

Semileptonic $D \rightarrow K$ decay from full lattice QCD with HISQ

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With HPQCD collaboration :

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- Flavour Physics is fertile ground for testing SM with high precision
- The flavour changing weak interactions can be parametrised in terms of the Cabbibo-Kobayashi-Maskawa (CKM) unitary matrix

$$V_{\text{CKM}} = \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix}$$

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9999 \pm 0.0006.$$

$$|V_{ud}|^2 + |V_{cd}|^2 + |V_{td}|^2 = 1.000 \pm 0.004.$$

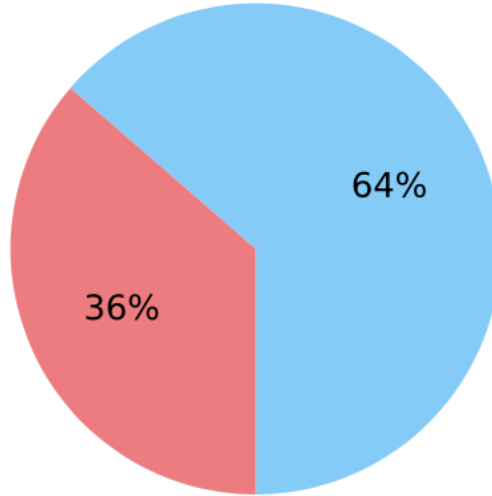
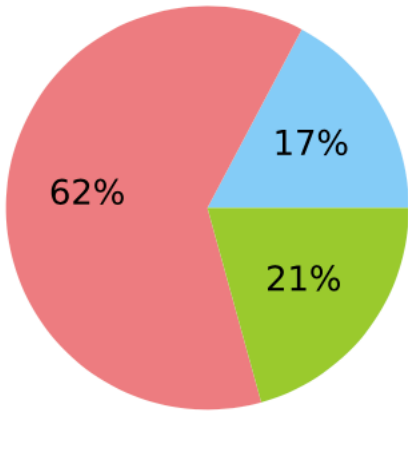
Not so precise -

$$|V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 = 1.024 \pm 0.032.$$

$$|V_{us}|^2 + |V_{cs}|^2 + |V_{ts}|^2 = 1.025 \pm 0.032.$$

Leptonic

Semileptonic

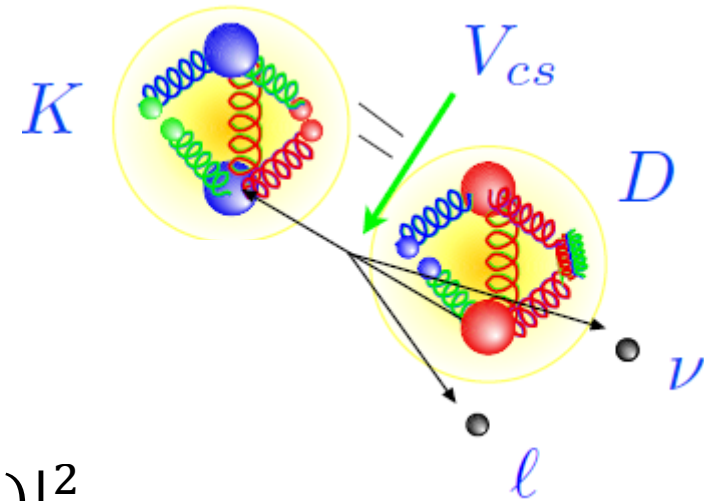


■ Lattice
 ■ Experiment
 ■ EM

[Diagram taken from PoS LATTICE2016 (2017) 305]

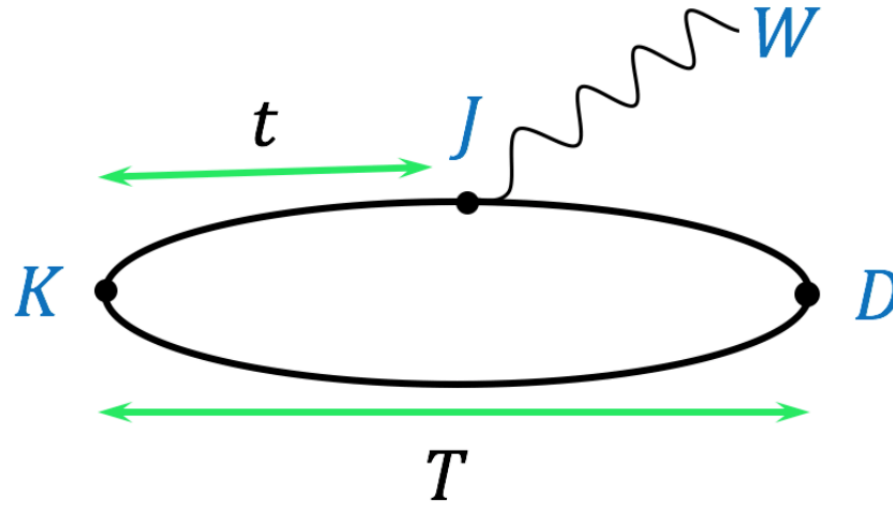
$$V_{\text{CKM}} = \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix}$$

