Lattice study on the twisted CP^{N-1} model on $R \times S^1$

Tatsuhiro Misumi Akita U. / Keio U.

Toshiaki Fujimori Keio U.
Etsuko Itou Keio U. / Kochi U. / RCNP
Muneto Nitta Keio U.
Norisuke Sakai Keio U.

06/18/2019@Lattice2019, Wuhan

CP^{N-1} sigma model

2D CP^{N-1} model is not only a toy model of QCD, but also effectively describes gauge theory !

- Effective theory on vortex in U(N) + Higgs model is CP^{N-1} Eto, et.al.(05)
- Effective theory on long strings in YM is CP^{N-1} Aharony, Komargodski(13)
- It is also notable that CP^1 describes spin chain systems Haldane(83)

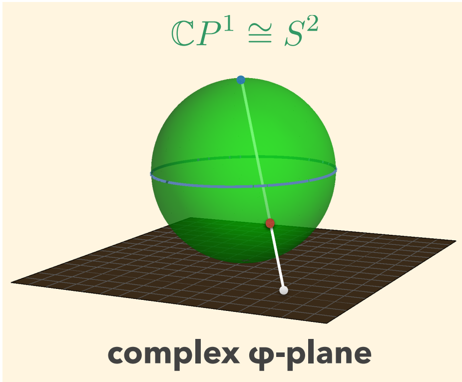
Lattice study on CP^{N-1} model is of physical significance

◆Lagrangian of CP^{N-1} models $\mathbb{C}P^{N-1} = \frac{SU(N)}{SU(N-1) \times U(1)}$

$$S = \frac{1}{2g^2} \int |D\phi|^2 \qquad |\phi|^2 = 1, \ D\phi = (d + ia)\phi, \ a = i\bar{\phi} \cdot d\phi$$

discretized on the lattice

$$S = -N\beta \sum_{n,\mu} \left(\bar{z}_{n+\mu} \cdot z_n \lambda_{n,\mu} + \bar{z}_n \cdot z_{n+\mu} \bar{\lambda}_{n,\mu} - 2 \right)$$



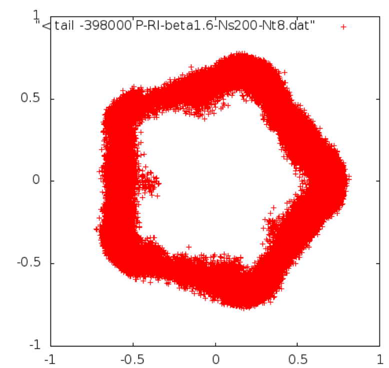
CP^{N-1} sigma model on R x S^1

- Global symmetry : PSU(N) flavor symmetry + Time reversal
- Z_N symmetry is not exact for periodic b. c. (cf. QCD)

- Z_N-twisted b.c.

$$\phi(x_1, x_2 + L) = \Omega\phi(x_1, x_2) \qquad \Omega = \mathrm{diag.}\left[1, e^{2\pi i/N}, e^{4\pi i/N}, \cdots, e^{2(N-1)\pi i/N}\right]$$

➡ Exact Z_N-symmetric model $\quad$ Z_N flavor shift + Z_N center
(irrelevant with decompactifying)



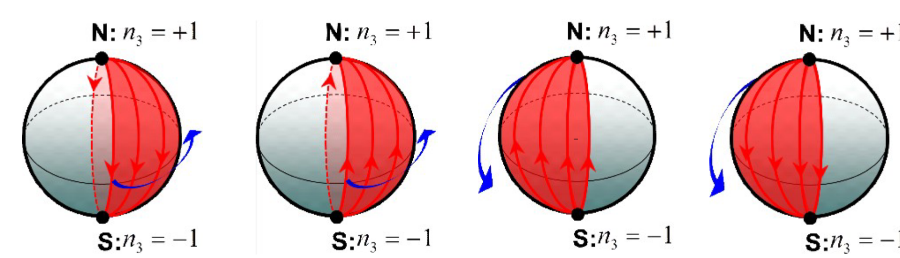
Distribution of P-loop for N=5

- Fractional instantons (Q=1/N, S=S$_I$/N)

BPS eq. $\quad D\phi \pm \mathrm{i} \star D\phi = 0$

BPS sol. $\quad \phi = \dfrac{(1, e^{z-z_0}, \ldots)}{\sqrt{1 + |e^{z-z_0}|^2}}$

cf.) N=2



Lee, Yi(97) Kraan, van Baal(97) Eto, et.al. (04~) Bruckmann,et.al. (05~)

＊It is shown to have resurgent structure (pert. vs non-pert. relation)

Dunne, Unsal(12) TM, Nitta, Sakai(14,15) Fujimori, et.al.(16~)

Resurgent structure in QM and QFT

$$\mathcal{S}_+\Phi_0(z) - \mathcal{S}_-\Phi_0(z) \approx \mathfrak{s}\, e^{-Az}\mathcal{S}\Phi_1(z) \qquad z = \frac{1}{g^2}$$

Perturbative imaginary ambiguity

Non-perturbative effect

Resurgent structure is expected to be in quantum theory, thus perturbative series could include nonpert. information !

Zinn-Justin(01), Marino(07) Marino, Schiappa, Weiss(09), Argyres, Unsal(12), Dunne, Unsal(12)

In a certain class of QFT as twisted CP^{N-1} models QFT can be defined based on the structure.

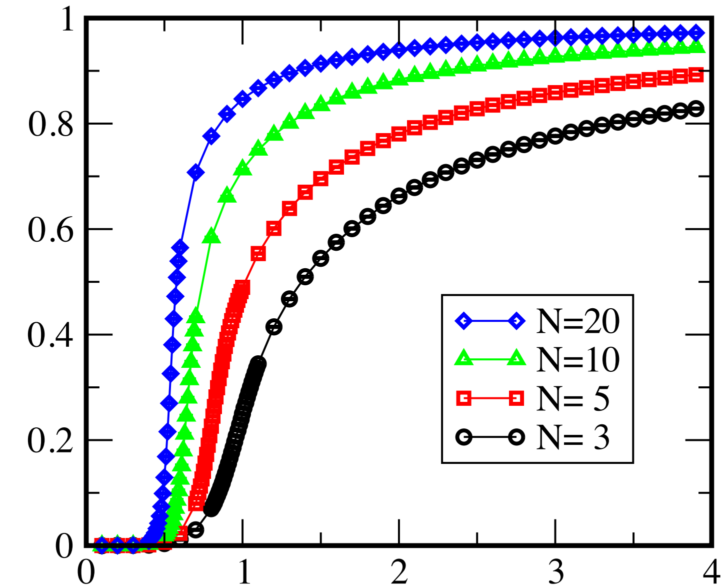
Main questions

◆Question 1 : Z_N (phase) transition for pbc

- 2nd-order phase transition expected in large-N
- it should be crossover for finite N since Z_N

$|\langle P \rangle| \sim 0$ **for small** $\beta \quad \xrightarrow{?} \quad |\langle P \rangle| \neq 0$ **for large** β

We will check it directly in numerical study



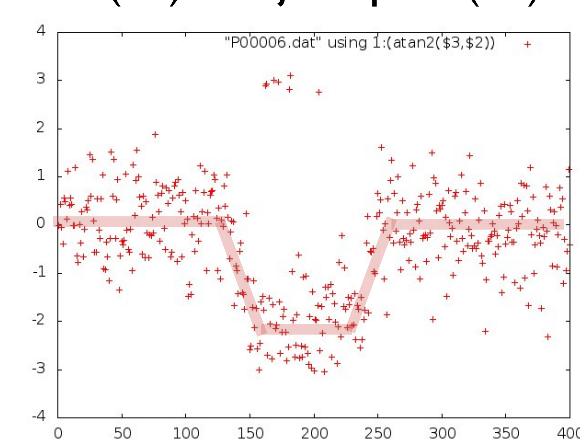
◆Question 2 : Continuity and fractional instantons for Z_N-tbc

- Fractional instantons yield transition between classical N-vacua
- makes Z_N stable, leading to volume indep. of vacuum structure

Dunne, Unsal(12) Sulejmanpasic(16)

$|\langle P \rangle| \sim 0$ **for small** $\beta \quad \xrightarrow{?} \quad$ **still** $|\langle P \rangle| \sim 0$ **for large** β

We will show quite suggestive results on fractional instantons and adiabatic continuity



Setup of lattice simulation

cf.) Berg,Luscher(81), Campostrini,et.al.(92),Alles,et.al.(00), Flynn,et.al.(15),Abe,et.al.(18)

- **Lattice formulation** $S = -N\beta \sum_{n,\mu} \left(\bar{z}_{n+\mu} \cdot z_n \lambda_{n,\mu} + \bar{z}_n \cdot z_{n+\mu} \bar{\lambda}_{n,\mu} - 2 \right)$

  Vector field Φ is introduced: $\begin{aligned} \phi_{2j} &= \Re[z_{n,j}], \quad \phi_{2j+1} = \Im[z_{n,j}], \quad j = 0, \cdots, N-1 \\ \phi_\mu^R &= \Re[\lambda_\mu], \quad \phi_\mu^I = \Im[\lambda_{n,\mu}], \end{aligned}$

  ➡ $s_\phi = -N\beta\phi \cdot F_\phi = -N\beta |F_\phi| \cos\theta$ updated just by updating θ

  Over heat-bath algorithm is adopted to update this θ

- **Parameters and quantities**

  N_x = 40-400, N_τ = 8,12, β= 0.1-4.0, N = 3-20, N_{sweep} = 200000,400000

  - Expectation values of Polyakov loop and its susceptibility
  - Thermal entropy $s = \beta(N\tau)^2(\langle T_{xx}\rangle - \langle T_{\tau\tau}\rangle)$

(1)Z_N transition(pbc)   (2)Z_N continuity(tbc)   (3)Thermal entropy

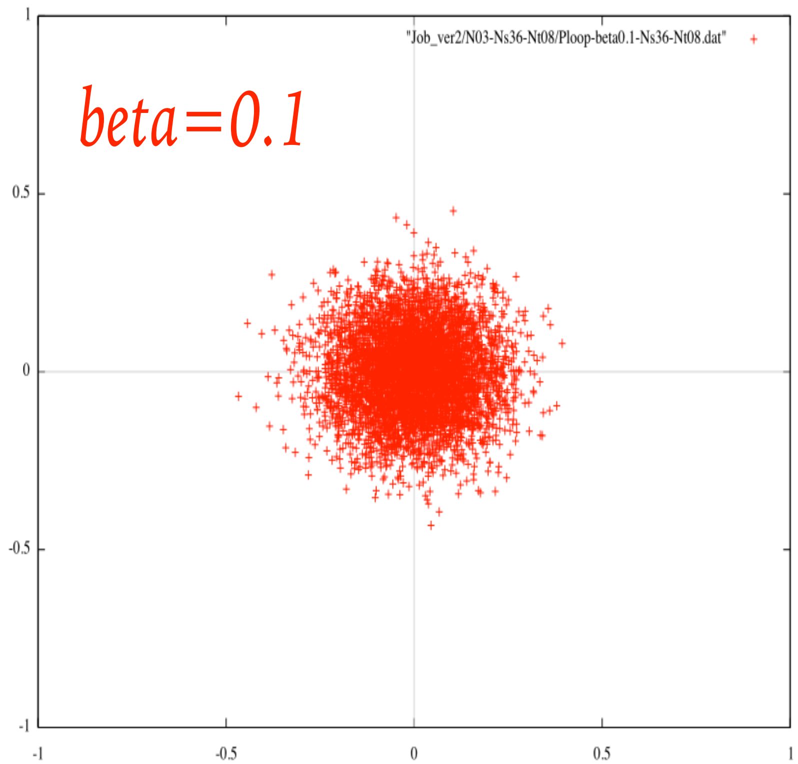
Polyakov-loop of CP^{N-1} models on R x S^1 with pbc.

N=3,5,10,20 (Nx,Nt) = (200, 8) Nsweep = 200,000

Distribution plot of P-loop

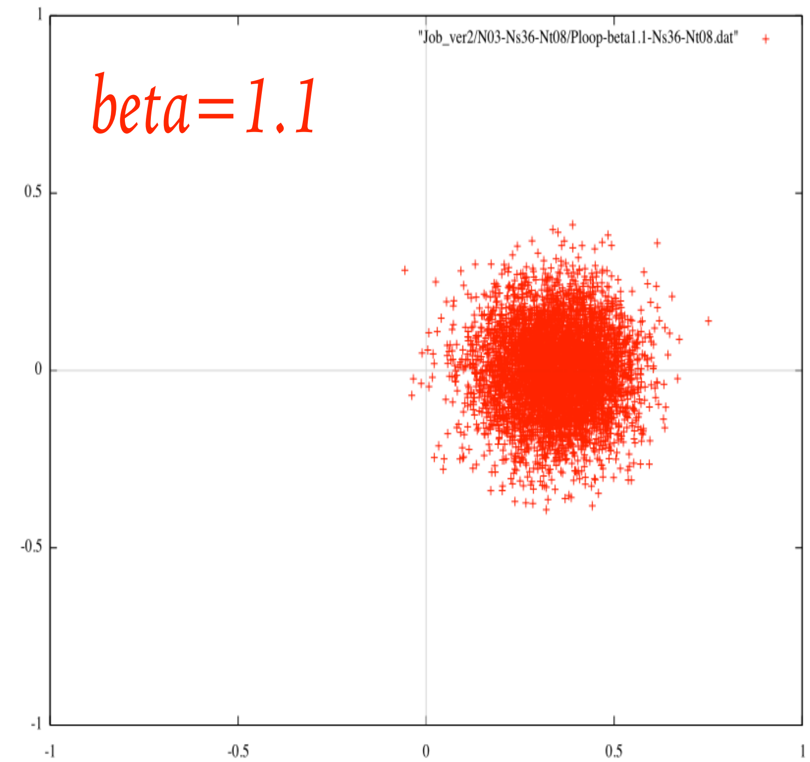
N=3 | N=3

Im[P]



Re[P]

