Continuous β function and anomalous dimensions in QCD and conformal systems

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Lattice 2019
Wuhan, June 17

with Oliver Witzel

Wuhan·China | June 16-22

Gradient flow vs real-space Wilsonian RG

A. Carosso, AH, E.Neal, PRL121,(2018)201601

GF is not an RG transformation, but

- GF fields act like RG blocked fields
- Wave function renormalization (η exponent) can be added
- Coarse graining done when calculating expectation values

→ GF acts like RG blocking with **continuous** scale change $b \propto \sqrt{t}$

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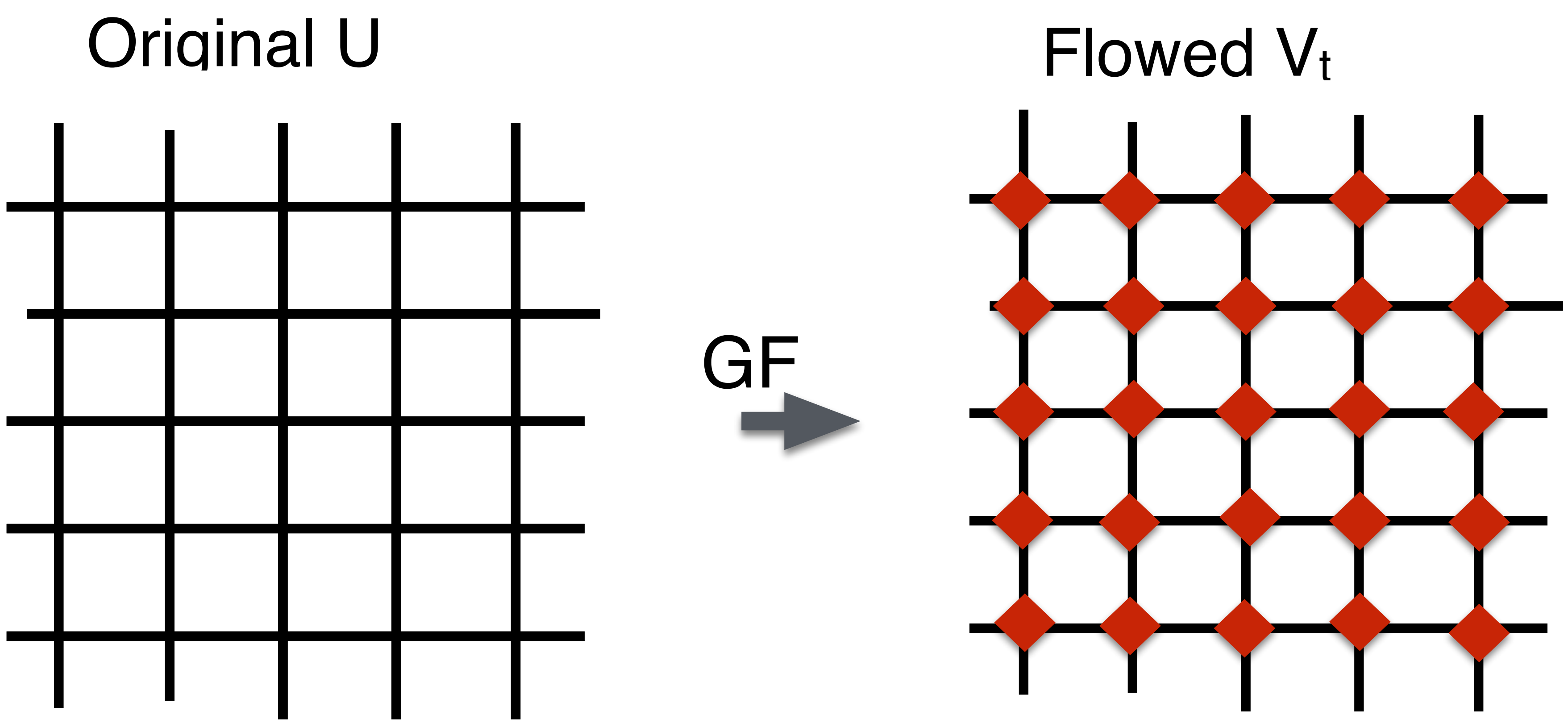

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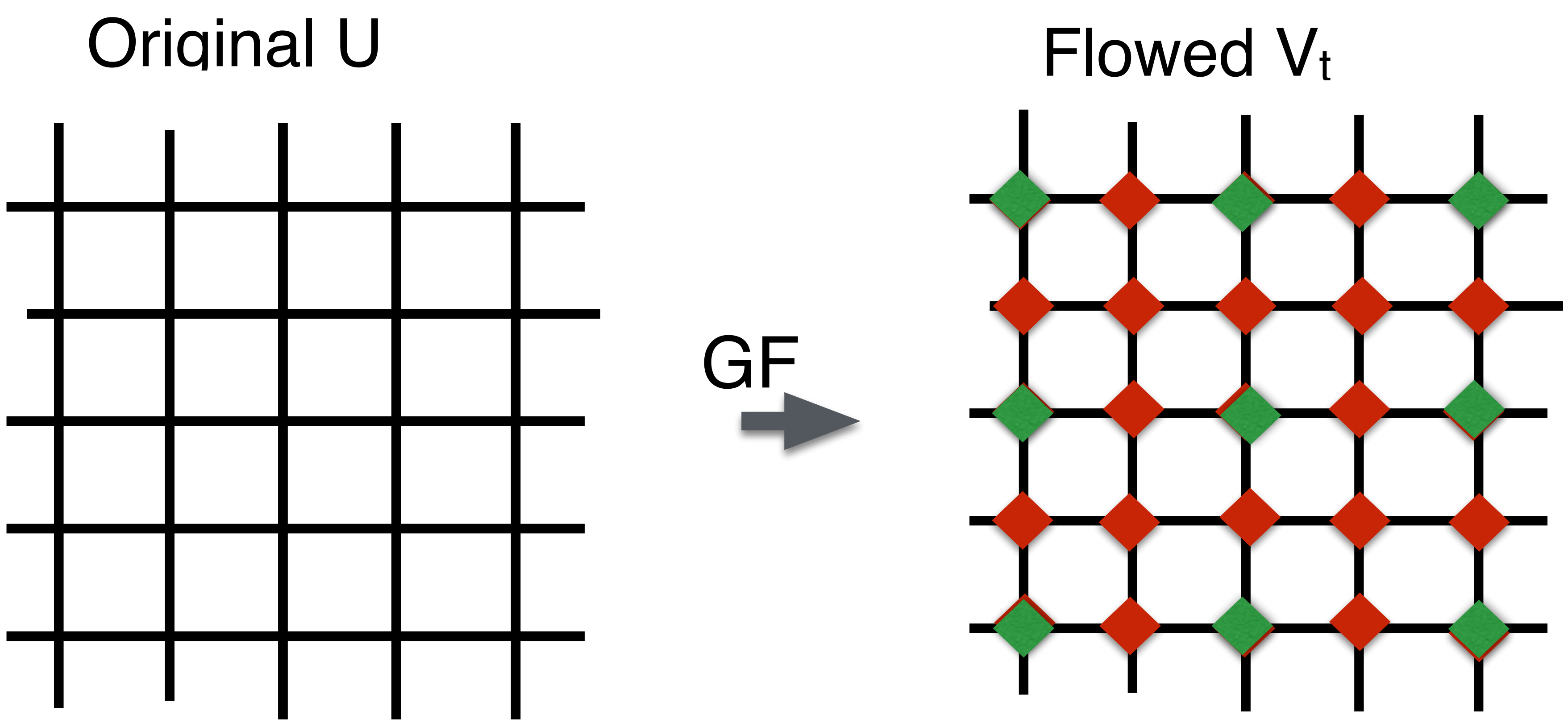