← (Red arrow points to the V_{cs} element in the matrix)



Comparison of $|V_{CS}|$ Error-
Radius proportional to total error

$$\frac{d\Gamma^{D \rightarrow K}}{dq^2} = \frac{G_F^2 p^3}{24\pi^3} |V_{CS}|^2 |f_+^{D \rightarrow K}(q^2)|^2$$



$$\langle K^- l^+ \nu | J_W | D^0 \rangle = \frac{G_F}{\sqrt{2}} V_{cs} \bar{v}(l) \gamma_\mu (1 - \gamma_5) u(\nu) \langle K^- | \bar{\psi}_s \gamma_\mu (1 - \gamma_5) \psi_c | D^0 \rangle.$$

$$\langle H_\mu \rangle = \langle K^- | \bar{s} \gamma_\mu (1 - \gamma_5) c | D^0 \rangle$$

- Only vector current would contribute here
- Calculate this matrix from lattice QCD

Lattice formalism

$$Z_{V,t} \times \langle K^- | V^\mu | D^0 \rangle = f_+^{D \rightarrow K}(q^2) \left[p_D^\mu + p_K^\mu - \frac{M_D^2 - M_K^2}{q^2} q^\mu \right] + f_0^{D \rightarrow K}(q^2) \frac{M_D^2 - M_K^2}{q^2} q^\mu$$

$$\langle K | V^\mu | D \rangle = f_+^{D \rightarrow K}(q^2) \left[p_D^\mu + p_K^\mu - \frac{M_D^2 - M_K^2}{q^2} q^\mu \right] + f_0^{D \rightarrow K}(q^2) \frac{M_D^2 - M_K^2}{q^2} q^\mu \rightarrow f_0(0) = f_+(0)$$

$$\langle K | S | D \rangle = f_0^{D \rightarrow K}(q^2) \frac{M_D^2 - M_K^2}{m_{0c} - m_{0s}}$$

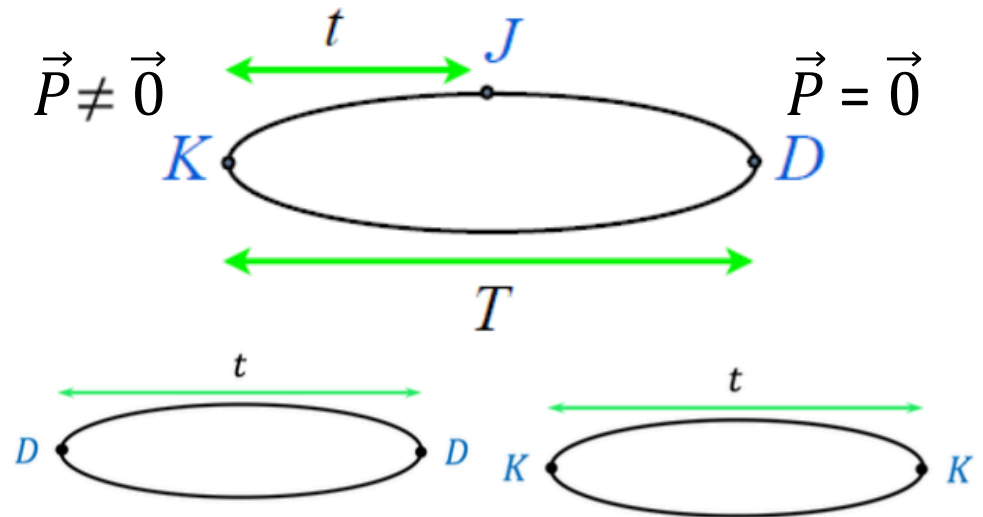
$$q^\mu = p_D^\mu - p_K^\mu$$

$$q^2 = q_{\max}^2 = (M_D - M_K)^2 :$$

$$\begin{aligned} Z_{V,t} \times \langle K^- | V^0 | D^0 \rangle &= f_+^{D \rightarrow K}(q^2) [M_D + M_K - (M_D + M_K)] \\ &\quad + f_0^{D \rightarrow K}(q^2) (M_D + M_K) \\ &= (M_D + M_K) \times f_0^{D \rightarrow K}(q_{\max}^2) \end{aligned}$$

Extract

- Local temporal vector current
- Non-goldstone D meson for that – local $\mathbf{Y}_0 \mathbf{Y}_5$
- “Sequential technique” for three-points correlators



Analysis Ingredients

MILC configurations: up/down, strange, charm quarks in the sea: $m_u = m_d$

❑ Physical m_u/d

❑ Three lattice spacings from 0.09 - 0.15 fm

❑ Valence strange and charm quark masses tuned accurately

Twisted boundary Conditions:

$$\psi(p + L_\mu \hat{\mu}) = e^{i\theta_\mu} \psi(p).$$

High statistics:
1,000 configurations,
4-16 time sources

Random wall sources:

$$\eta(i_t) = \begin{cases} e^{i\theta} & \text{for } i_t = t_0, \\ 0 & \text{for } i_t \neq t_0. \end{cases}$$

