
**Resurgence and fractional
instanton of the $SU(3)$ gauge
theory in weak coupling regime**

Etsuko Itou
(Keio U./Kochi U./RCNP,
Osaka U.)

JHEP 1905 (2019) 093
(arXiv:1811.05708[hep-th])

Toward

I search for

**Resurgence and fractional
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theory in weak coupling regime**

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What's new

Fractional instanton of SU(3) gauge theory in weak coupling regime

Spacetime structure is

$$\mathbb{T}_{L_s}^3 \times S_{L_\tau}^1 \quad \text{where} \quad L_s \ll L_\tau$$

with center-twisted boundary condition to (only) two-dimensions in spatial directions

B.C. for $(x,y,z,\tau)=(\text{TBC},\text{TBC},\text{PBC},\text{PBC})$

The deformations of the spacetime structure are motivated by several recent works of “resurgence scenario”

cf) The classical solution ($Q=1/Nc$) is known on $\mathbb{T}^3 \times \mathbb{R}$ with the same twisted boundaries (in the SUSY breaking context by E.Witten NPB202(1982)253)

The property of the fractional instanton:

If fractional instanton appears, the complex phase of Polyakov loop in z-direction rotates.

Motivation

Resurgence structure: Cancellation of the imaginary ambiguity of the physical observables

Perturbative series naively diverge even in weak coupling
=> Borel resummation
=> Several singular points exist, and **give an imaginary ambiguity** coming from the path of integration

Contribution from the nonperturbative effects gives imaginary ambiguity

$$\mathbb{B}(g^2) = \text{Re}\mathbb{B} \pm i\pi \frac{1}{\beta_0} e^{\frac{8\pi}{\beta_0 g^2(\mu)}} \sim e^{-S_I/N_c}$$

Weight of the singularity is 1/N of instanton action

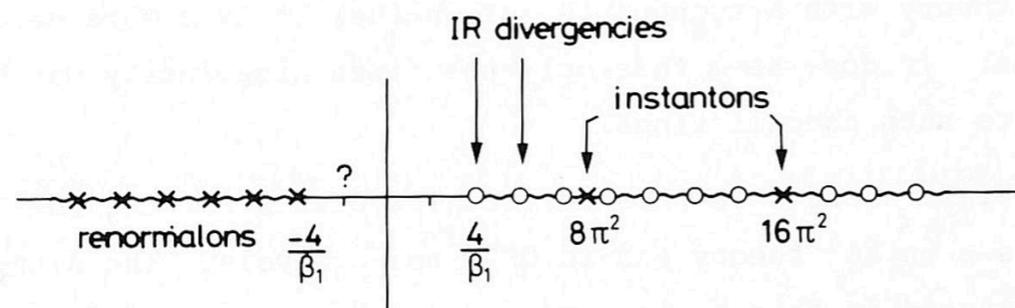


Fig. 6 Borel z plane for QCD. The circles denote IR divergencies that might vanish or become unimportant in colour-free channels.

A calculation of the plaquette in numerical stochastic perturbation up to 35-th power of α finds a Borel singularity

Bali et al, Phys. Rev. D 89, 054505 (2014)

't Hooft, Subnucl. Ser. 15 (1979) 943.

Motivation

Resurgence structure: Cancellation of the imaginary ambiguity of the physical observables

Perturbative series naively diverge even in weak coupling
 => Borel resummation
 => Several singularities
 ambiguity comes from

Contribution from the nonperturbative effects gives imaginary ambiguity

If it does not work, then a uniqueness could not be given in the continuum limit of the lattice calculation

$$\mathbb{B}(g^2) =$$

Weight of the singularity is $1/N$ of instanton action

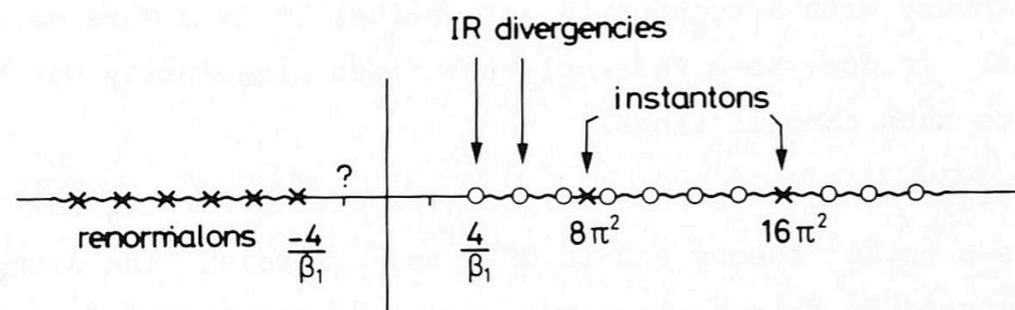


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The resurgence structure is proven in
several quantum mechanical models and low-dim. QFT.

G.V.Dunne and M.Unsal JHEP1211(2012) 170, PRD87(2013)025015, PRD89(2014)0141701, JHEP1612(2016)002,

T.Misumi, M.Nitta and N.Sakai, JHEP1509,157 (2015)

T.Fujimori, S.Kamata, T.Misumi, M.Nitta and N.Sakai PRD94 (2016) 105002, PRD95 (2017) 105001, PTEP 2017 no.8 083B02

A key is the deformed spacetime.

That gives a novel type of saddle points of the action
(1/N topological background field, bion configurations)

Lattice setup

What is a good lattice setup to study YM in weak coupling regime?

Remind us about a calculation of running coupling constant.

- (1) Focus on weak coupling regime
- (2) Renormalization scale ($\sim 1/L_s$) is large. Weak coupling \Rightarrow **Small L_s , Large L_t ($T=0$)**
- (3) Consider standard perturbative vacuum (avoid the toron problem)

Nontrivial boundary condition (SF, twisted)

- (4) Self-dual (anti-self-dual) solution of YM eq. on hypertorus

Twisted b.c. $U_\mu(n + \hat{\nu}N_s) = \Omega_\nu U_\mu(n) \Omega_\nu^\dagger$ 't Hooft, NPB153(1979)141, Commun. Math.Phys.81 (1981) 267

Topological charge on twisted hypertorus $Q = \frac{1}{32\pi^2} \int d^4x \text{Tr} \epsilon_{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma} = \text{integer} - \frac{\kappa}{N_c}$

κ denotes non-commutativity of spacetime directions $\kappa = \frac{1}{8} \epsilon_{\mu\nu\rho\sigma} n_{\mu\nu} n_{\rho\sigma}$

Only if all 4dim b.c. are twisted, then Q can be fractional.

M.Garcia Perez, A.Gonzalez-Arroyo and B.Soderberg, PLB235(1990) 117

At that time, the global symmetry becomes $SU(N_c)/Z_{N_c}$ not $SU(N_c)$.

We want to study the theory with $SU(N_c)$ global symmetry.

Impose the twisted b.c. only for two-dimensions (Note that **Q is integer on hypertorus**)

Lattice setup

Lattice action:

Wilson-Plaquette gauge action

Lattice parameter:

beta=16.0, N_s=12, N_τ=60

$g^2(1/L_s) \approx 0.7$ in TPL scheme: 1-loop consistent

Boundary condition:

x,y : twisted b.c.

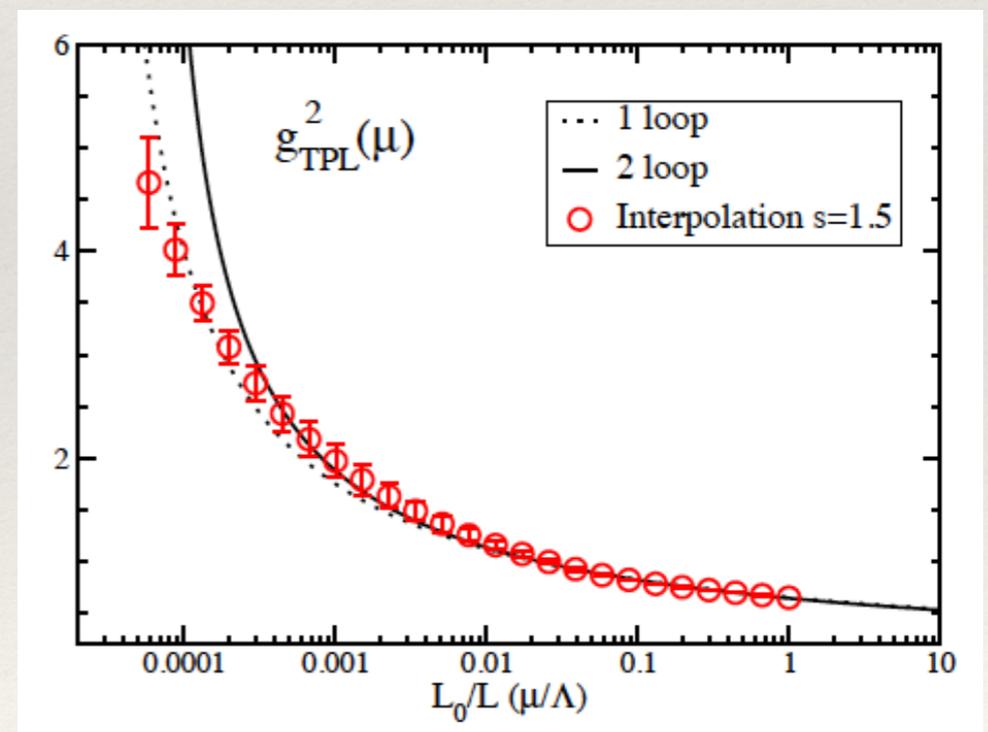
z,τ : periodic b.c.

$$L_s \Lambda \approx 1.5^{-24} \quad \Lambda = 200[\text{MeV}]$$

$$a \approx 5.0 \times 10^{-6} [\text{fm}]$$

We expect a long autocorrelation

We prepare 100 independent random seeds, and generate 100conf. by independent update.



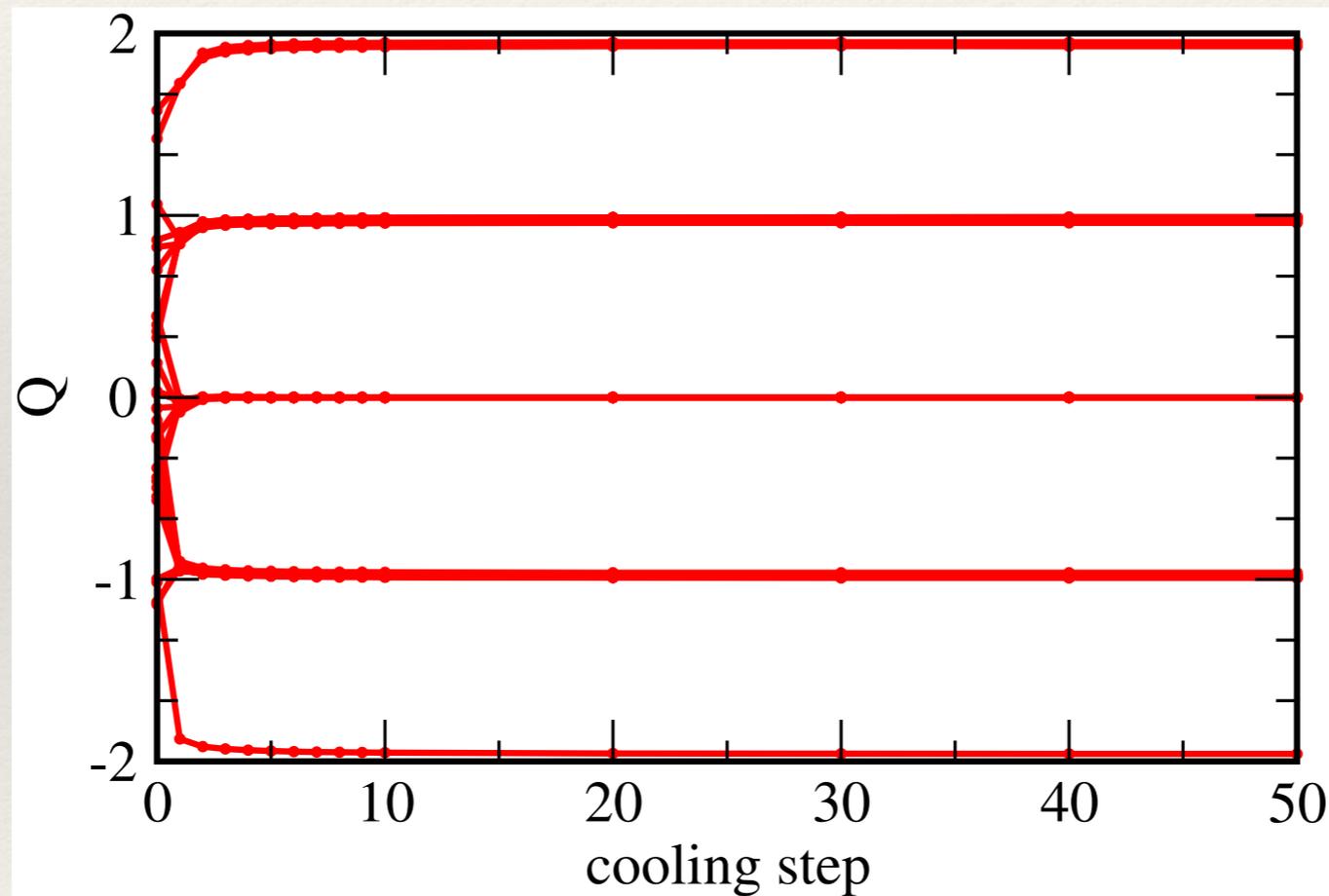
E.I. PTEP(2013) no.8, 083B01

Results

Q using cooling method

Total topological charge

$$Q = \frac{1}{32\pi^2} \int d^4x \text{Tr} \epsilon_{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma}$$