$|\langle P \rangle| \sim 0$



$|\langle P \rangle| \neq 0$

Low-β : around the origin

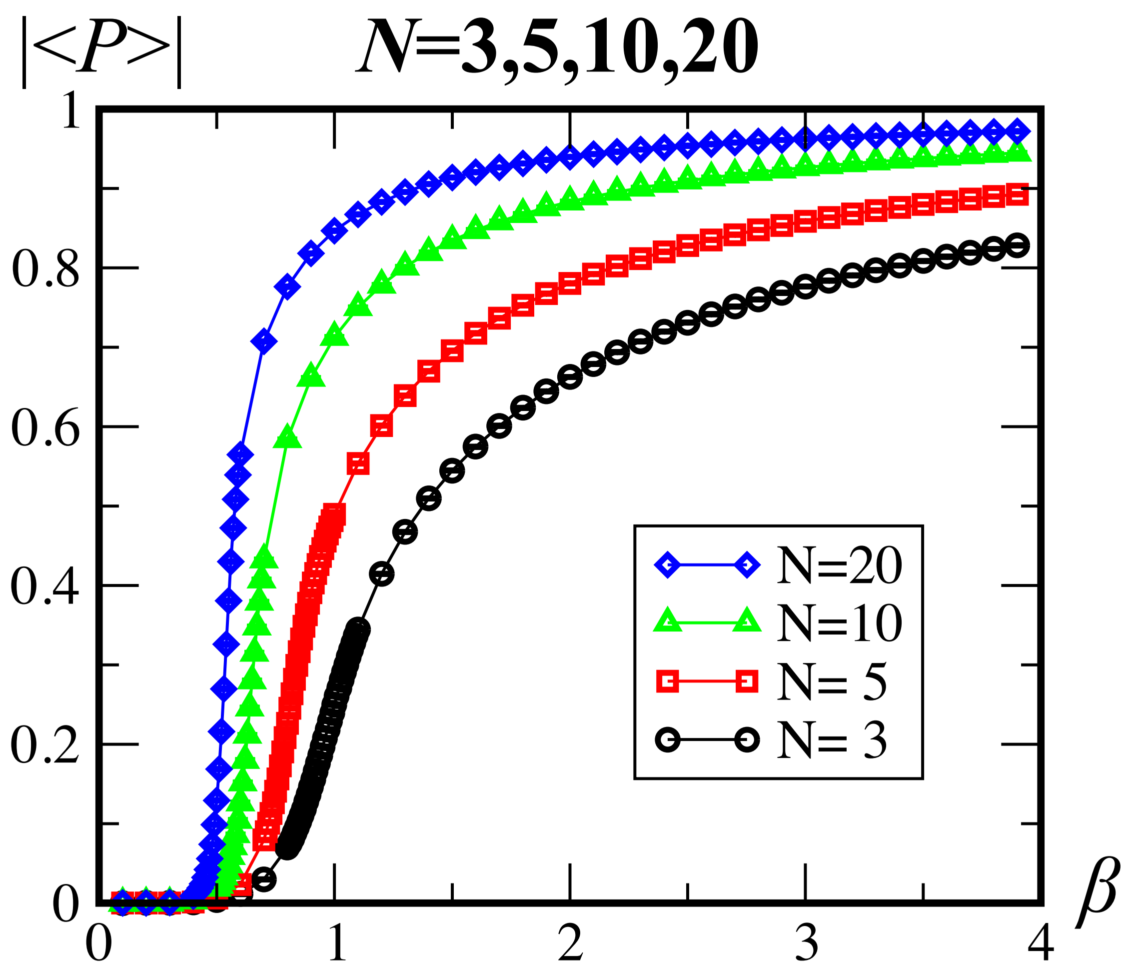
→ approximate Z_N symmetry

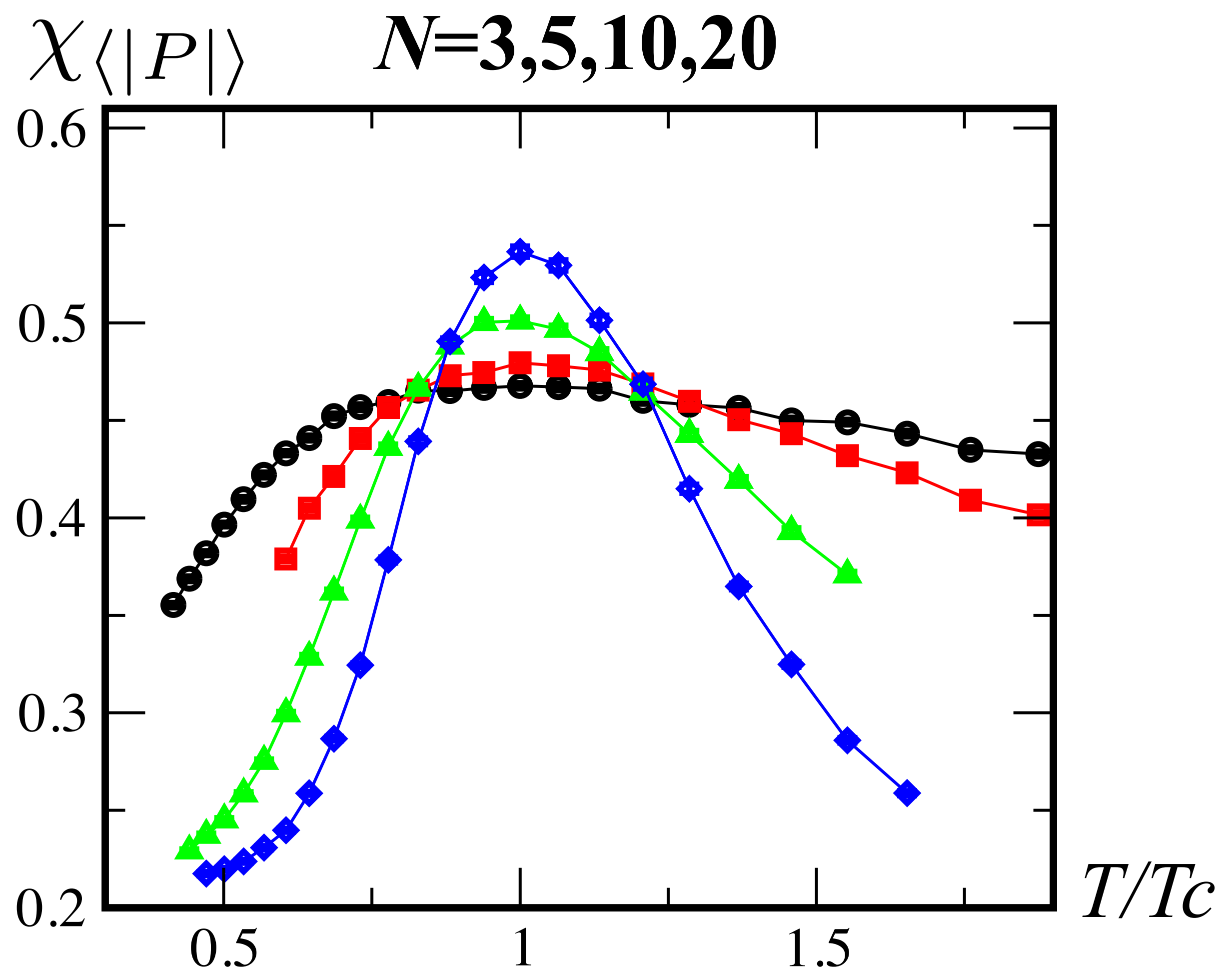
High-β : moves to one of Z_N vacua

→ Z_N breaking transition

Note that Z_N symmetry is not exact for PBC

VEV of Polyakov loop |<P>|

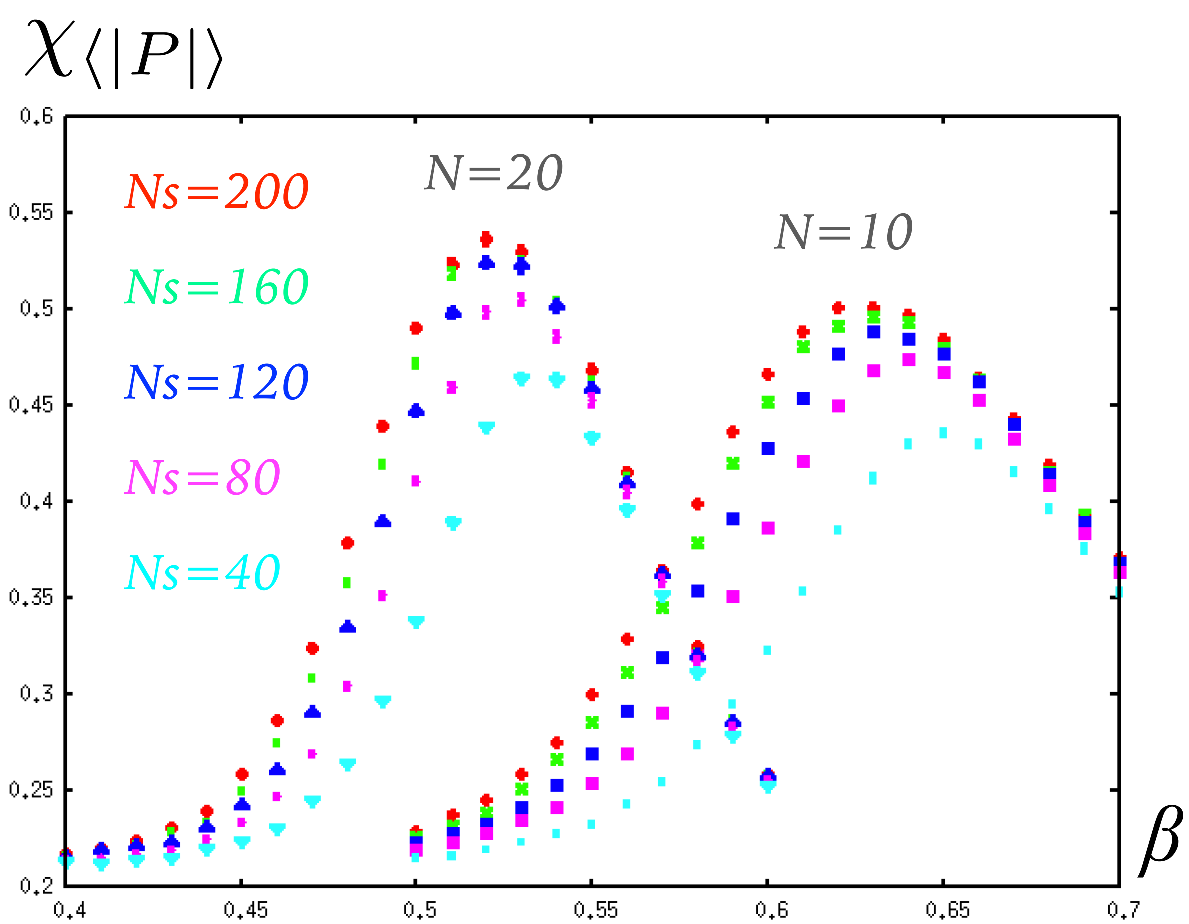


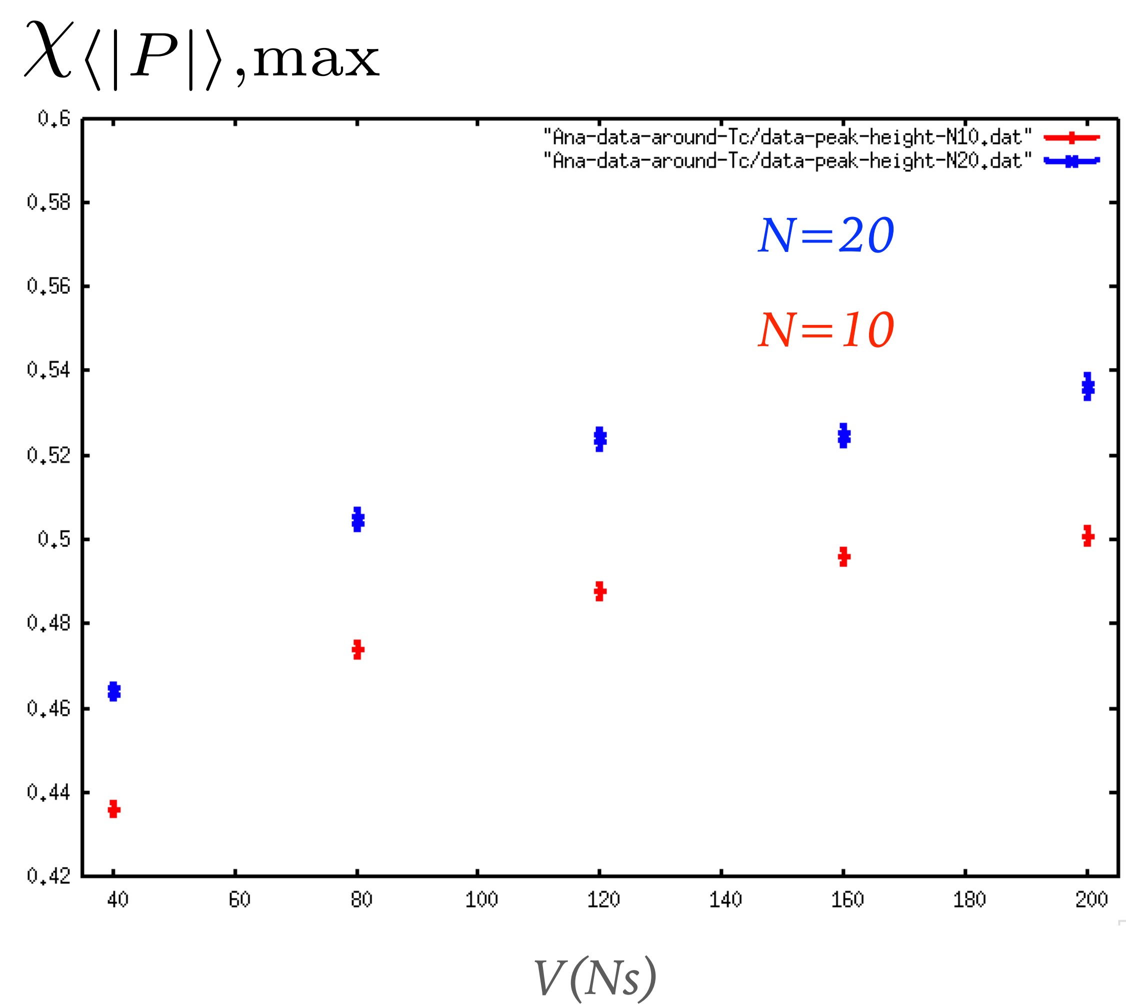


- |<P>| ~ 0 at low β, then |<P>| undergoes crossover-like transition
- Peak of Polyakov-loop susceptibility χ gets sharper with N

Crossover transition for finite N is checked, which would be 2nd-order phase transition for large N limit

Volume dependence of χ-peak





$$\chi_{\max} = c + aV^p$$ **p=1 : 1st, 0<p<1: 2nd or crossover**

Fukugita,et.al.(90)

- Volume dependence of the peak is not linear → not 1st-order
- χ for N=20 is larger than that for N=10 → 2nd-order in large N?

