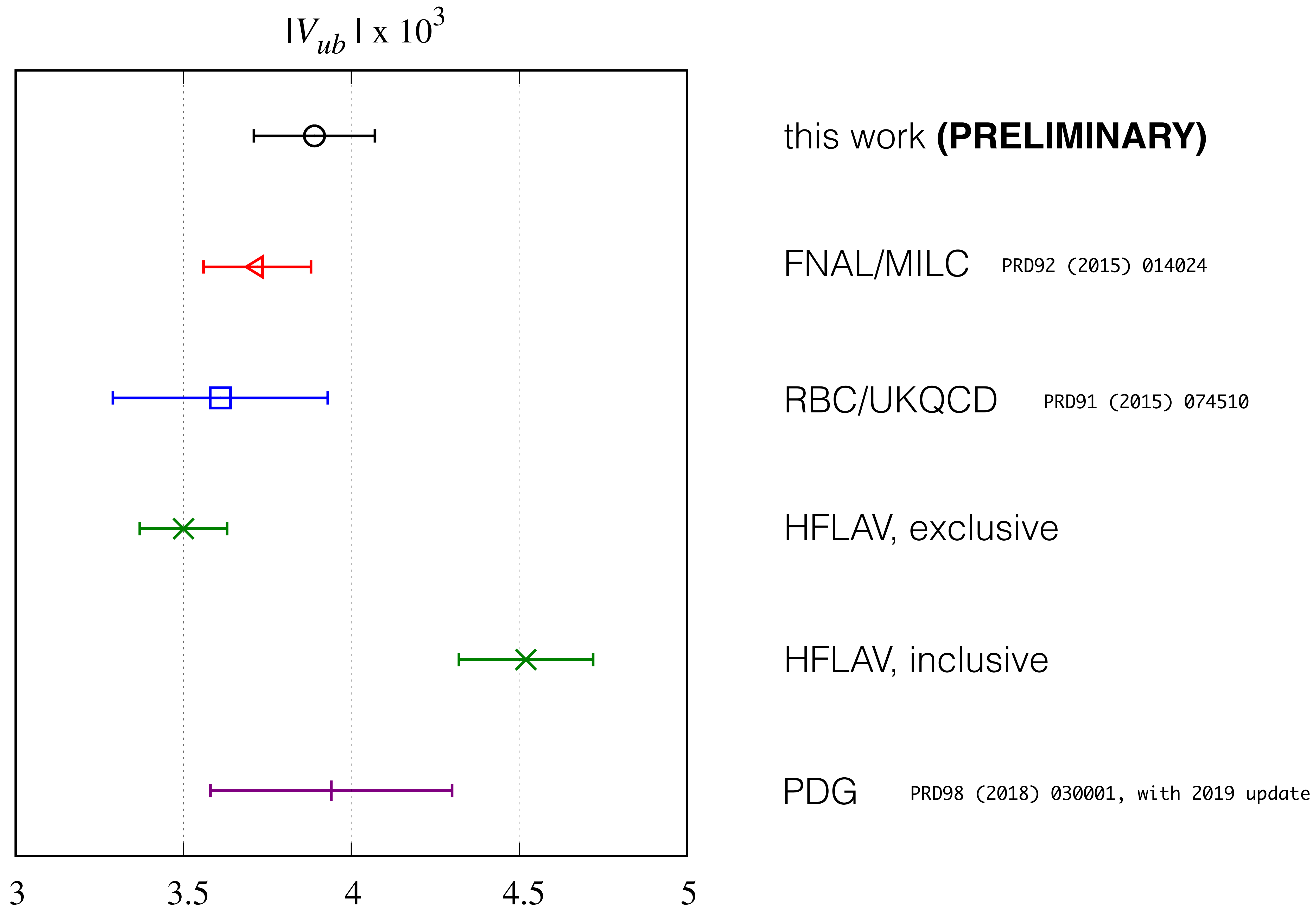
BaBar, PRD83, 032007 (2011)
BaBar, PRD86, 092004 (2012)
Belle, PRD83, 071101 (2011)
Belle, PRD88, 032005 (2013)

$$\frac{d\mathcal{B}(B \to \pi \ell \nu)}{dq^2} = \frac{\tau_B G_F^2 |V_{ub}|^2}{24\pi^3} \frac{(q^2 - m_\ell^2)^2 \sqrt{E_\pi^2 - M_\pi^2}}{q^4 M_B^2} \times \left[\left(1 + \frac{m_\ell^2}{2q^2}\right)^2 M_B^2 (E_\pi^2 - M_\pi^2) |f_+(q^2)|^2 + \frac{3m_\ell^2}{8q^2} \left(M_B^2 - M_\pi^2\right)^2 |f_0(q^2)|^2 \right]$$

- output: improved form factors & $|V_{ub}|$



$O(z^3),\ \chi^2/\mathrm{dof} = 152.4/127 = 1.2,\ Q = 0.05$



$|V_{ub}|$ x 10^3
this work **(PRELIMINARY)**
FNAL/MILC PRD92 (2015) 014024
RBC/UKQCD PRD91 (2015) 074510
HFLAV, exclusive
HFLAV, inclusive
PDG PRD98 (2018) 030001, with 2019 update
3
3.5
4
4.5
5