$$\langle \eta^\dagger(i') \eta(i) \rangle = \delta_{ii'}$$

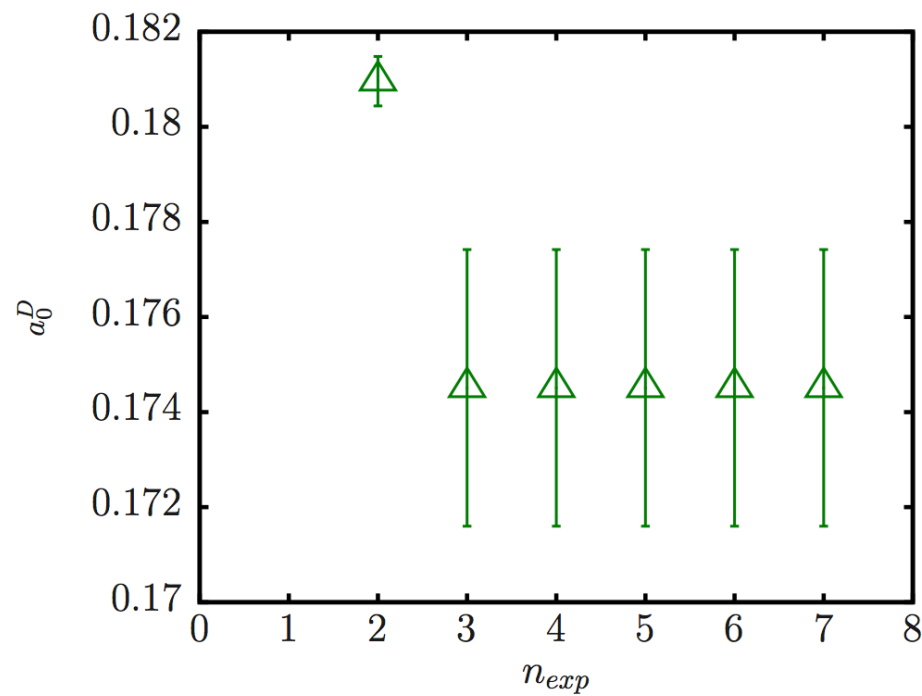
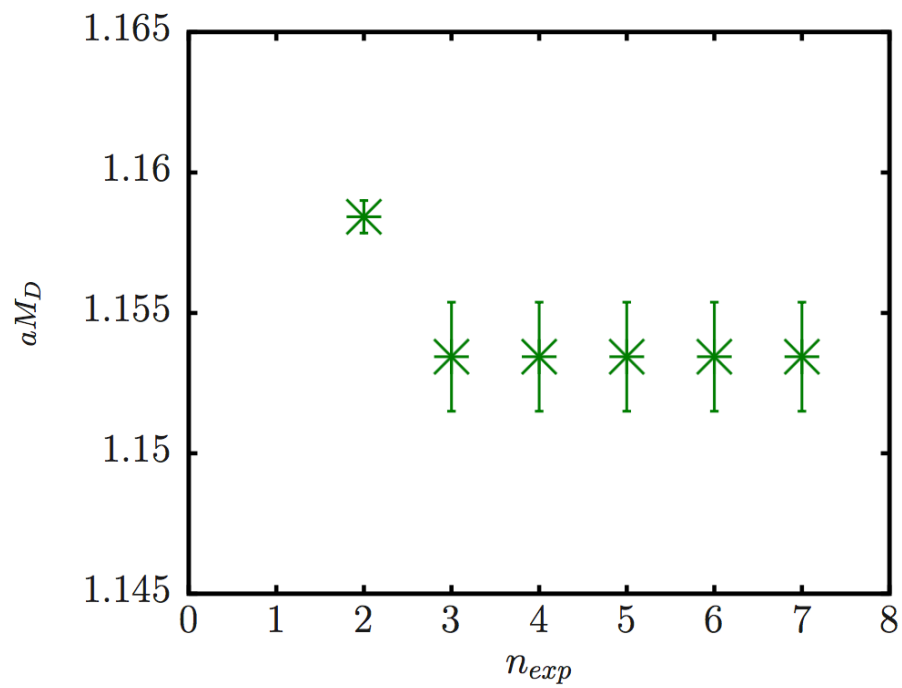
Previous HPQCD work with 2+1 Asqtad:
J. Koponen *et. al.*
arXiv:1305.1462

