Phase structure and real-time dynamics of the massive Thirring model in 1+1 dimensions using tensor-network methods

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Lattice 2019
Wuhan
18/06/2019

The 1+1 dimensional Thirring model and its duality to the sine-Gordon model

$$S_{\text{Th}}\left[\psi,\bar{\psi}\right] = \int d^2x \left[\bar{\psi} i \gamma^{\mu} \partial_{\mu} \psi - m_0 \bar{\psi}\psi - \frac{g}{2}\left(\bar{\psi}\gamma_{\mu}\psi\right)^2\right]$$

strong-weak duality $g \leftrightarrow t$

Thirring	sine-Gordon
$\bar{\psi}\gamma_{\mu}\psi$	$\frac{1}{2\pi}\epsilon_{\mu\nu}\partial_{\nu}\phi$
$\bar{\psi}\psi$	$\frac{\Lambda}{\pi}cos\phi$

$$S_{\text{SG}}\left[\phi\right] = \int d^2x \left[\frac{1}{2}\partial_{\mu}\phi(x)\partial^{\mu}\phi(x) + \frac{\alpha_0}{t}\cos\left(\sqrt{t}\phi(x)\right)\right]$$

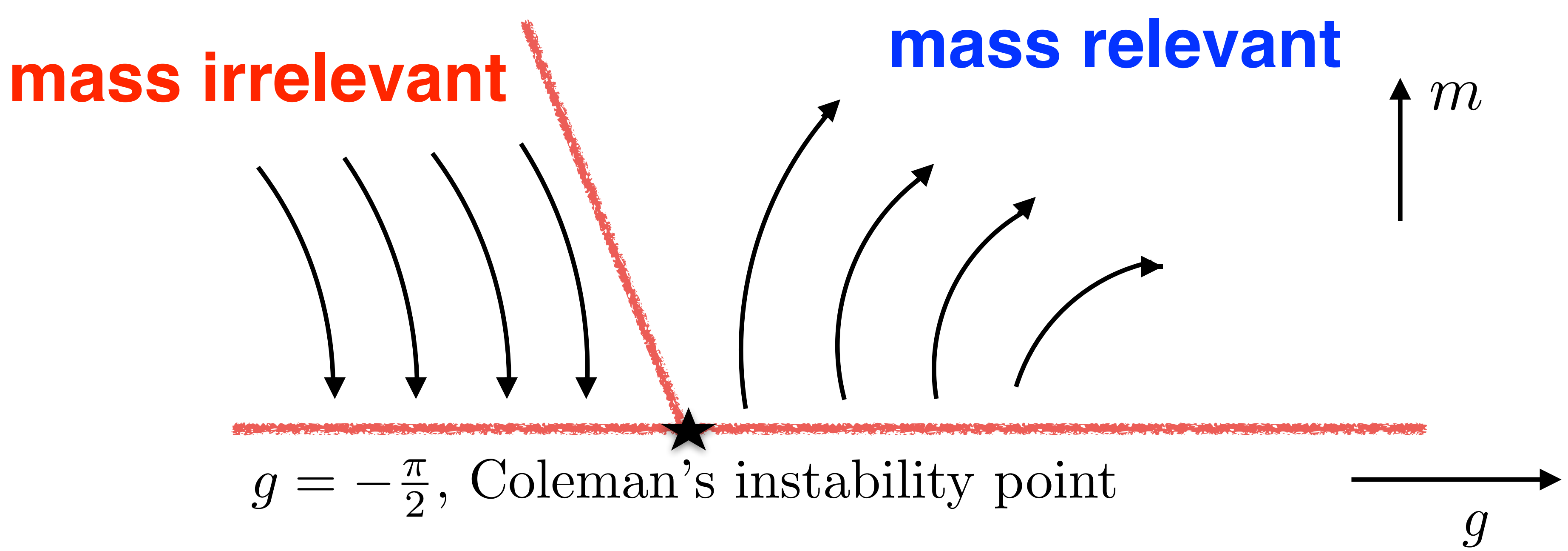
$$\xrightarrow{\phi\to\phi/\sqrt{t}} \frac{1}{t}\int d^2x \left[\frac{1}{2}\partial_{\mu}\phi(x)\partial^{\mu}\phi(x) + \alpha_0\cos\left(\phi(x)\right)\right]$$

Works in the zero-charge sector

RG flows of the Thirring model

$$\beta_g \equiv \mu \frac{dg}{d\mu} = -64\pi \frac{m^2}{\Lambda^2},$$
$$\beta_m \equiv \mu \frac{dm}{d\mu} = \frac{-2(g+\frac{\pi}{2})}{g+\pi} m - \frac{256\pi^3}{(g+\pi)^2\Lambda^2} m^3$$

★ Massless Thirring model is a conformal field theory



Operator formalism and the Hamiltonian

- Operator formaliam of the Thirring model Hamiltonian

C.R. Hagen, 1967

$$H_{\rm Th} = \int dx \left[-i\bar{\psi}\gamma^1\partial_1\psi + m_0\bar{\psi}\psi + \frac{g}{4}\left(\bar{\psi}\gamma^0\psi\right)^2 - \frac{g}{4}\left(1+\frac{2g}{\pi}\right)^{-1}\left(\bar{\psi}\gamma^1\psi\right)^2 \right]$$

- Staggering, J-W transformation ($S_j^{\pm} = S_j^x \pm iS_j^y$):

J. Kogut and L. Susskind, 1975; A. Luther, 1976

$$\bar{H}_{XXZ} = \nu(g)\left[-\frac{1}{2}\sum_n^{N-2}\left(S_n^+ S_{n+1}^- + S_{n+1}^+ S_n^-\right) + a\tilde{m}_0 \sum_n^{N-1}(-1)^n\left(S_n^z + \frac{1}{2}\right) + \Delta(g)\sum_n^{N-1}\left(S_n^z+\frac{1}{2}\right)\left(S_{n+1}^z + \frac{1}{2}\right)\right]$$

$$\nu(g) = \frac{2\gamma}{\pi\sin(\gamma)},\ \tilde{m}_0 = \frac{m_0}{\nu(g)},\ \Delta(g) = \cos(\gamma),\ \text{with } \gamma = \frac{\pi - g}{2}$$

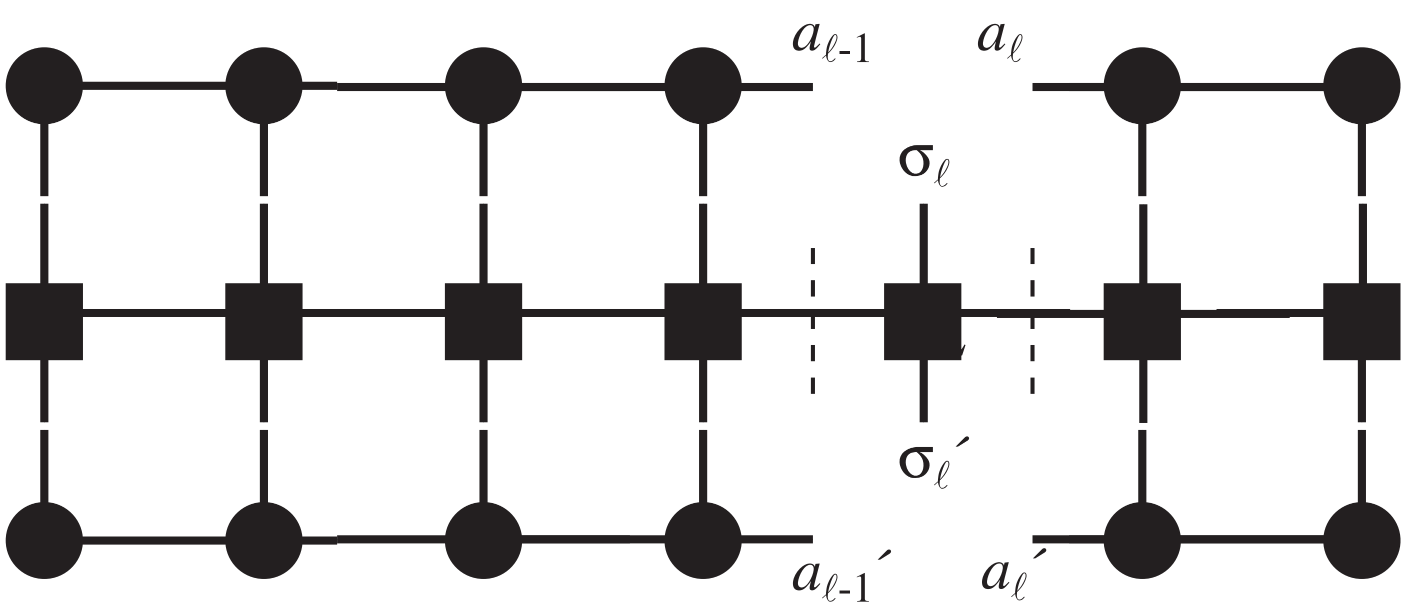
$$\bar{H}_{\rm sim} = \frac{\bar{H}_{XXZ}}{\nu(g)} + \lambda\left(\sum_{n=0}^{N-1} S_n^z - S_{\rm target}\right)^2$$