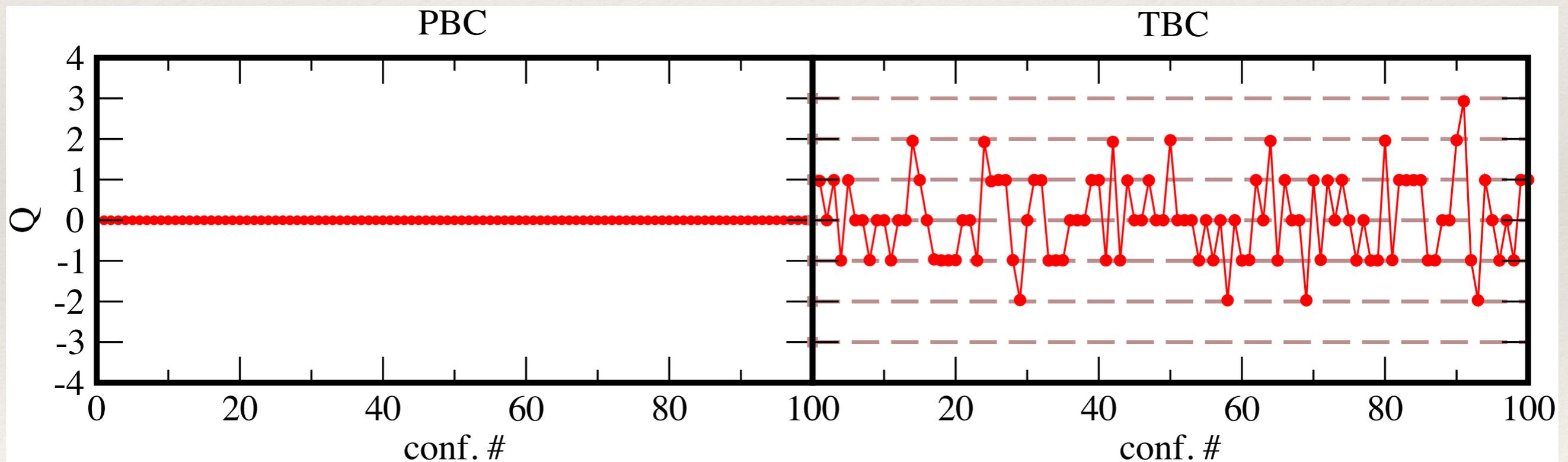


Neglect a small discrepancy from the exact integer $\Delta Q/Q \approx 0.04$

Total charge(Q)

$$Q = \frac{1}{32\pi^2} \int d^4x \text{Tr} \epsilon_{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma}$$

beta=16.0, Ns=12, Nτ=60



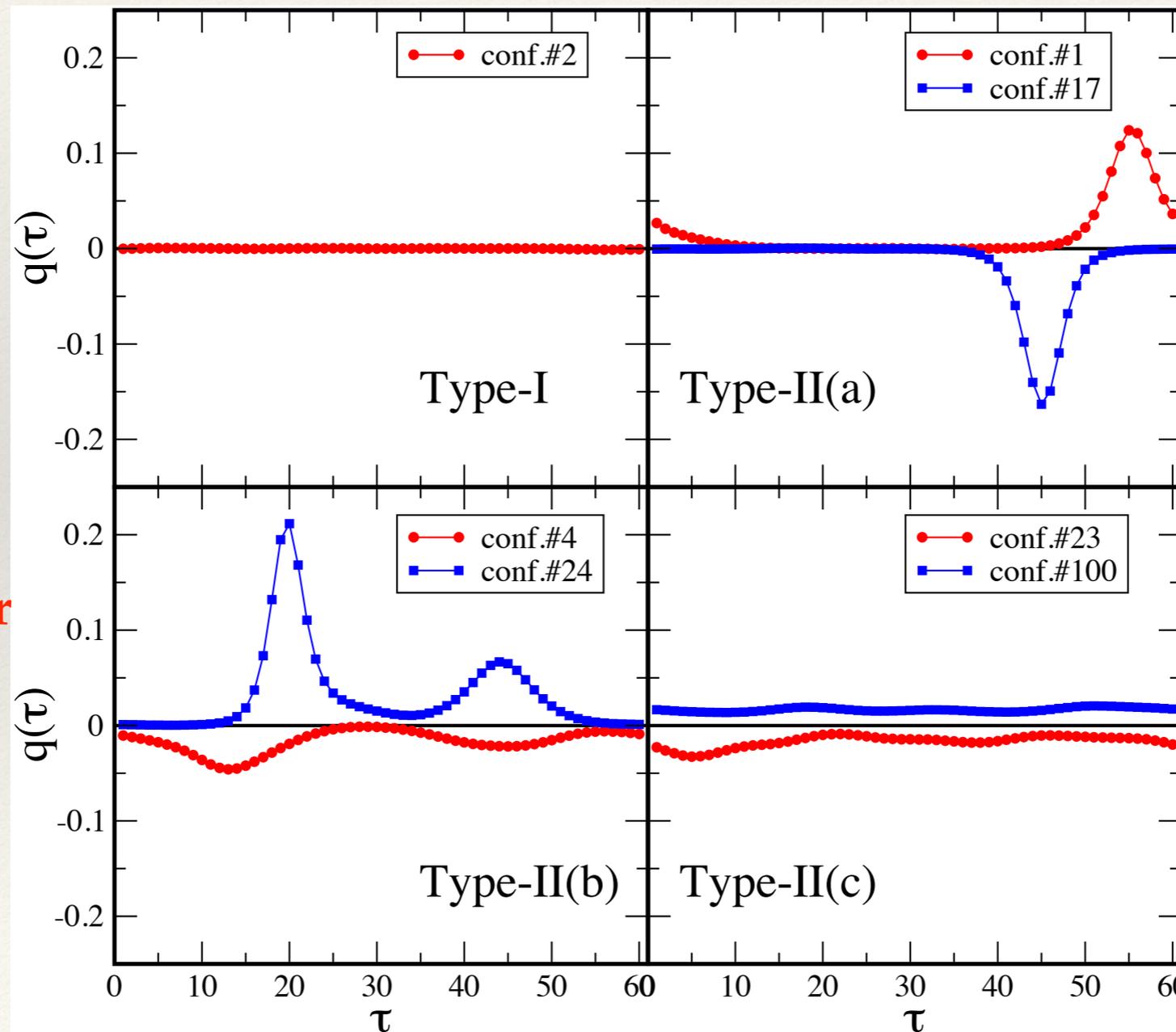
B.C. for $(x,y,z,\tau)=(\text{PBC},\text{PBC},\text{PBC},\text{PBC})$

B.C. for $(x,y,z,\tau)=(\text{TBC},\text{TBC},\text{PBC},\text{PBC})$

Local charge(q)

$$q(\tau) = \frac{1}{32\pi^2} \sum_{x,y,z} \text{Tr} \epsilon_{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma}(x,y,z,\tau)$$

Q=0



Q is nonzero integer

Sum of q for single peak
is also integer
=> integer instanton

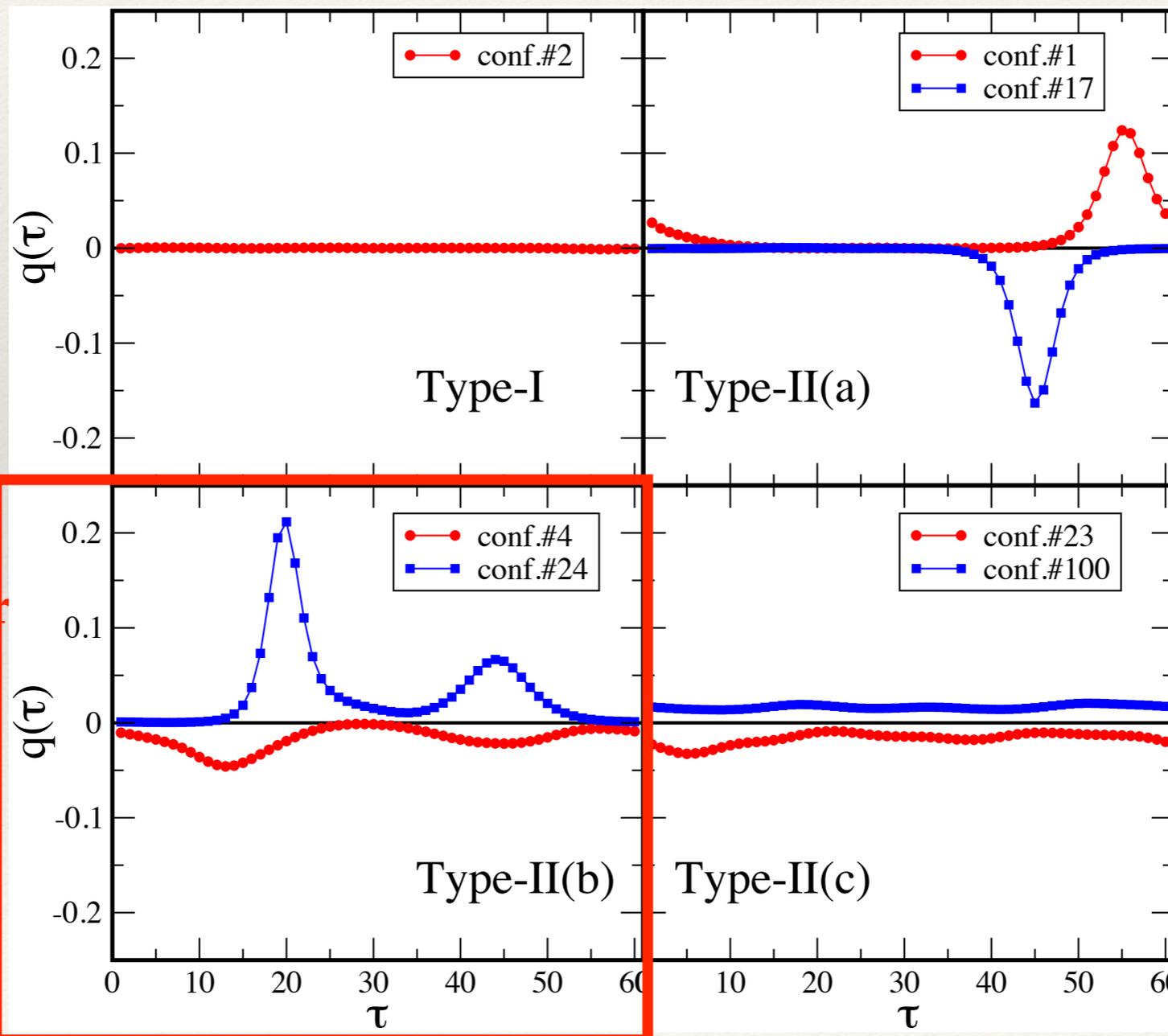
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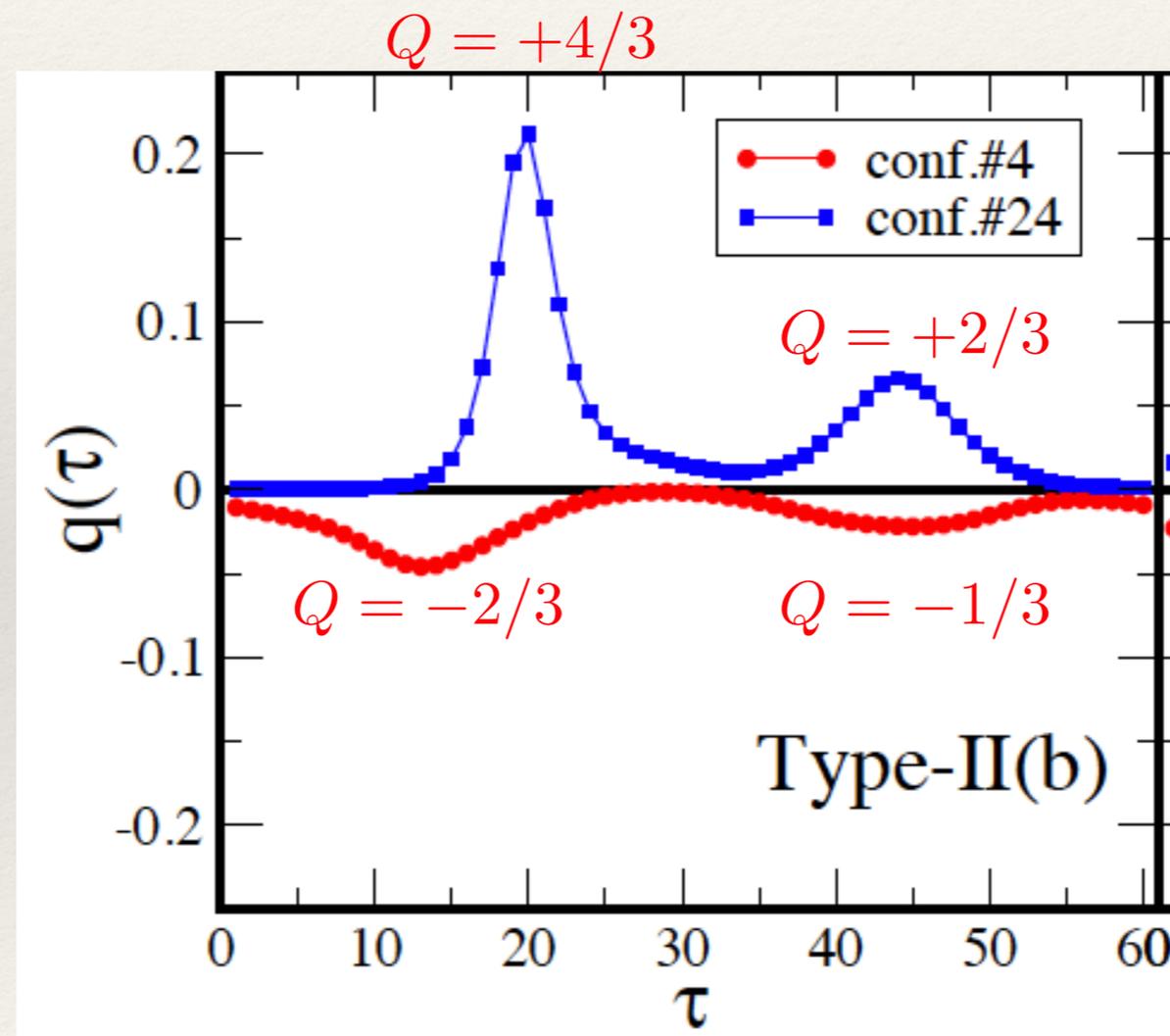
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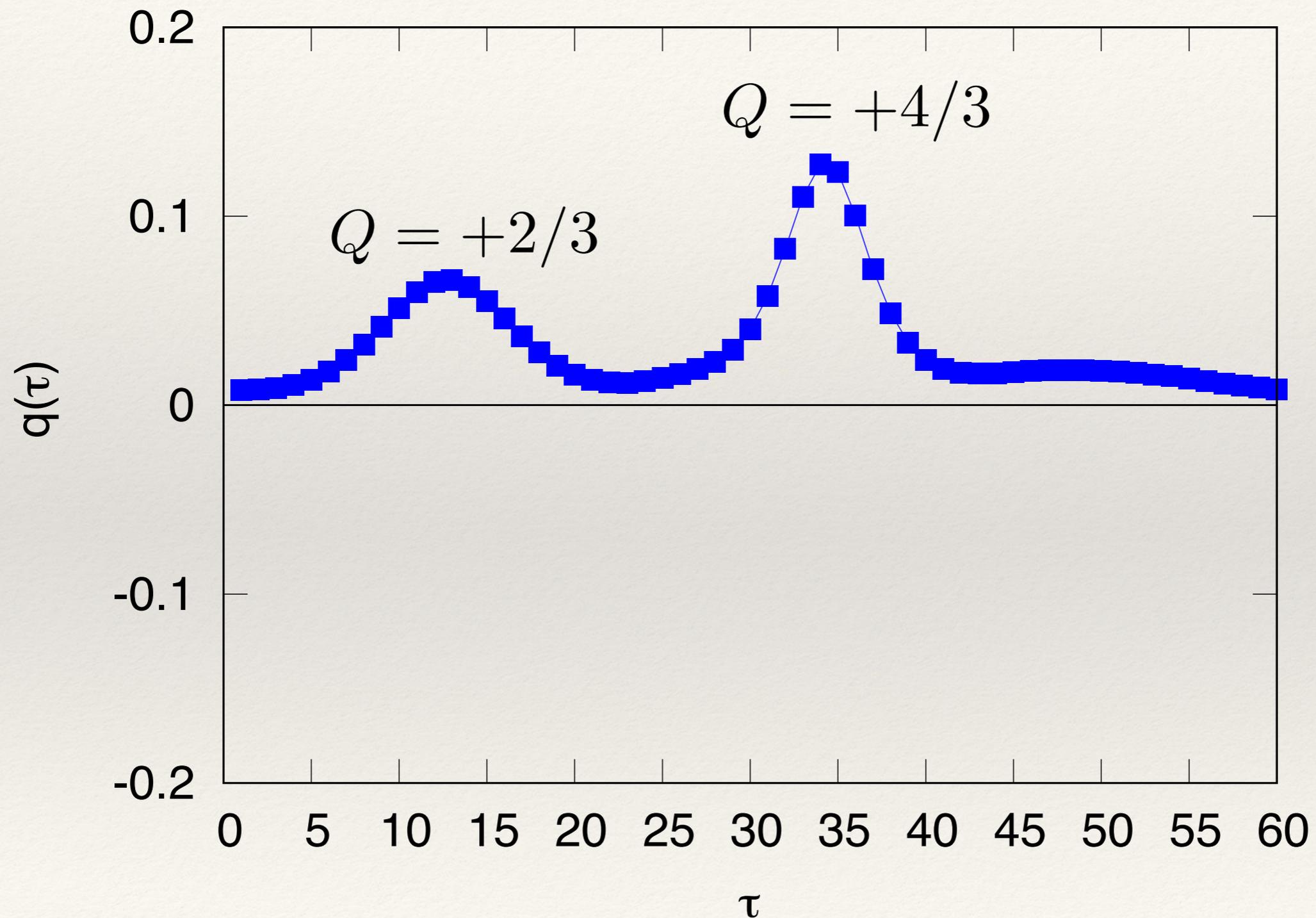


Taking the sum for each peak, the result gives a fractional number!!
Total Q is integer, but locally fractional instantons exist.