Multi-exponential Bayesian fitting

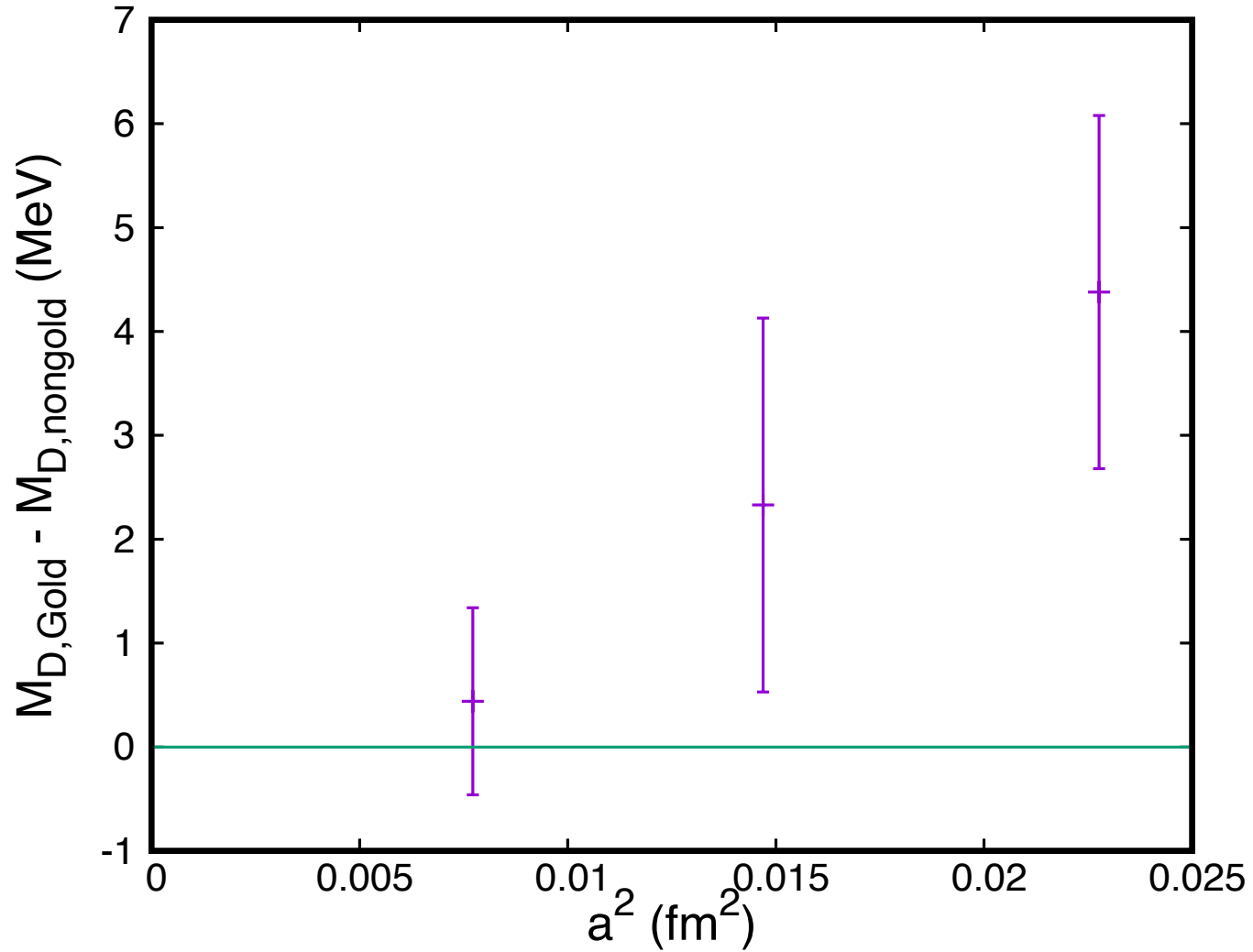
$$G^{2\text{pt}}(t; \vec{p}) = \sum_n a_n^2 (e^{-E_n t} + e^{-E_n(T-t)}) \\ + (-1)^t \sum_{n_o} a_{n_o}^2 (e^{-E_{n_o} t} + e^{-E_{n_o}(T-t)})$$

$$G^{3\text{pt}}(t; T) = \sum_{n_1, n_2} a_{n_1} a_{n_2} V_{n_1 n_2}^{nn} (e^{-E_{n_1} t} + e^{-E_{n_2}(T-t)}) \\ + (-1)^t \sum_{n_{1o}, n_2} a_{n_{1o}} a_{n_2} V_{n_{1o} n_2}^{on} (e^{-E_{n_{1o}} t} + e^{-E_{n_2}(T-t)}) \\ + (-1)^T \sum_{n_1, n_{2o}} a_{n_1} a_{n_{2o}} V_{n_1 n_{2o}}^{no} (e^{-E_{n_1} t} + e^{-E_{n_{2o}}(T-t)}) \\ + (-1)^{t+T} \sum_{n_{1o}, n_{2o}} a_{n_{1o}} a_{n_{2o}} V_{n_{1o} n_{2o}}^{oo} (e^{-E_{n_{1o}} t} + e^{-E_{n_{2o}}(T-t)})$$