projected to a sector of total spin

JW-trans of the total fermion number,
Bosonise to topological index in the SG theory.

Practice of finite MPS

One step in a sweep of finite-size DMRG



- ★ Open BC
- ★ Random tensors for the smallest bond dim

Simulation details for the phase structure

- Matrix product operator for the Hamiltonian (bulk)

$$W^{[n]} = \begin{pmatrix} 1_{2\times 2} & -\frac{1}{2}S^{+} & -\frac{1}{2}S^{-} & 2\lambda S^{z} & \Delta S^{z} & \beta_n S^{z} + \alpha 1_{2\times 2} \\ 0 & 0 & 0 & 0 & 0 & S^{-} \\ 0 & 0 & 0 & 0 & 0 & S^{+} \\ 0 & 0 & 0 & 1 & 0 & S^{z} \\ 0 & 0 & 0 & 0 & 0 & S^{z} \\ 0 & 0 & 0 & 0 & 0 & 1_{2\times 2} \end{pmatrix}$$

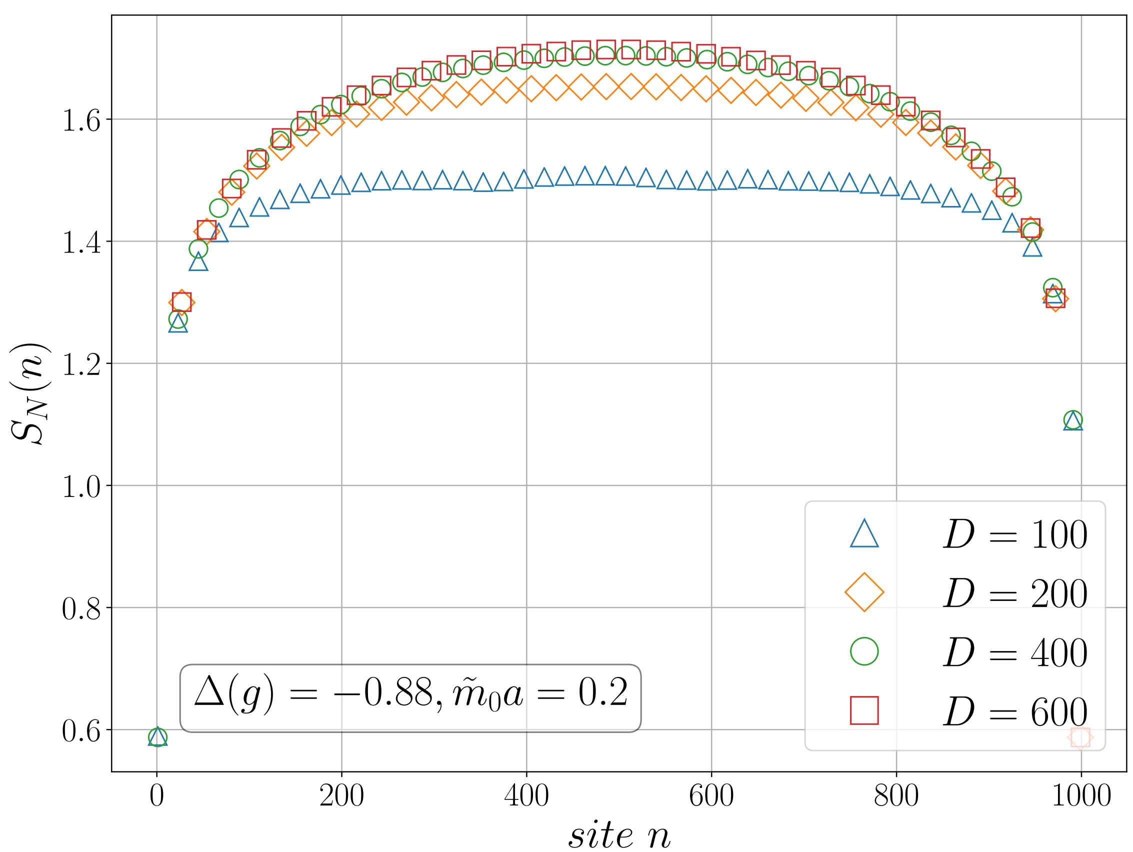
$$\beta_n = \Delta + (-1)^n \tilde{m}_0 a - 2\lambda\, S_{\text{target}}\,,\ \alpha = \lambda\left(\frac{1}{4} + \frac{S^2_{\text{target}}}{N}\right) + \frac{\Delta}{4}$$

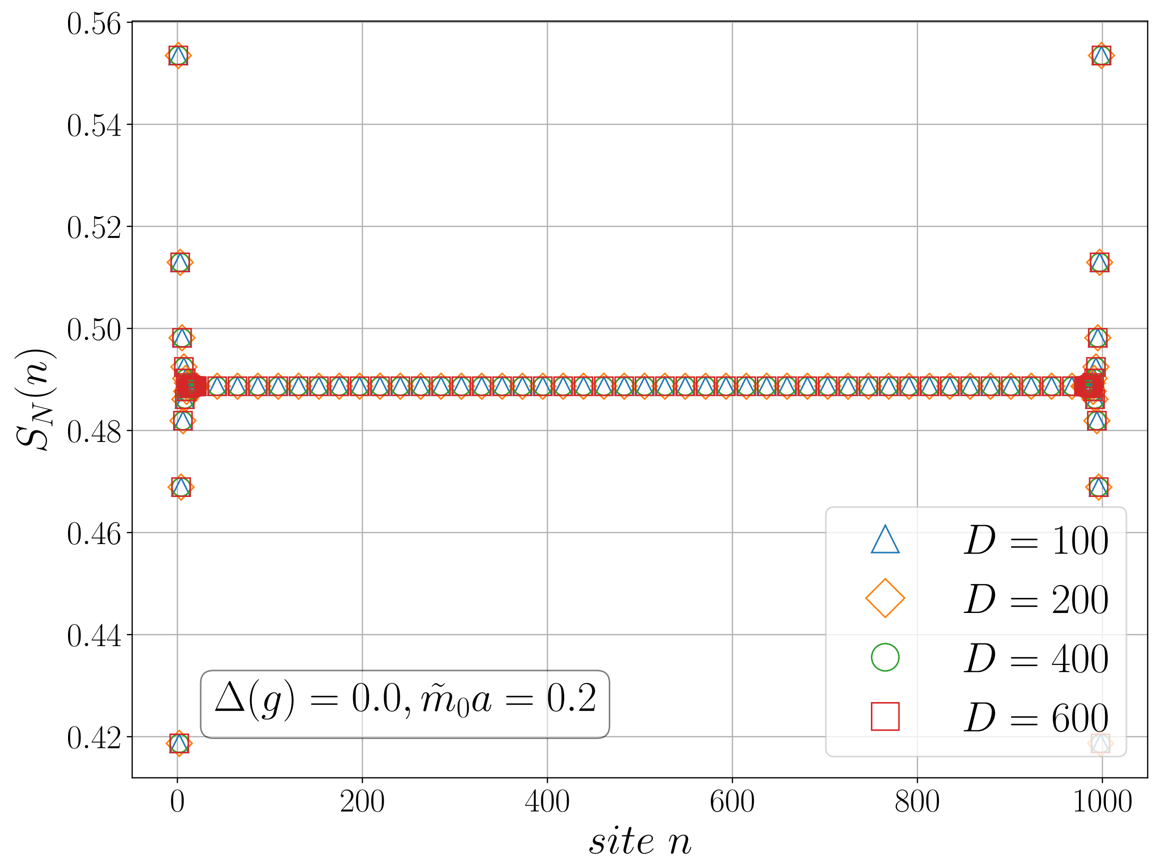
- Simulation parameters
  - ★ Twenty values of $\Delta(g)$, ranging from -0.9 to 1.0
  - ★ Fourteen values of $\tilde{m}_0 a$, ranging from 0 to 0.4
  - ★ Bond dimension $D = 50, 100, 200, 300, 400, 500, 600$
  - ★ System size $N = 400, 600, 800, 1000$