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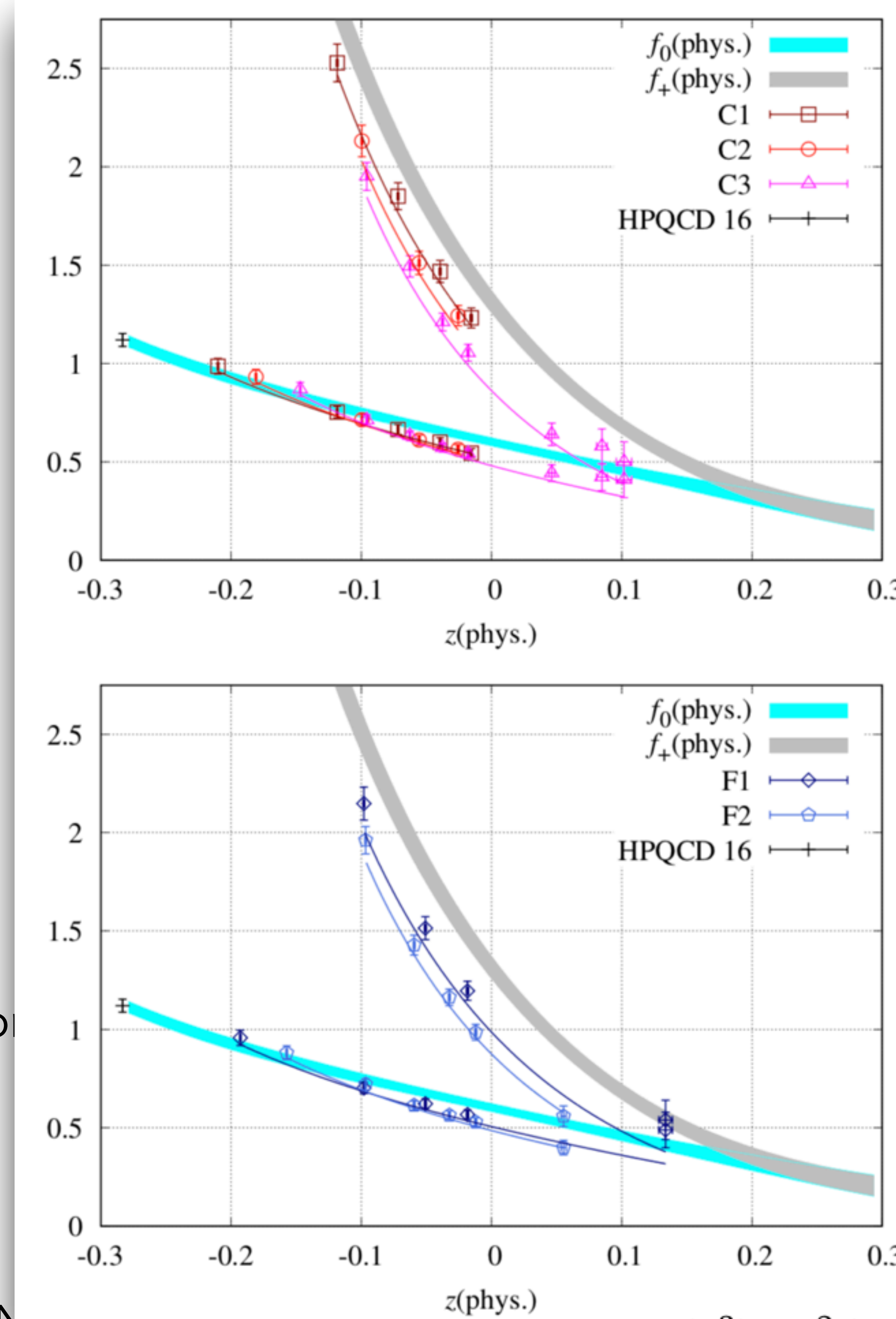
Summary/outlook …

- **preliminary results for $f_{0,+,T}$**
  - momenta over full kinematic range
  - hard pion ChPT modified z expansion

- preliminary |Vub|
  - ~equal error from LQCD & expt

- To do:
  - Cross-checks (e.g. vs. 2-step extrapolatio
  - More phenomenology, including FCNC
  - Full error budget
  - Uncertainty dominated by statistics and N



Summary/outlook ...