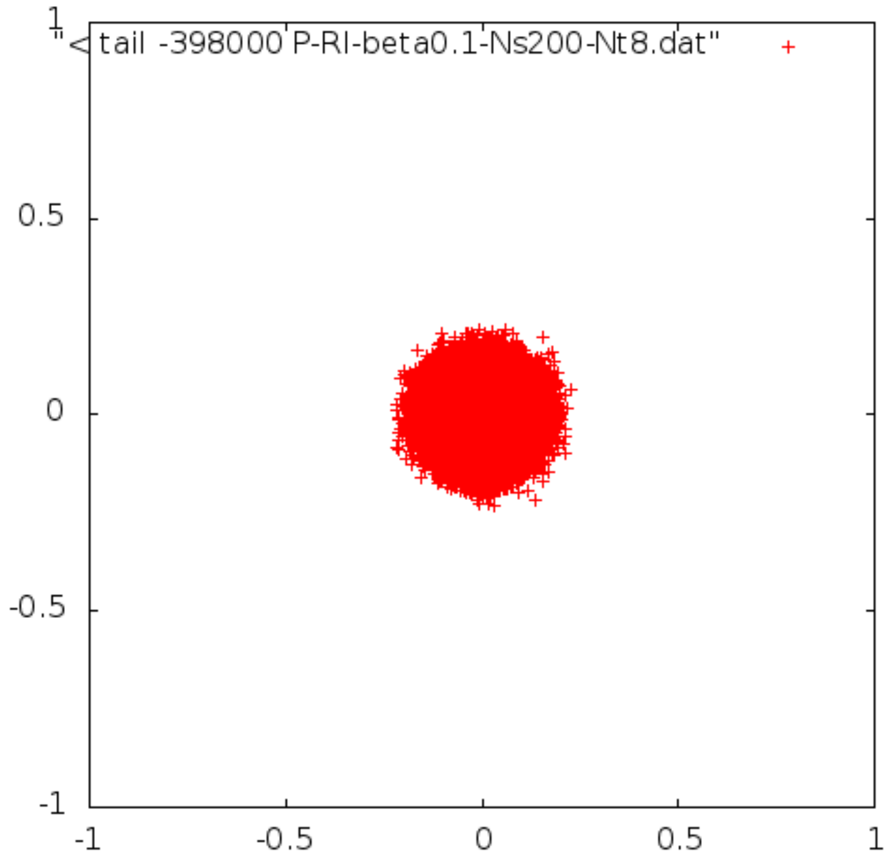
it supports crossover transition for finite N (2nd-order in large-N)

Polyakov loop of CP^{N-1} models on R x S^1 with Z_N tbc.

N=3,5,10,20 (Nx,Nt) = (200, 8), (400,12) Nsweep=200000, 400000

Distribution plot of P-loop

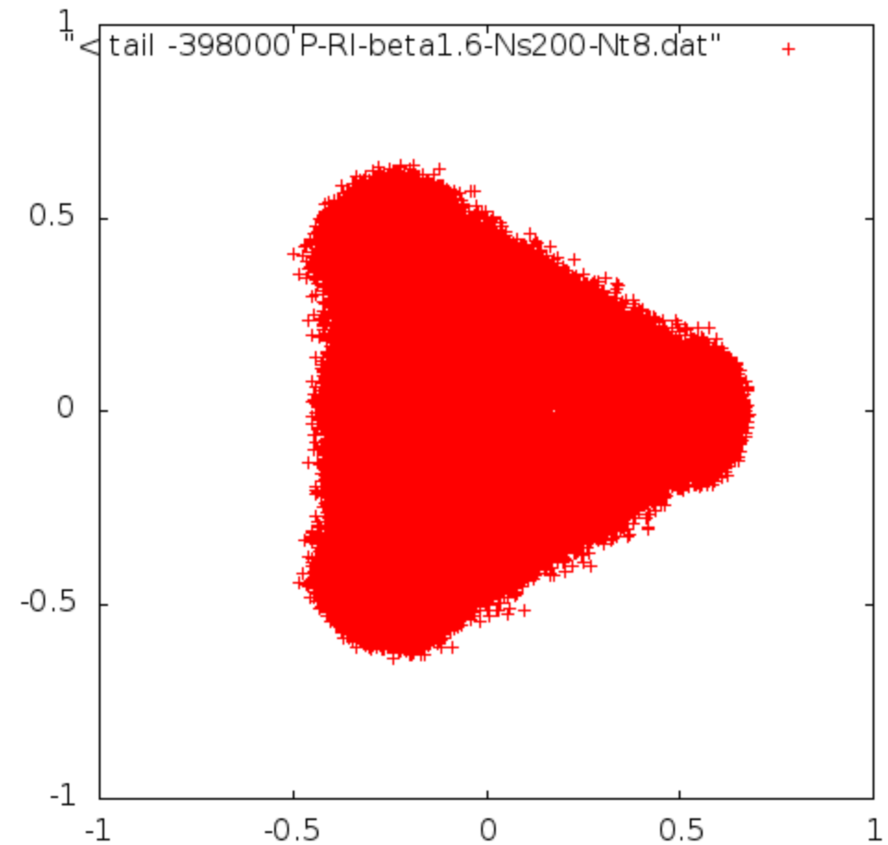
Im[P] N=3, β=0.1



Re[P]

$|\langle P \rangle| \sim 0$

N=3, β=1.6



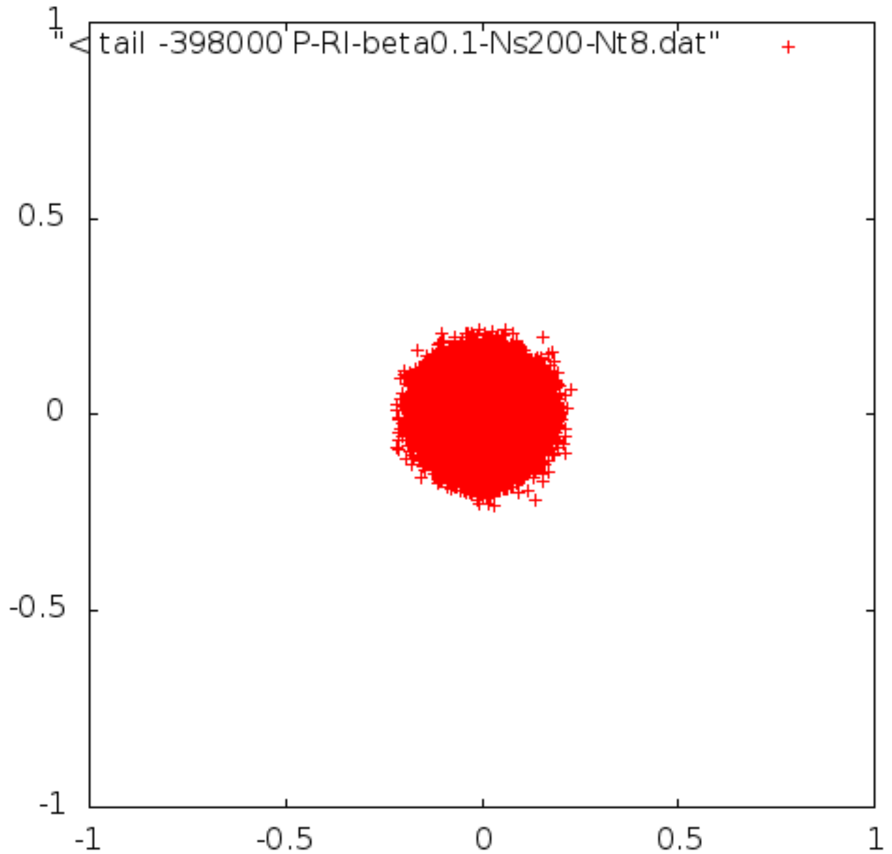
$|\langle P \rangle| \sim 0$

Low-β : around the origin → Z_N symmetry at the action level

Intermediate-β : Transition between N vacua → quantum Z_N symmetry

Distribution plot of P-loop

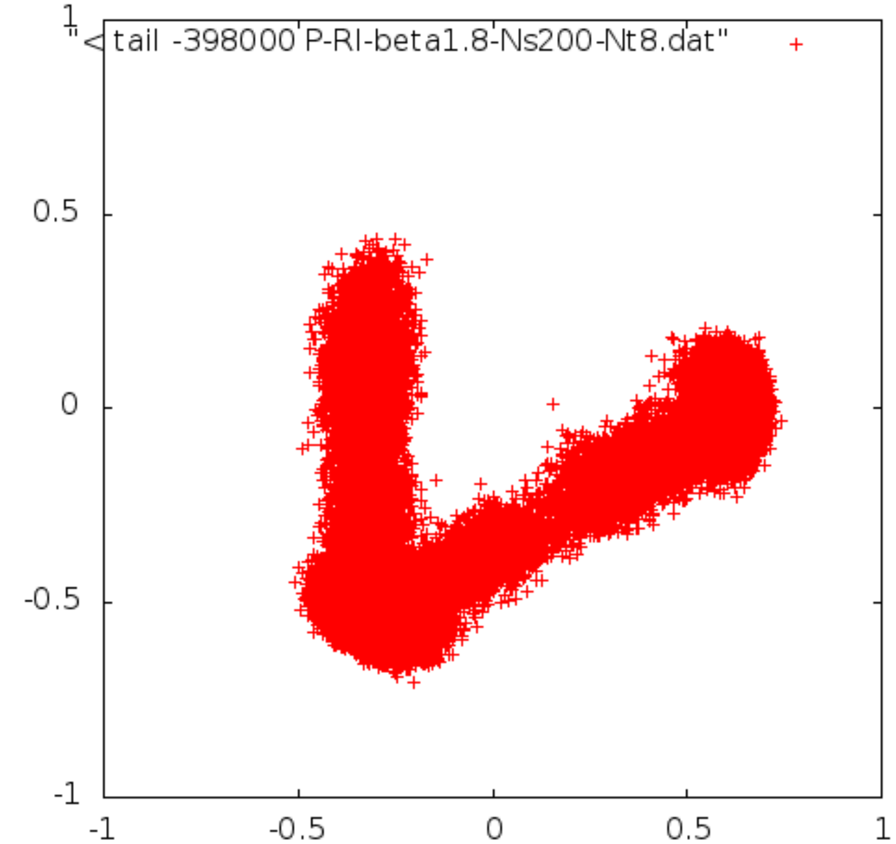
Im[P] N=3, β=0.1



Re[P]

$|\langle P \rangle| \sim 0$

N=3, β=1.8



$|\langle P \rangle| \neq 0$

Low-β : around the origin →

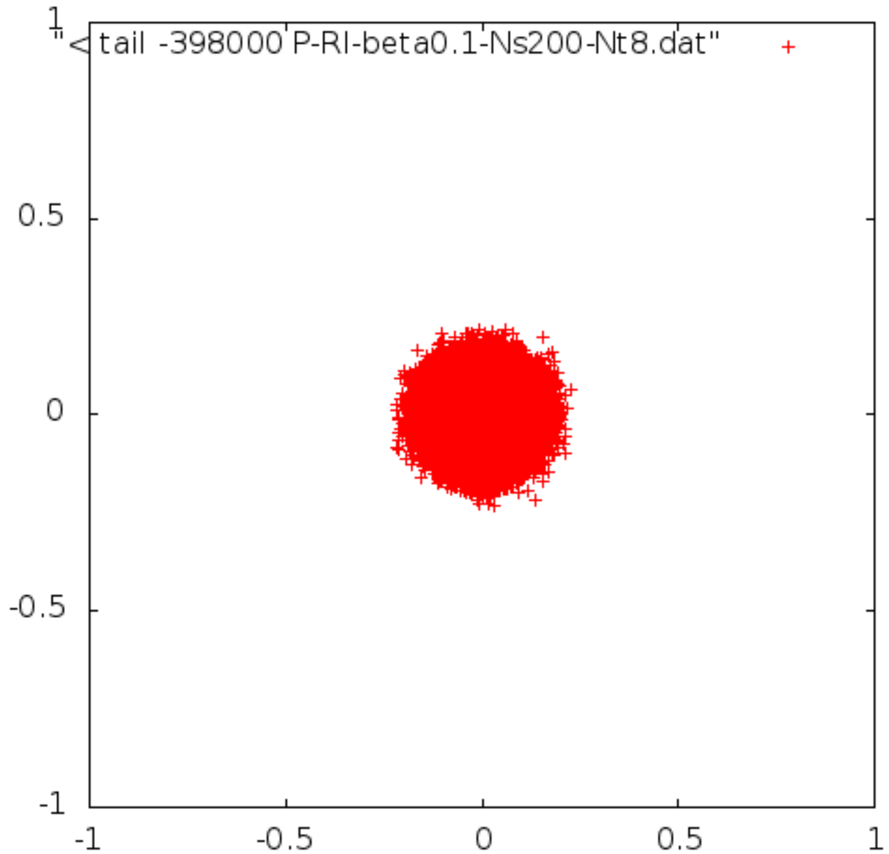
Z_N symmetry at the action level

High-β : One of Z_N vacua selected

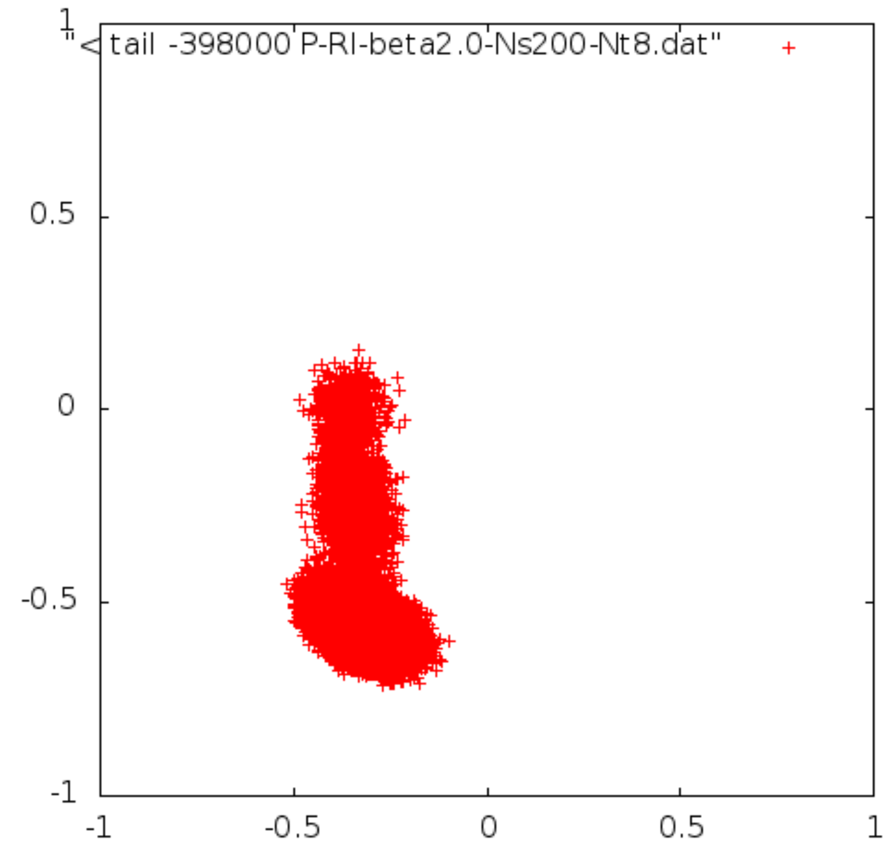
→ SSB of Z_N symmetry….?