Monte Carlo step dependence

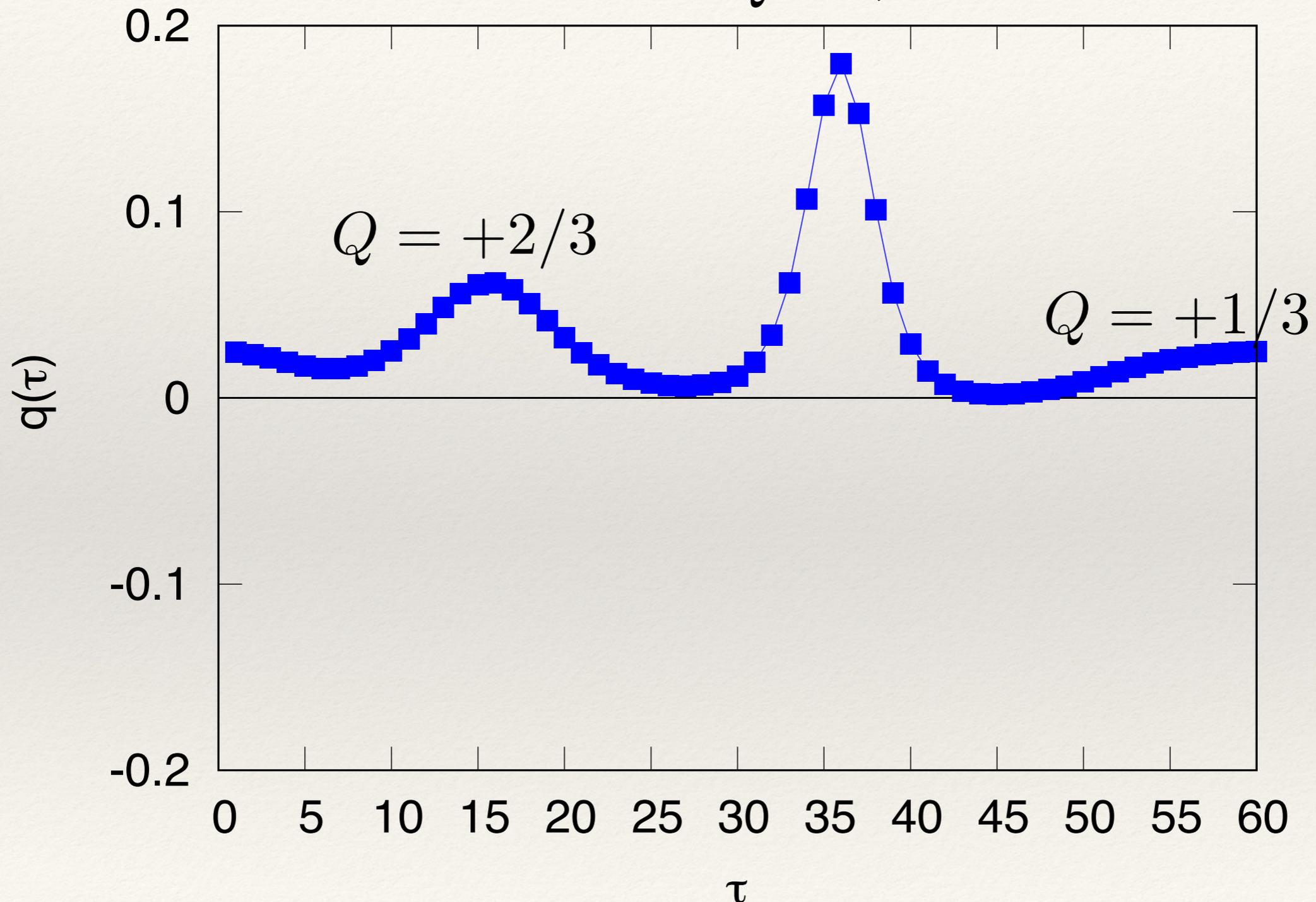
In ordinary QCD, the topology of integer instanton changes during MC steps.
How about the fractional instantons?

Monte Carlo step dependence



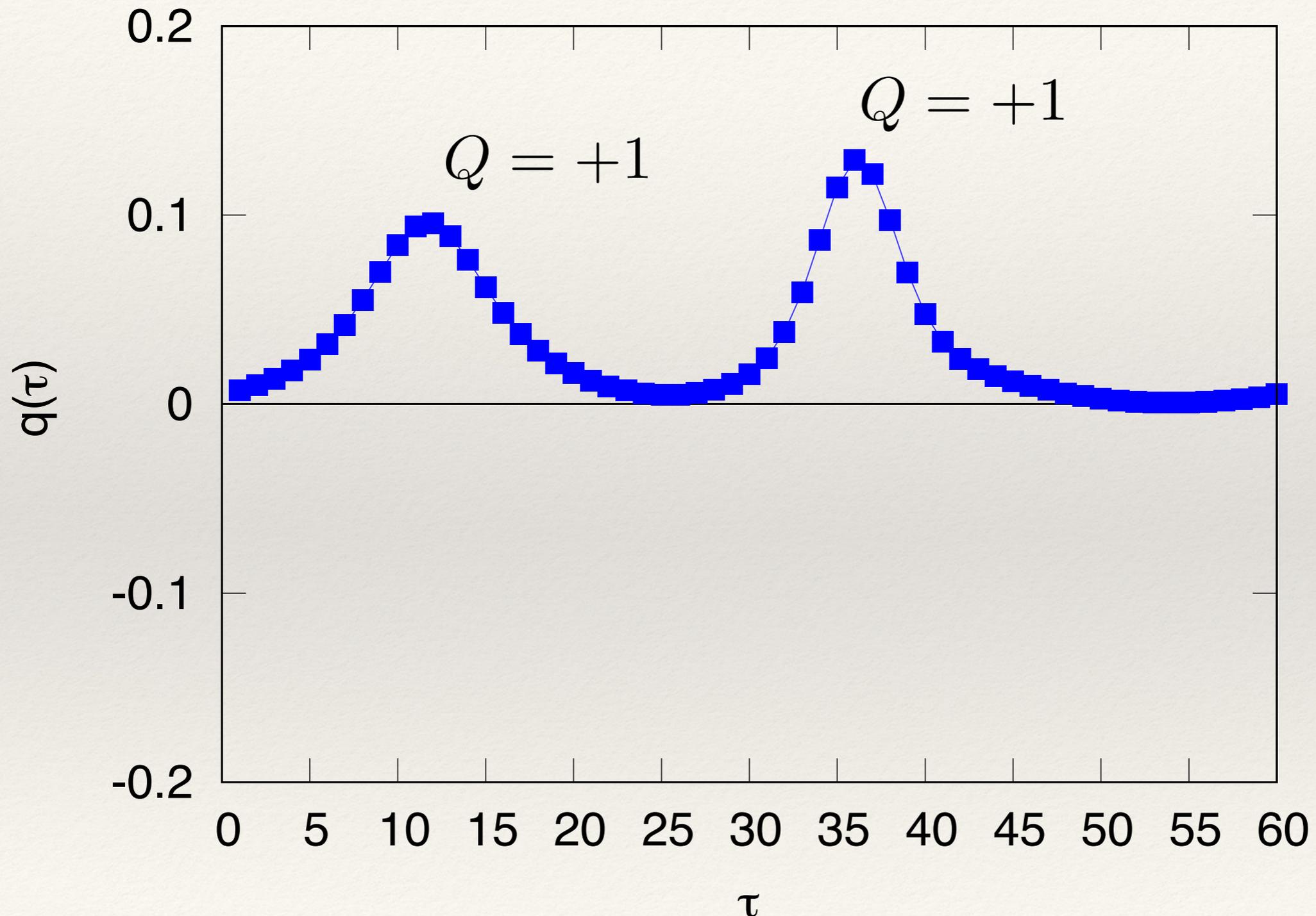
Monte Carlo step dependence

$$Q = +1$$



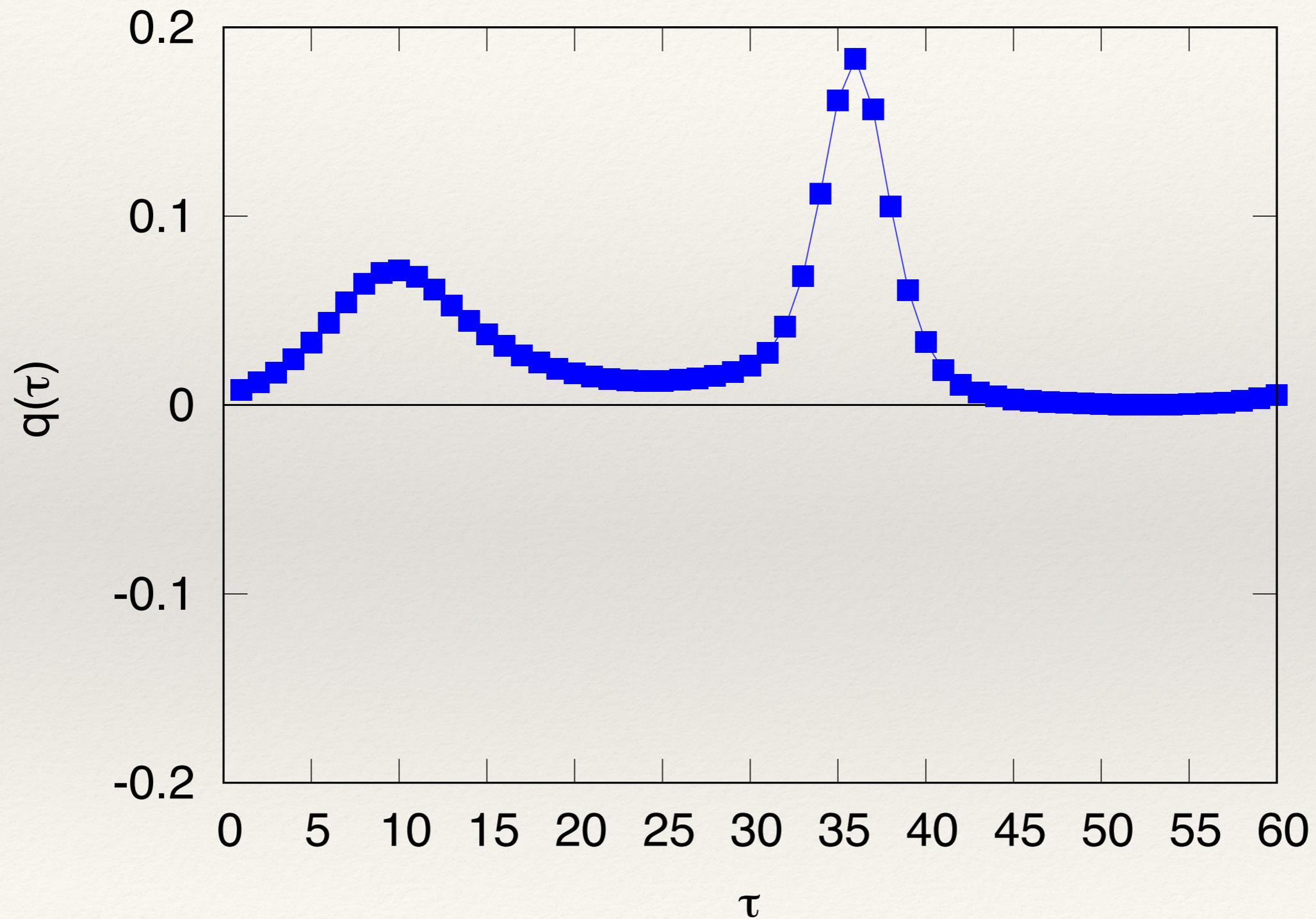
1 fractional instanton ($Q=4/3$) \rightarrow 1 integer instanton and 1 fractional instanton

Monte Carlo step dependence

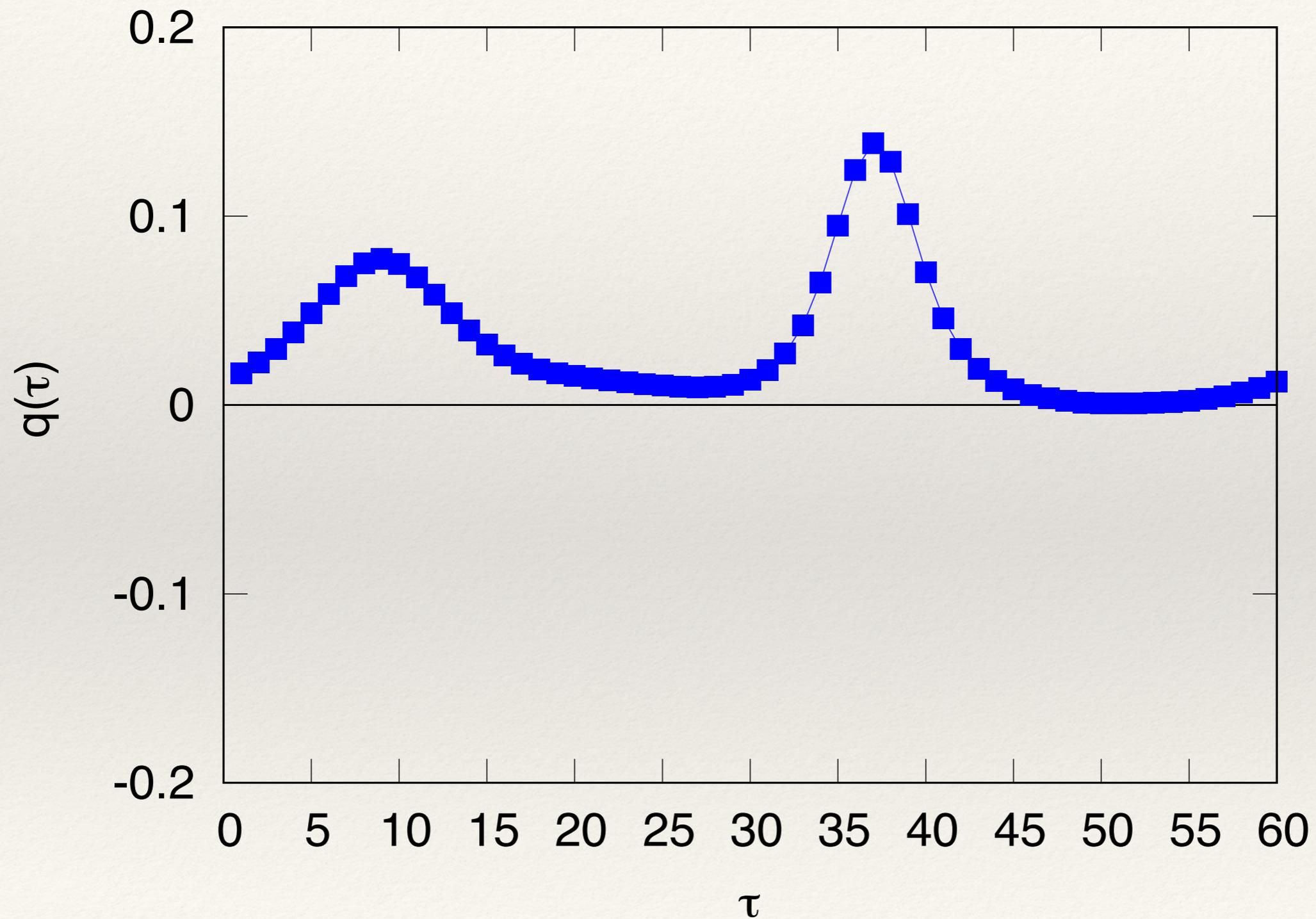


2 fractional instantons ($Q=2/3 + 1/3$) \rightarrow 1 integer instantons ($Q=1$)