Check 1: Stability of the fits with multiple exponentials

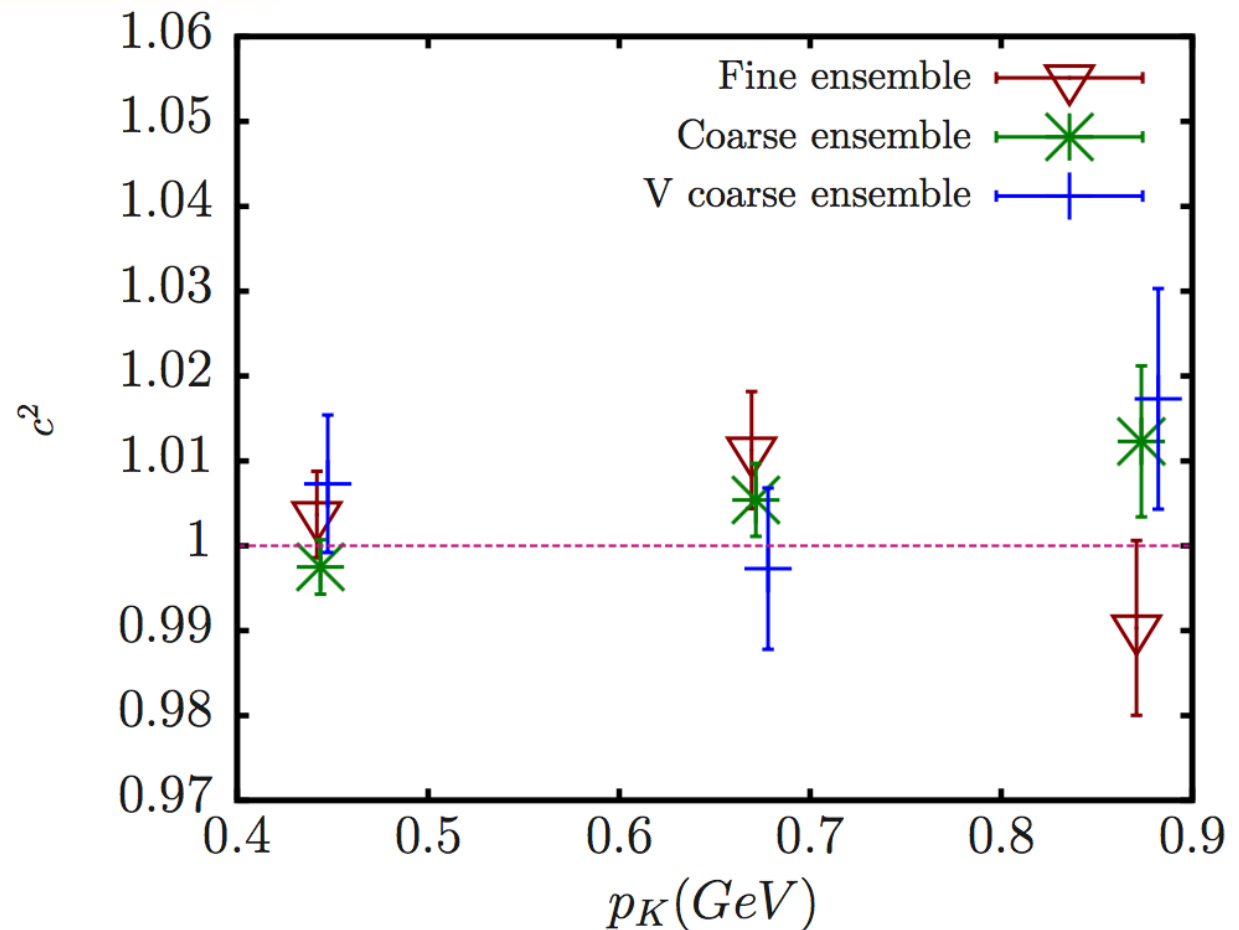


Check 2: Mass difference: Goldstone and non-goldstone D mesons



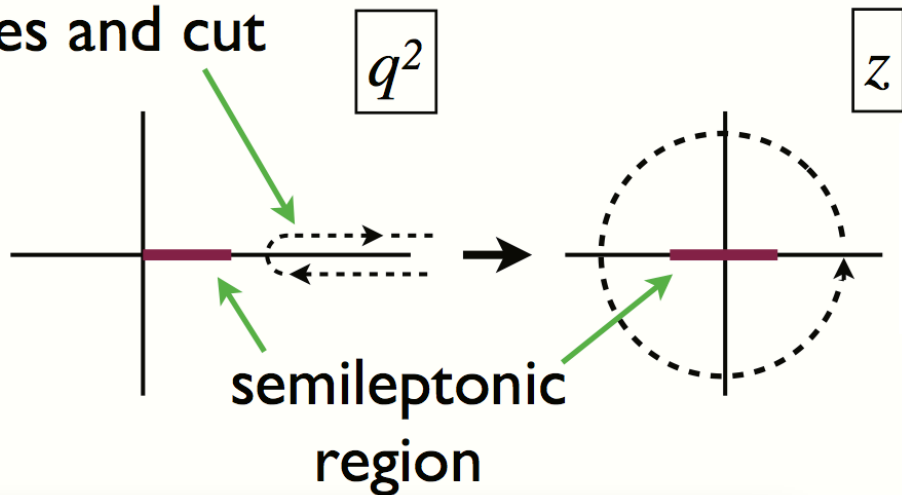
Check 3 : Relativistic dispersion relation on lattice