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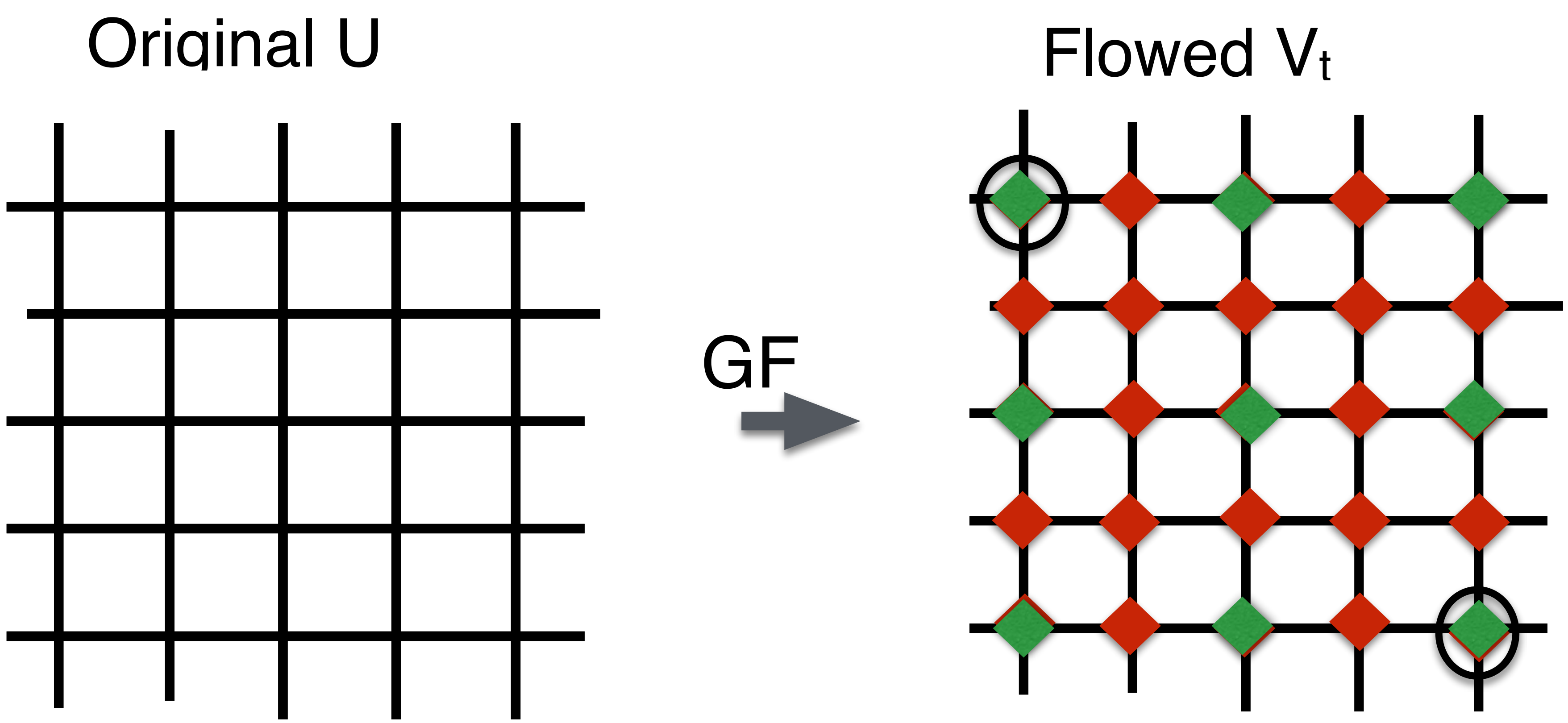
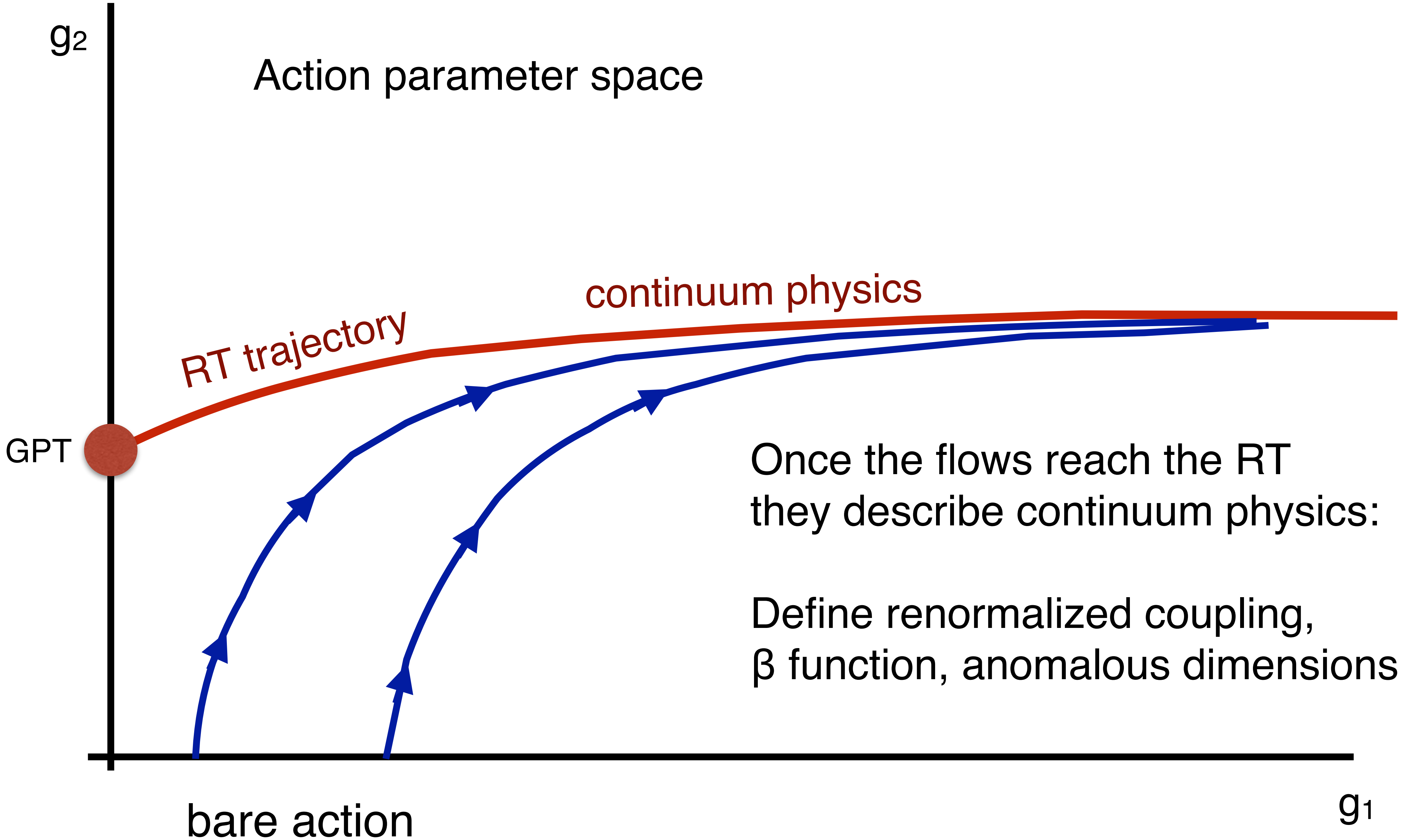