Distribution plot of P-loop

Im[P] N=3, β=0.1



N=3, β=2.0



Re[P]

$|\langle P \rangle| \sim 0$

$|\langle P \rangle| \neq 0$

Low-β：around the origin →

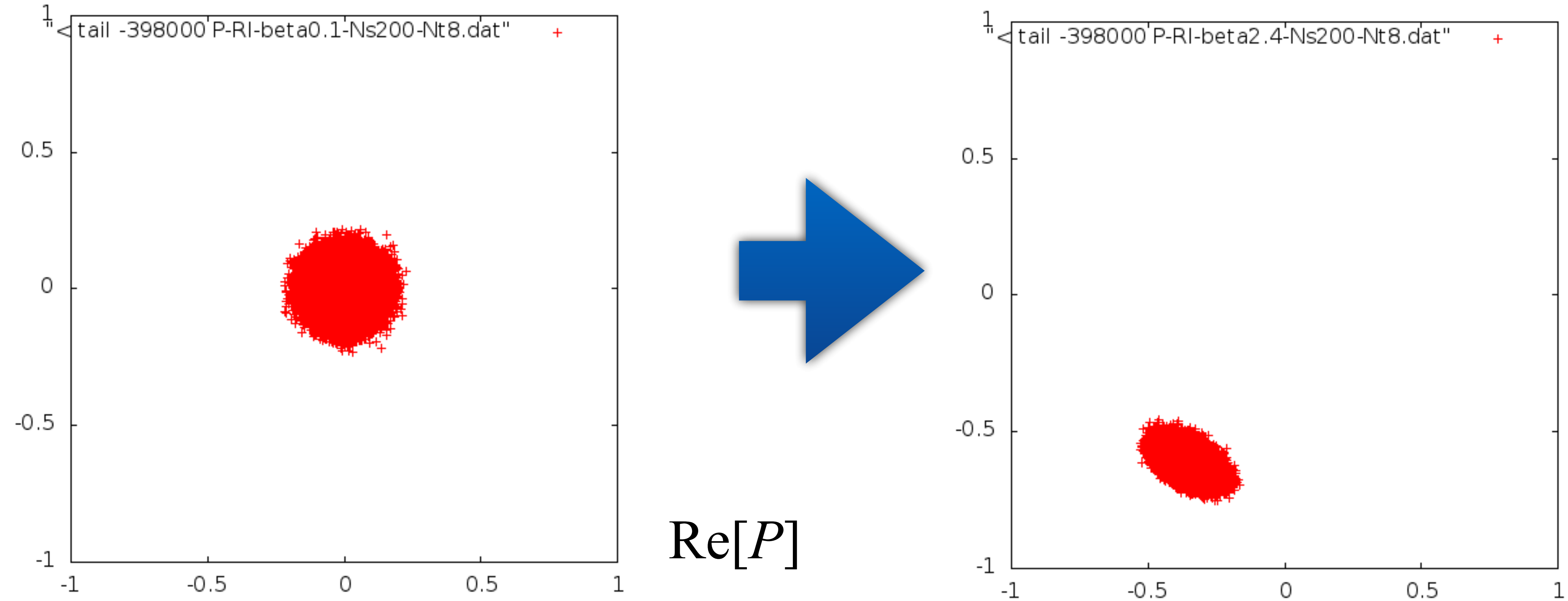
Z_N symmetry at the action level

High-β：One of Z_N vacua selected

→ SSB of Z_N symmetry….?

Distribution plot of P-loop

Im[P] **N=3, β=0.1** **N=3, β=2.4**

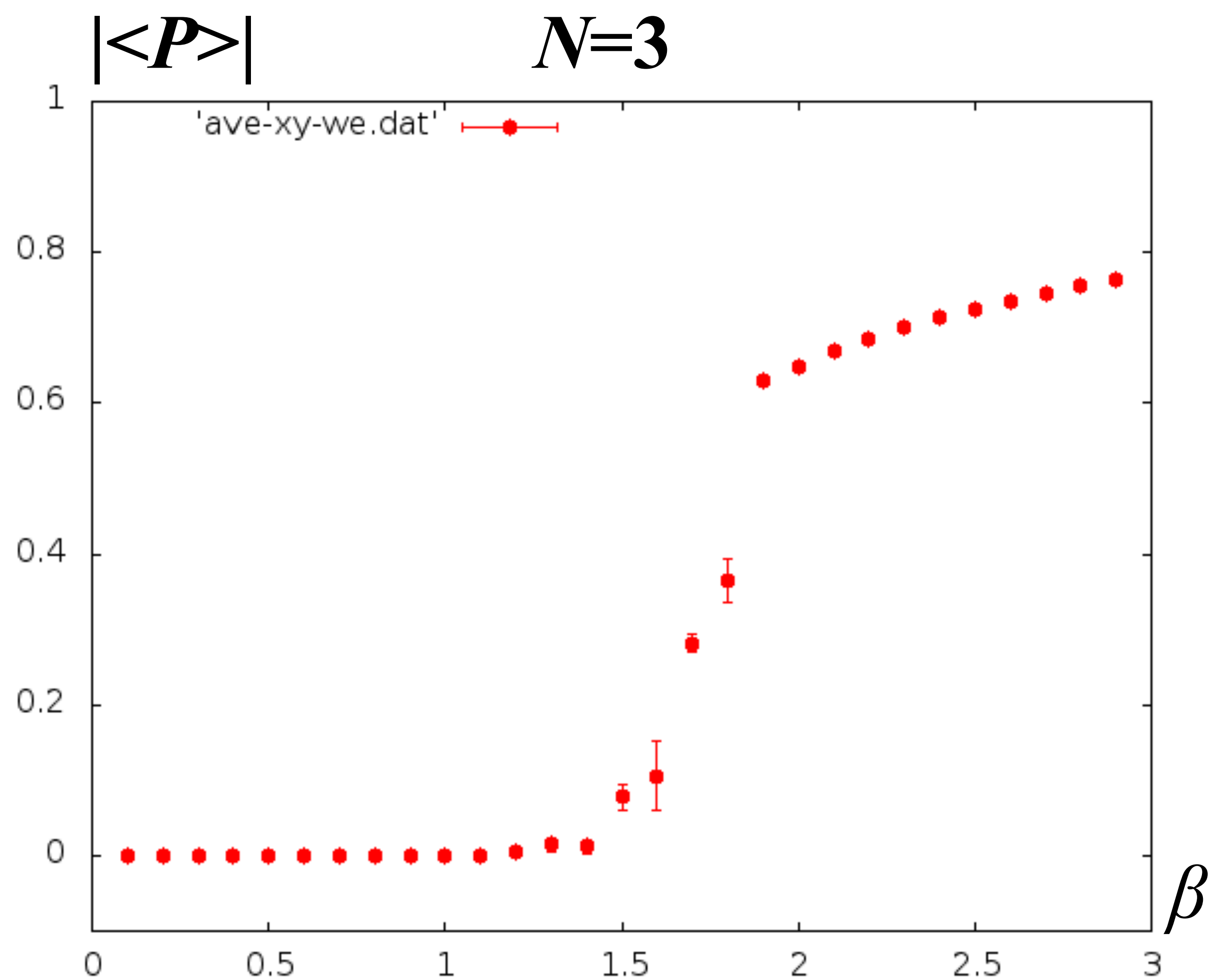


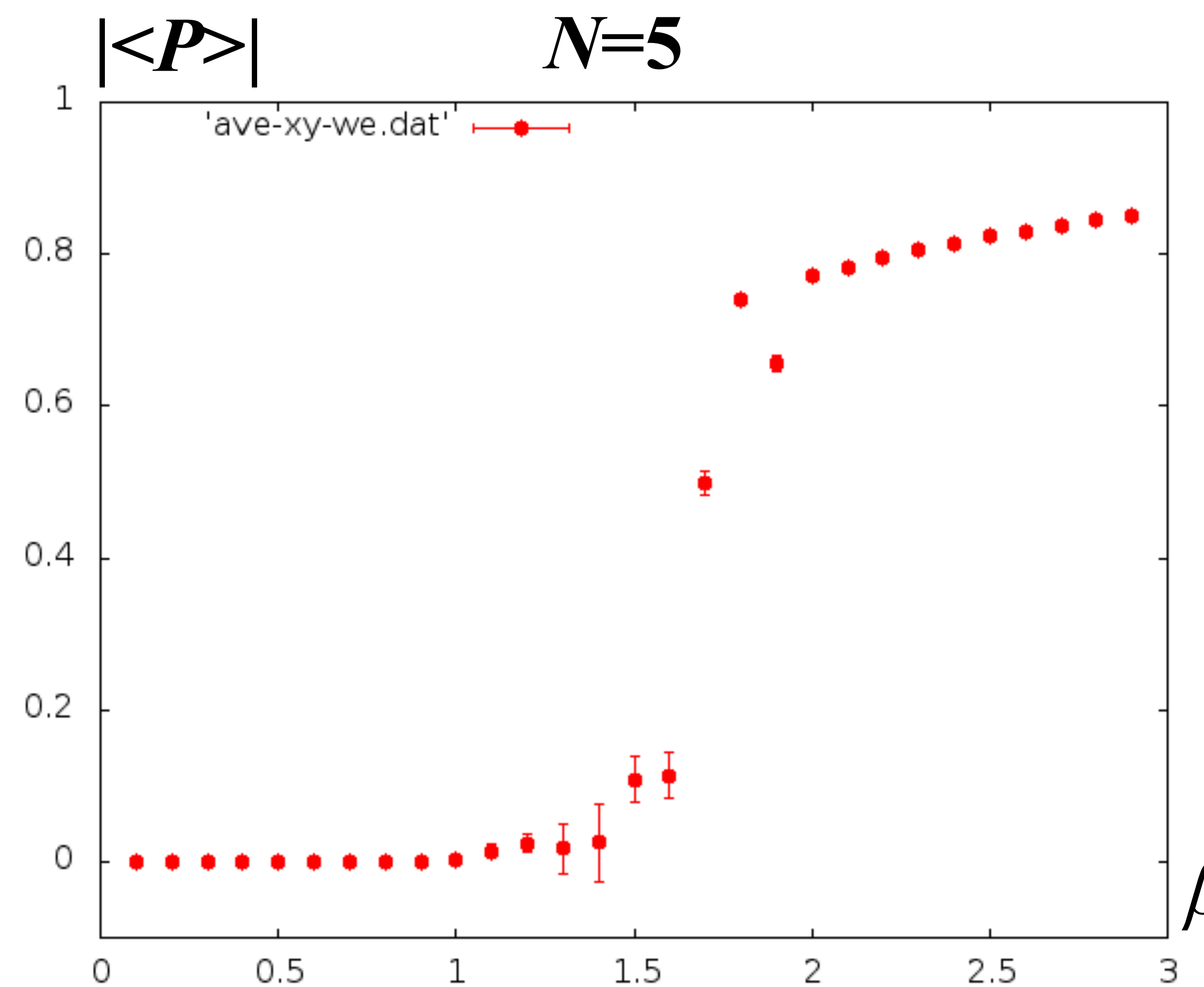
$|\langle P\rangle| \sim 0$ **$|\langle P\rangle| \neq 0$**

Low-β : around the origin →
Z_N symmetry at the action level

High-β : One of Z_N vacua selected
→ SSB of Z_N symmetry….?

VEV of Polyakov loop |<P>|

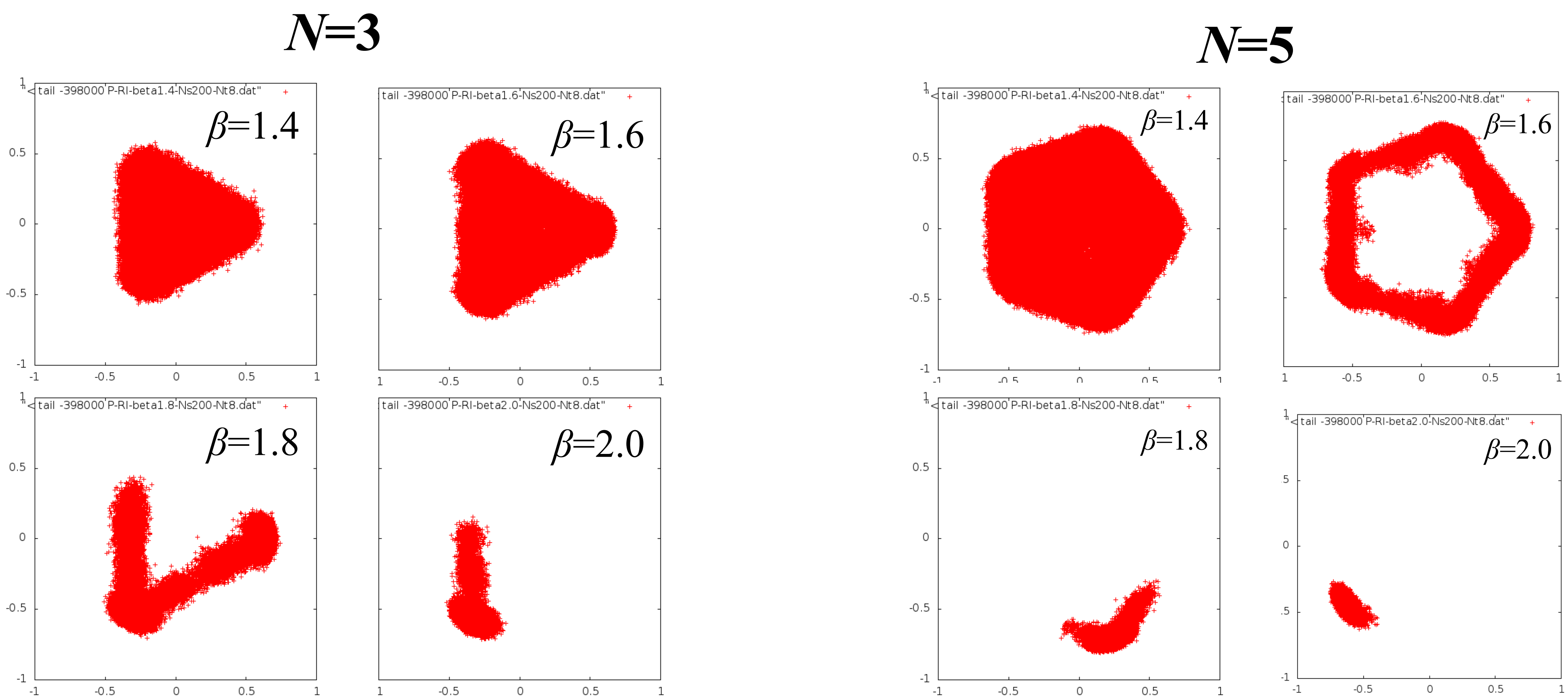




- Low β → |<P>|= 0 : distribution around origin
- Mid β → |<P>| highly fluctuates : distribution forms polygons
- High β → suddenly gets |<P>|≠ 0 : but more stat. can form polygon

This peculiar P-loop could imply something special (Z_N stability?).
We still need larger volume or more statistics to judge continuity.