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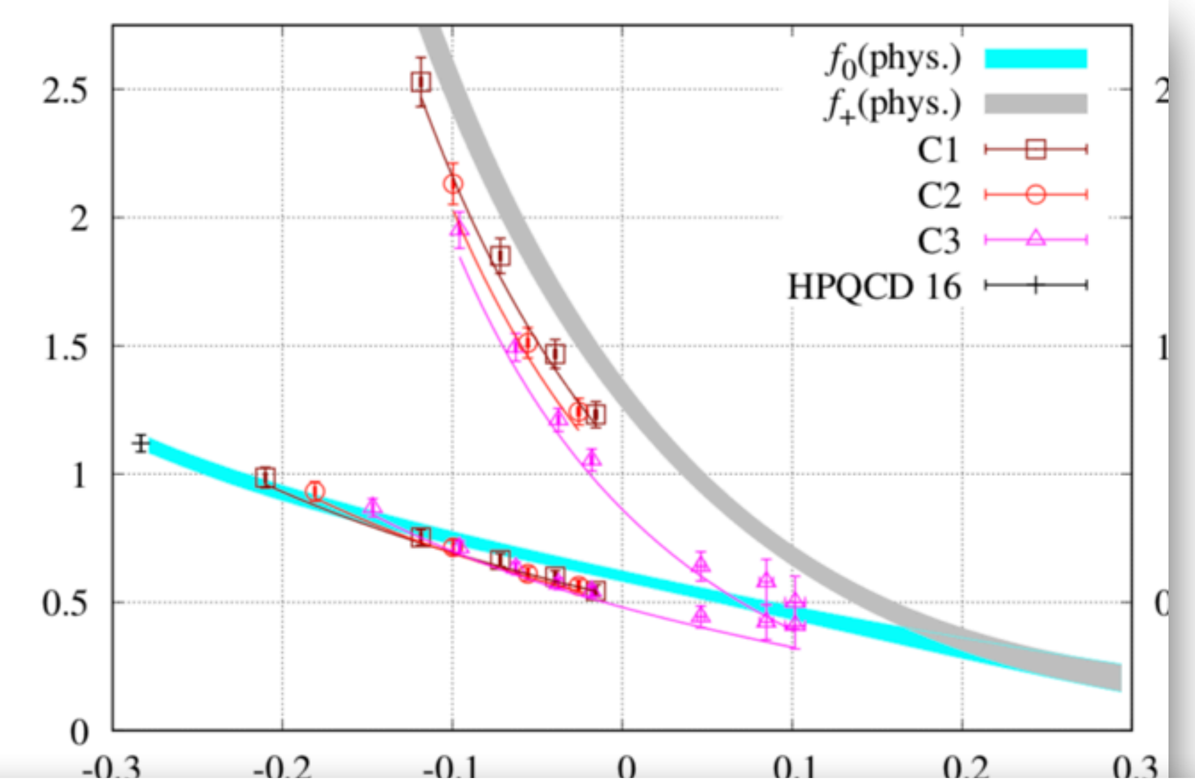
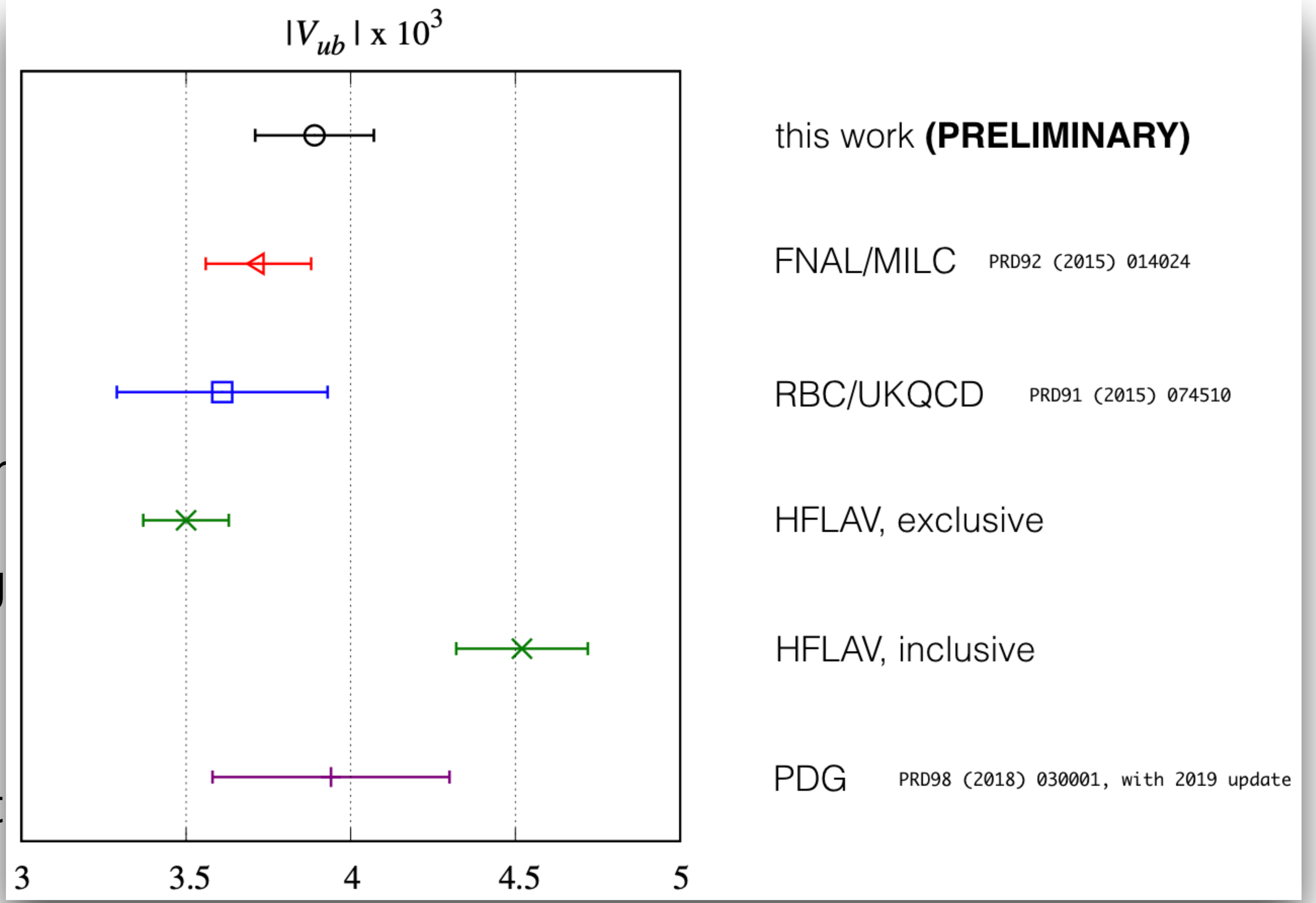
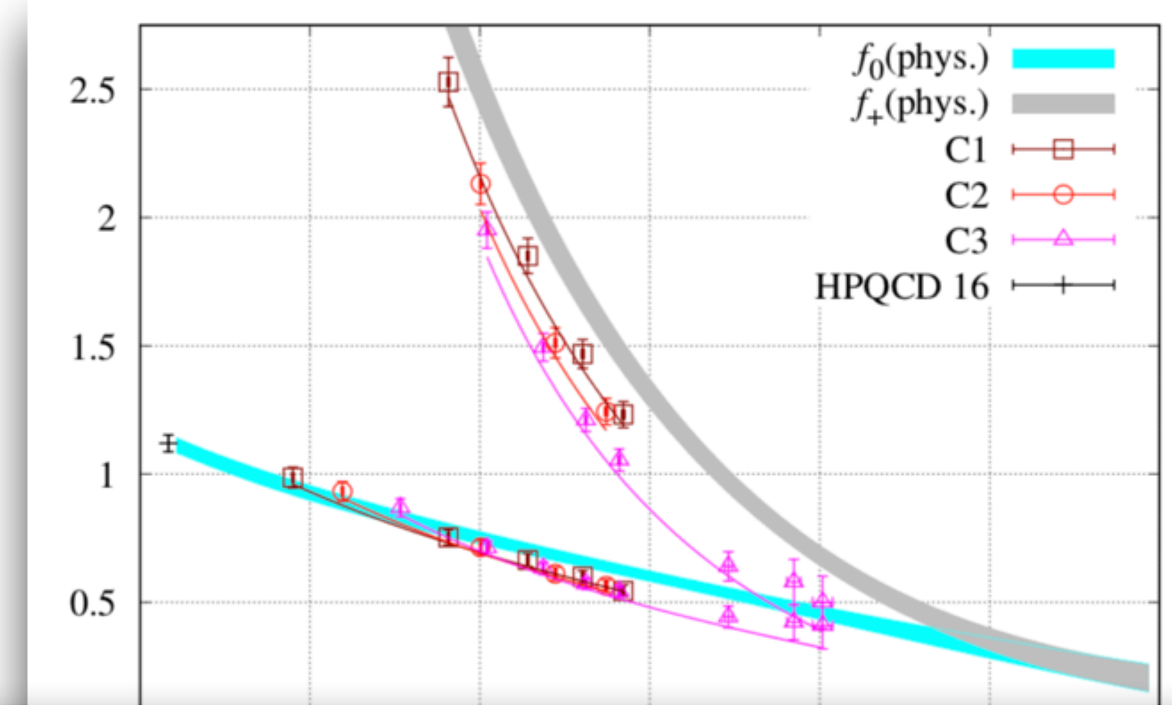




Summary/outlook ...