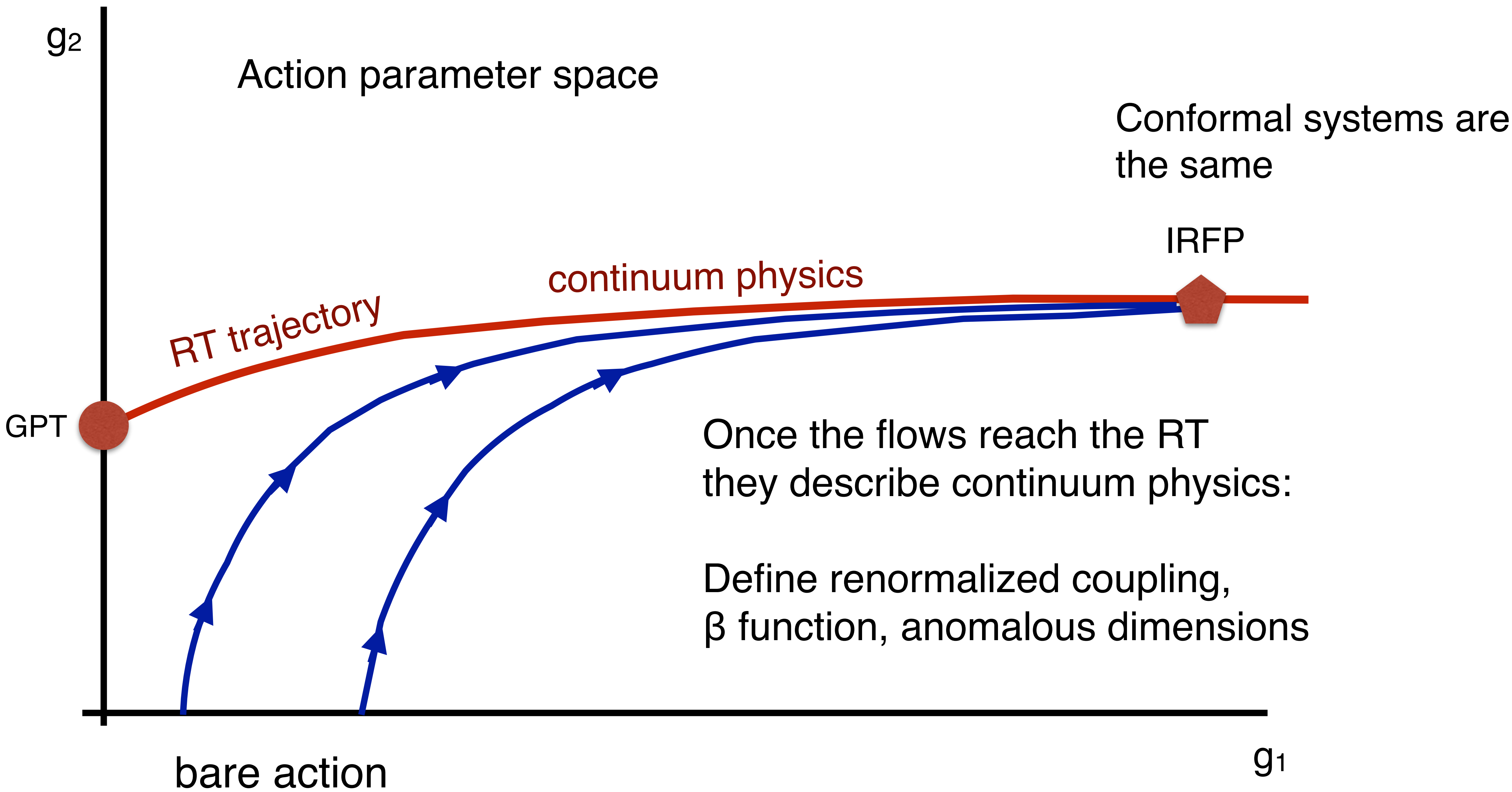
Topology of RG flows



Once the flows reach the RT
they describe continuum physics:

Define renormalized coupling,
β function, anomalous dimensions

Topology of RG flows



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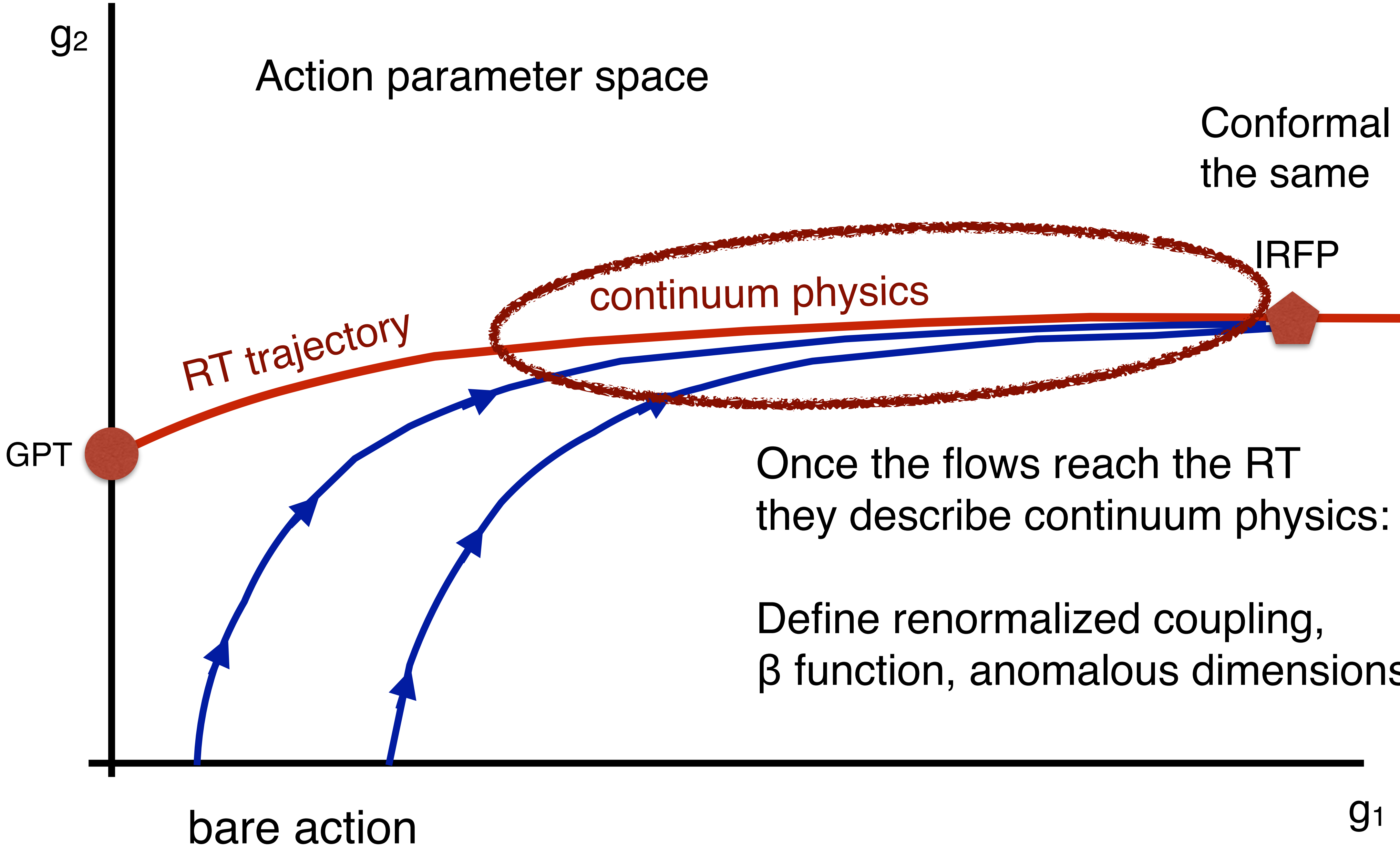


Conformal systems are the same

Once the flows reach the RT they describe continuum physics:

Define renormalized coupling, β function, anomalous dimensions

Different bare couplings will overlap on RT!

RG properties from a single simulation?

Yes, in infinite volume and $g_0^2 \to 0$

In a realistic simulation only if

- flow time is large to reduce cutoff effects
- but small to avoid finite volume effects

$\to$ chain together several flows

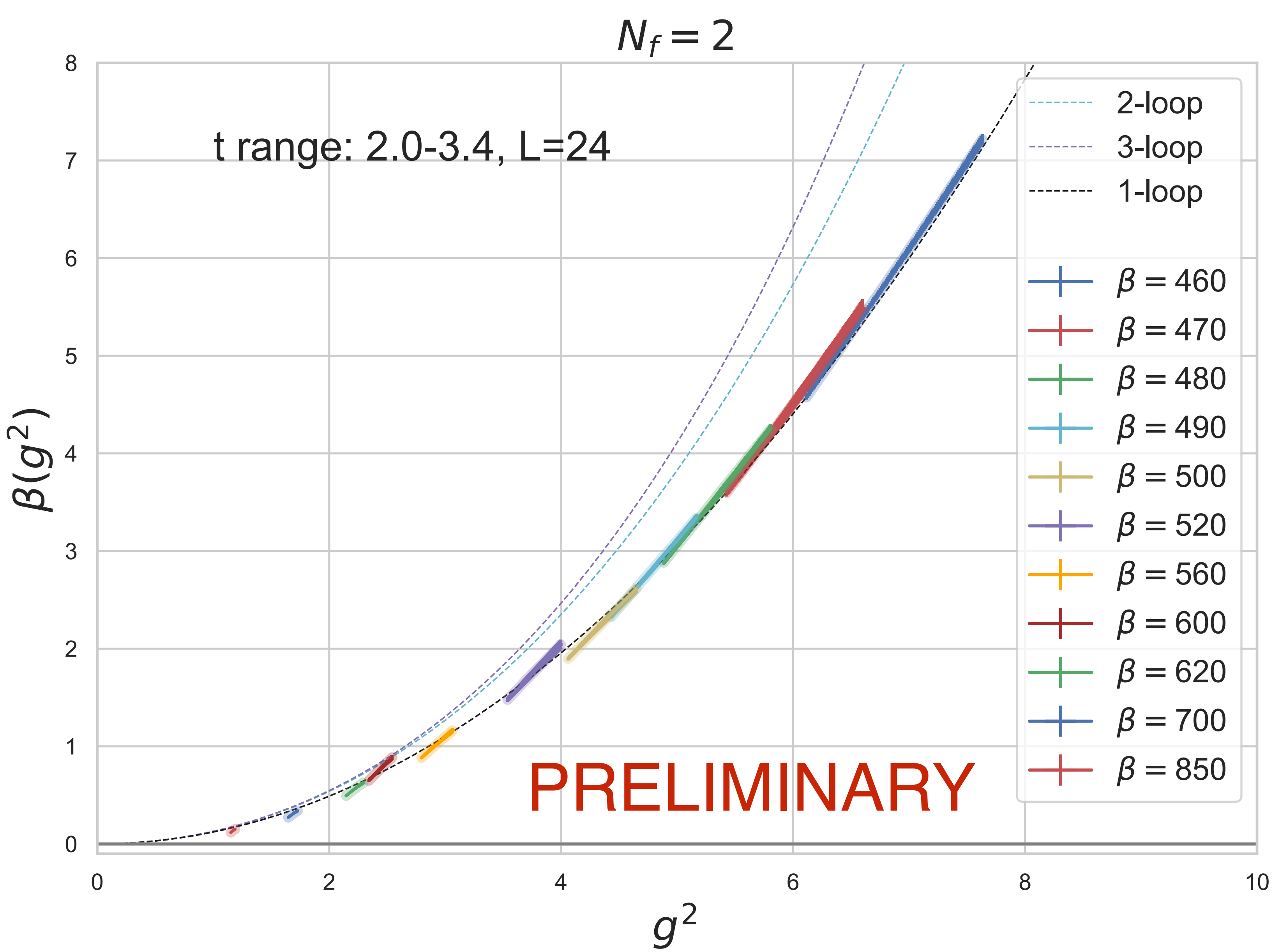
Example: continuous β function in N_f=2 flavor massless QCD