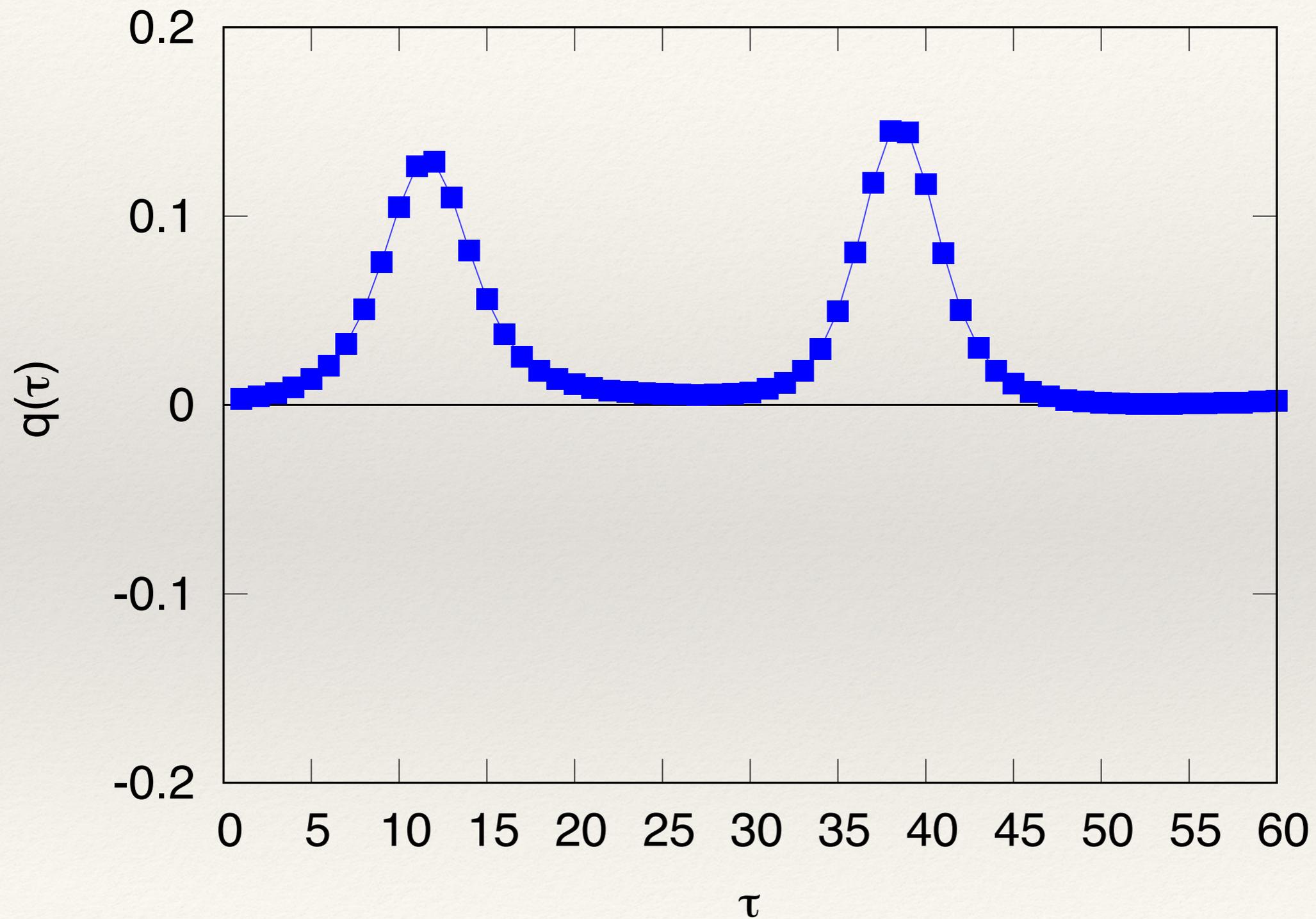
Monte Carlo step dependence



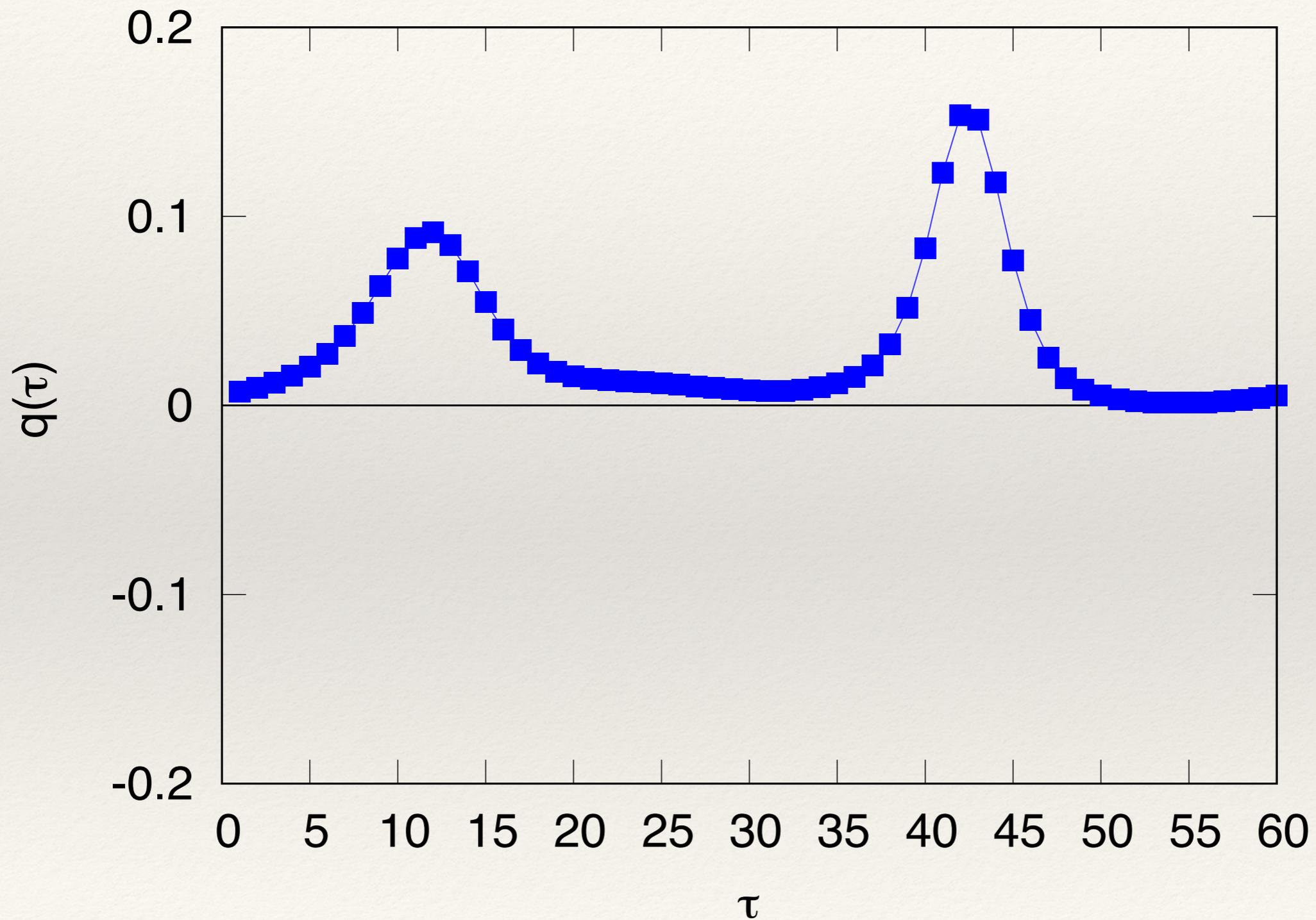
Monte Carlo step dependence



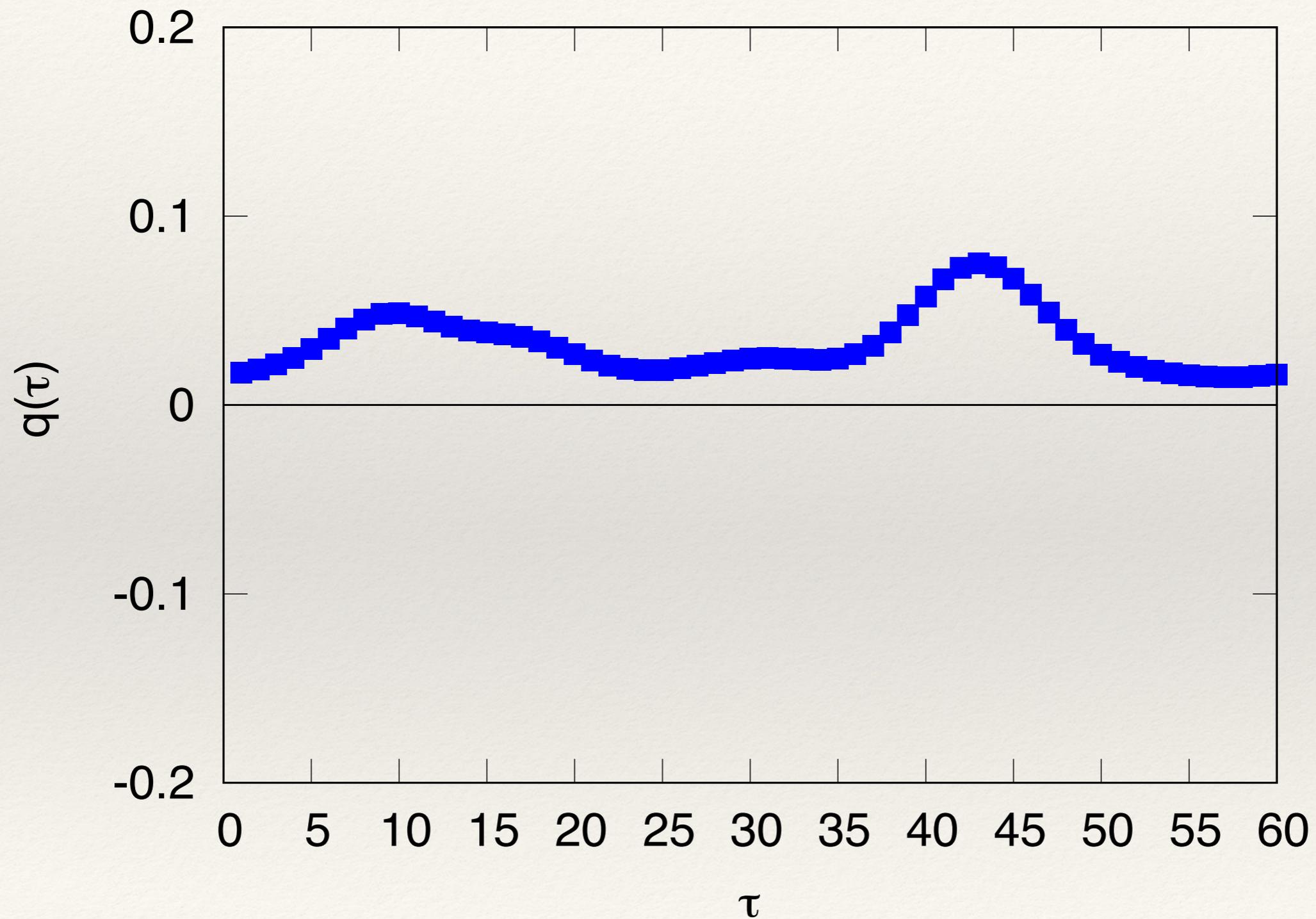
Monte Carlo step dependence



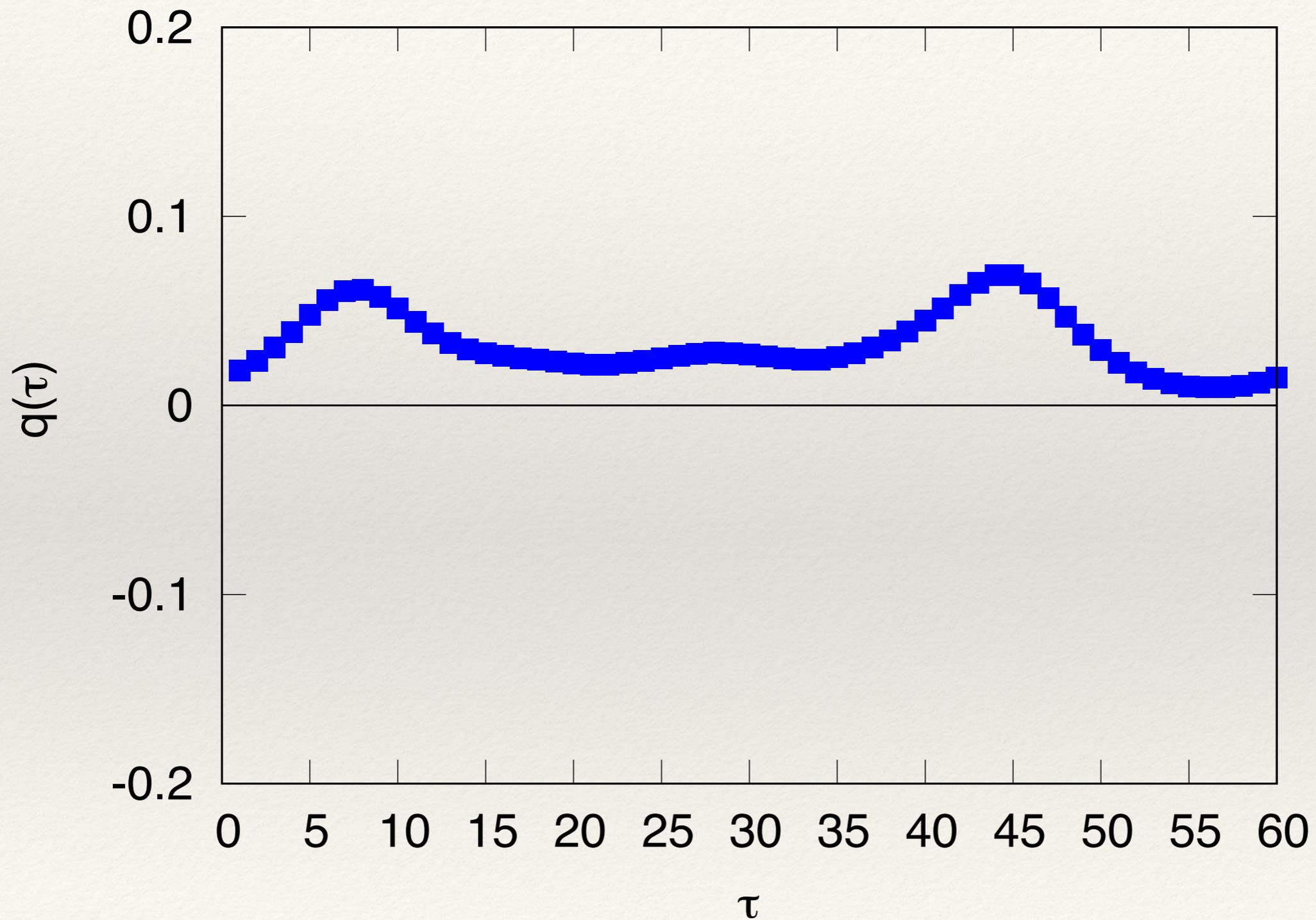
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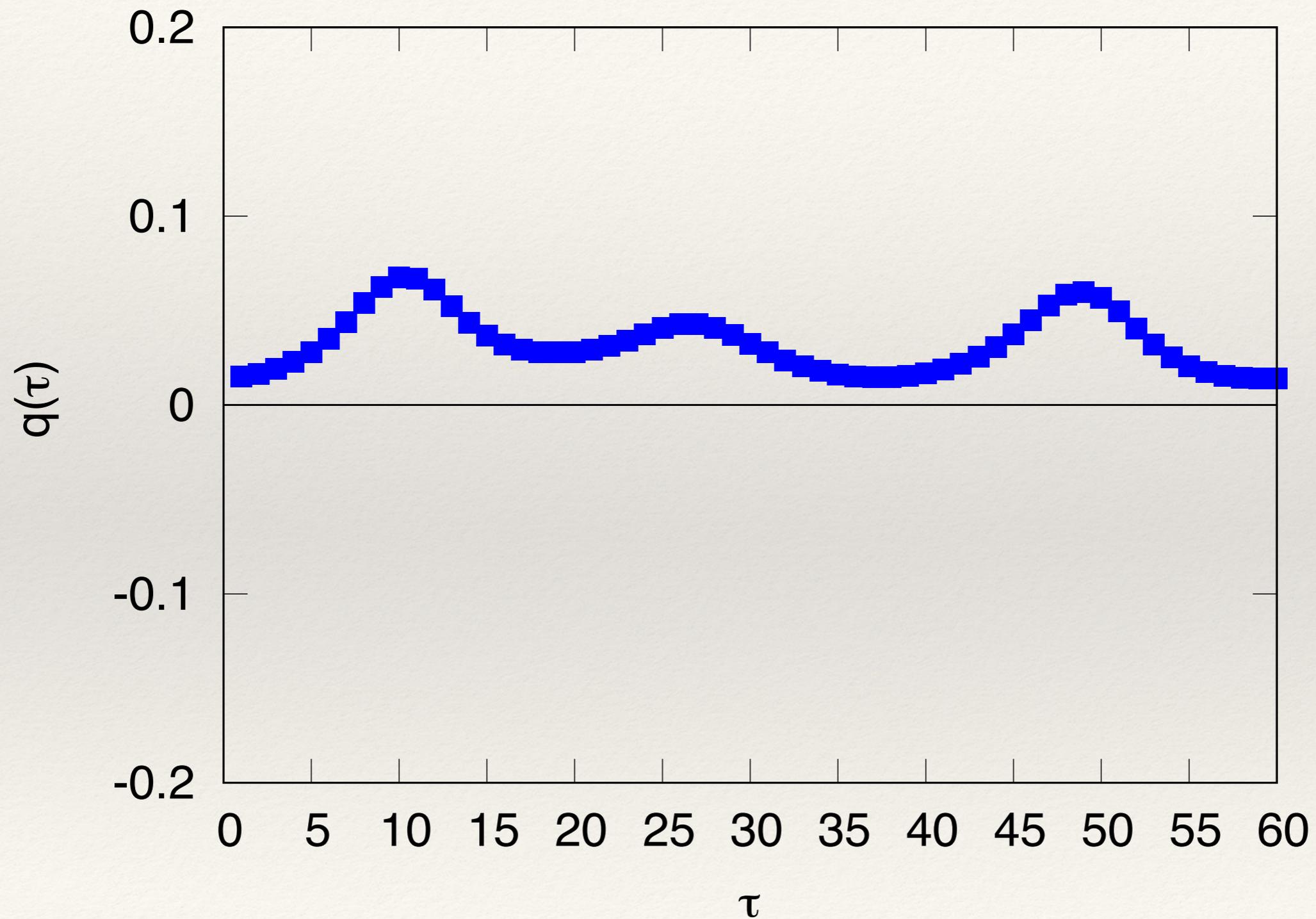
Monte Carlo step dependence