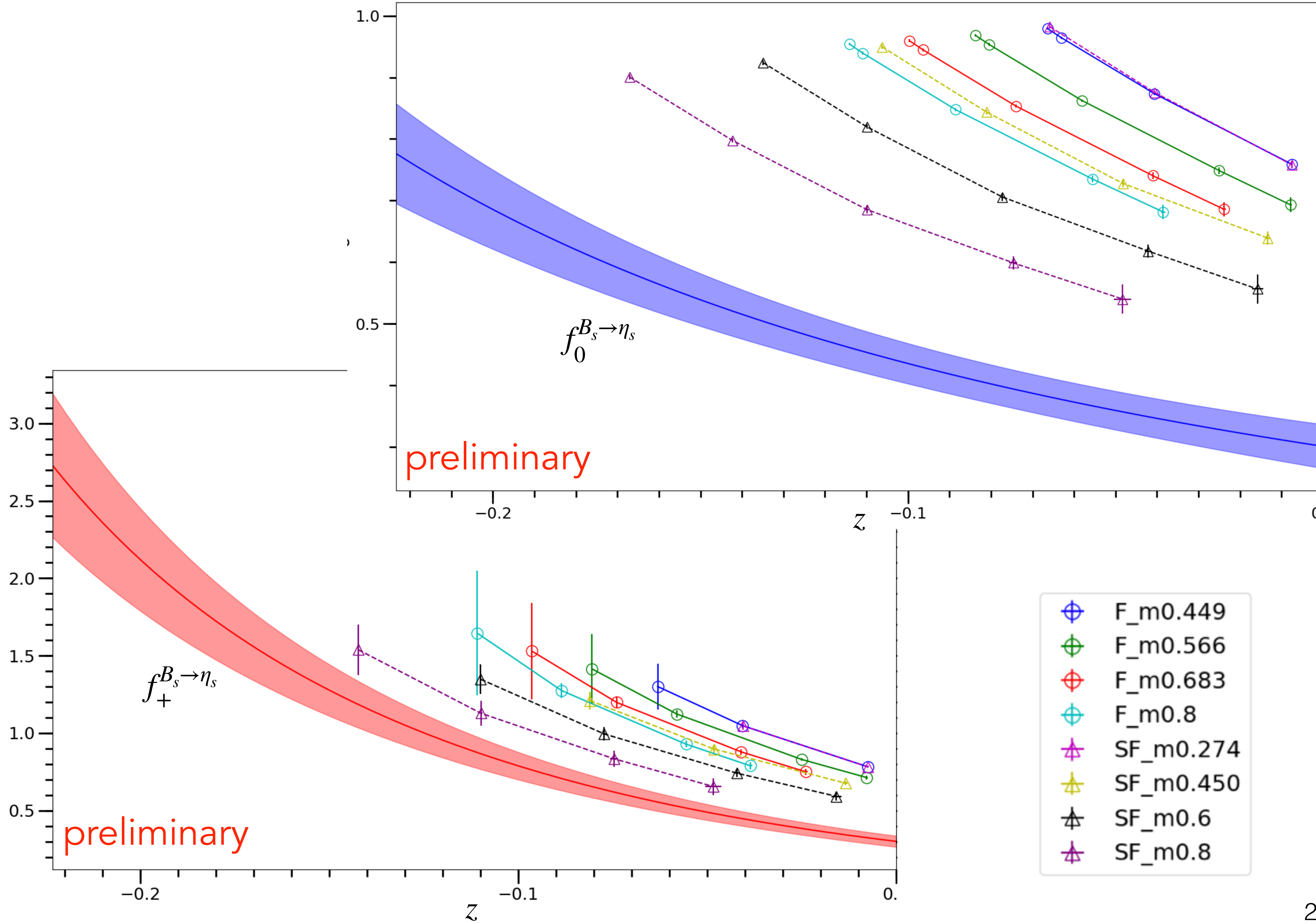


- preliminary results for $f_{0,+,T}$
  - momenta over full kinematic range
  - hard pion ChPT modified z expansion

- preliminary |Vub|
  - ~equal error from LQCD

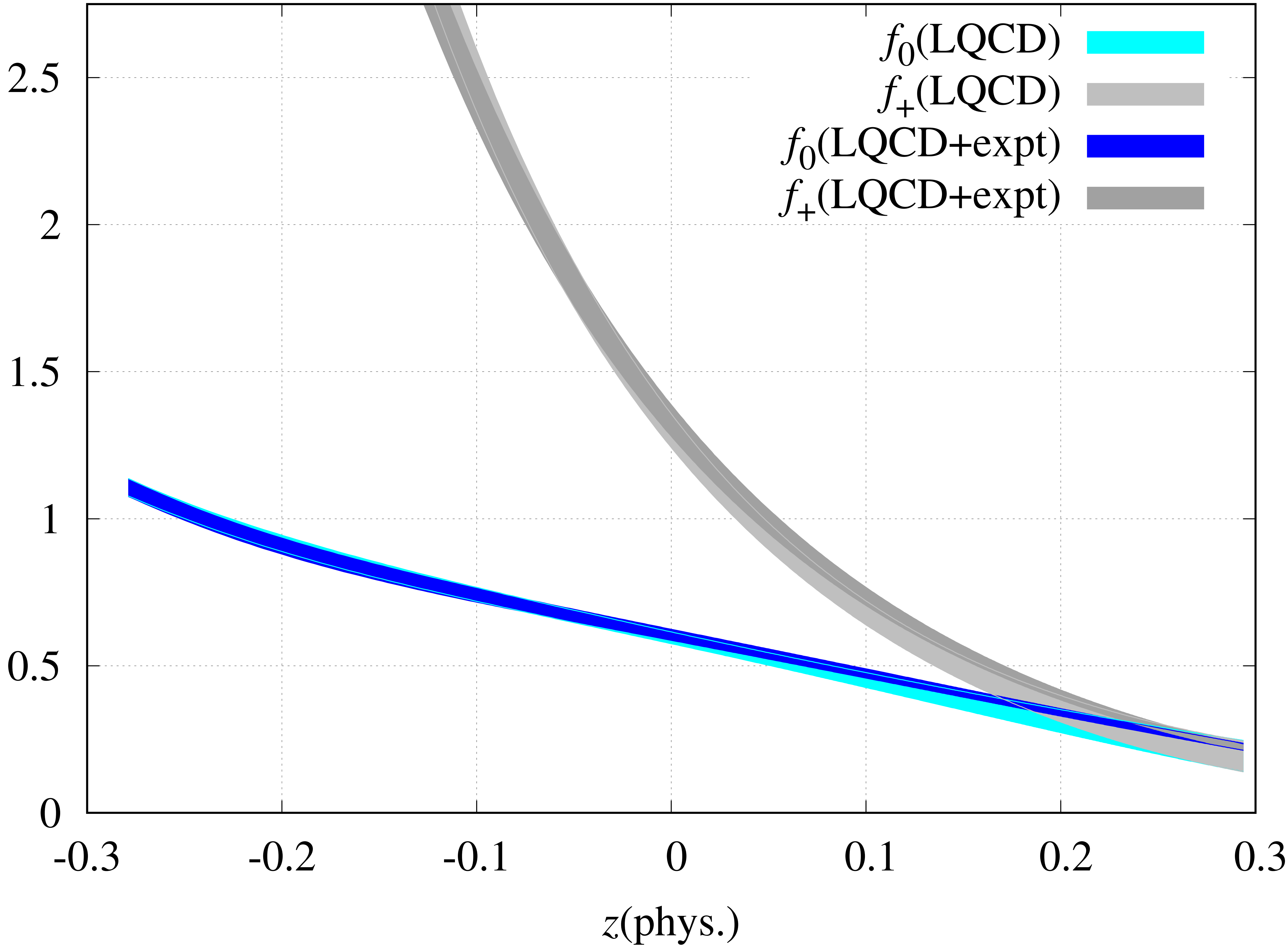
- **To do:**
  - Cross-checks (e.g. vs. 2-step extrapolations)
  - More phenomenology, including FCNC
  - Full error budget
  - Uncertainty dominated by statistics and NRQCD matching...