$$\beta(g^2) = -2t\frac{dg^2}{dt}$$

Demonstration:

- Symanzik gauge action, stout smeared Möbius domain wall fermions
- Zeuthen flow, Symanzik operator
- 24^3 x 64 (and 16^3 x 64) volumes, standard BC
- 11 (x 2) ensembles, β = 4.60 - 8.50 (chirally symmetric!)

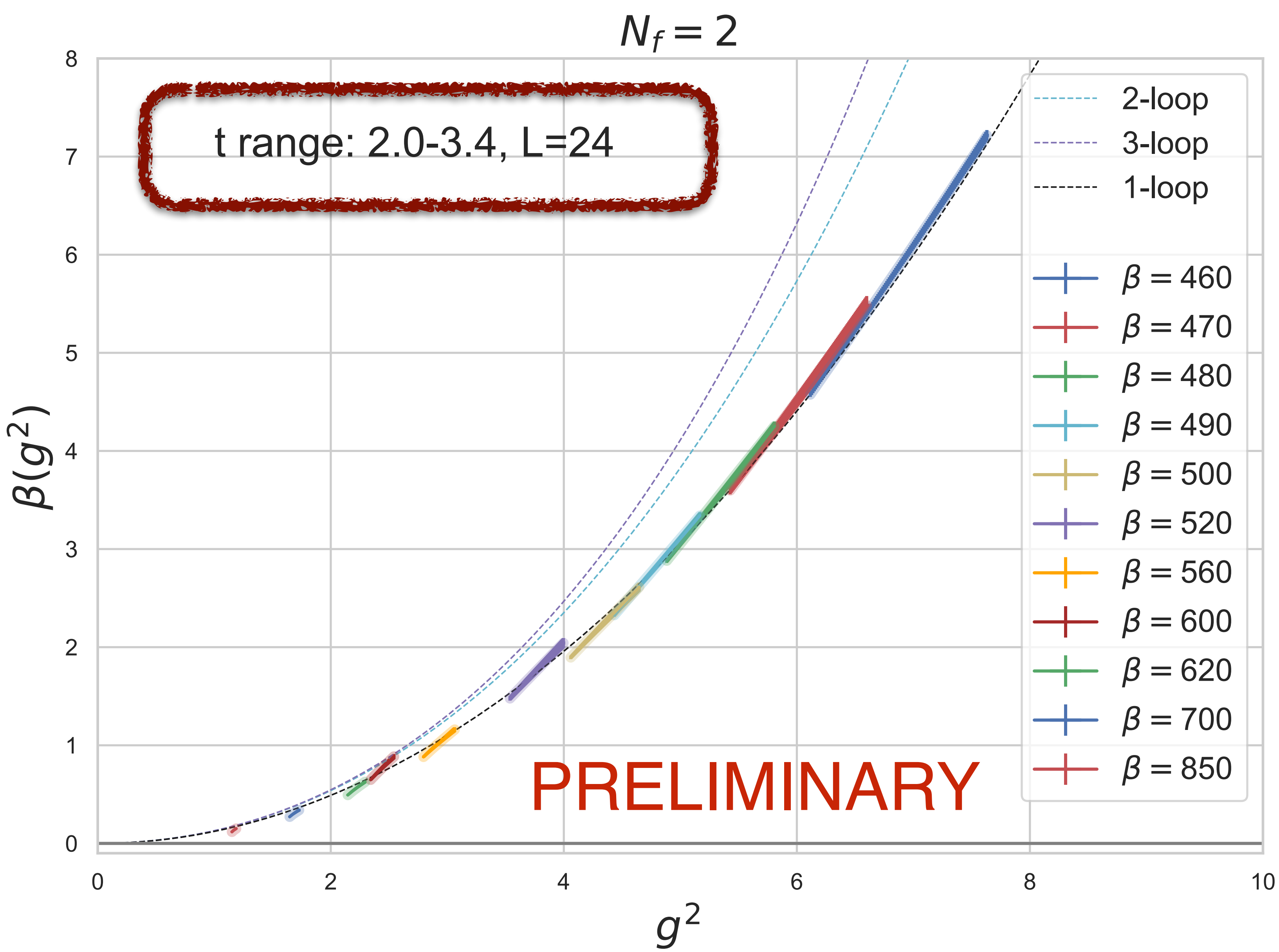
Continuous β function



color bands:
predictions of the β function from various single ensembles

Predictions of different bare coupling values overlap, as RG considerations suggests