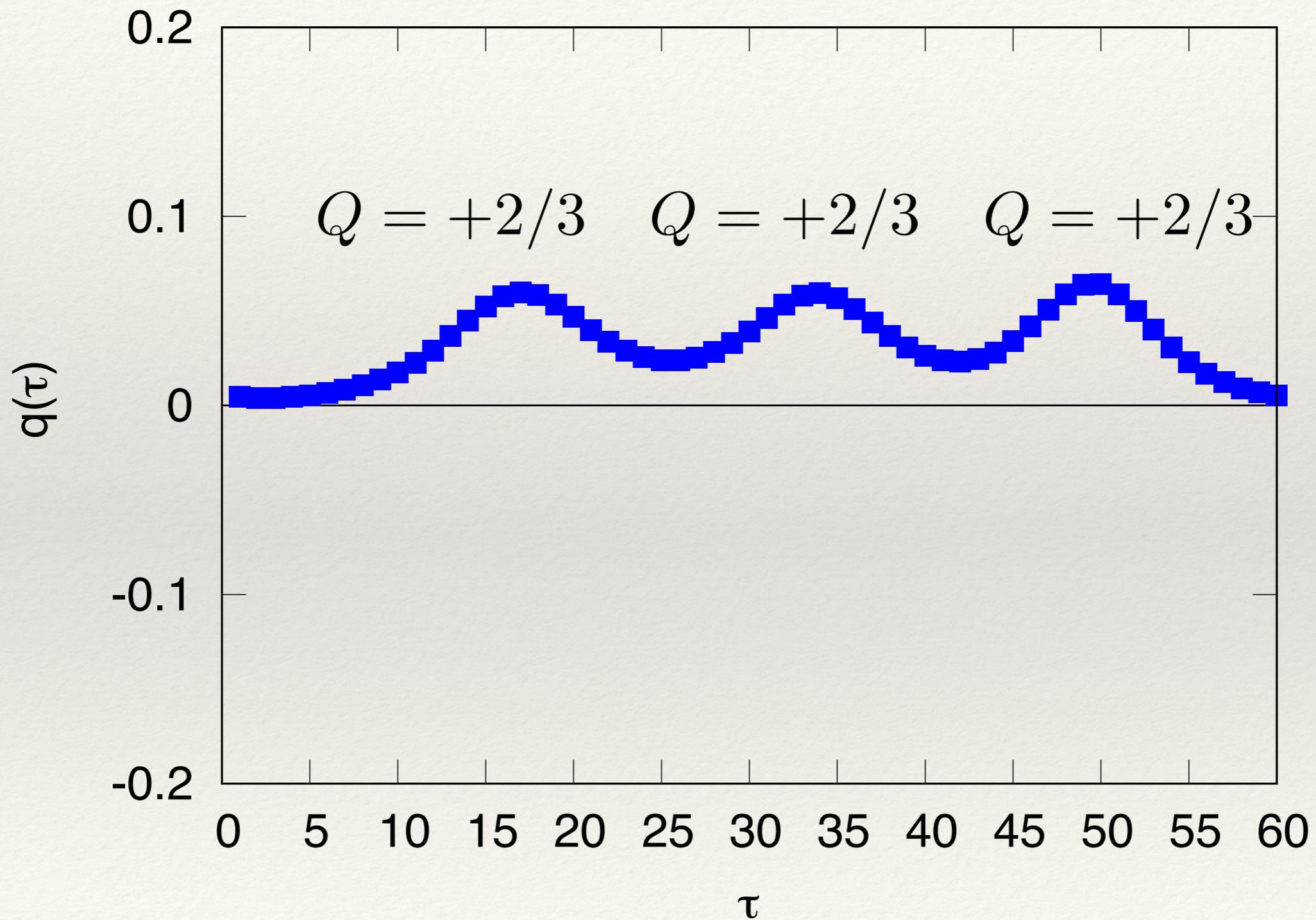
Monte Carlo step dependence



Monte Carlo step dependence

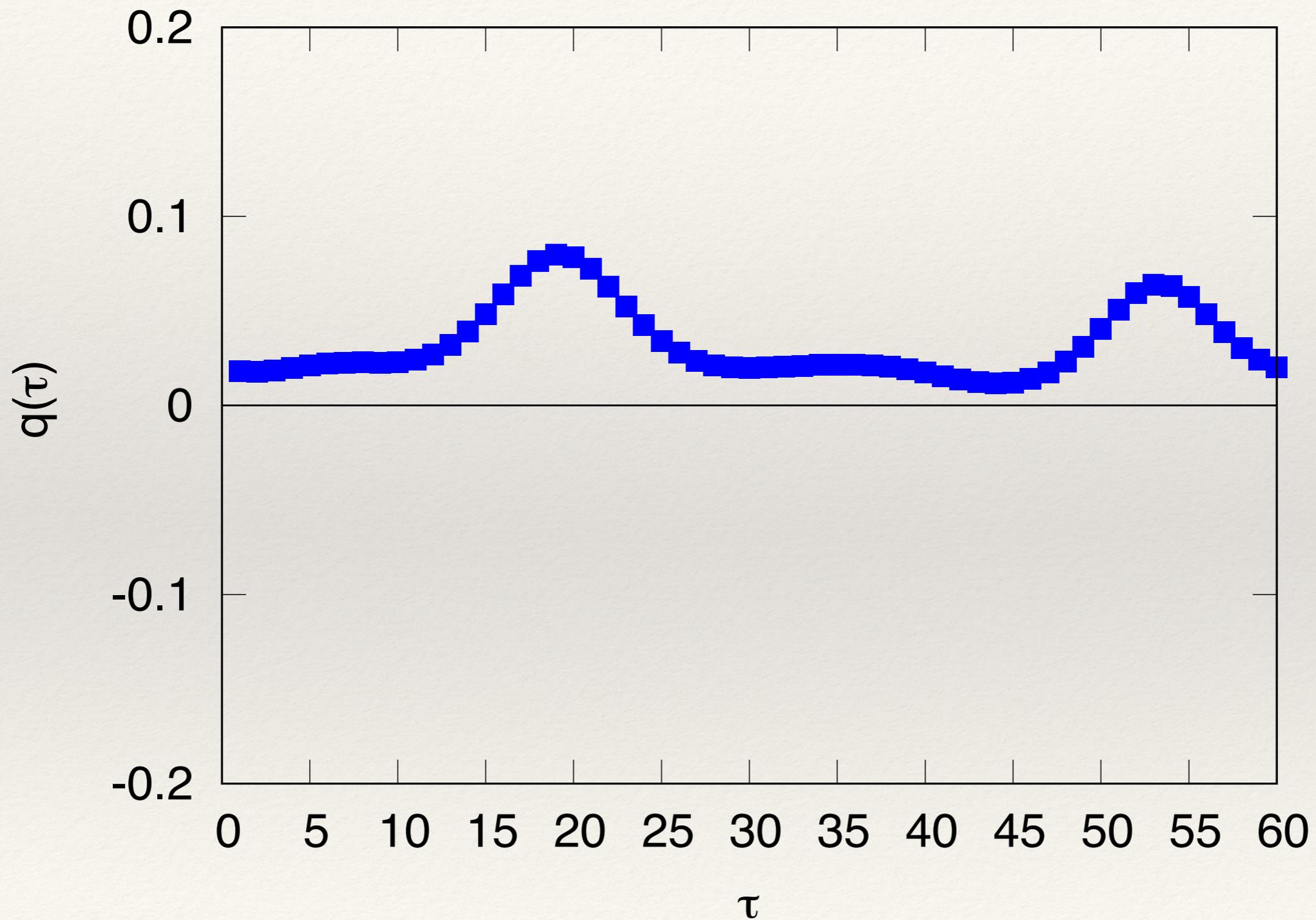


Monte Carlo step dependence

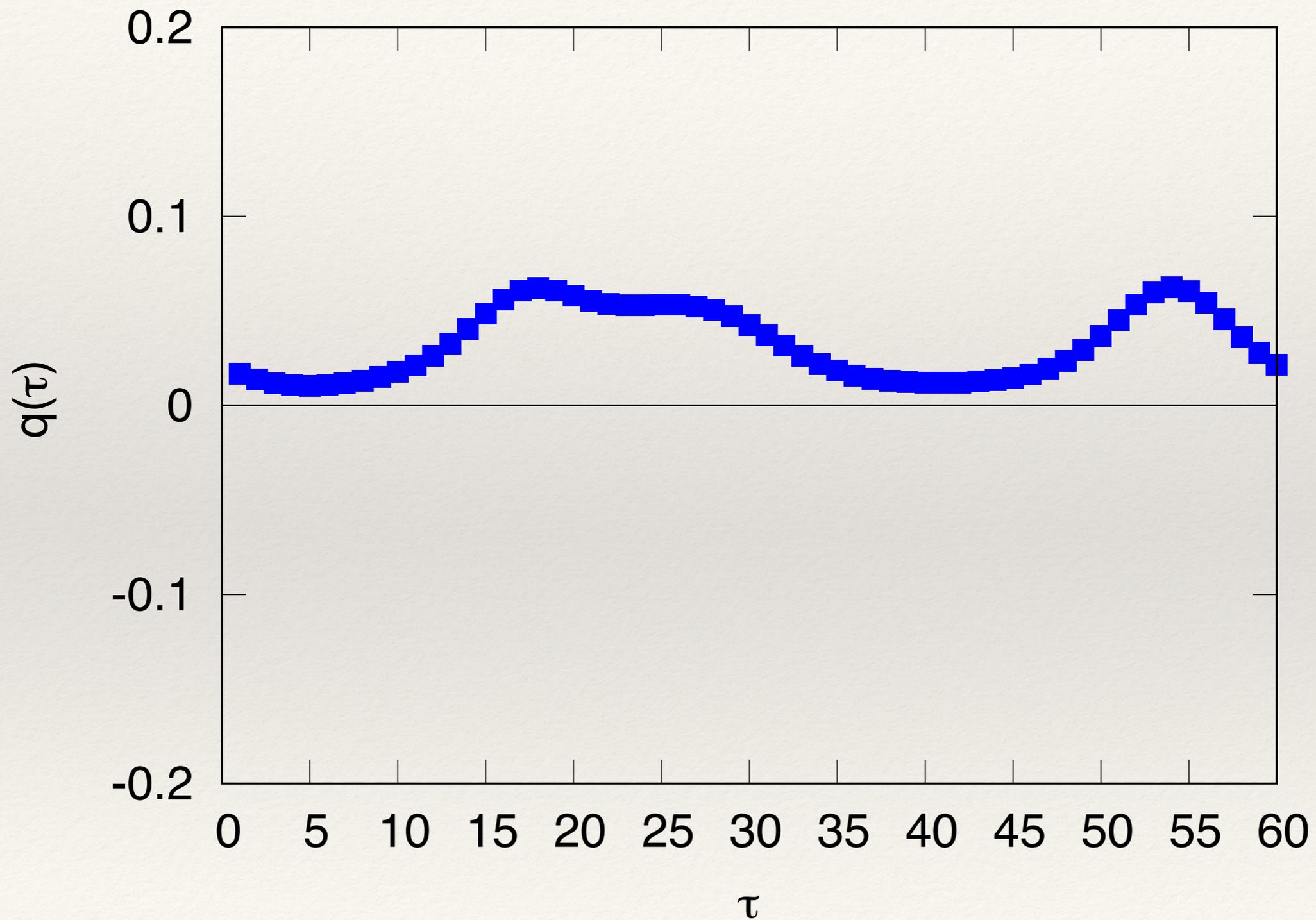


3 fractional instantons

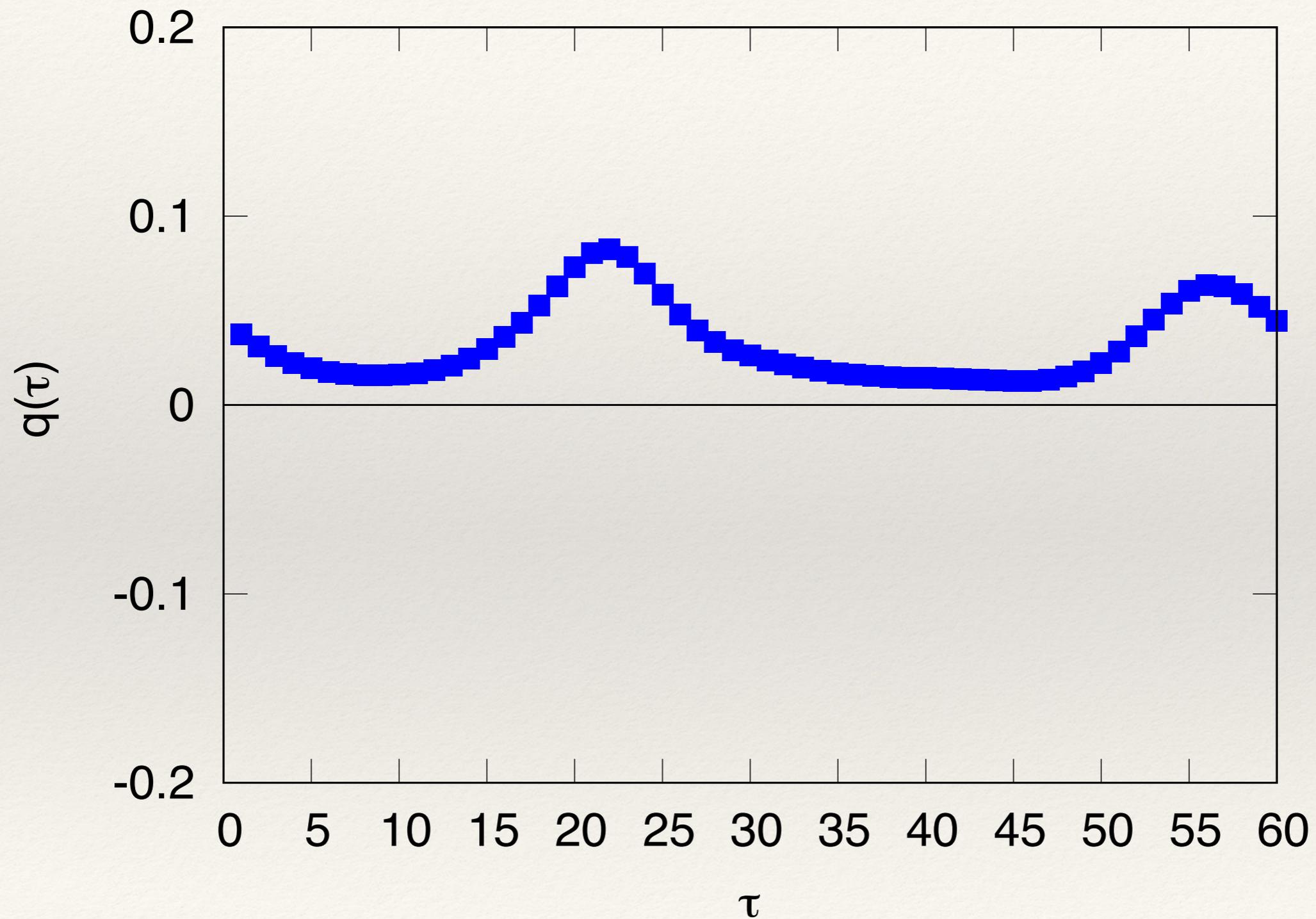
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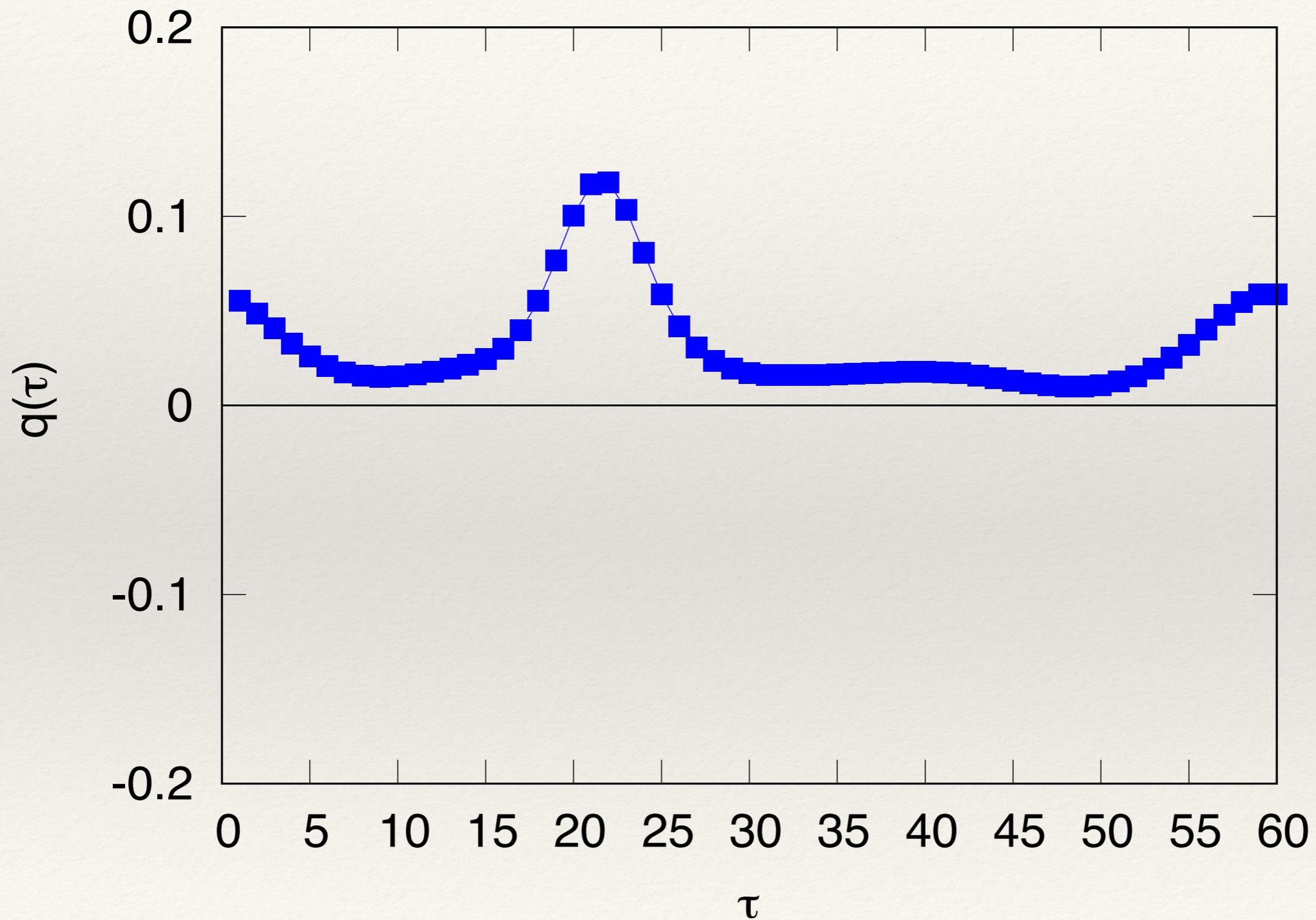
Monte Carlo step dependence