All-HISQ b decays (Will Parrot)



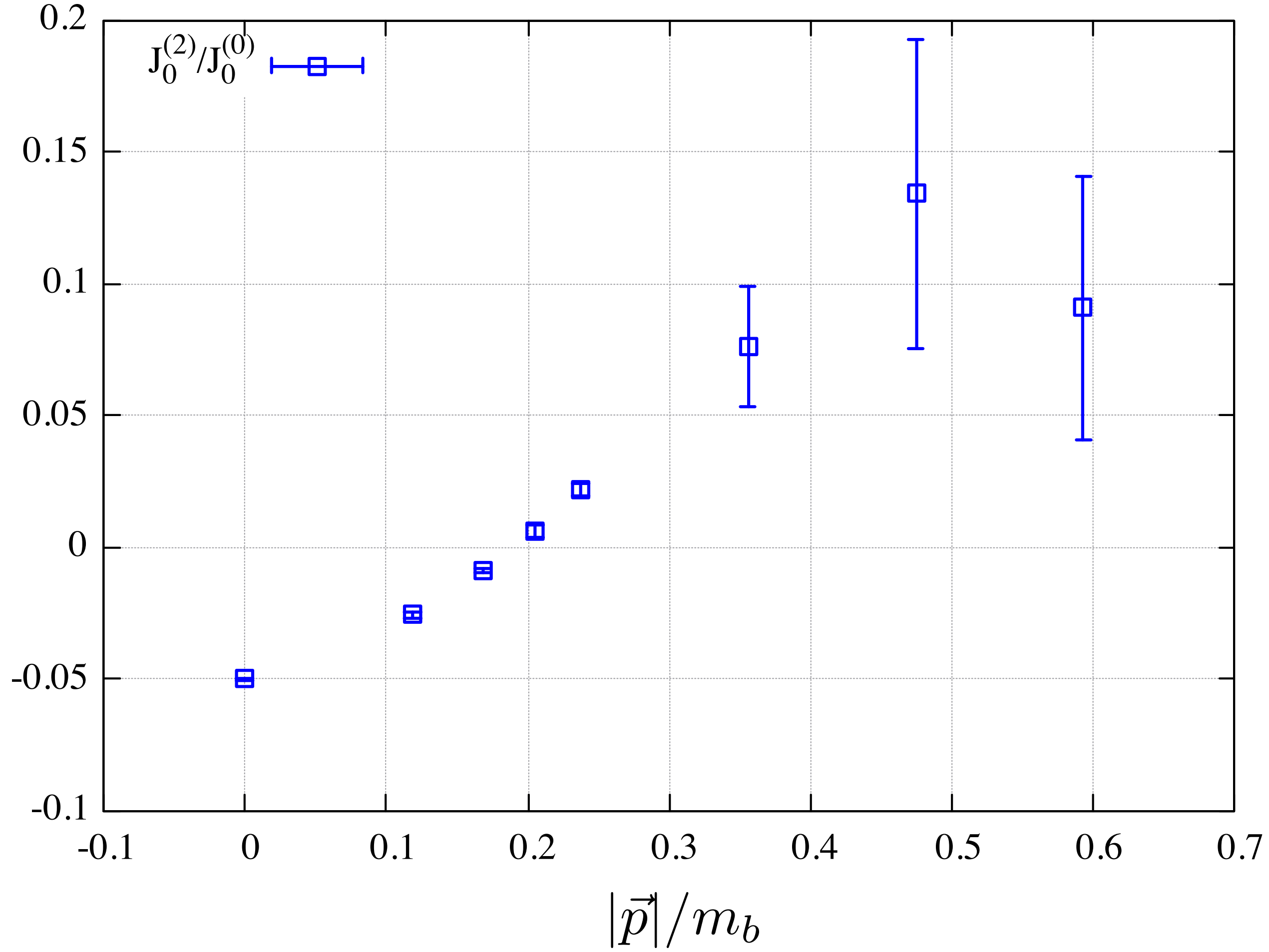
Thank you.

Back up slides ...



f_0(LQCD)
f_+(LQCD)
f_0(LQCD+expt)
f_+(LQCD+expt)
2.5
2
1.5
1
0.5
0
-0.3
-0.2
-0.1
0
0.1
0.2
0.3
z(phys.)