Entanglement entropy (Lattice 2018)

Calabrese-Cardy scaling and the central charge

$$S_N(n) = \frac{c}{6} \ln\left[\frac{N}{\pi}\sin\left(\frac{\pi n}{N}\right)\right] + k$$





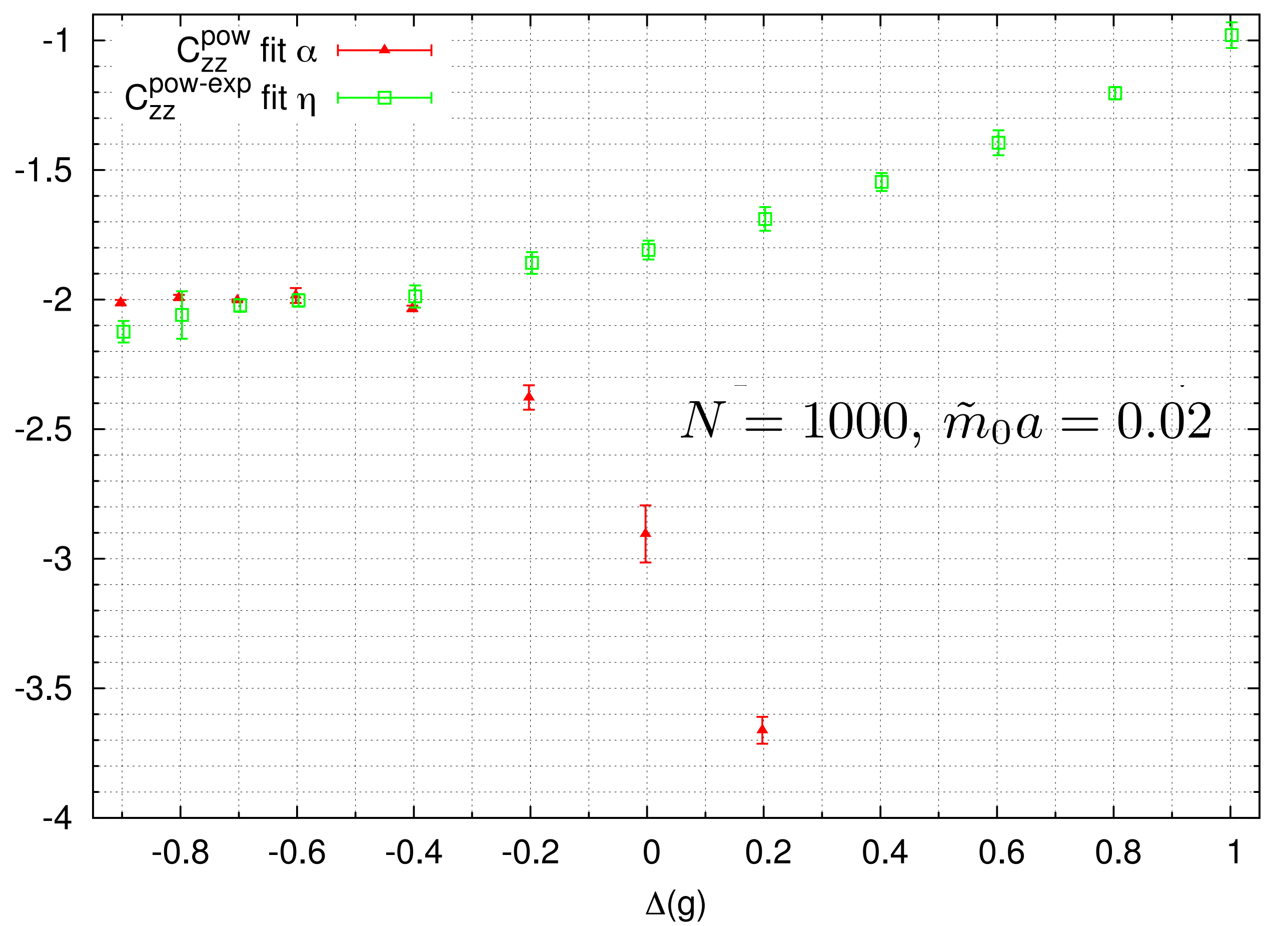
- ★ Scaling observed at $\Delta(g) \lesssim -0.7$ for $\tilde{m}_0 a \neq 0$, and for all values of $\Delta(g)$ at $\tilde{m}_0 a = 0$
- ★ In the critical phase, $c = 1$