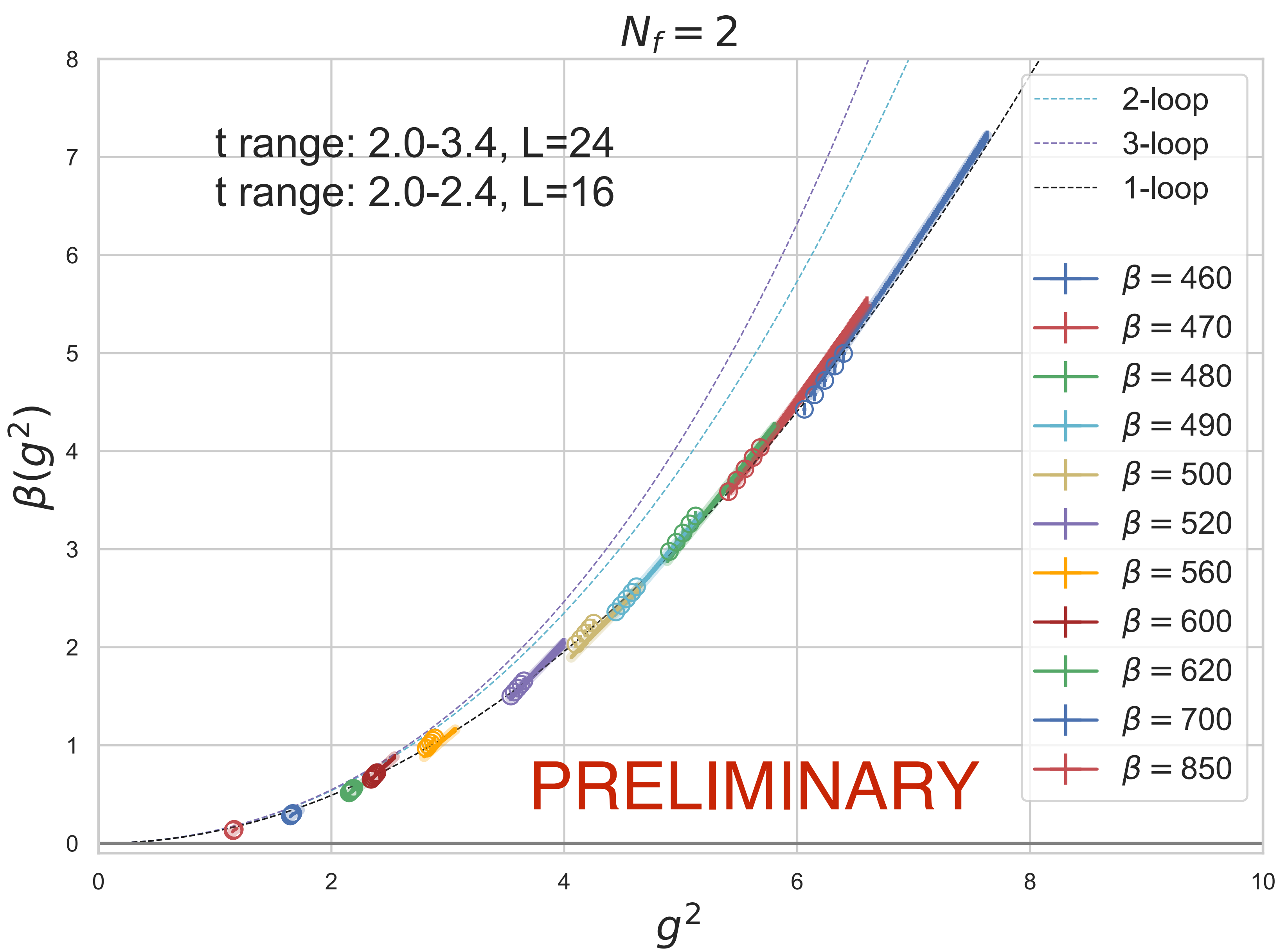
Continuous β function



t_{min} = 2.0 is large enough to avoid most cutoff effects. Corresponds to c=0.166 in step scaling

t_{max} = 3.4 is small enough to avoid most finite volume effects. Corresponds to c=0.22 in step scaling

Continuous β function



16^3 x 64 (open circles) is very similar (t_{max} is smaller)

Finite volume effects are small

Why does it follow 1-loop PT?

- β function is scheme dependent
- N_f=3 GF β function from step scaling also follows 1-loop PT

Continuous RG β function, step-by-step

Definitions:

$$\beta(g^2) = -2t\frac{dg^2}{dt}$$

in the $L/a \to \infty, \quad t/L^2 \to 0,$ limit with

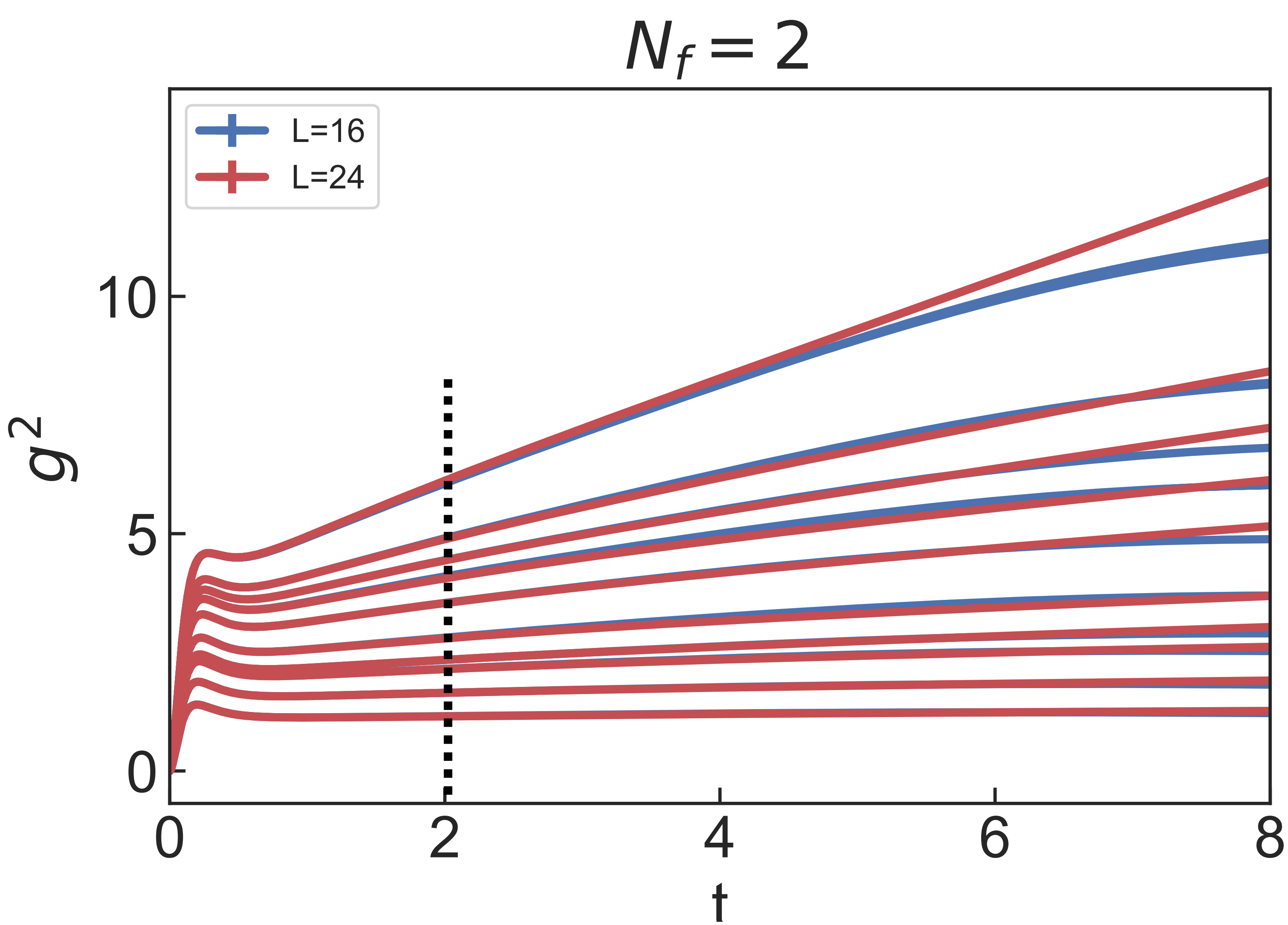
$$g^2_{GF}(t/a^2; L/a, g_0^2) = \frac{128\pi^2}{3(N^2-1)}\frac{1}{1+\delta(t/L^2)}\langle t^2 E(t)\rangle$$

$\beta(g^2)$ is defined for each ensemble

- $1+\delta(t/L^2)$ corrects for gauge zero modes
- Correlated t values reduce statistical uncertainty
- Only large volumes used, reducing cut-off effects
- Flow time t/a^2 does not grow with volume
- (Definition of continuous $\beta(g^2)$ differs from step scaling function by a factor -2)

Volume dependence of $g^2_{\rm GF}$

- Finite volume effects are controlled by t/L^2
- Simulations at $am_0=0$ in the chirally symmetric regime
  - limits the accessible g^2 range (absolute limit)

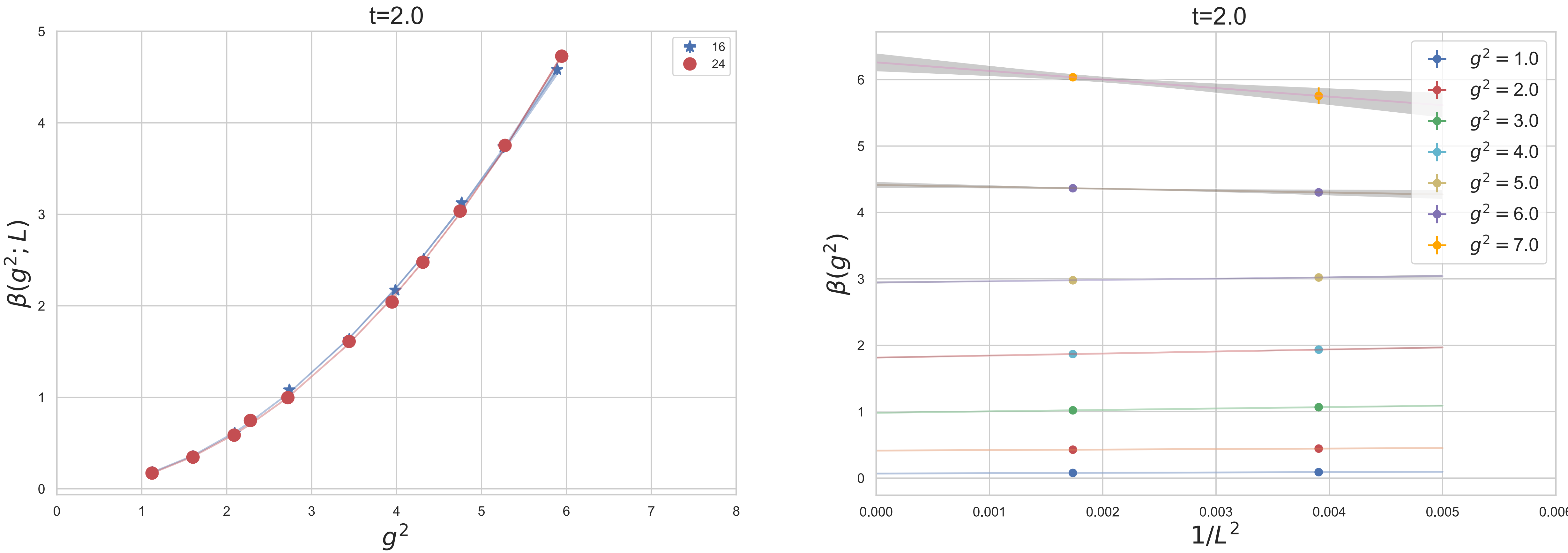


Simulations at
β = 4.60 - 8.50

Continuum limit: L⟶∞

1) Take L/a = ∞ while keeping t/a^2 fixed :