$$c^2(\vec{p}) = \frac{E_K^2(\vec{p}) - M_K^2}{\vec{p}^2}$$



Z-expansion

poles and cut



$$z = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}$$

0

$$t_{\pm} = (m_D \pm m_K)^2$$

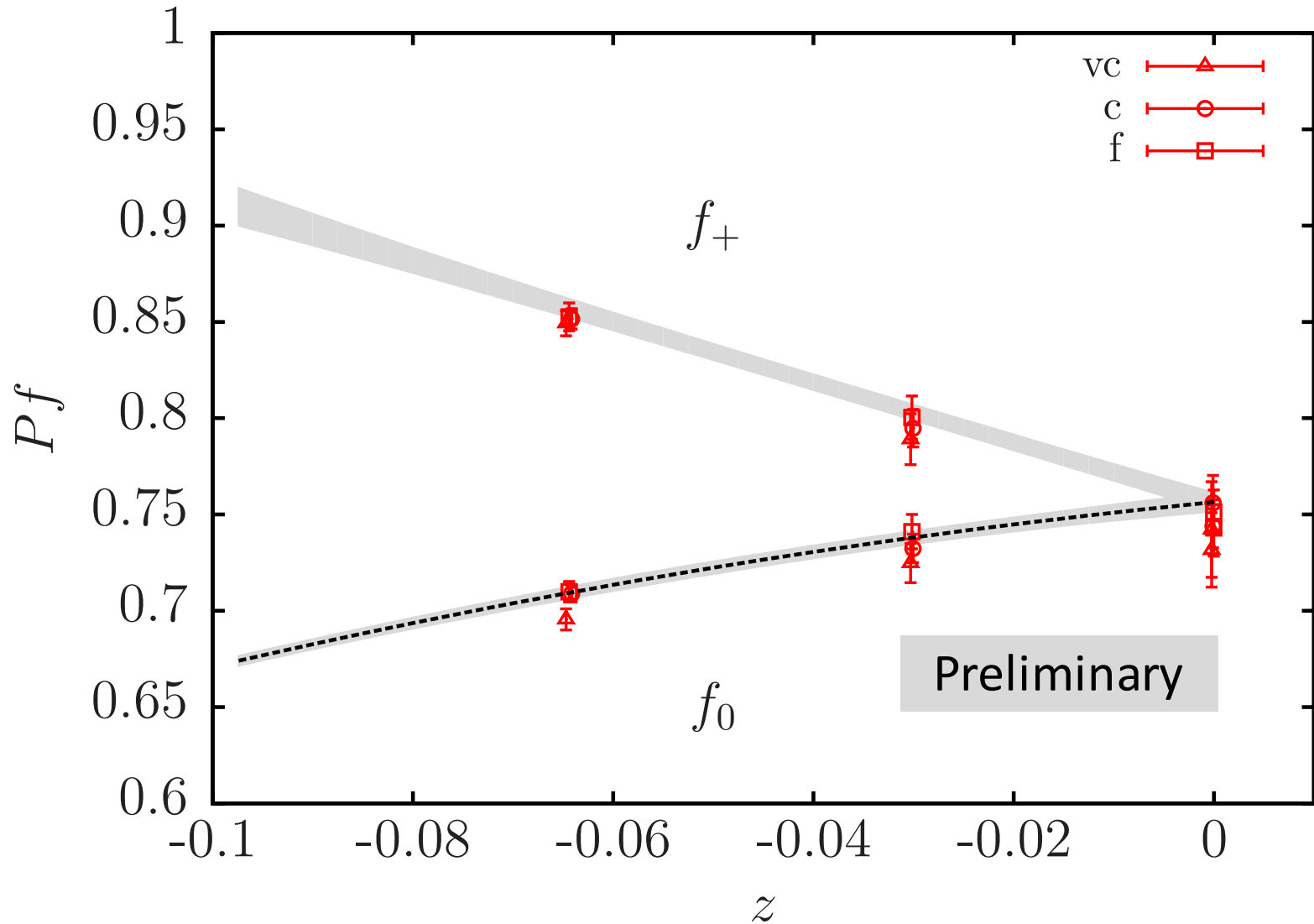
$$f(q^2) = \frac{1}{P(q^2)\Phi(q^2)} \sum_{n=0}^N b_n z^n$$