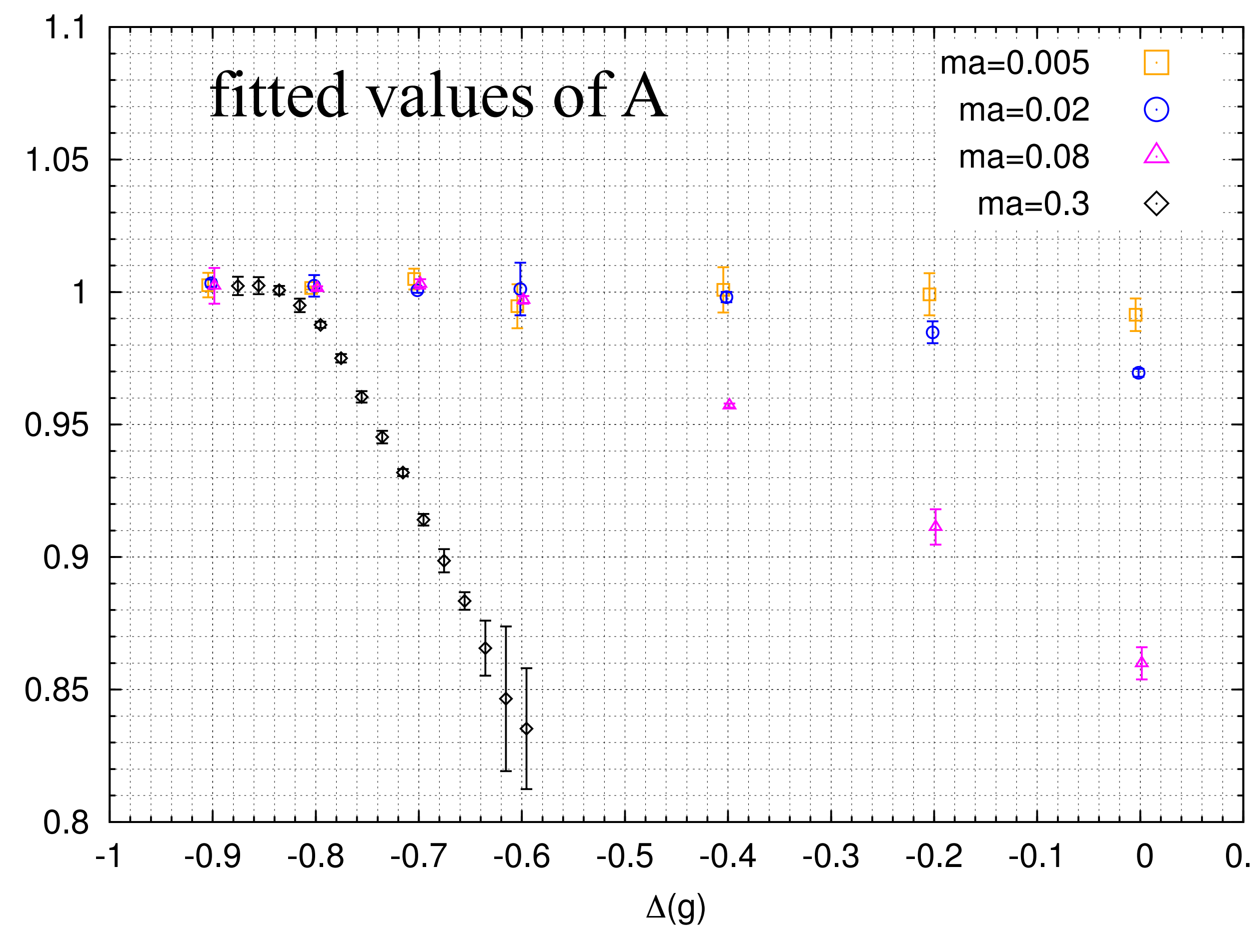
Density-density correlators

$$C_{zz}(x) = \langle \bar{\psi}\psi(x_0 + x)\bar{\psi}\psi(x_0)\rangle_{\text{conn}} \xrightarrow{\text{JW trans}} \frac{1}{N_x}\sum_n S^z(n)S^z(n+x) - \frac{1}{N_0}\sum_n S^z(n)\sum_n S^z(n+1)$$

try fitting to

$$C_{zz}^{\text{pow}}(x) = \beta x^{\alpha} \text{ and } C_{zz}^{\text{pow}-\text{exp}}(x) = B x^{\eta} A^x$$