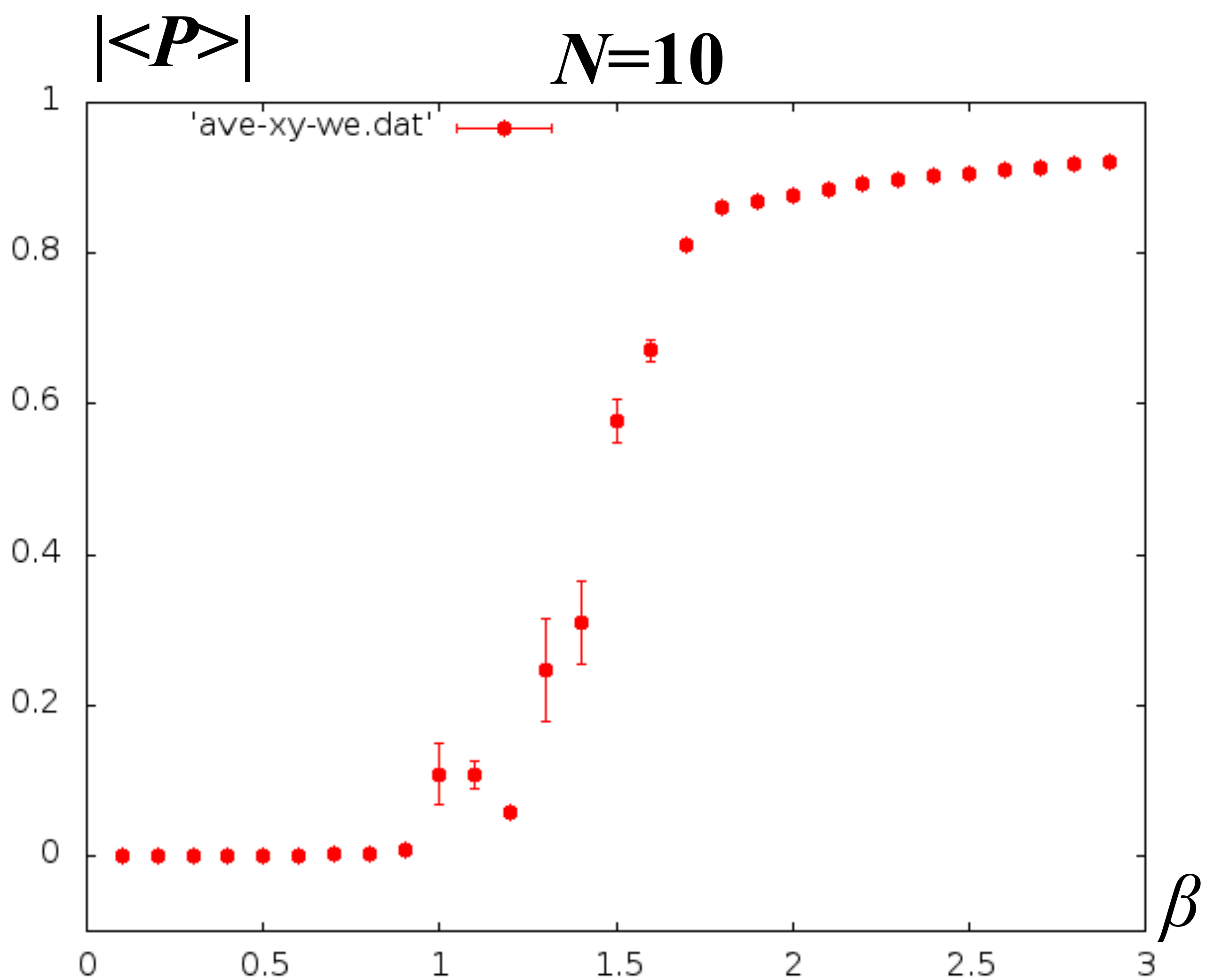
VEV of Polyakov loop |<P>|

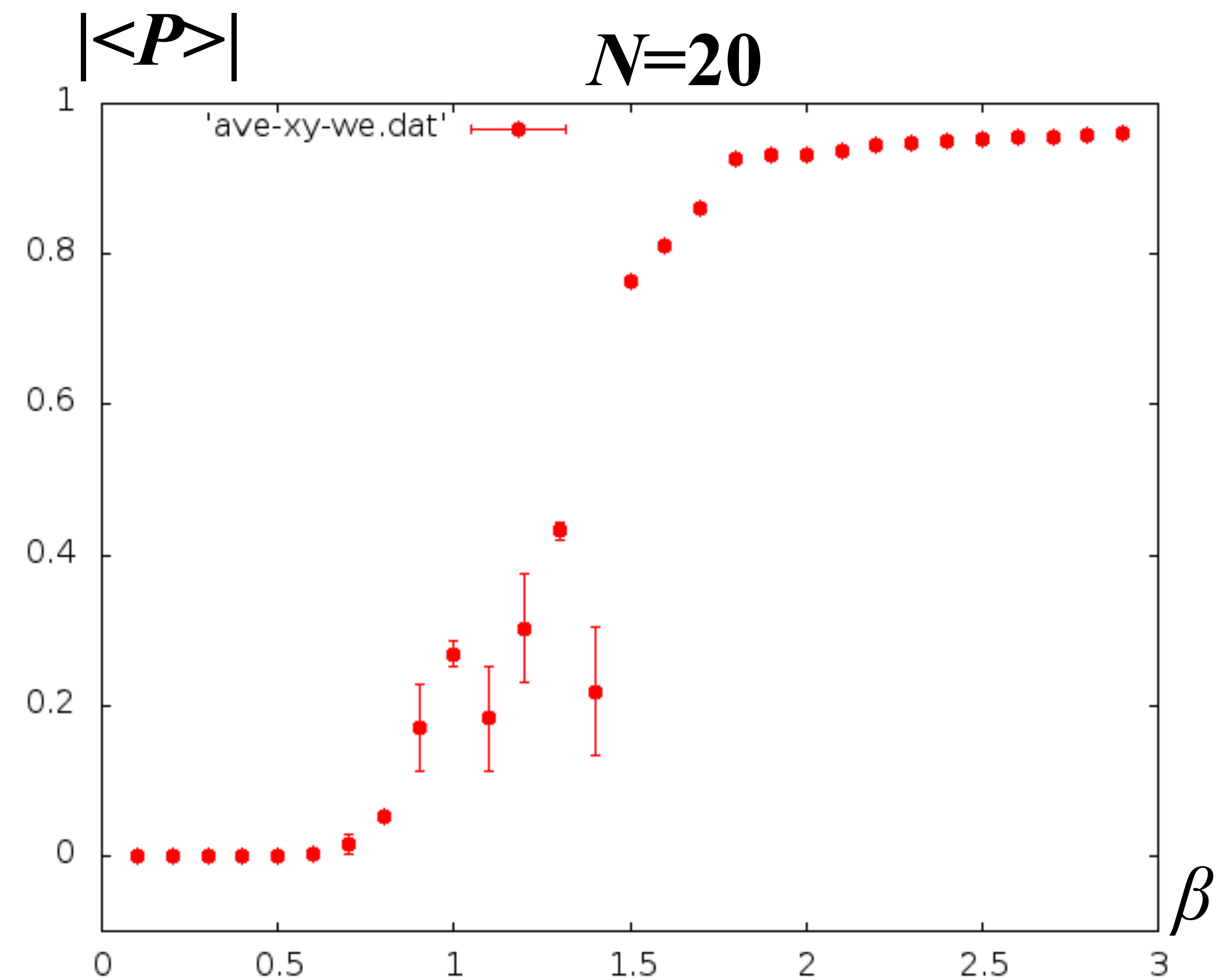


- Low β → |<P>|= 0 : distribution around origin
- Mid β → |<P>| highly fluctuates : distribution forms polygons
- High β → suddenly gets |<P>|≠ 0 : but more stat. can form polygon

This peculiar P-loop could imply something special (Z_N stability?).
We still need larger volume or more statistics to judge continuity.

VEV of Polyakov loop |<P>|

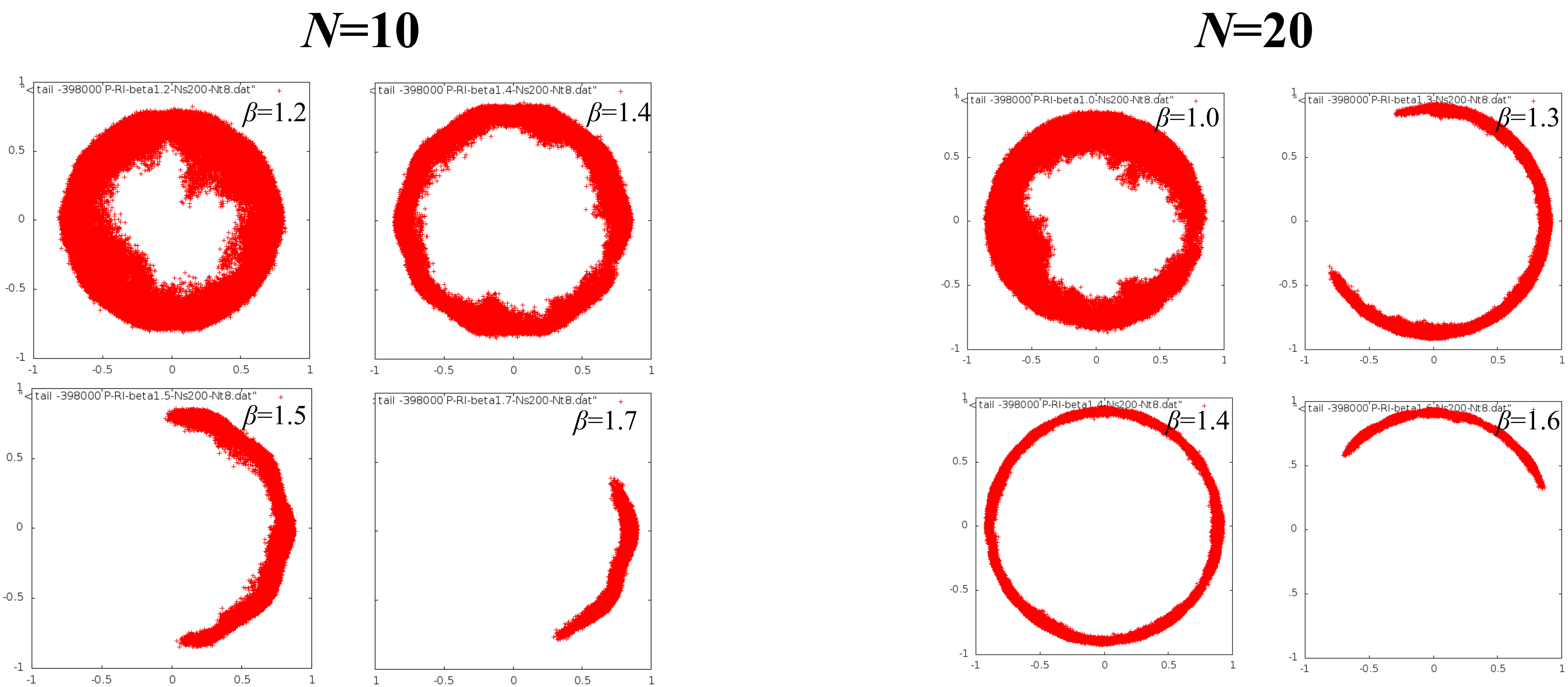




- Low β → |<P>|= 0 : distribution around origin
- Mid β → |<P>| highly fluctuates : distribution forms polygons
- High β → suddenly gets |<P>|≠ 0 : but more stat. can form polygon

This peculiar P-loop could imply something special (Z_N stability?).
We still need larger volume or more statistics to judge continuity.

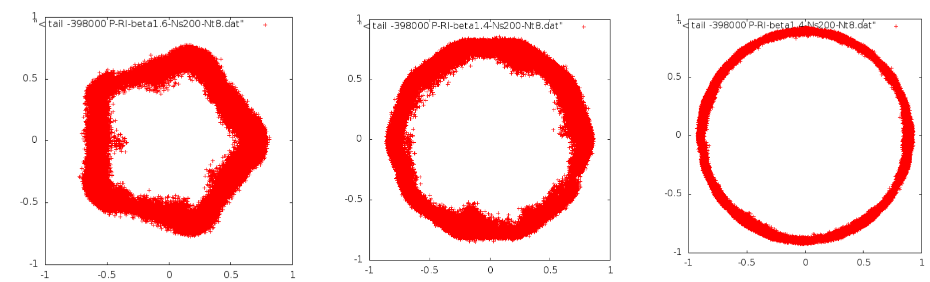
VEV of Polyakov loop |<P>|



- Low β → $|\langle P\rangle| = 0$: distribution around origin
- Mid β → $|\langle P\rangle|$ highly fluctuates : distribution forms polygons
- High β → suddenly gets $|\langle P\rangle| \neq 0$: but more stat. can form polygon

This peculiar P-loop could imply something special (Z_N stability?).
We still need larger volume or more statistics to judge continuity.