- interpolate $\beta(g^2; L)$ vs g^2 on every volume, fixed flow time
- extrapolate to L = ∞ at selected (g^2 , t)



If $t << L^2$, linear extrapolation in $1/L^2$ is sufficient. Add more/larger volumes to check systematic effects.

Continuum limit: L⟶∞

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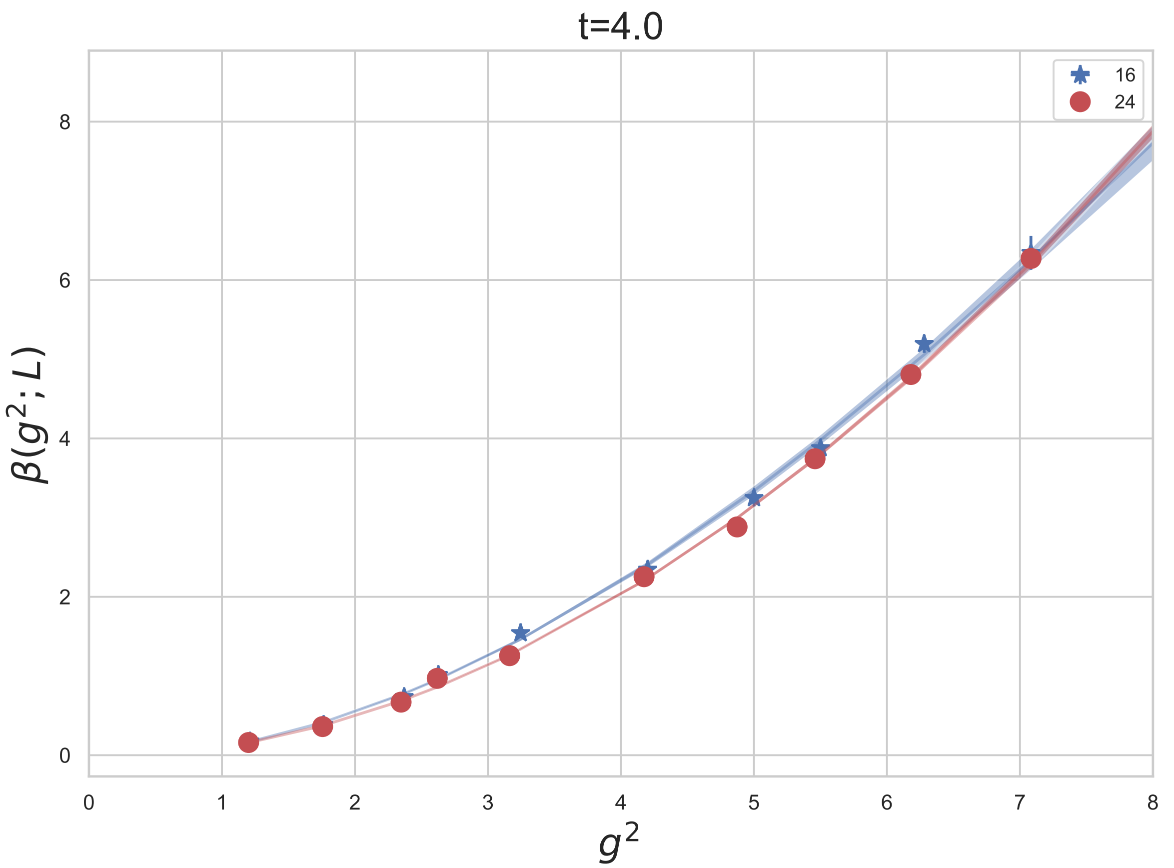
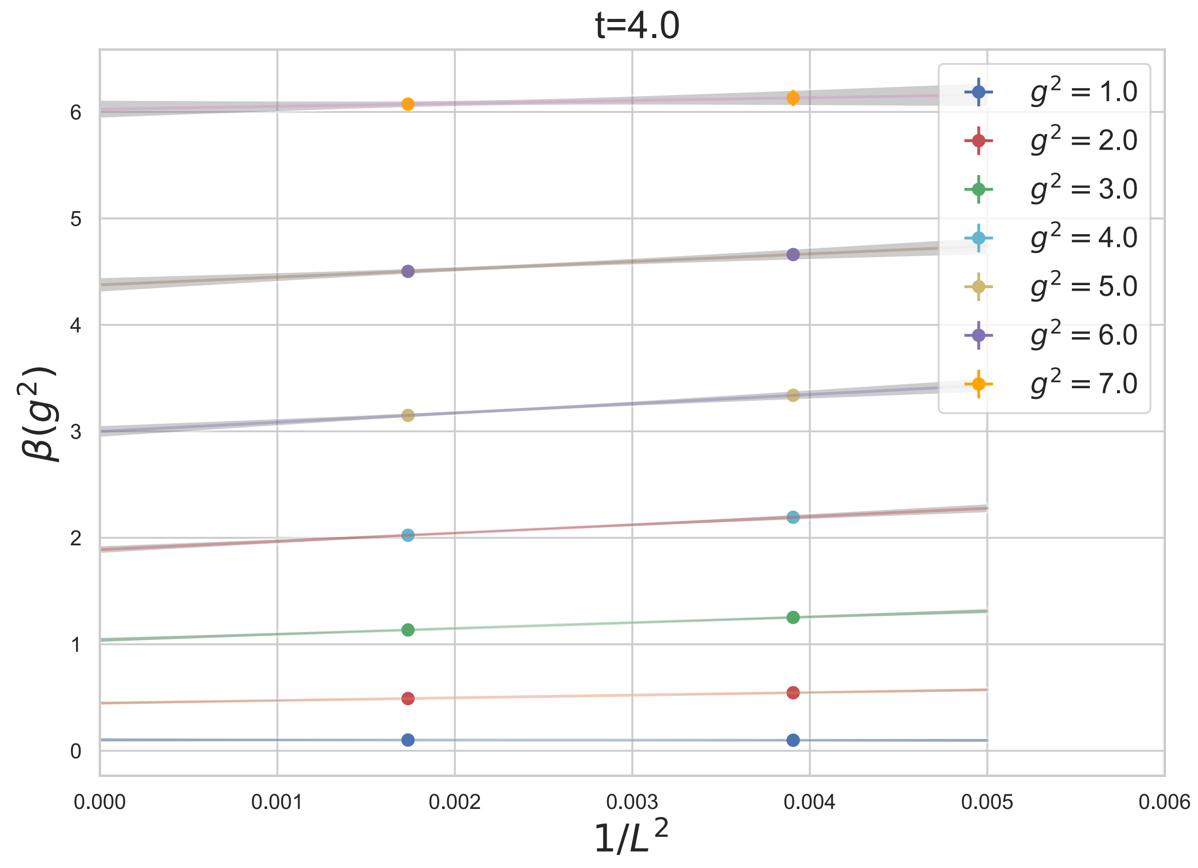