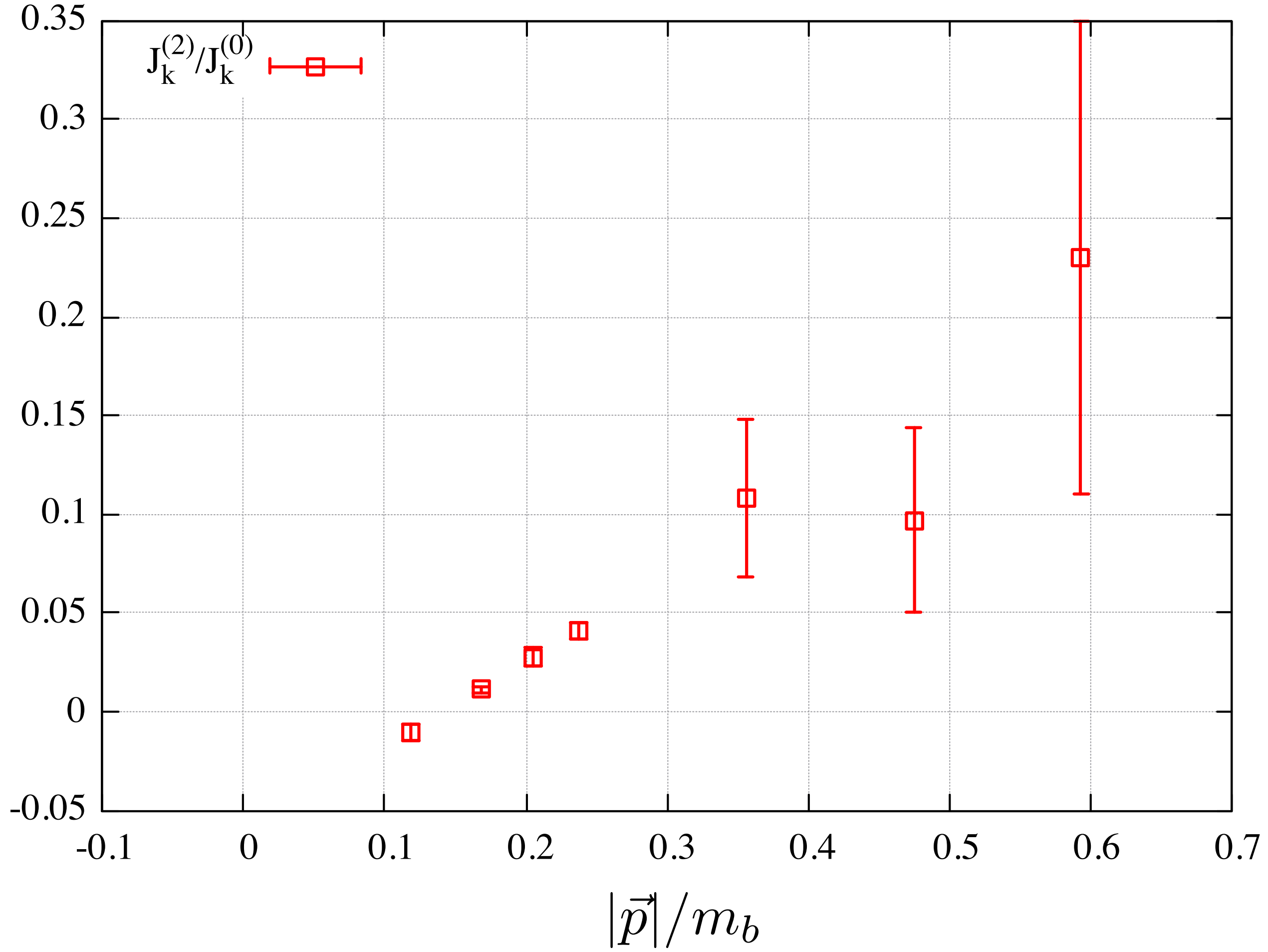
i



<u>ensemble C3</u>

$$am_b = 2.650$$
$$\alpha_s = 0.3047$$
$$a|\vec{p}|_{\max} = \pi/2$$

$$\tilde{m}_{||} \sim \mathcal{O}\left(\frac{J_0^{(2)}}{J_0^{(0)}}\right) \times \mathcal{O}\left(\alpha_s \frac{|\vec{p}|^2}{m_b^2}, \alpha_s^2 \frac{|\vec{p}|}{m_b}\right)$$
$$\sim 0.15 \times (0.11, 0.055)$$
$$\implies \text{prior}[\tilde{m}_{||}] = 0(0.02)$$