Pole masses

$$(1 - q^2/M_X^2)$$

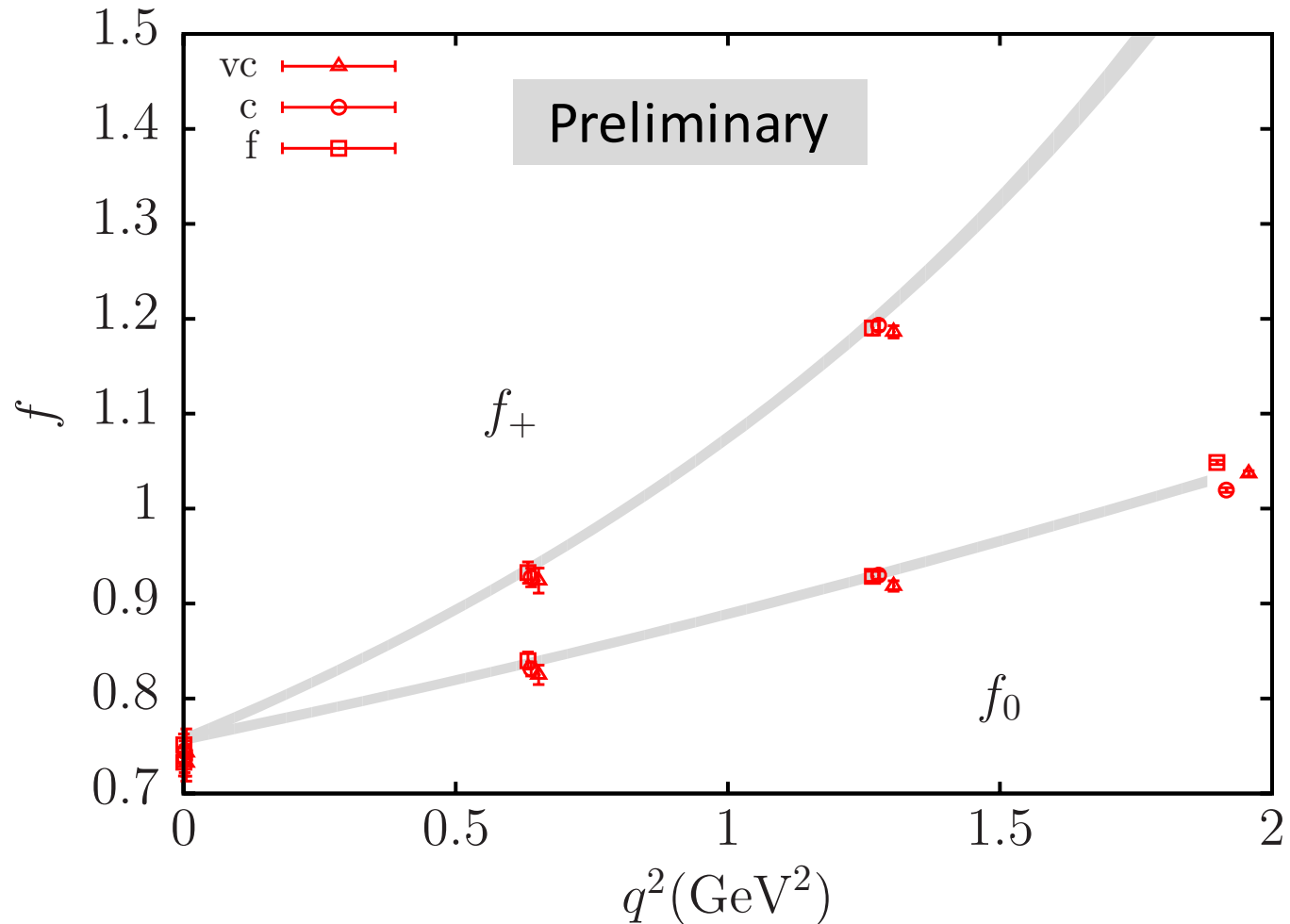
$$b_n(a, m_l) = A_n \{ 1 + B_n a^2 + C_n a^4 + D_n \delta_l + E_n (\delta_l \ln[\delta_l] + F_n a^2 \delta_l) \}$$

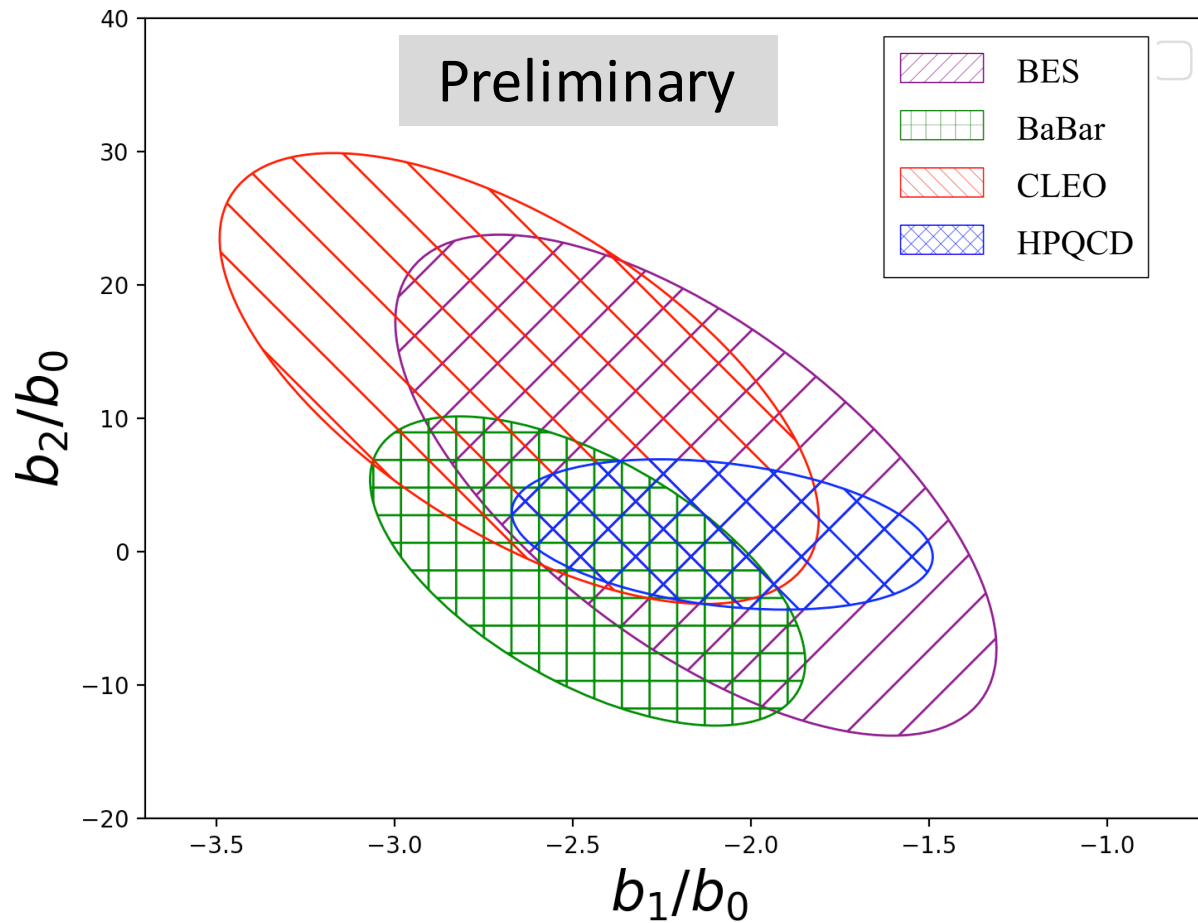
Shape of the form factors: $D \rightarrow Kl\nu$