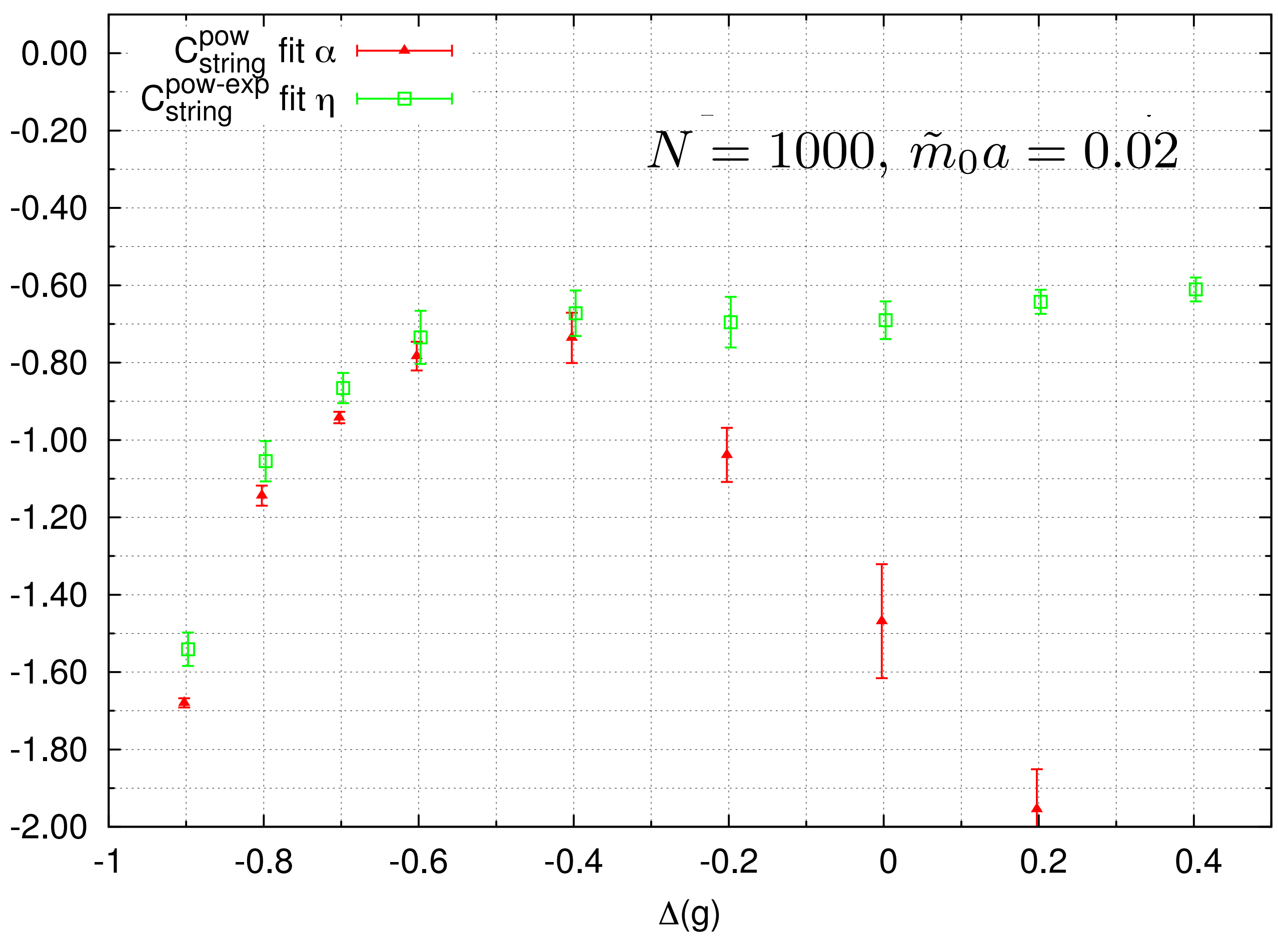
★ Evidence for a critical phase

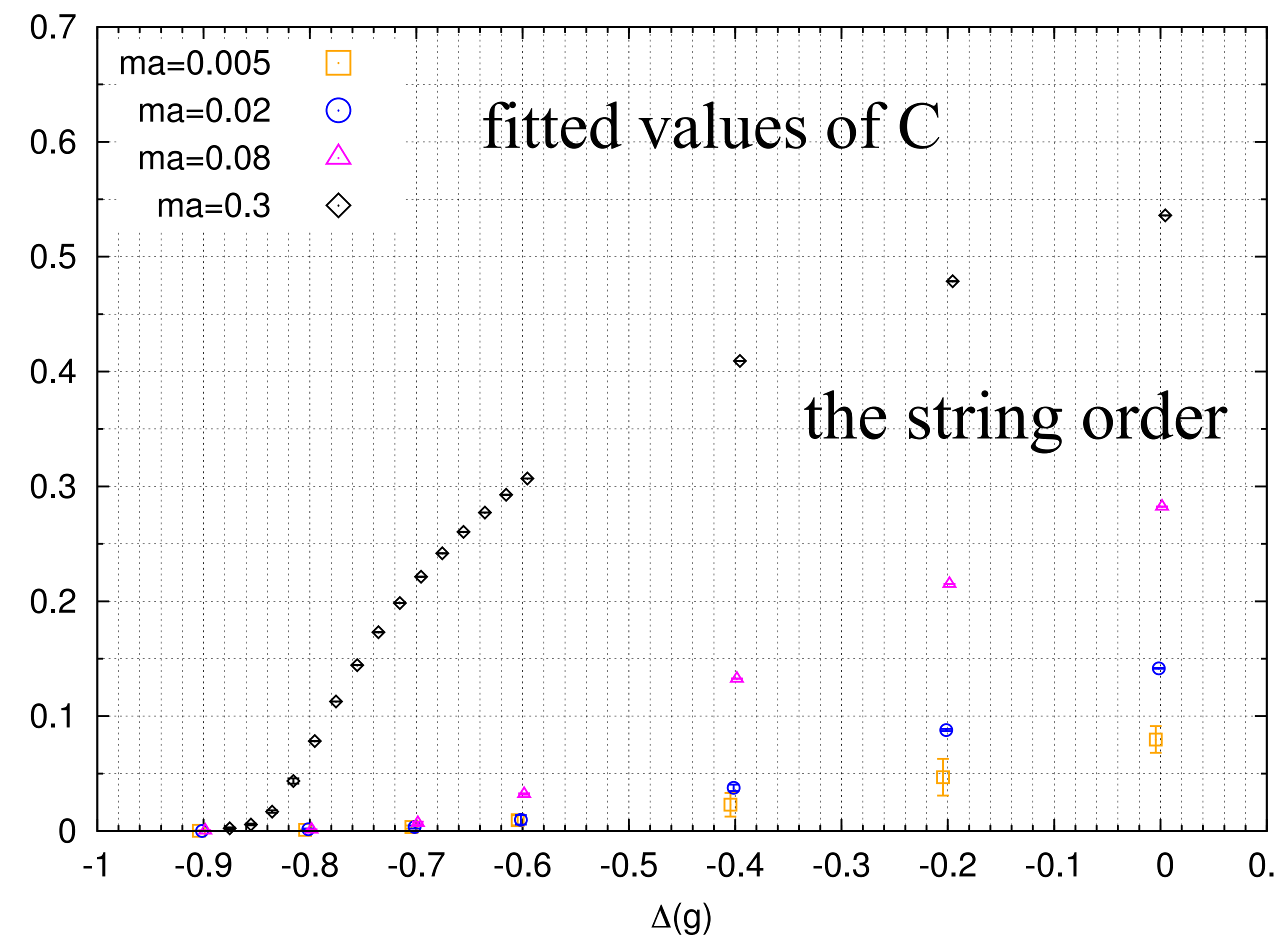
Soliton (string) correlators

$$C_{\text{string}}(x) = \langle \psi^\dagger(x_0+x)\psi(x_0)\rangle \xrightarrow{\text{JW trans}} \frac{1}{N_x}\sum_n S^+(n)S^z(n+1)\cdots S^z(n+x-1)S^-(n+x)$$

try fitting to

$$C^{\text{pow}}_{\text{string}}(x) = \beta x^\alpha + C \quad \text{and} \quad C^{\text{pow-exp}}_{\text{string}}(x) = Bx^\eta A^x + C$$

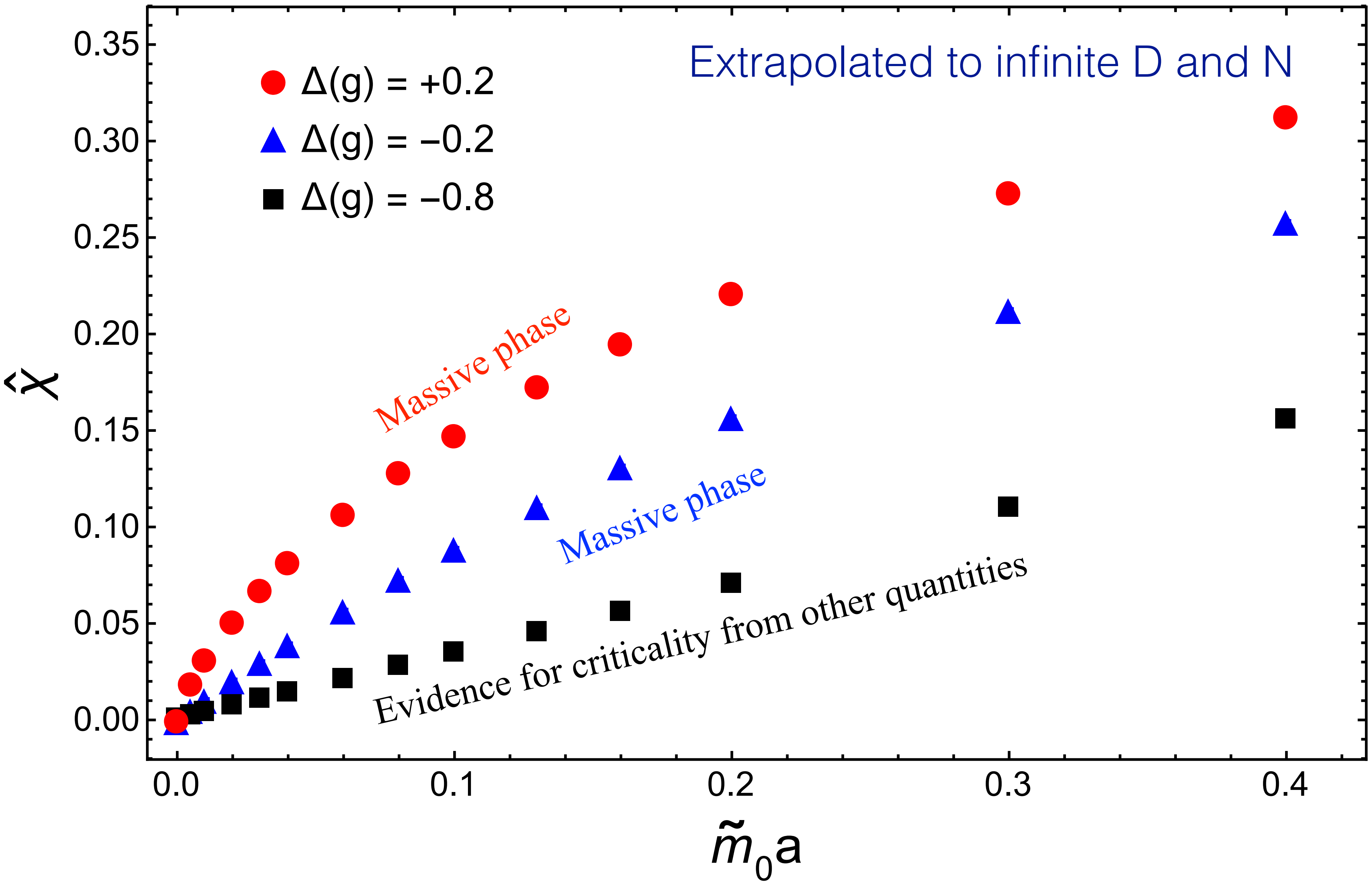




★ Similar behaviour in A. Evidence for a critical phase

Chiral condensate

$$\hat{\chi} = a\left|\langle\bar{\psi}\psi\rangle\right| = \frac{1}{N}\left|\sum_n(-1)^n S_n^z\right|$$