For systematic L→∞ and more details see N_f=12 poster

Continuum limit: t⟶∞

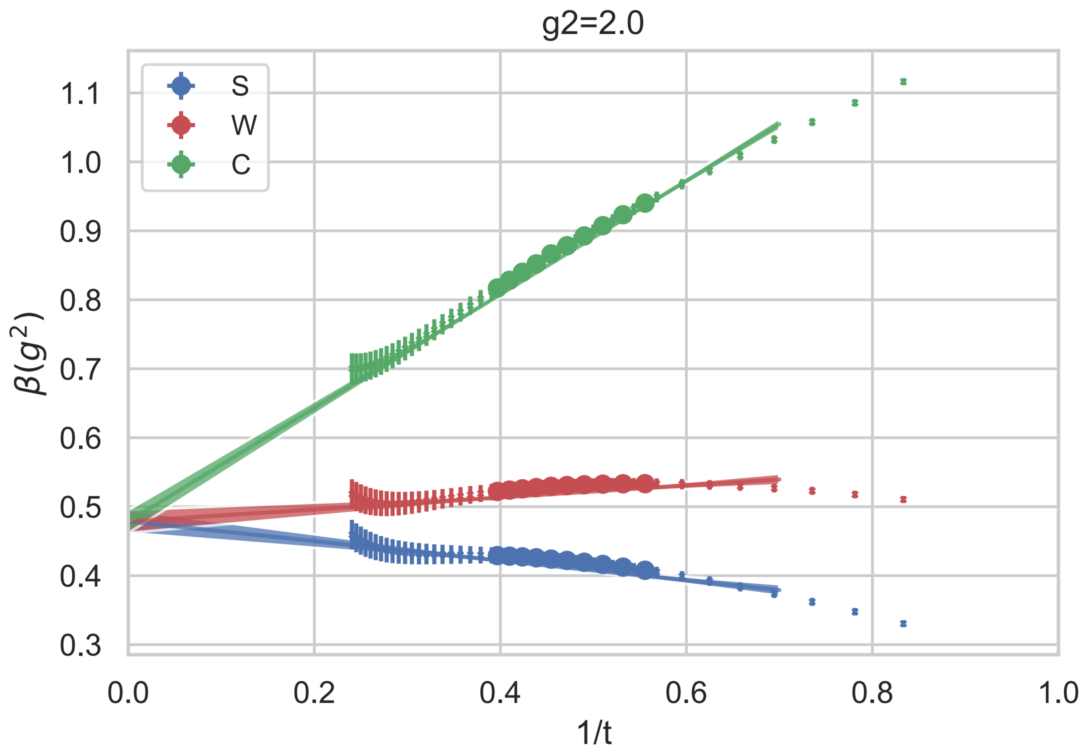
2) Take $t/a^2 = \infty$ at fixed g^2 :

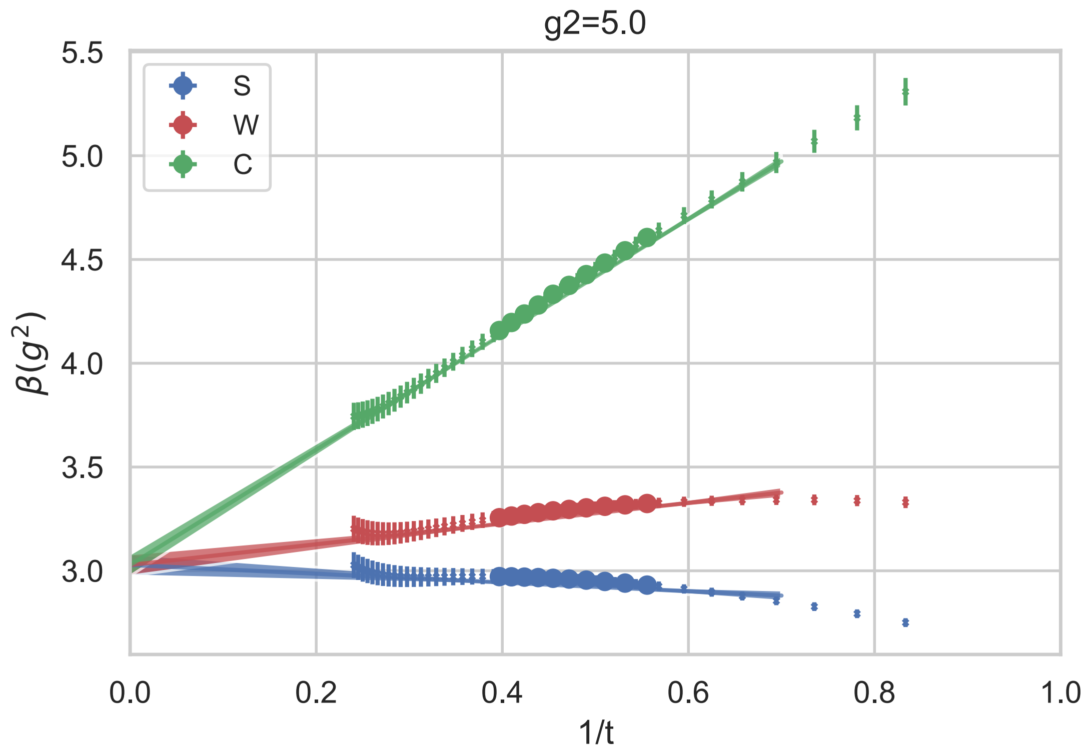
- flow approaches RT, irrelevant operators die out

$$\beta(g^2) = \beta(g^2; t/a^2) + \xi(a^2/t)^{1+p} + \text{h.o.t}$$

$1/t^{1+p}$ describes the leading irrelevant operator (p=0 at GFP)

Consider different operators that approximate the energy density





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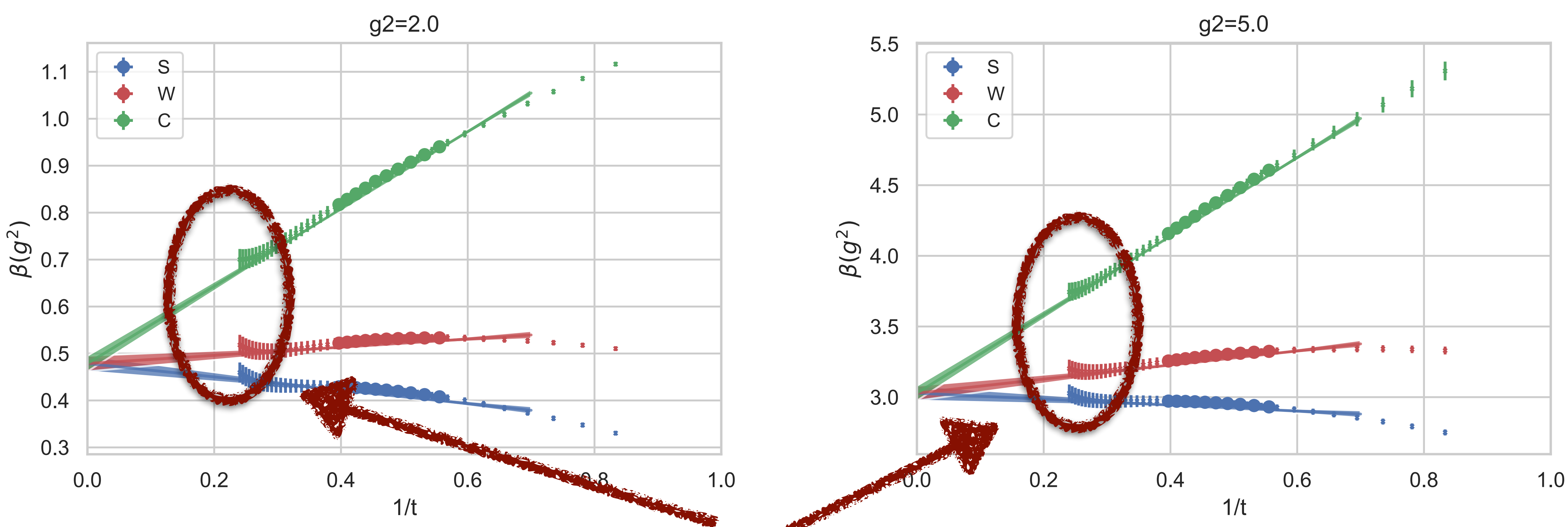
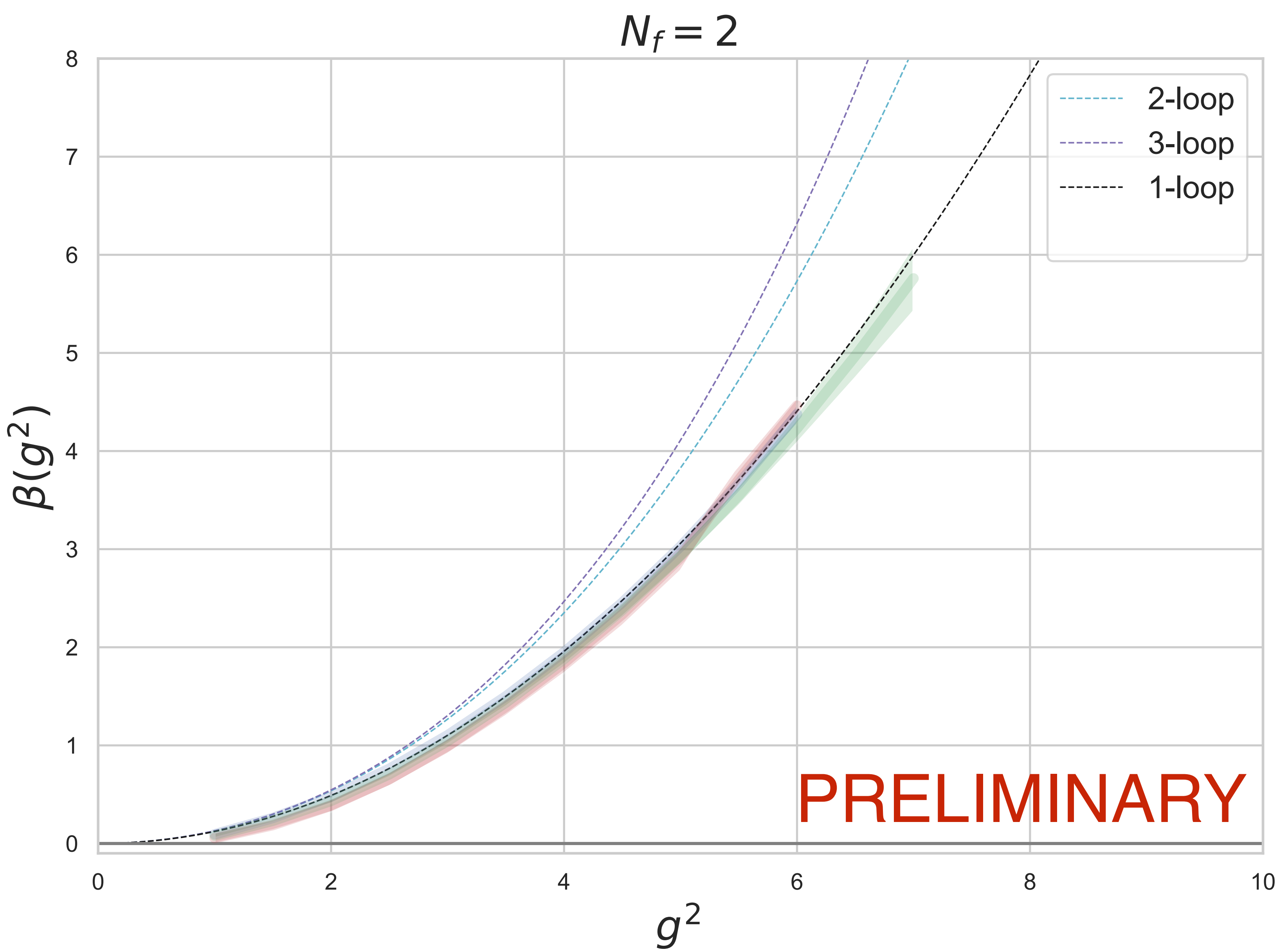