Polygon-shaped distributions of Polyakov loop (|<P>|~0)
appear more often with more statistics



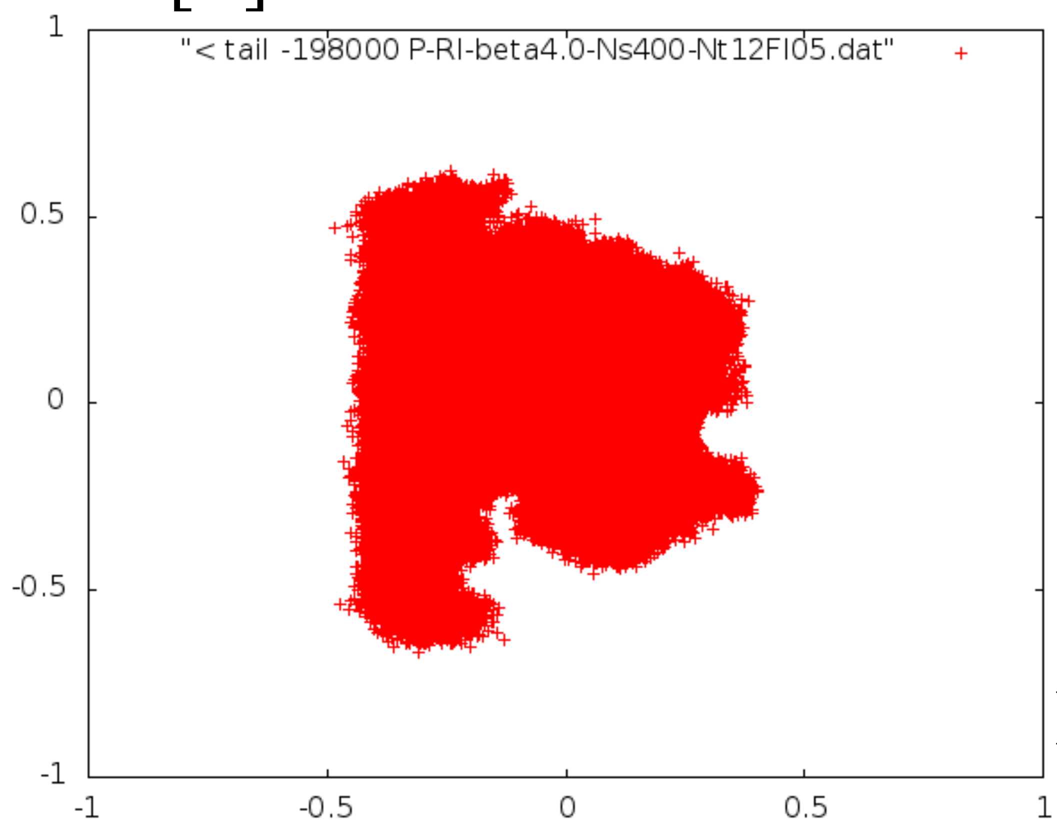
It may indicate Z_N stability (continuity)….

Furthermore,

Distribution plot of P-loop (very high β, large volume)

Independent configurations for very high β (β=4.0) with large volume include a quantum Z_N symmetric case as below !

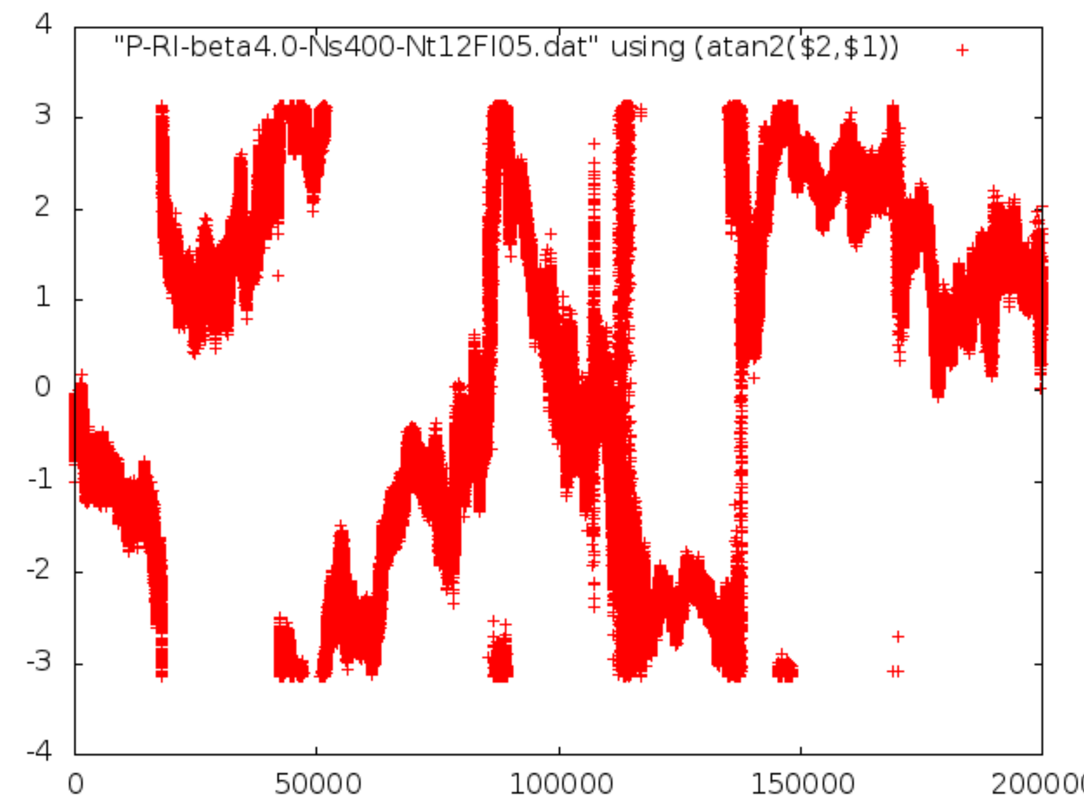
Im[*P*] ***N*=3, β=4.0, (400×12)**



Re[*P*]

|<*P*>| ~ small

Hysteresis of arg[*P*]



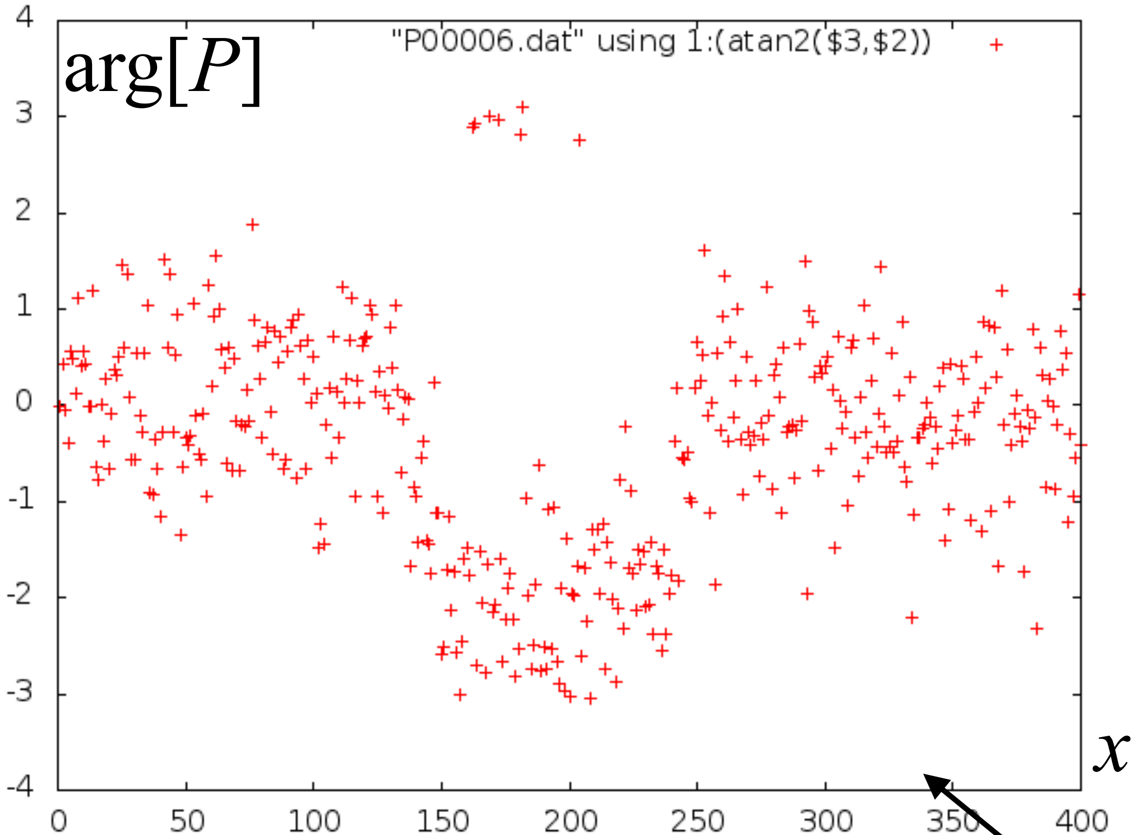
Any of Z_N vacua is not selected

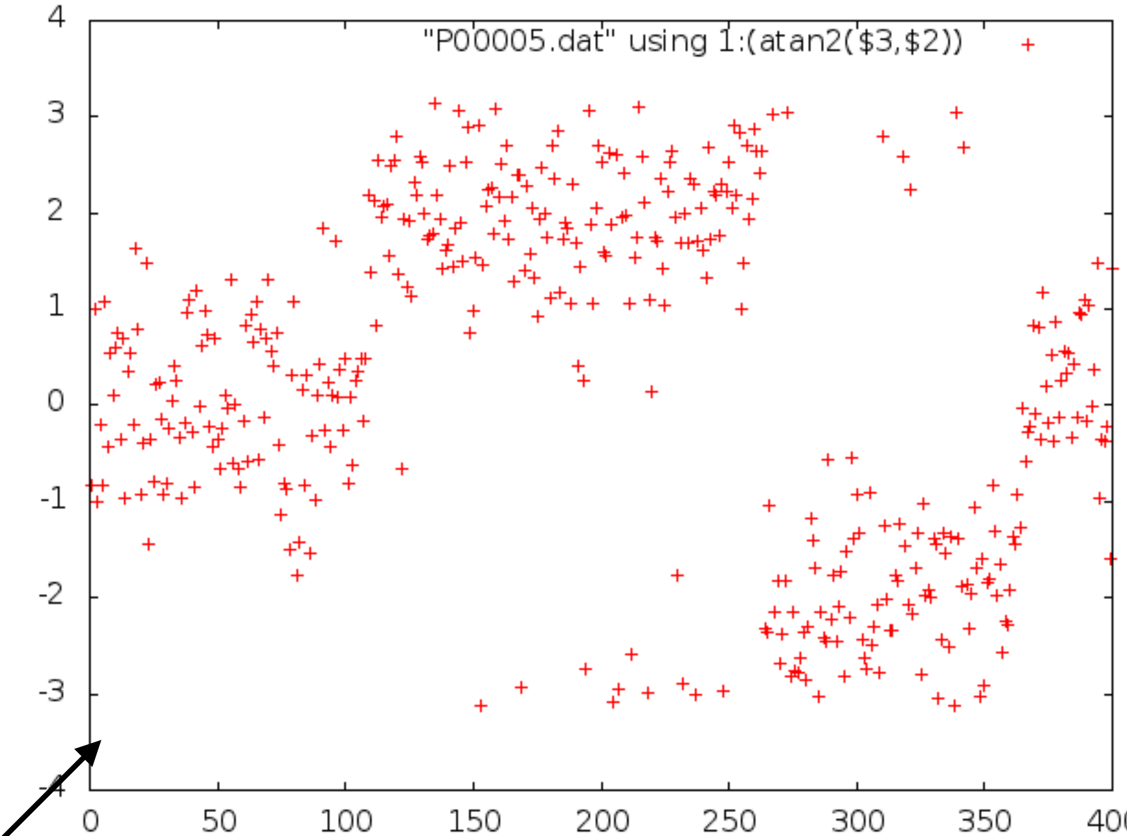
Very high-β : quantum Z_N symmetric case found with certain probability

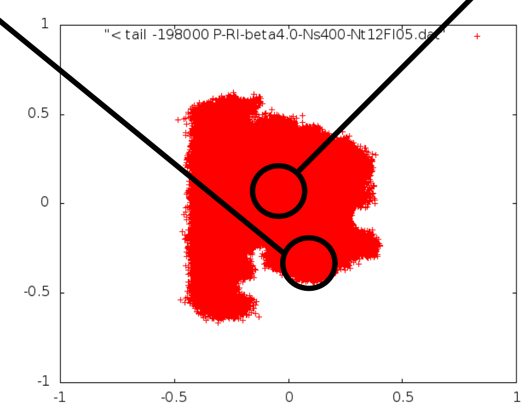
it seems we need larger volume or more statistics for Z_N continuity….