★ Chiral condensate is not an order parameter

Phase structure of the Thirring model

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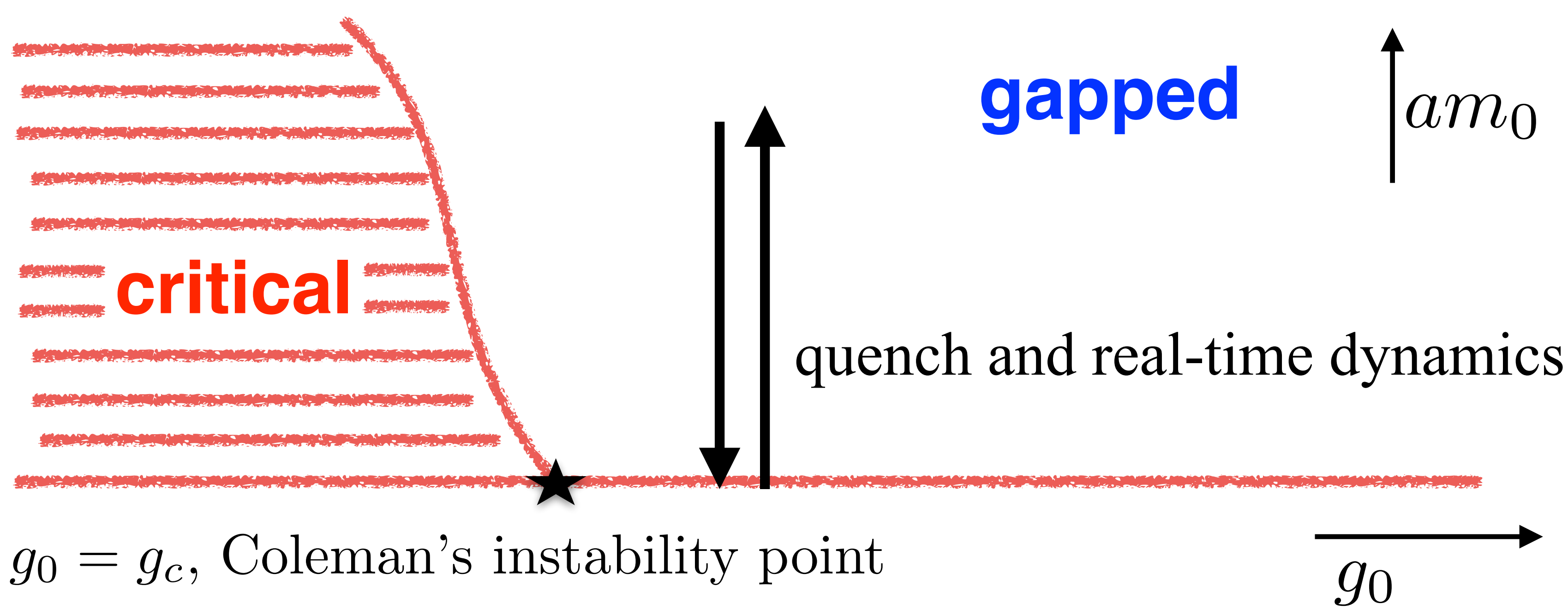
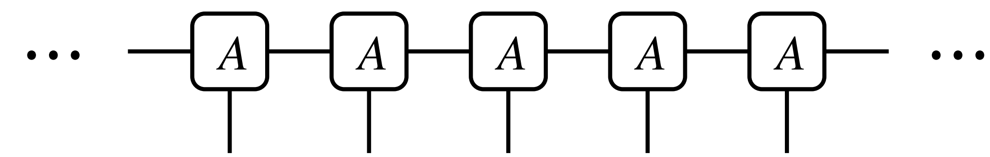


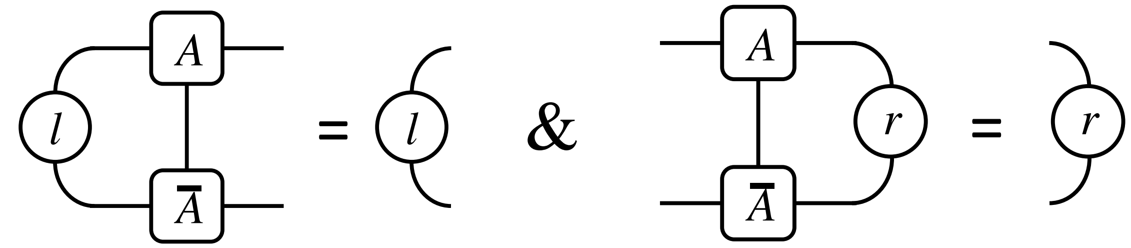
Uniform MPS and real-time evolution

★ Translational invariance in MPS



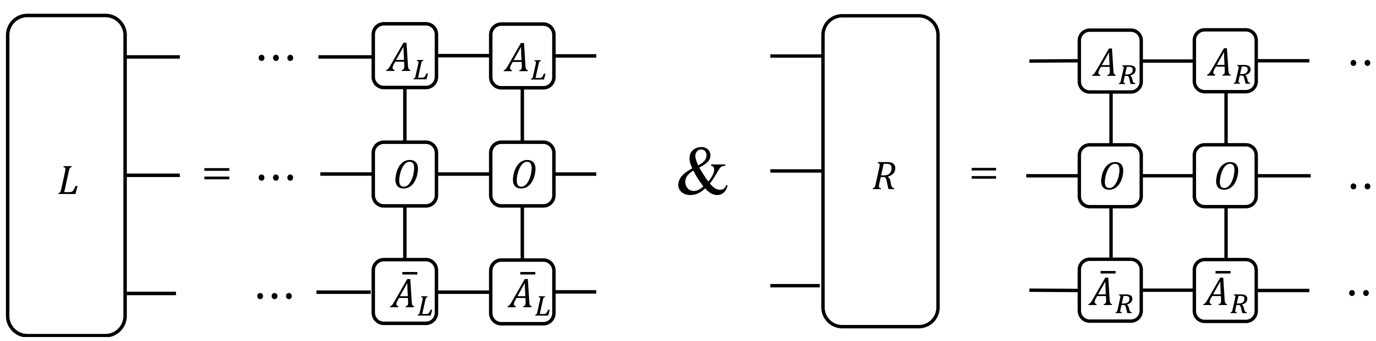
★ Finding the infinite BC for amplitudes

(largest eigenvalue normalised to be 1)



H.N. Phien, G. Vidal and I.P. McCulloch, Phys. Rev. B86, 2012

★ Similar (more complicated) procedure in the variation search for the ground state



…V. Zauner-Stauber *et al*, Phys. Rev. B97, 2018

★ Real-time evolution *via* time-dependent variational principle

➡ Key: projection to MPS in $i\frac{d}{dt}|\Psi(A(t))\rangle = P_{|\Psi(A)\rangle}\hat{H}|\Psi(A(t))\rangle$

J. Haegeman *et al*, Phys. Rev. Lett.107, 2011

Dynamical quantum phase transition

★ "Quenching" : Sudden change of coupling strength in time evolution

$$H(g_1)|0_1\rangle = E_0^{(1)}|0_1\rangle \quad \text{and} \quad |\psi(t)\rangle = \mathrm{e}^{-iH(g_2)t}|0_1\rangle$$

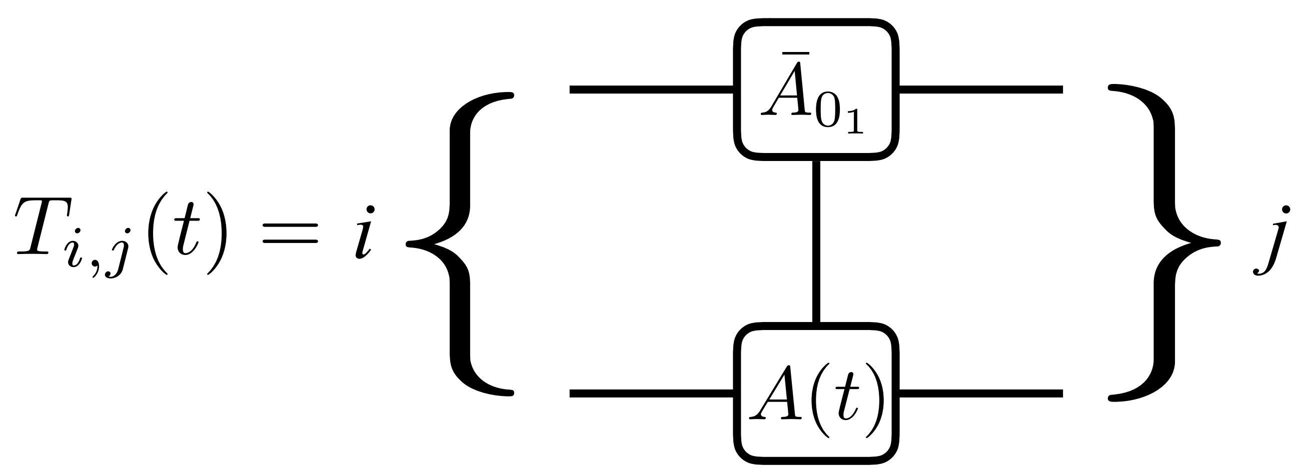
★ Questions: Any singular behaviour? Related to equilibrium PT?