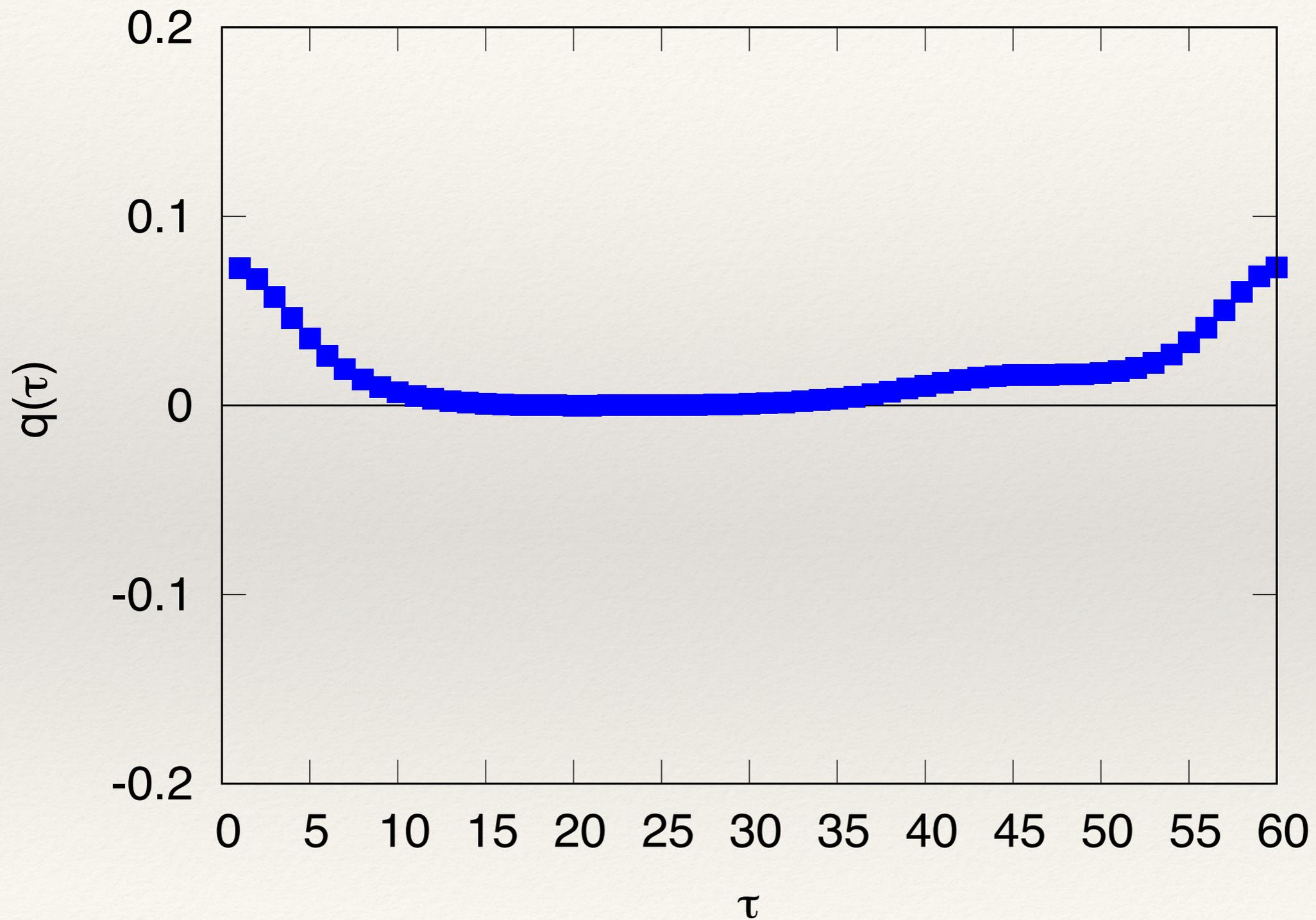
Monte Carlo step dependence



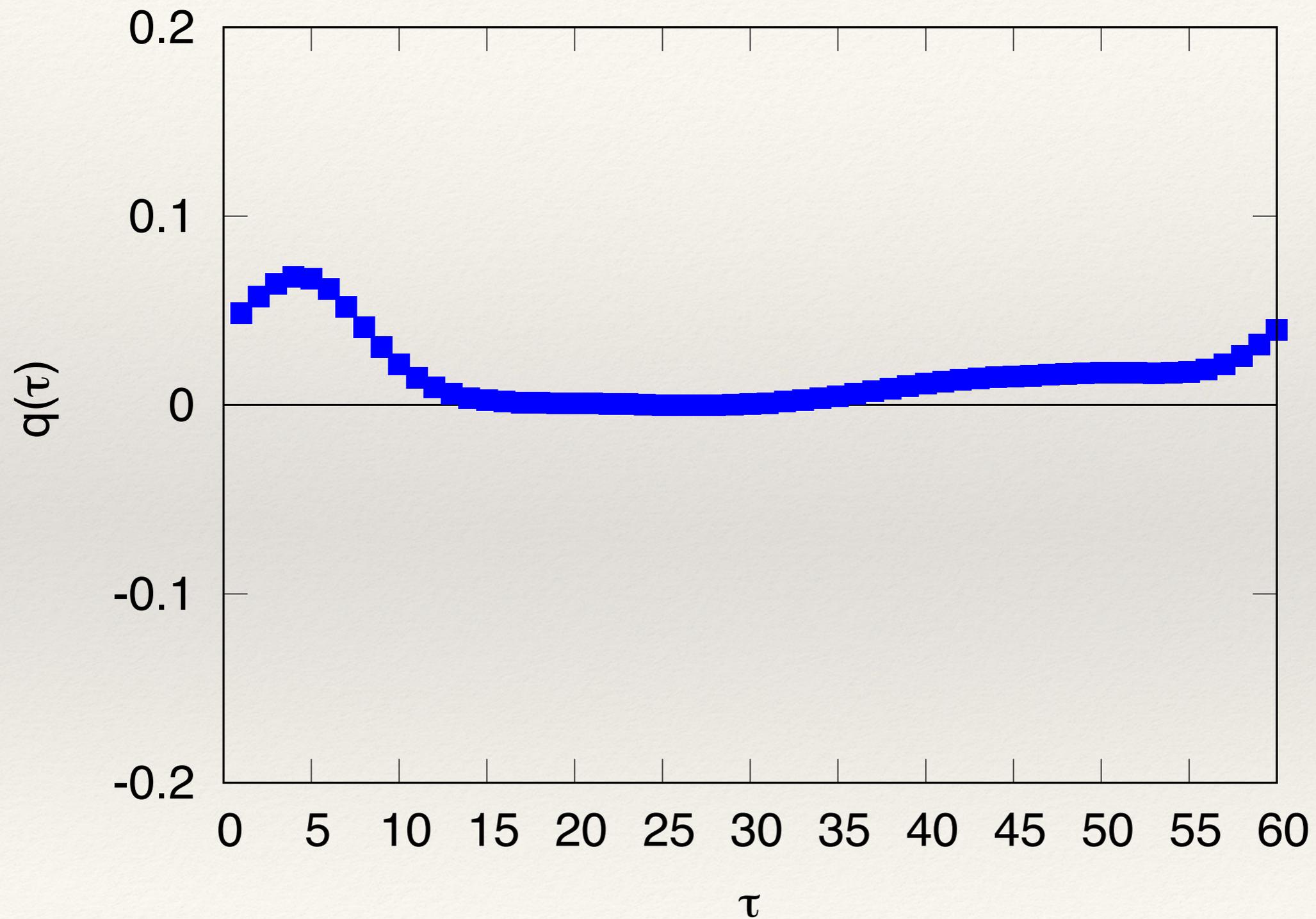
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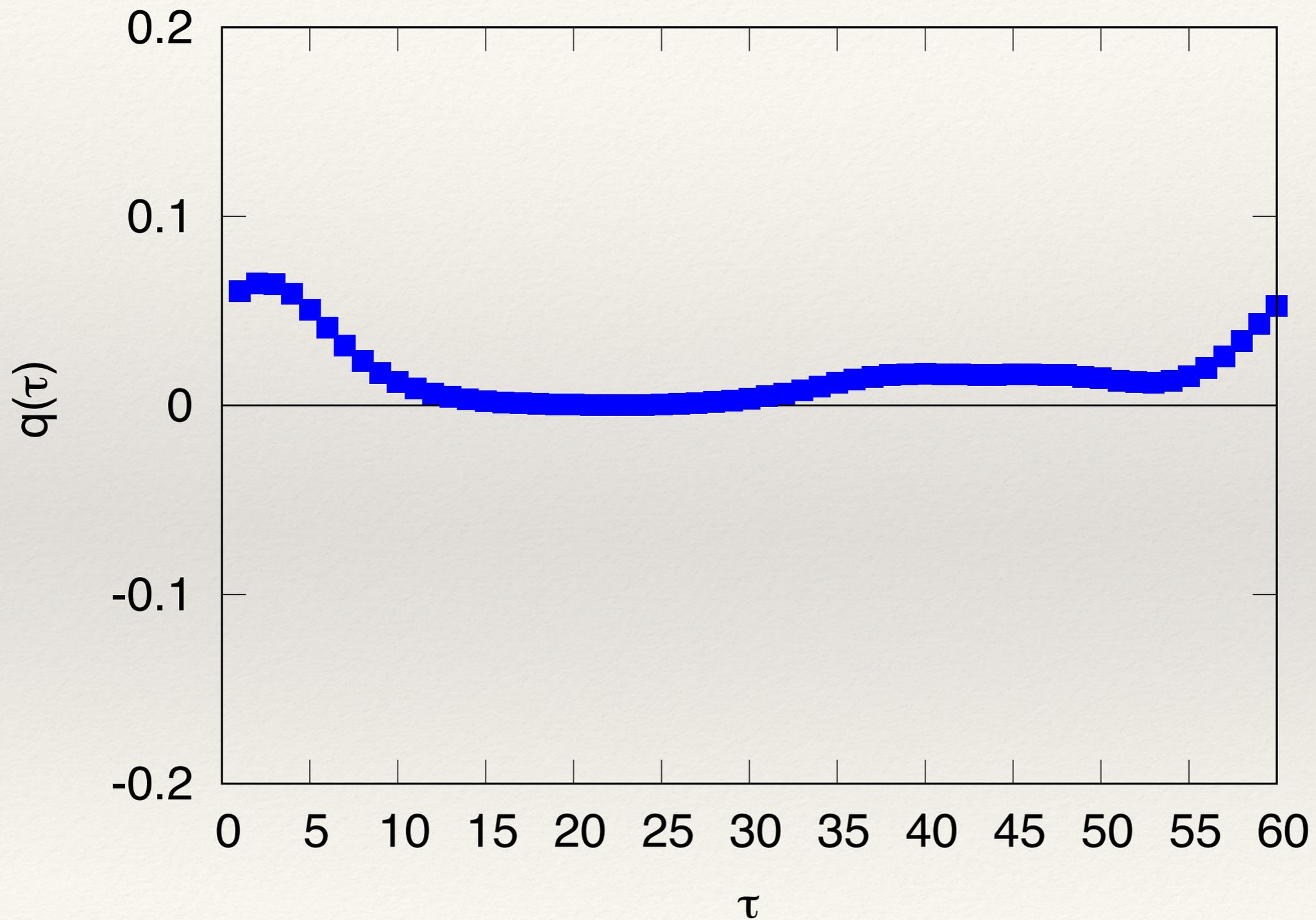
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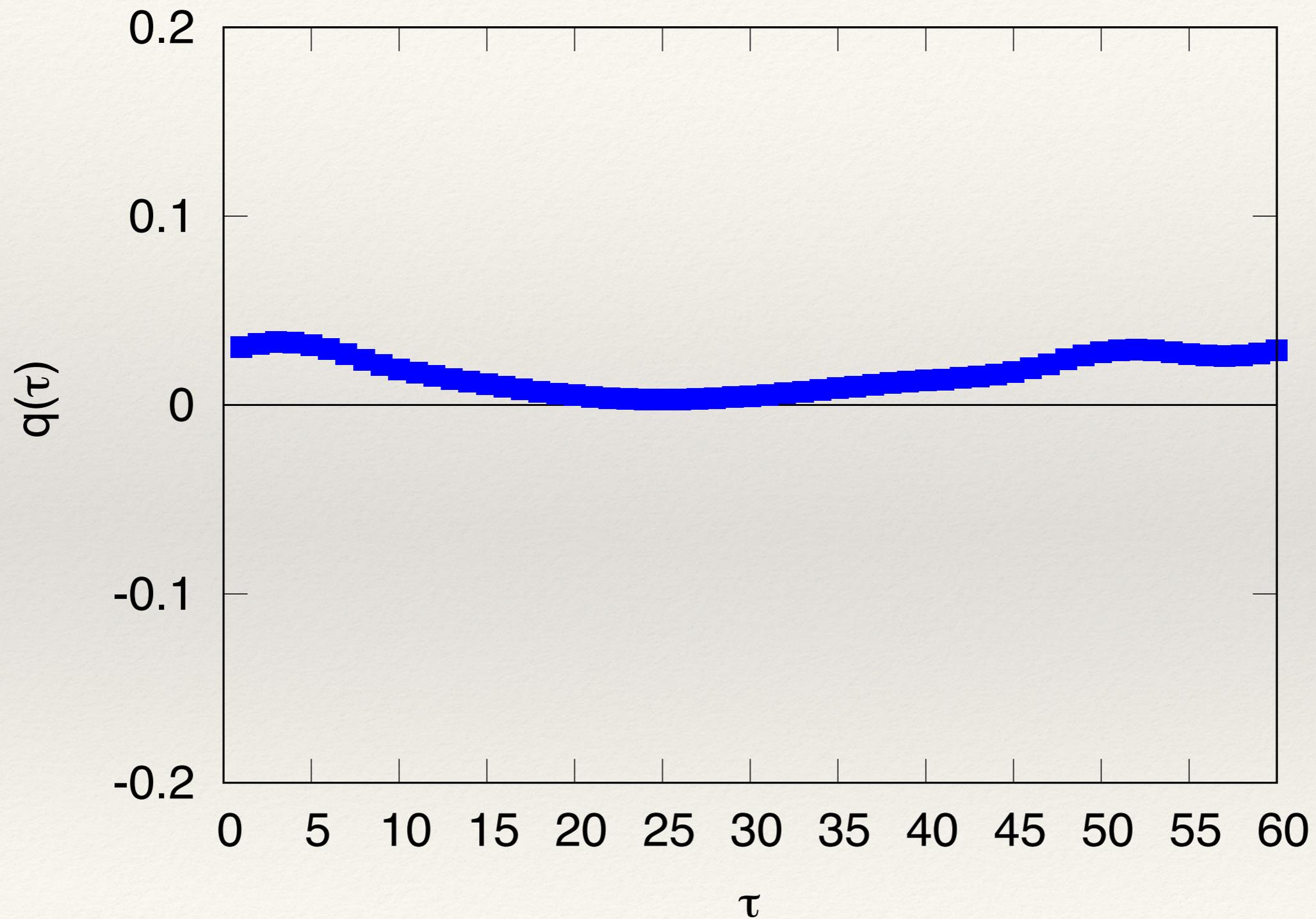
Monte Carlo step dependence



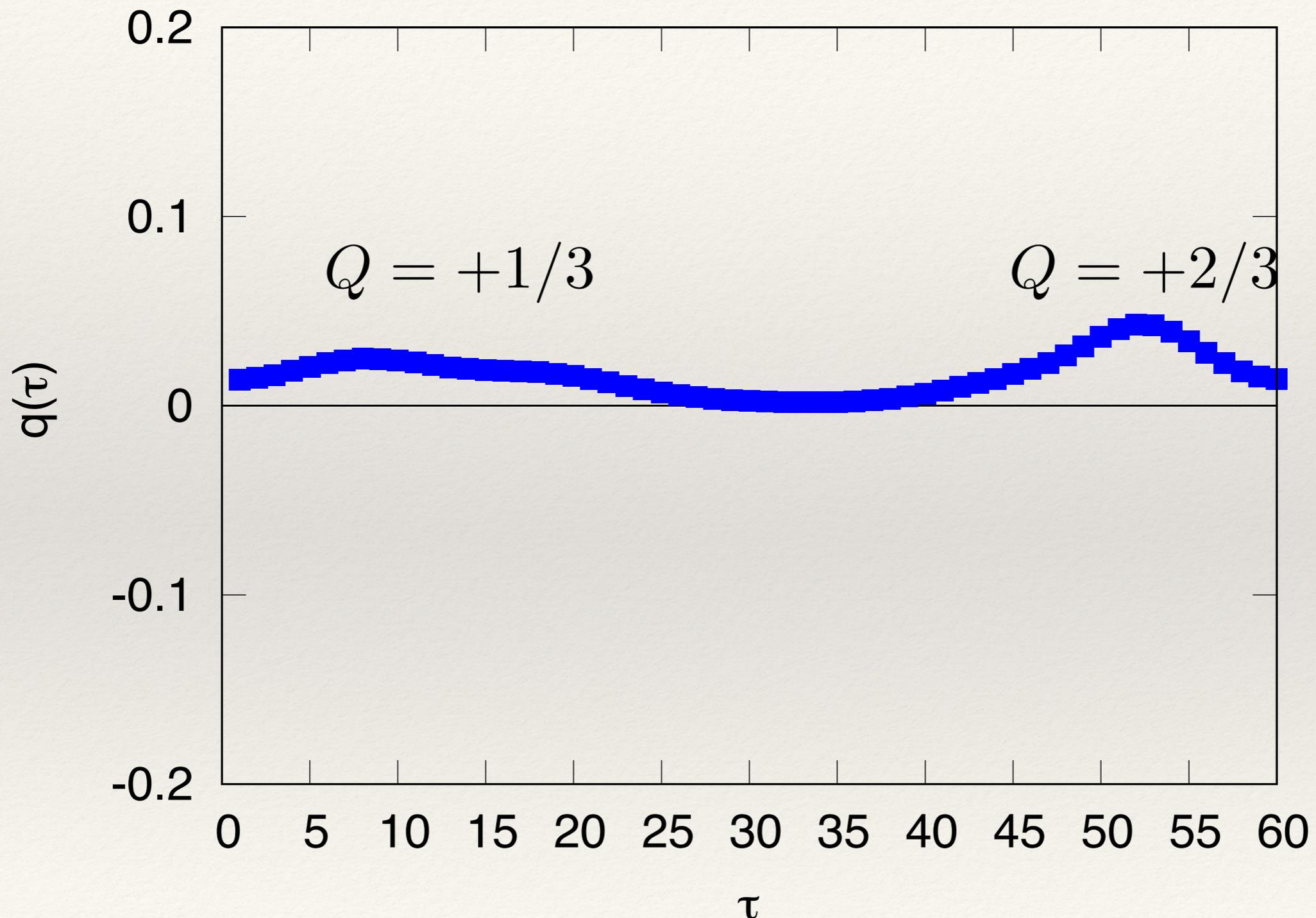