Fractional instantons

Pick up two of configurations and look into the x-dependence of arg[P]







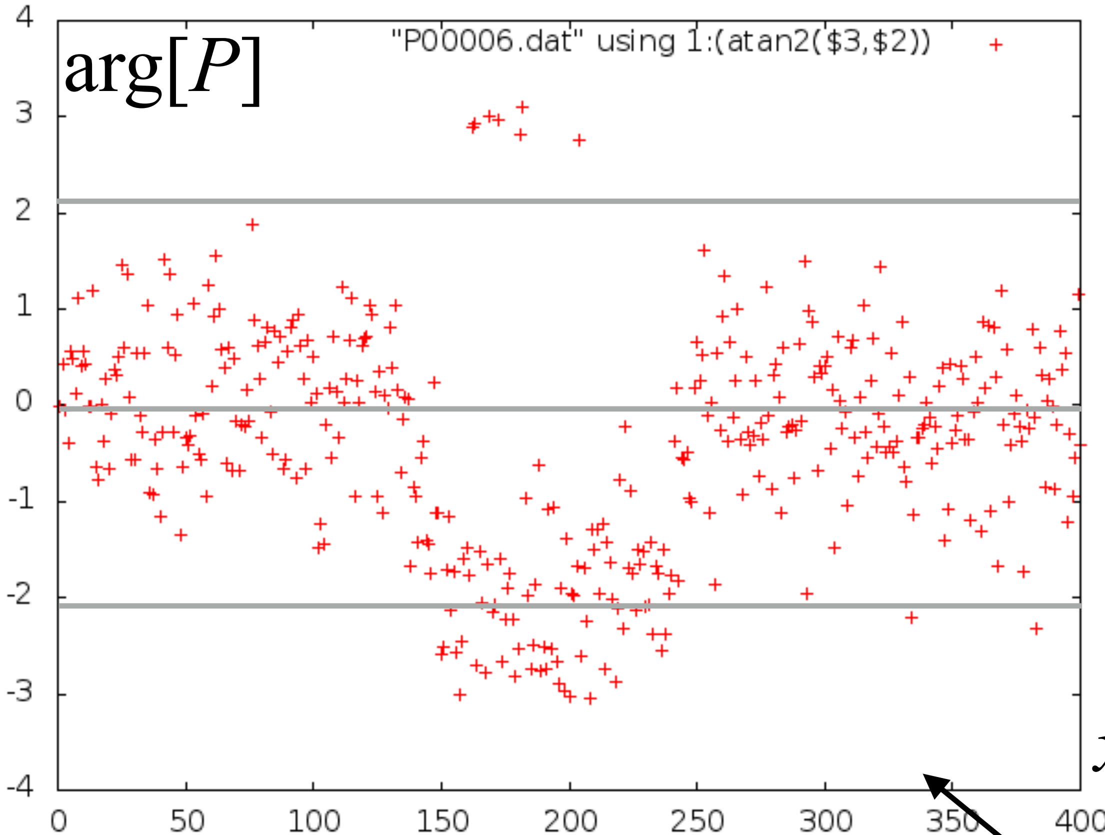
1/3 fractional antiintanton +
1/3 fractional instanton
= **bion**

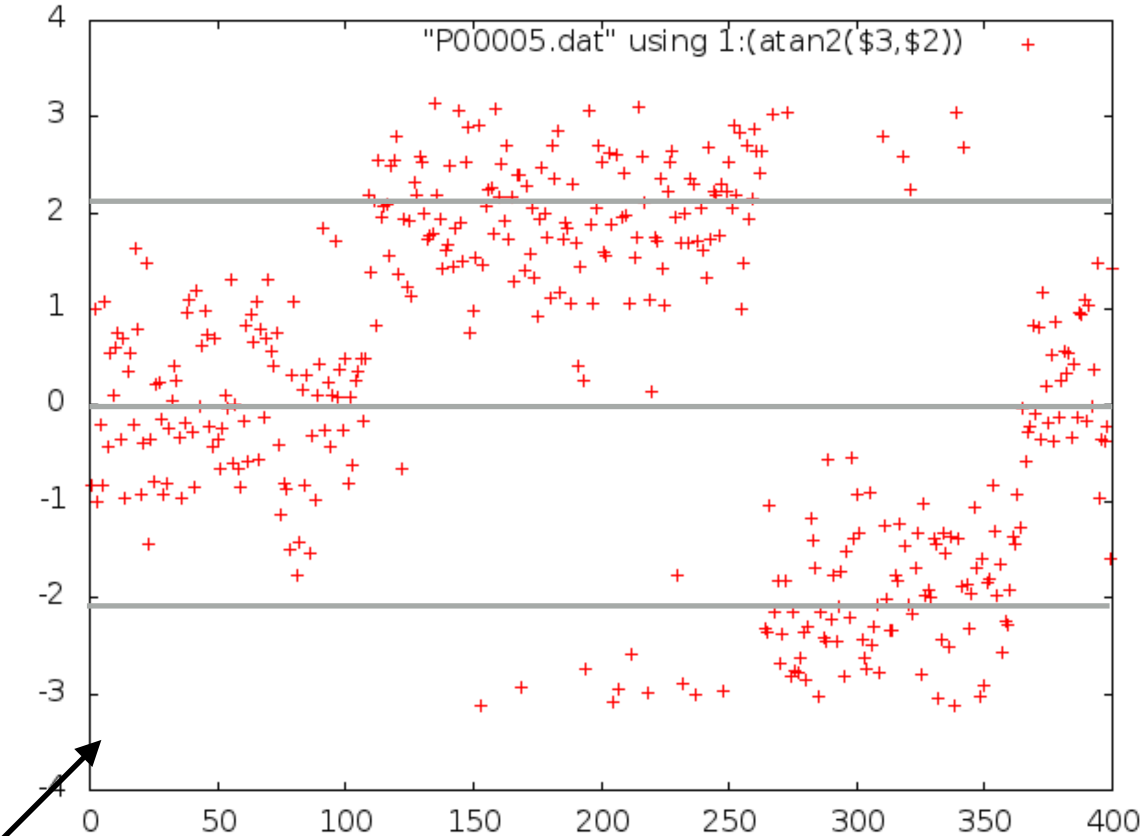
3 × 1/3 fractional intantons
= **instanton**

implies fractional instantons cause transition between classical vacua at high β, which lead to quantum Z_N symmetry and could yield adiabatic continuity

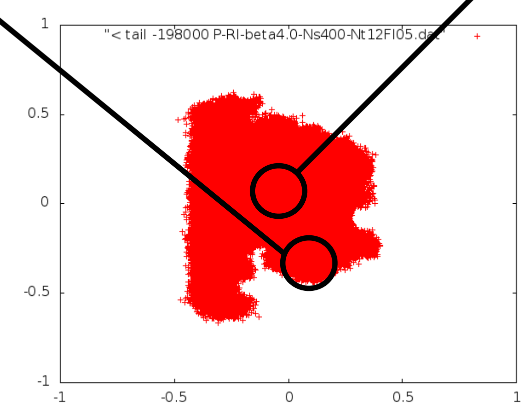
Fractional instantons

Pick up two of configurations and look into the x-dependence of arg[P]





1/3 fractional antiintanton +
1/3 fractional instanton
= **bion**

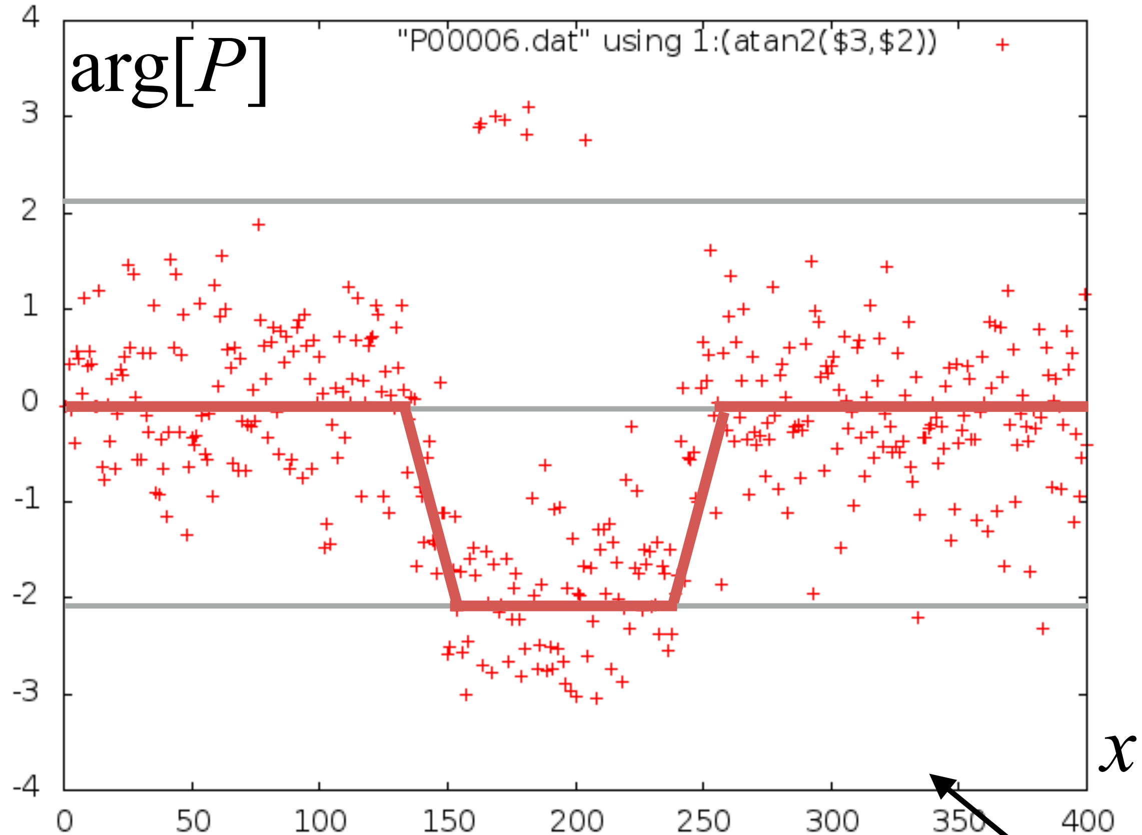


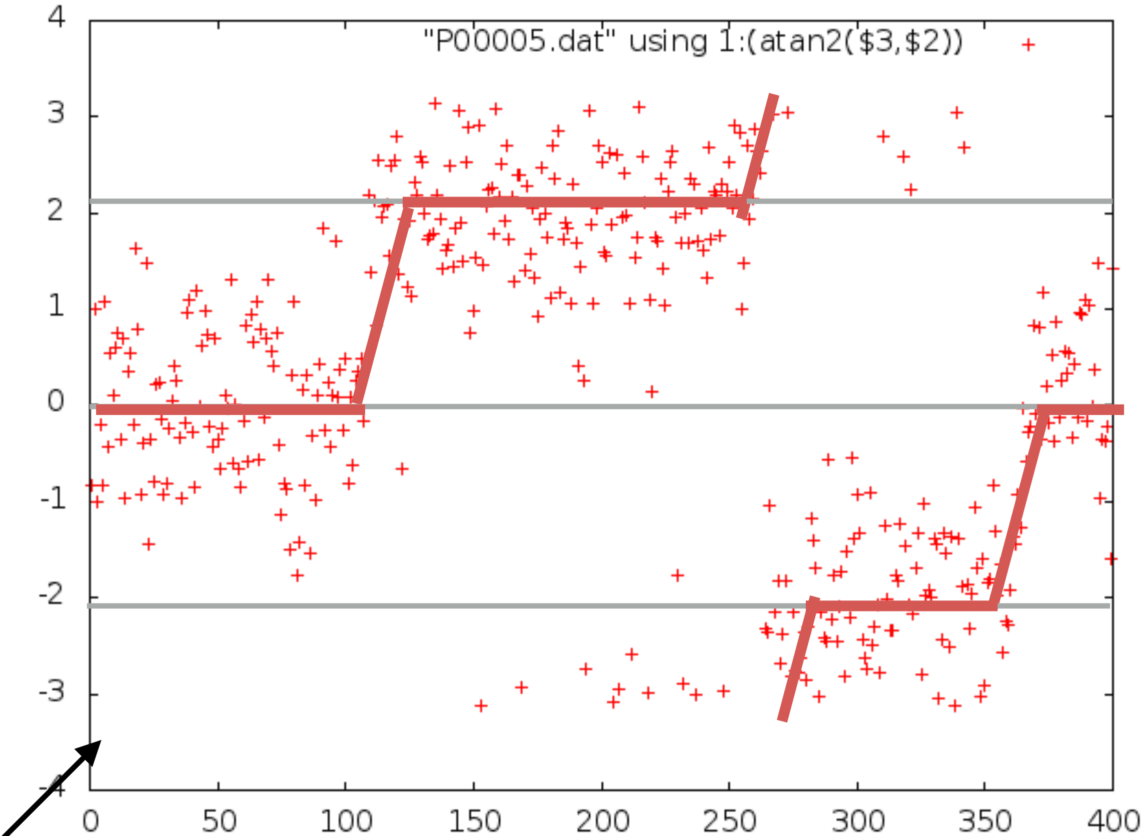
3 × 1/3 fractional intantons
= **instanton**

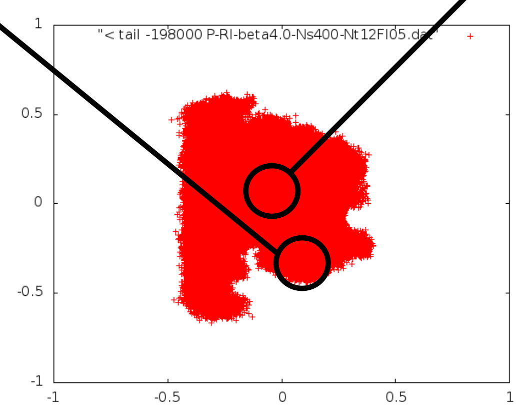
implies fractional instantons cause transition between classical vacua at high β, which lead to quantum Z_N symmetry and could yield adiabatic continuity

Fractional instantons

Pick up two of configurations and look into the x-dependence of arg[P]







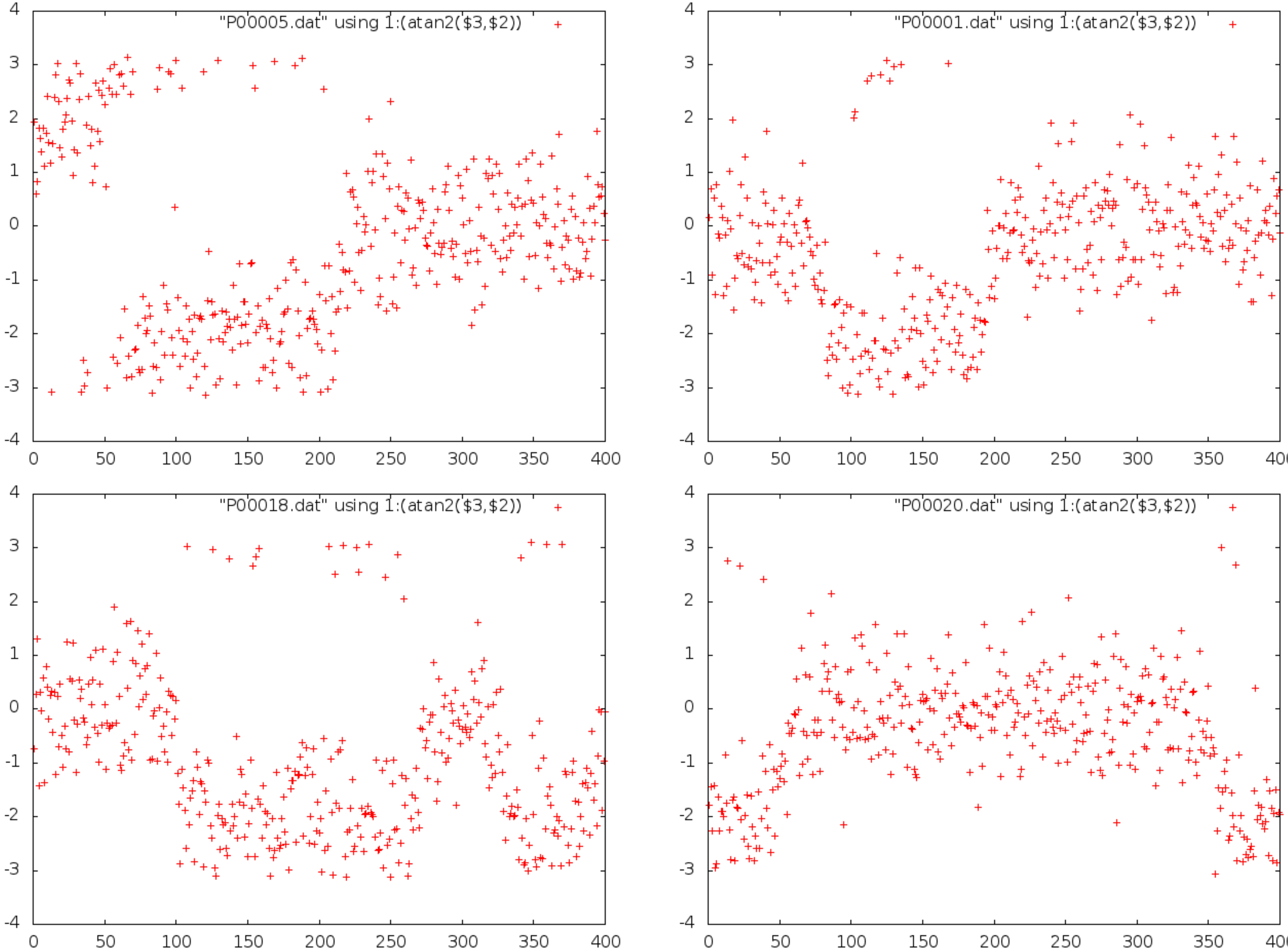
1/3 fractional antiintanton +
1/3 fractional instanton
= **bion**

3 × 1/3 fractional intantons
= **instanton**

implies fractional instantons cause transition between classical vacua at high β, which lead to quantum Z_N symmetry and could yield adiabatic continuity

＊we are on the way of calculating topological charge density directly.