★ The Loschmidt echo and the return rate

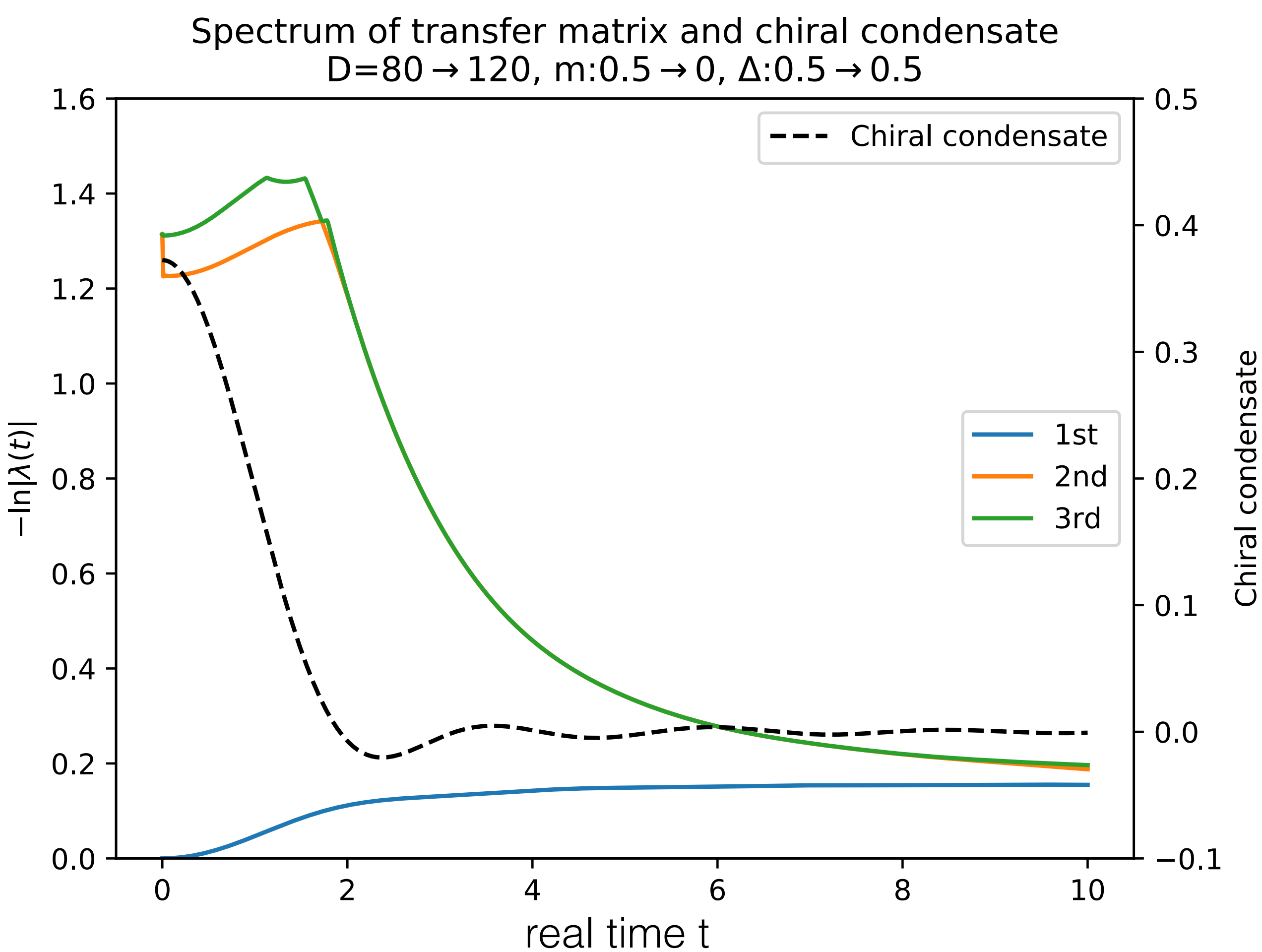
$$L(t) = \langle 0_1|\mathrm{e}^{-iH(g_2)t}|0_1\rangle \quad \& \quad g(t) = -\lim_{N\to\infty}\frac{1}{N}\mathrm{ln}L(t)$$

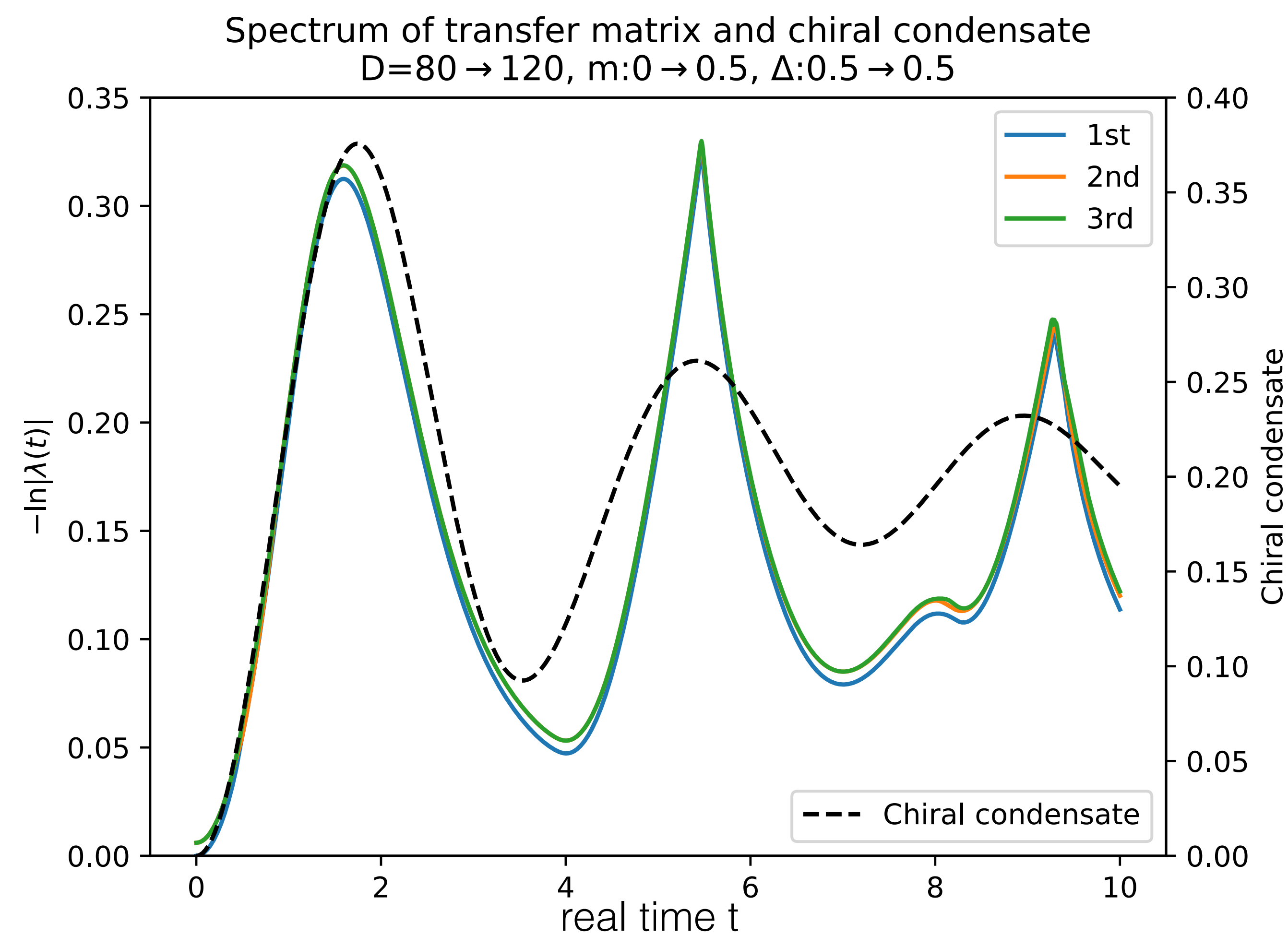
➤ *c.f.*, the partition function and the free energy