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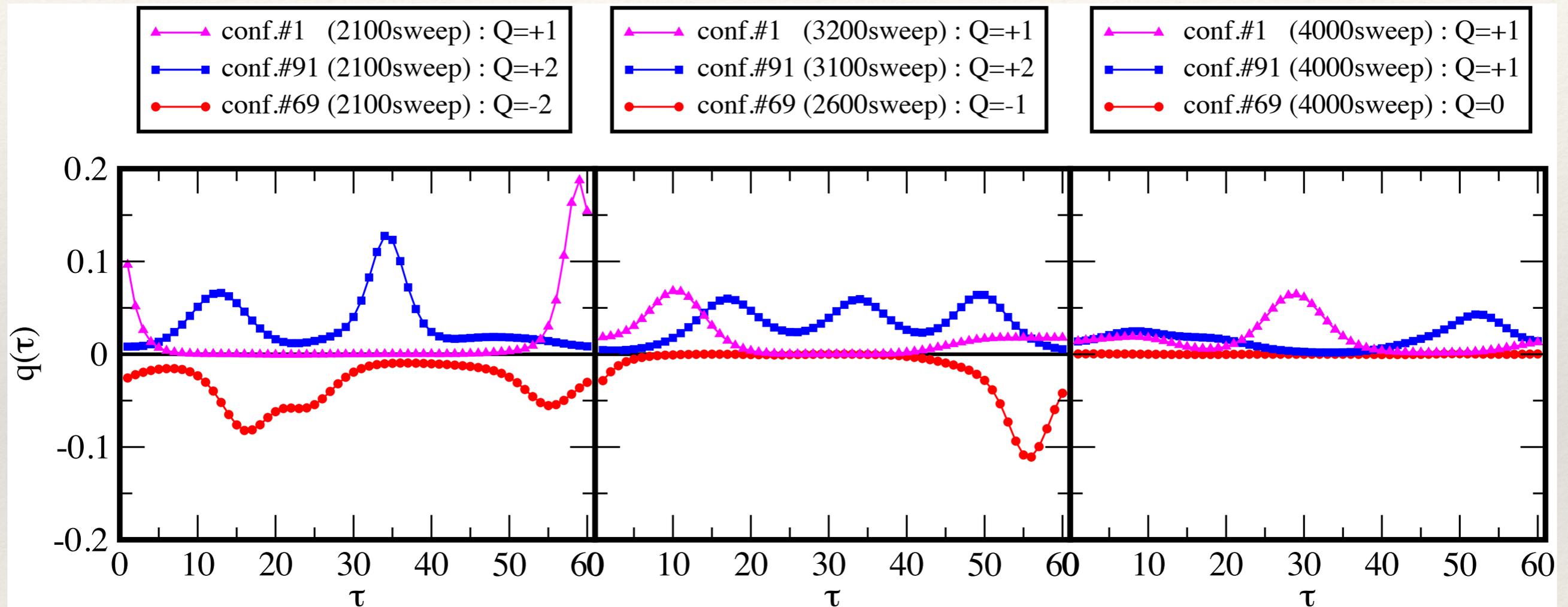


Monte Carlo step dependence



Total topological charge $Q = 2$ to $Q = 1$

Monte Carlo step dependence



The fractional-instantons can merge into the integer-instanton and vice versa during the Monte Carlo update processes.
The total instanton number can be also changed.