Fractional instantons



Thermal entropy for pbc and Z_N tbc.

N=3,5,10,20 (Nx,Nt) = (200, 8) Nsweep=200000, 400000

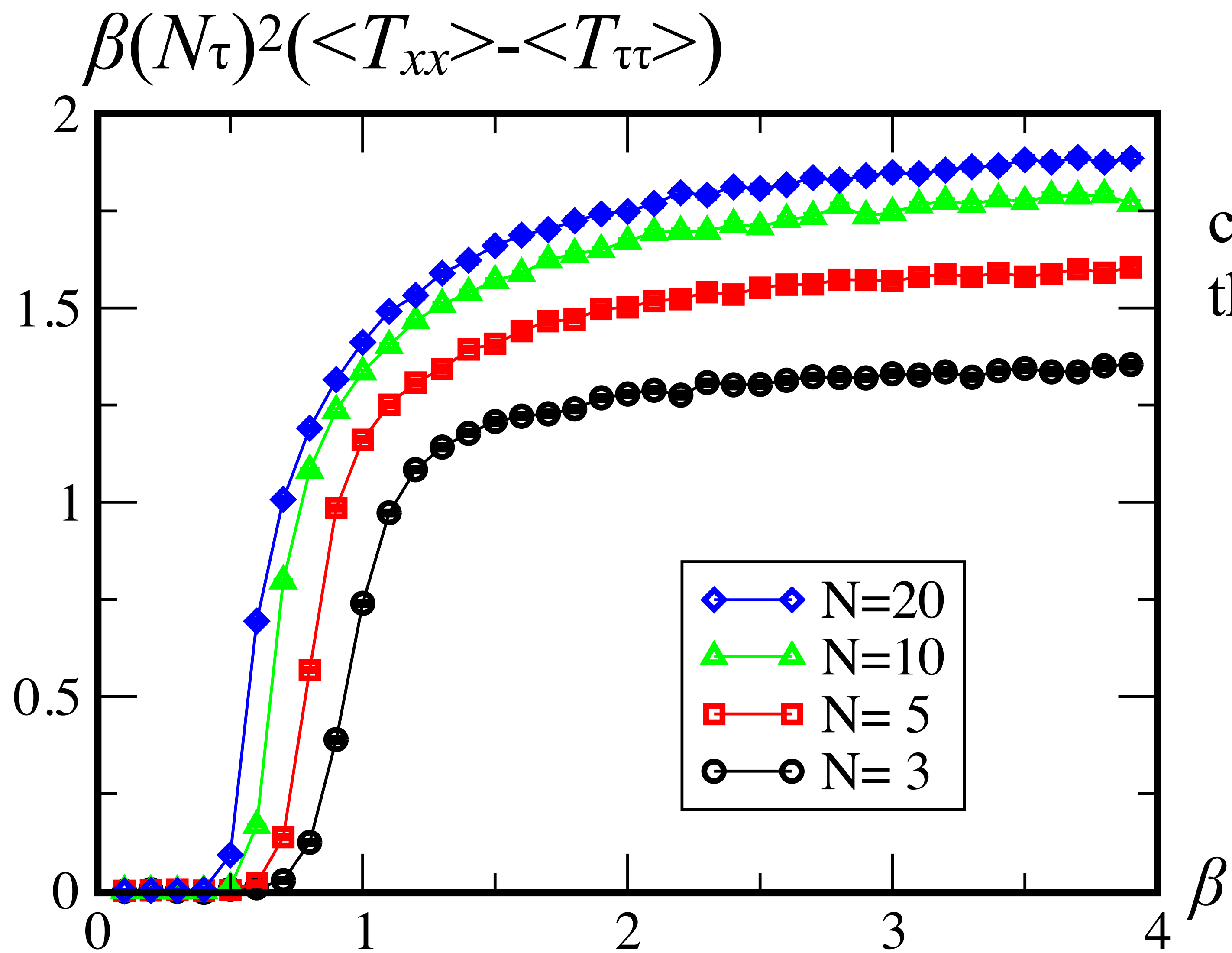
◆Question 3 : Thermal entropy

- Free energy of CP^∞ and free energy of free scalar for small L also indicate Monin, Shifman, Yung(15)

$$s = \beta N_\tau^2 (\langle T_{xx} \rangle - \langle T_{\tau\tau} \rangle) = \frac{2\pi(N-1)}{3N}$$

We will check thermal entropy for large β (small L)

Thermal entropy for pbc



consistent with the prediction $\frac{2\pi(N-1)}{3N}$

↓

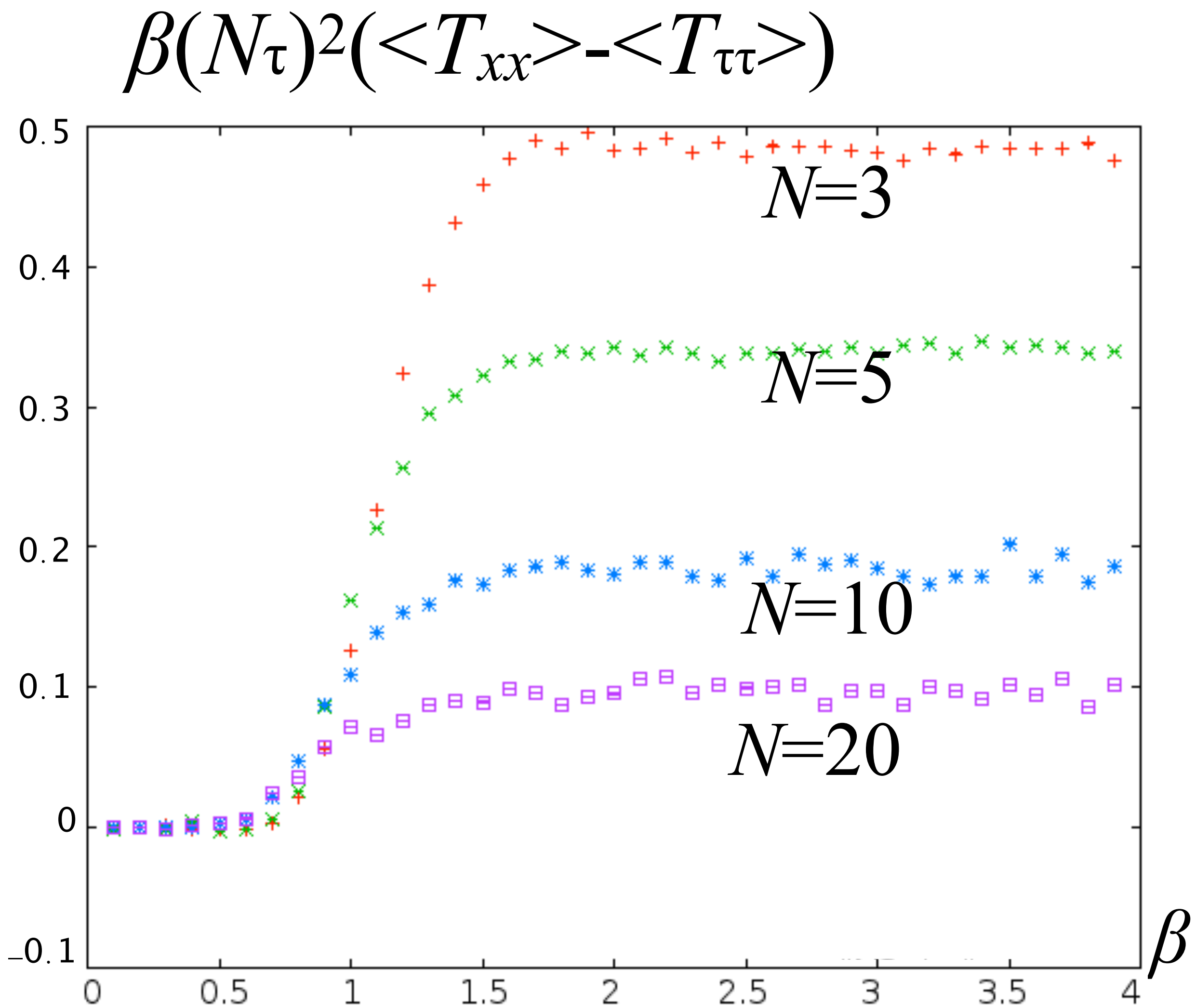
$$N = 3 \ : \ 1.396$$
$$N = 5 \ : \ 1.675$$
$$N = 10 \ : \ 1.885$$
$$N = 20 \ : \ 1.990$$

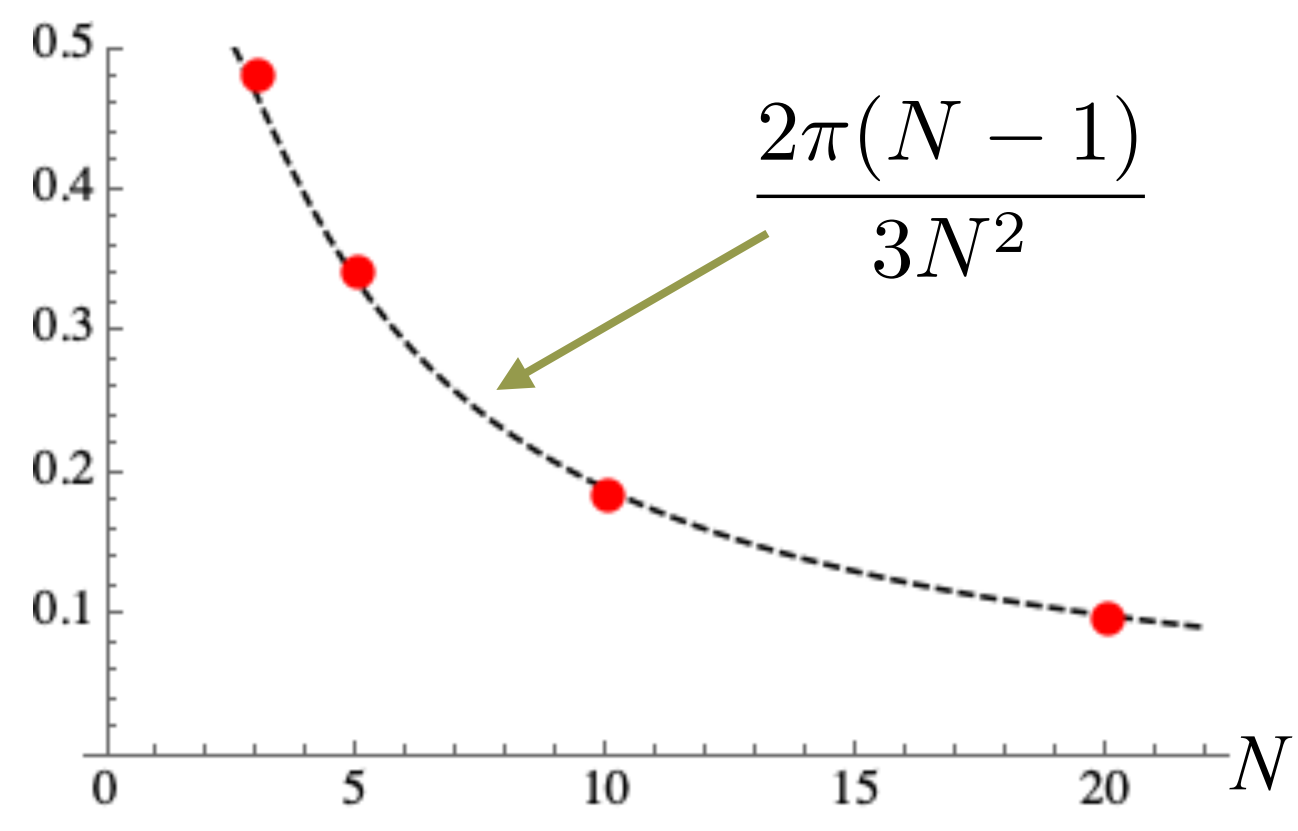
- Thermal entropy is in agreement with the analytical prediction.
- This is also consistent with the prediction from YM+Higgs model.

Monin, Shifman, Yung(15)

Our numerical results successfully confirm the predicted thermal entropy

Thermal entropy for tbc





- Thermal entropy behaves 1/N smaller than that of PBC.
- This observation should be checked analytically.

Prediction from numerical study which should be reproduced analytically

Summary

- Lattice simulation of CP^{N-1} model on R x S^1
- Z_N crossover transition is confirmed for pbc
- Thermal entropy agrees with the prediction for pbc
- Characteristic β dependence of P-loop for tbc, which inspires more study on adiabatic continuity
- A pivotal role of fractional instantons is implied for tbc