➤ In uMPS computed from the largest eigenvalue of the “transfer matrix"



Observing DQPT



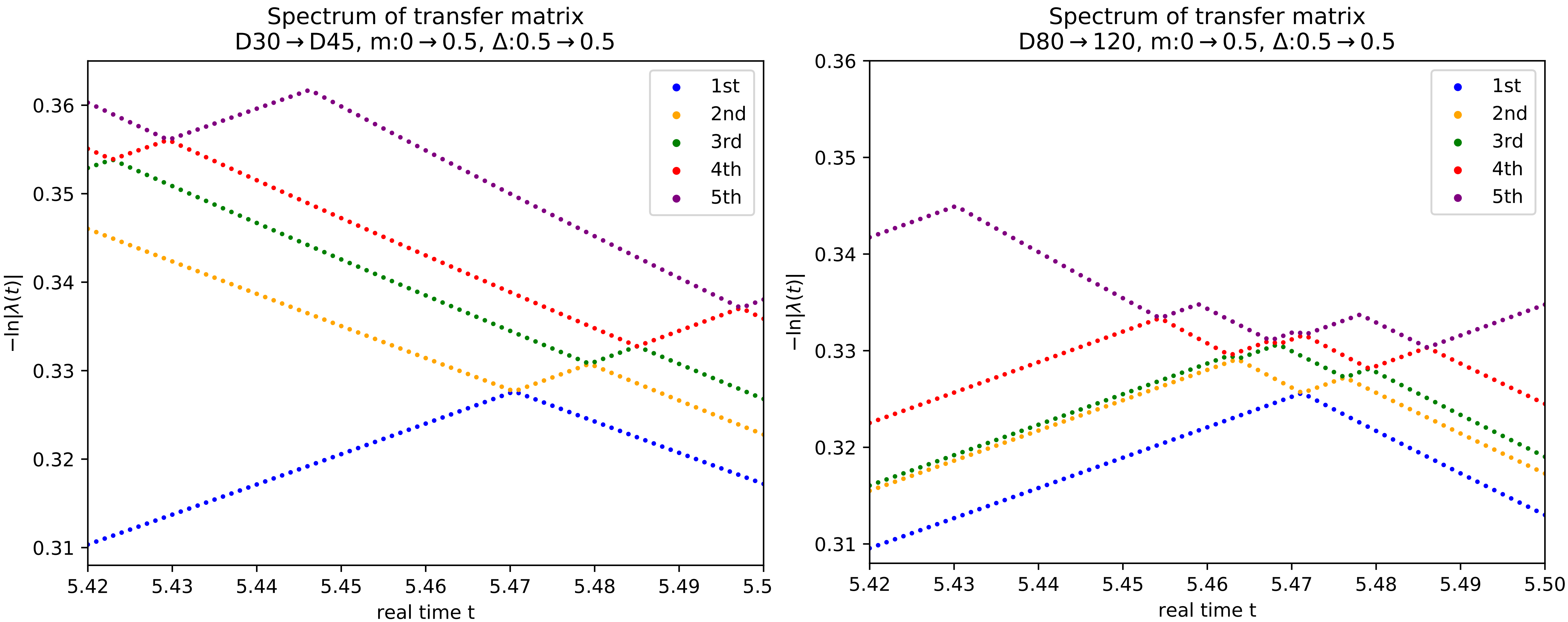


massive → critical

critical → massive

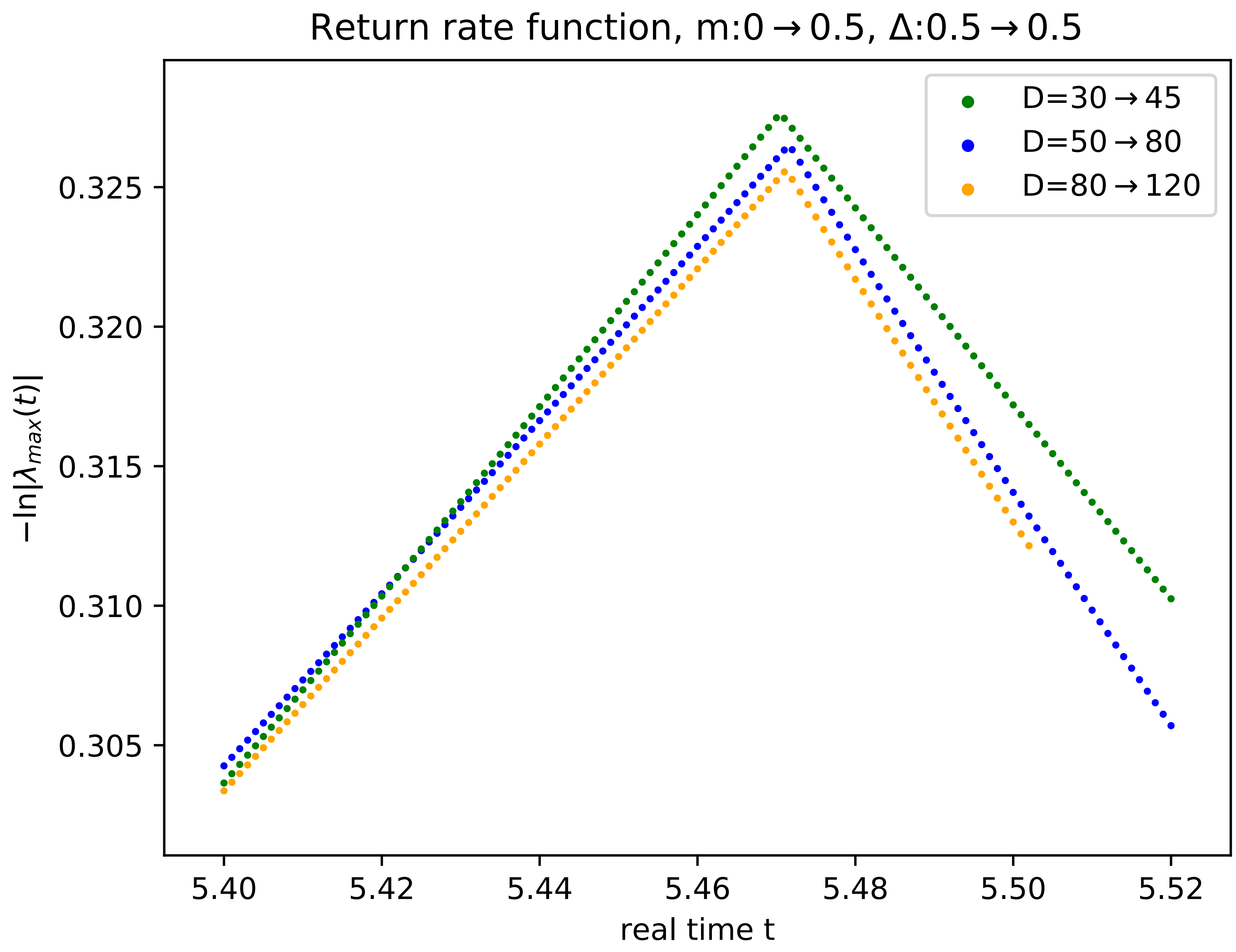
★ DQPT is a "one-way" transition…

DQPT and eigenvalue crossing



★ D-dependence in the crossing points

“Universality” in DQPT?



Conclusion and outlook

- Concluding results for phase structure
  - ★ KT-type transition observed
- Exploratory results for real-time dynamics
  - ★ DQPT observed
  - ★ Relation to equilibrium KT phase transition?