Polyakov loop

Classical solution on $\mathbb{T}^3 \times \mathbb{R}$ with twist

Witten, NPB202(1982)253 (section7)

Gauge transf. under twisted b.c. $U_\mu(n) \rightarrow \Lambda(n)U_\mu(n)\Lambda^\dagger(n + \hat{\mu})$

The boundary condition is satisfied if extended \mathbb{Z}_{N_c} transf. is added in z direction
(compact, PBC)

$$\Lambda(n + \hat{z}N_s) = e^{2\pi i l_z / N_c} \Lambda(n) \quad l_z = 0, 1, \dots, N_c - 1$$

The gauge equivalent conf. with standard perturbative conf. has

Topological charge

$$\begin{aligned} Q &= \frac{1}{8\pi^2} \int \text{Tr}(F \wedge F) \\ &= -\frac{1}{24\pi^2} \int \text{Tr}(\Lambda^{-1}d\Lambda) \wedge (\Lambda^{-1}d\Lambda) \wedge (\Lambda^{-1}d\Lambda) \\ &= \frac{l_z n'}{N_c} + \text{integer} \end{aligned}$$

Polyakov loop in z-direction

$$\begin{aligned} P_z &= \frac{1}{N_c} \text{Tr} \exp \left[i \int A_z dx \right] \\ \text{Gauge transf. } A_z &\rightarrow \Lambda^{-1} A_z \Lambda - i \Lambda^{-1} (\partial_z \Lambda) \\ P_z &\rightarrow \frac{1}{N_c} \text{Tr} \exp \left[i \int A_z dx + 2\pi l_z / N_c + 2\pi n \right] \\ &= e^{2\pi i l_z / N_c} P_z \end{aligned}$$

If l_z is not a multiple number of N_c , then Q can be fractional.
If fractional instanton appears, the P_z rotates in complex plane.

Complex phase of Pz

If fractional instanton appears, the Pz rotates in complex plane.

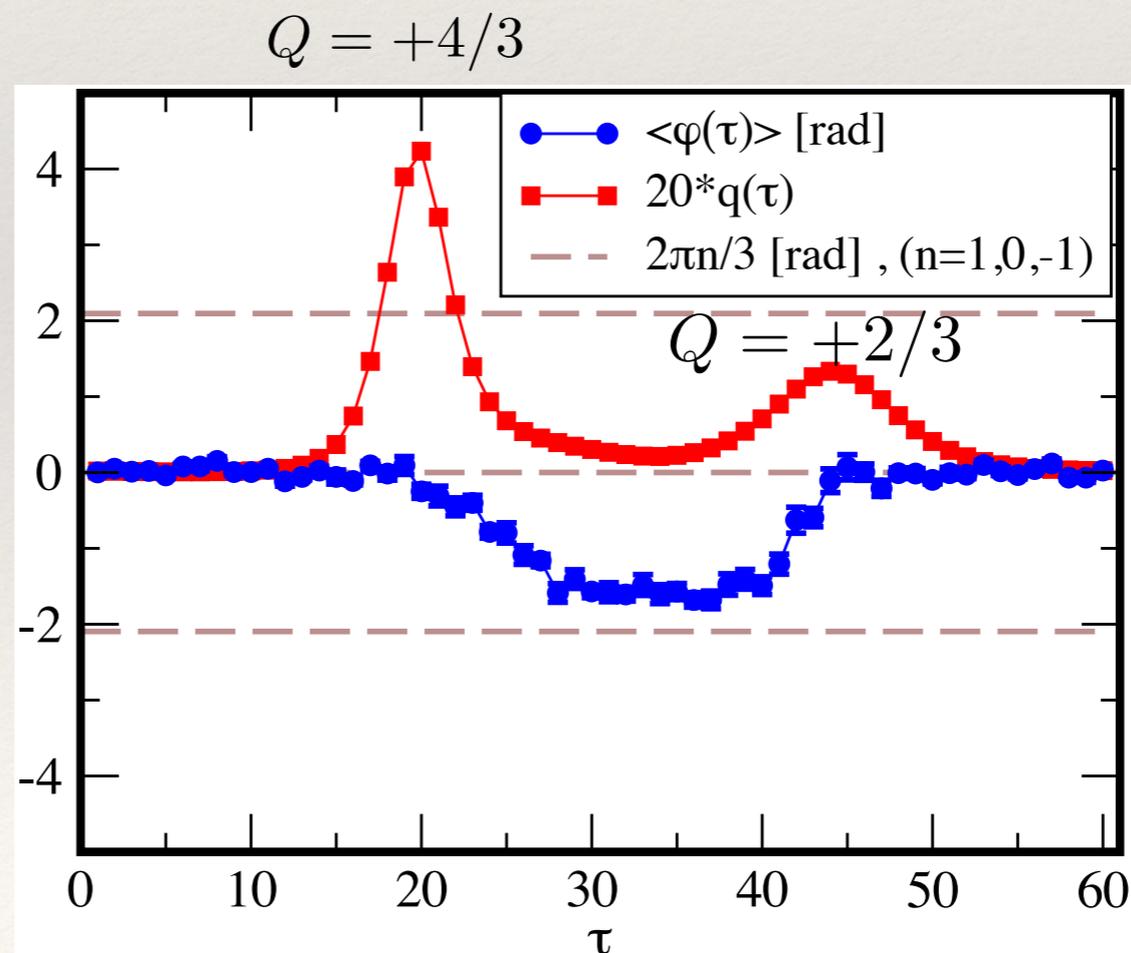
Let us focus on complex phase of Pz for a configuration including frac. inst.

$$\begin{aligned} \tilde{P}_z(x, y, \tau) &= \frac{1}{N_c} \text{Tr} \left[\prod_j U_z(x, y, z = j, \tau) \right] \\ &\equiv |\tilde{P}_z(x, y, \tau)| e^{i\varphi(x, y, \tau)}. \end{aligned}$$

Sum up them for spatial coordinates

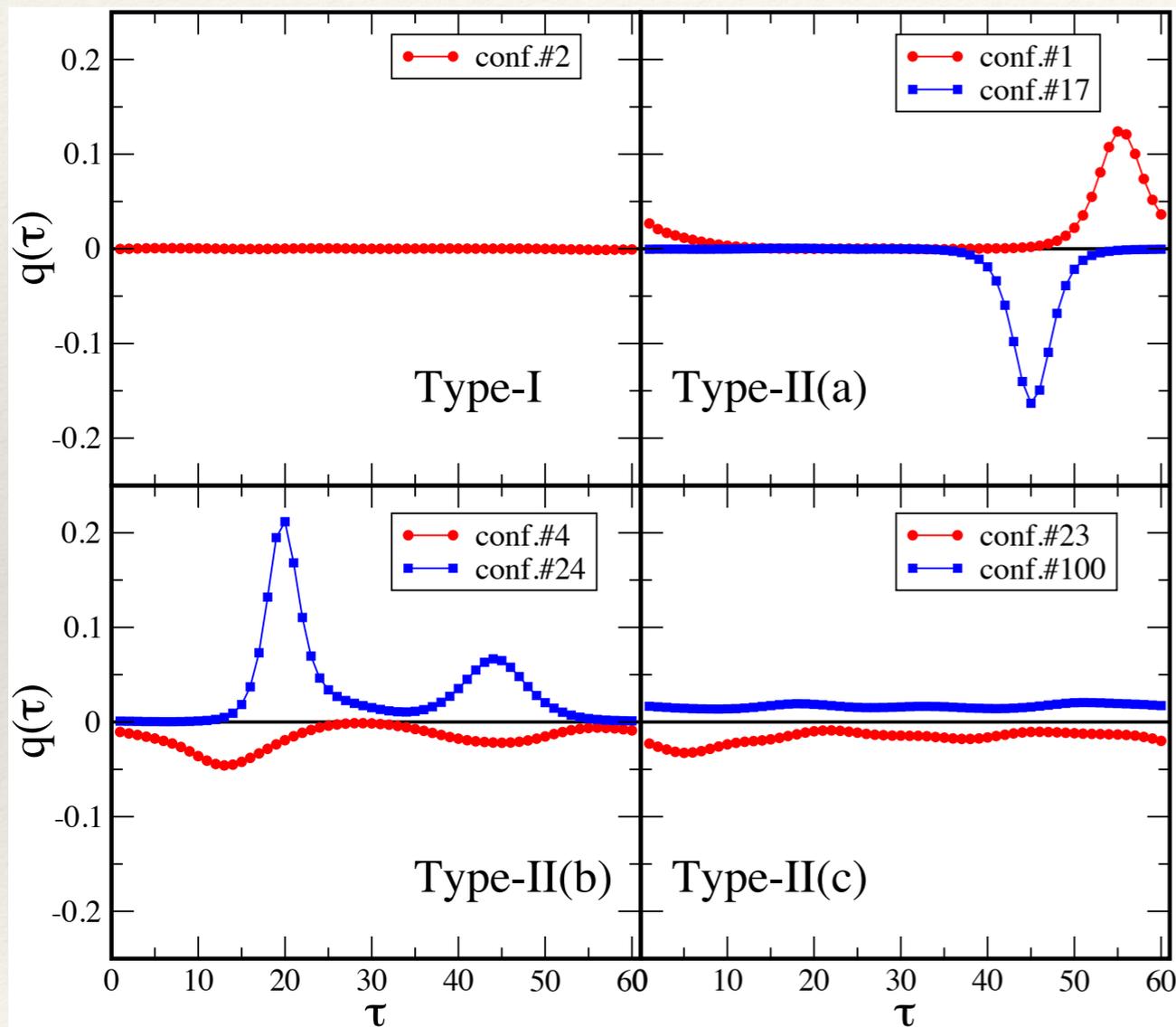
$$\langle \varphi(\tau) \rangle = \sum_{x, y} \varphi(x, y, \tau)$$

$\langle \varphi(\tau) \rangle$ [rad]

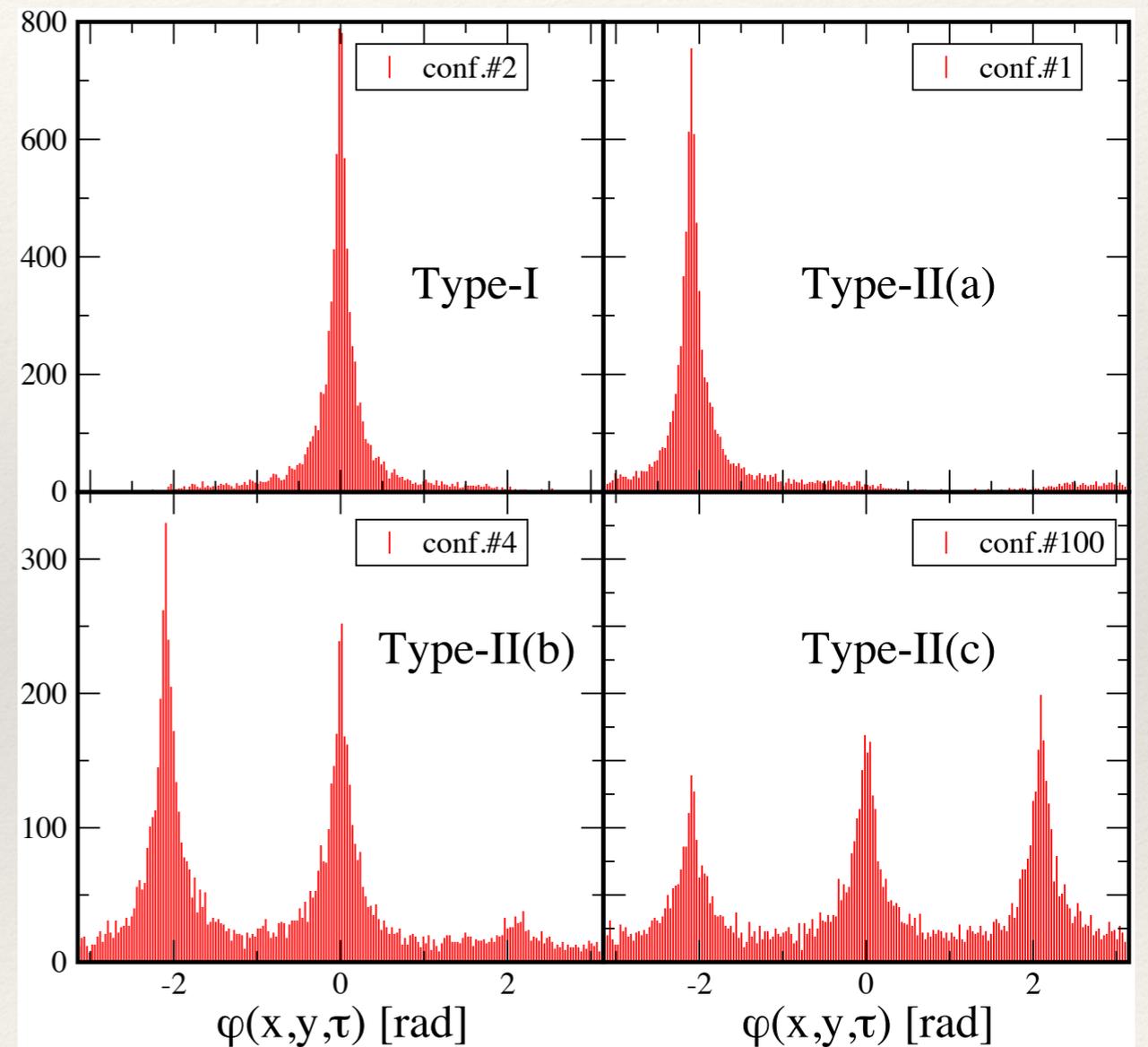


Relationship between q and Pz

Local charge(q)



Histogram of complex phase of Pz



The type of local topological charge is strongly related with the distribution of Pz .

Summary

- (1) We find a fractional-instanton in the weak coupling regime on a promising deformed spacetime toward the resurgence of the $SU(3)$ gauge theory
- (2) The fractional-instantons have the same properties as the ones of the classical solutions given by the gauge equivalent of the standard perturbative vacua under the extended Z_3 gauge symmetry in the $S^1 \rightarrow \mathbb{R}$ limit.
(Rotation of Polyakov loop , size moduli is absent)
- (3) Center symmetry breaking becomes milder than the one on PBC lattice
- (4) The scaling law of Polyakov loop shows the deconfinement property (as expected)

Backup

Towards the resurgence of QCD.....

SU(3) gauge theory on \mathbb{R}^4 or \mathbb{T}^4 (T=0) in weak coupling regime
(Q: trivial, perturbative exp. not to converge)

Regularize by introducing compactification and twisted boundary condition

T=0 (weak coupling)
 $\mathbb{T}^3 \times S^1$ Lattice
with $L_s \ll L_\tau$, TBC
(Q: integer,
locally fractional)

Investigate the resurgence structure

- (1) contributions to some physical observables (e.g. plaquette) from frac. inst. configurations (g will be complex)
- (2) cancelation of imaginary ambiguity between renormalon and (1) occur?

(Bali et al, $O(\alpha^3)$ calculation, [Phys. Rev. D 89, 054505 \(2014\)](#))

Can we take a decompactified limit without any phase transition?
(Adiabatically continue?)

SU(3) gauge theory on \mathbb{R}^4 or \mathbb{T}^4 (T=0) in strong coupling regime
(Q: integer)