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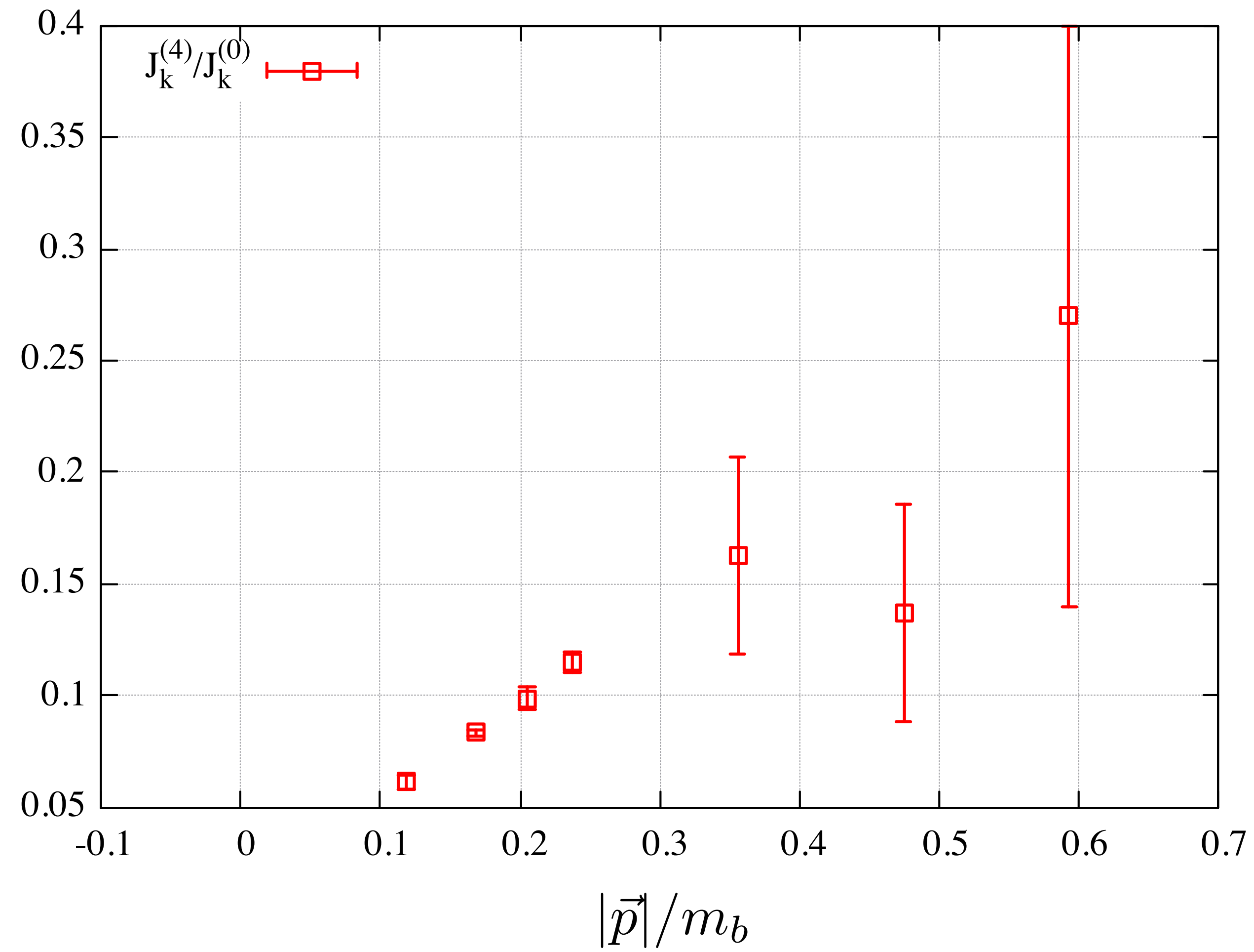
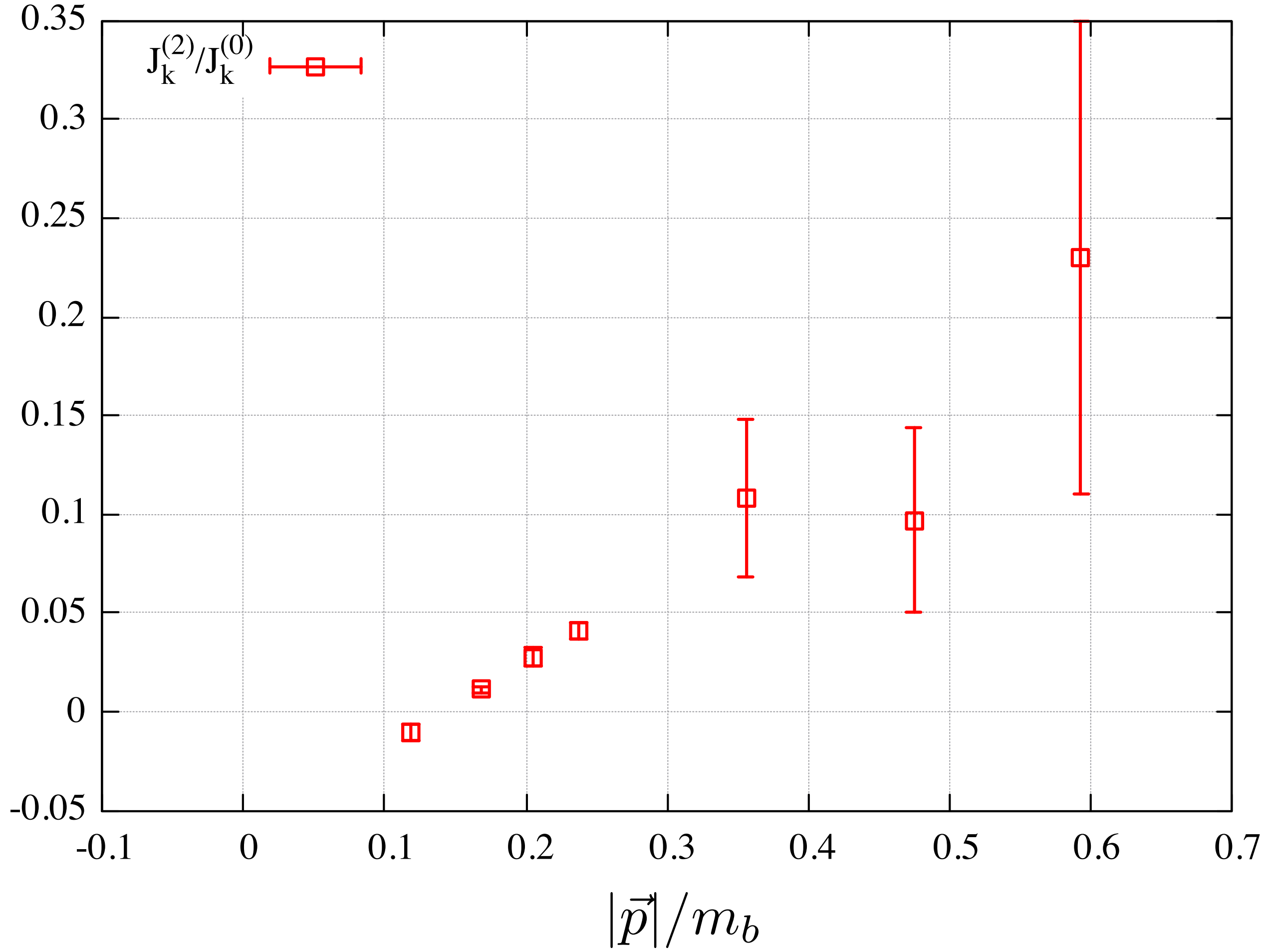
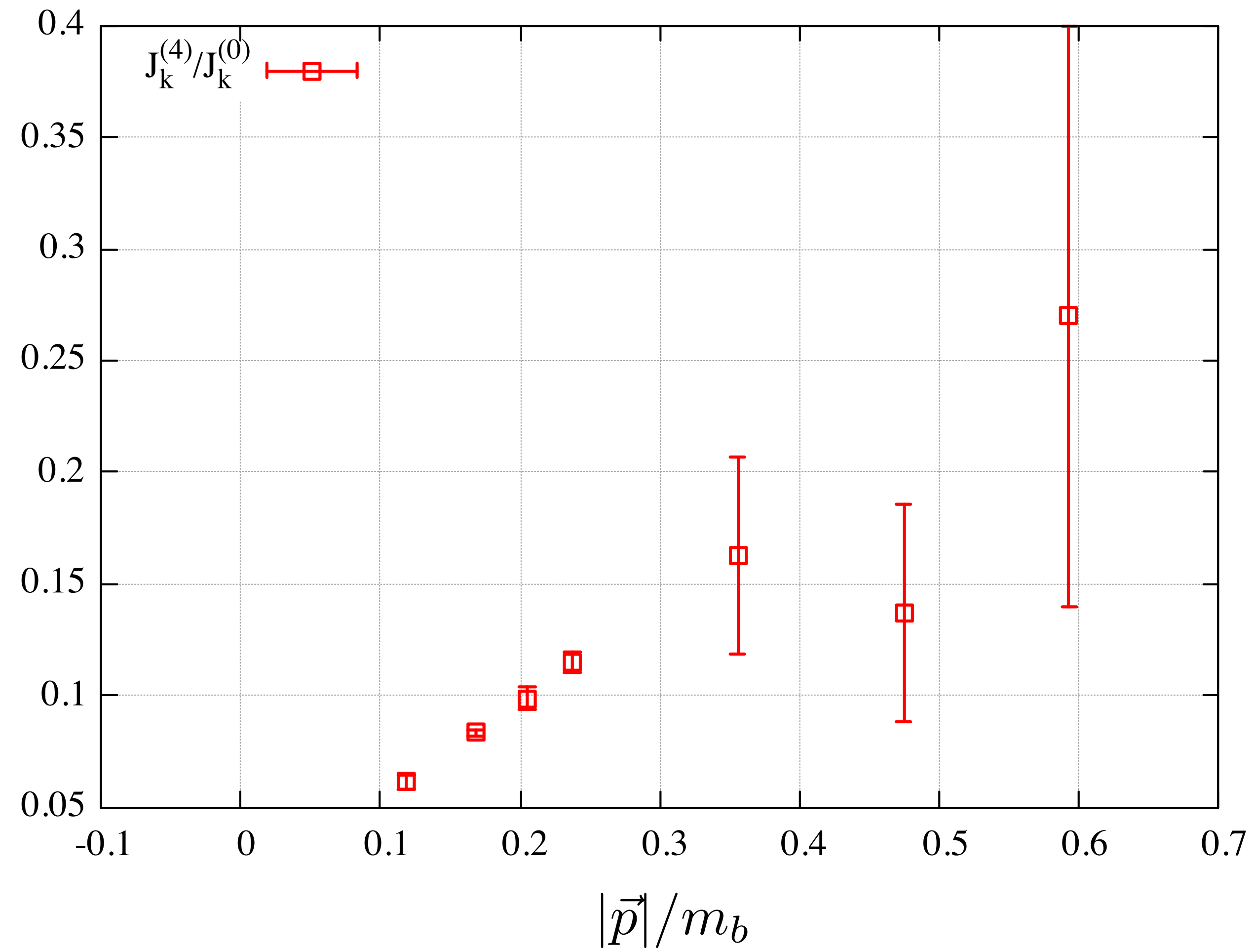