Shape of the form factors: $D \rightarrow Klv$

Converting back to 'q' space

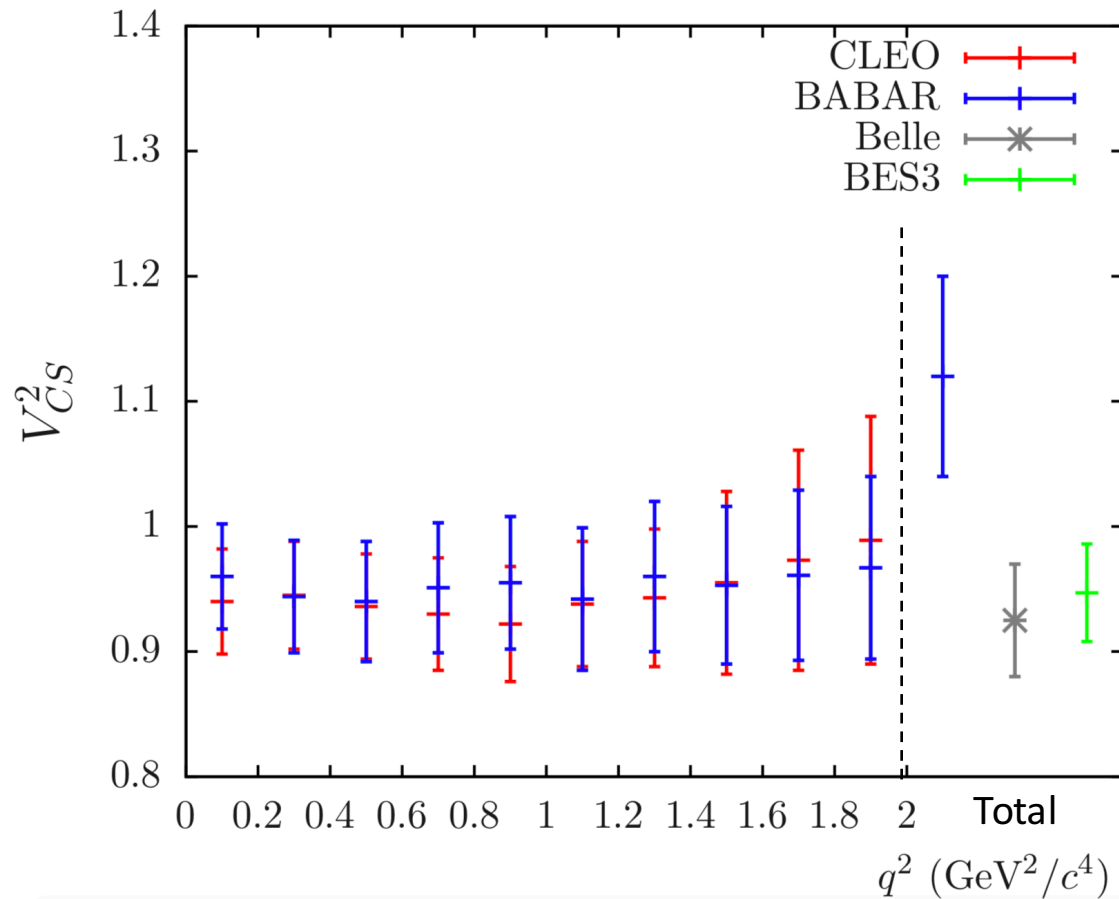




$$z = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}$$

Comparison of $f_+(q^2)$
shape parameters

$$t_0 = t_+ (1 - (1 - t_-/t_+)^{1/2})$$



Preliminary

ETMC:
Phys. Rev. D96, 054514

MILC/Fermilab:

PoS LATTICE2016 (2017) 305

JLQCD:
arXiv:1701.00942

- Ratio : Expt/Lattice
- Looking for bin-to-bin correlation
- Extracting V_{CS} from fitting all bins

Thank you