$$\tilde{m}_\perp \sim \mathcal{O}\left(\frac{J_k^{(2)}}{J_k^{(0)}}, \frac{J_k^{(4)}}{J_k^{(0)}}\right) \times$$
$$\mathcal{O}\left(\alpha_s \frac{|\vec{p}|^2}{m_b^2}, \alpha_s^2 \frac{|\vec{p}|}{m_b}\right)$$
$$\sim (0.25, 0.27) \times (0.11, 0.055)$$
$$\Longrightarrow \text{prior}[\tilde{m}_\perp] = 0(0.03)$$





<u>ensemble C3</u>

$$am_b = 2.650$$

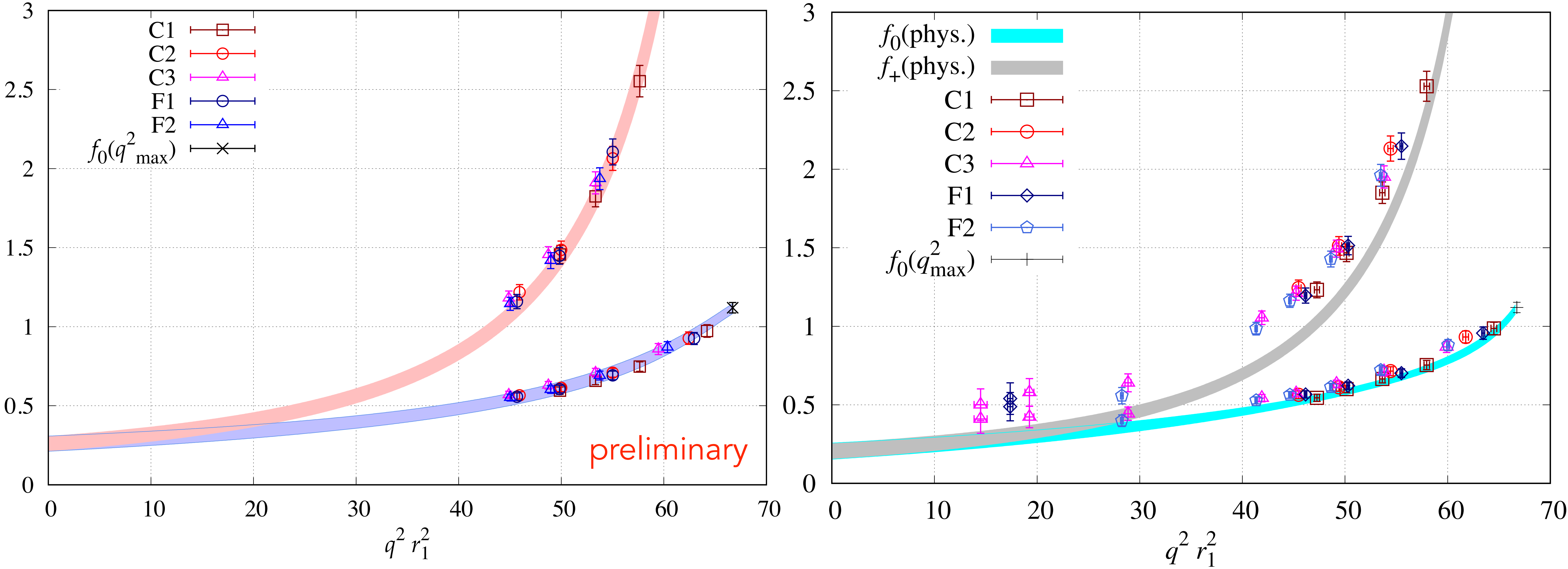
$$\alpha_s = 0.3047$$

$$a|\vec{p}|_{\max} = \pi/2$$

$$\tilde{m}_T \sim \mathcal{O}\left(\frac{J_k^{(2)}}{J_k^{(0)}}, \frac{J_k^{(4)}}{J_k^{(0)}}\right) \times \mathcal{O}\left(\alpha_s \frac{|\vec{p}|}{m_b}\right)$$

$$\sim (0.25, 0.27) \times 0.18$$

$$\Longrightarrow \text{prior}[\tilde{m}_T] = 0(0.05)$$



C1
C2
C3
F1
F2
$f_0(q^2_{max})$
preliminary
$q^2\ r_1^2$
f_0(phys.)
f_+(phys.)
C1
C2
C3
F1
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$q^2\ r_1^2$

V