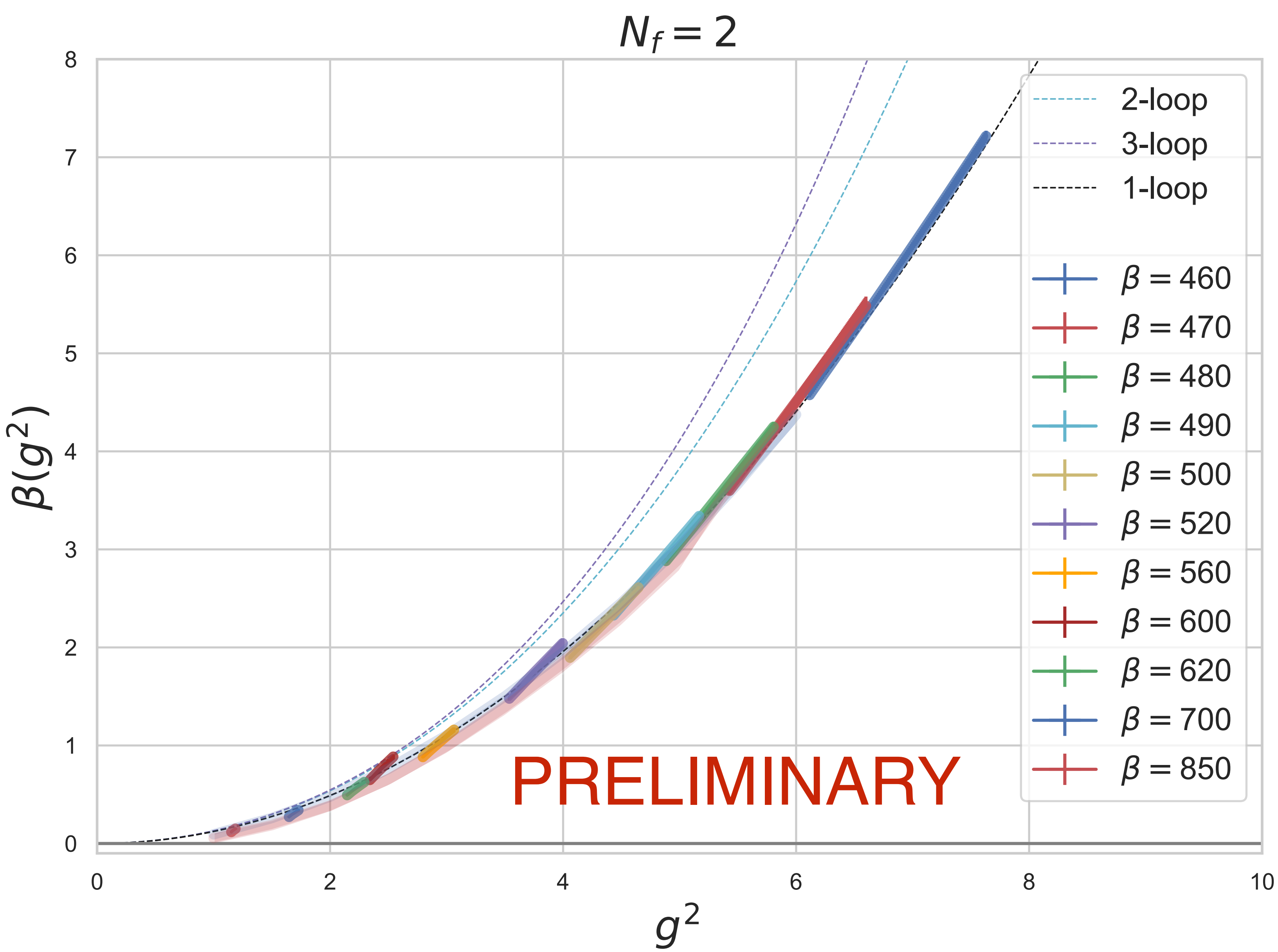
Finite volume extrapolation is unreliable : limit t range

N_f = 2 continuous β function in continuum limit:



Various flow ranges, operator combinations

N_f =2 continuous β function in continuum limit:



Compare full continuum limit prediction to L=24

Why was this so easy?

- DW fermions have small cutoff effects
- We used Symanzik gauge and Zeuthen flow: RT is close
- Symanzik and plaquette operators are closest to scaling operator

Continuum limit of continuous β function

Systematic effects:

- consider different L = ∞ extrapolations
- vary t_{min}, t_{max}
- allow higher order operators

$$\beta(g^2) = \beta(g^2; t/a^2) + \xi_1(a^2/t) + \xi_2(a^2/t)^{1+p}$$

- combine different operators and force $\beta(g^2)$, p to be common

- In 2-flavor QCD systematic effects are small
- In slowly running conformal / near-conformal systems $\beta(g^2)$ is
  is more difficult

GF as RG: anomalous dimension

Along the RT all cut-off effects are removed.
Ratio of flowed & unflowed correlators predict the anomalous dimension

$$\frac{\langle O_t(0) O_t(x_0)\rangle}{\langle O(0) O(x_0)\rangle} = b^{2\Delta_O - 2n_O\Delta_\phi} \qquad x_0 \gg b$$

Use an operator with no anomalous dimension to remove wave function renormalization (ex. vector)

Double-ratio

$$\mathcal{R}_t^O(x_0) = \frac{\langle O(0) O_t(x_0)\rangle}{\langle O(0) O(x_0)\rangle}\left(\frac{\langle A(0) A(x_0)\rangle}{\langle A(0) A_t(x_0)\rangle}\right)^{n_O/n_A} = t^{\gamma_O}$$

predicts anomalous dimension

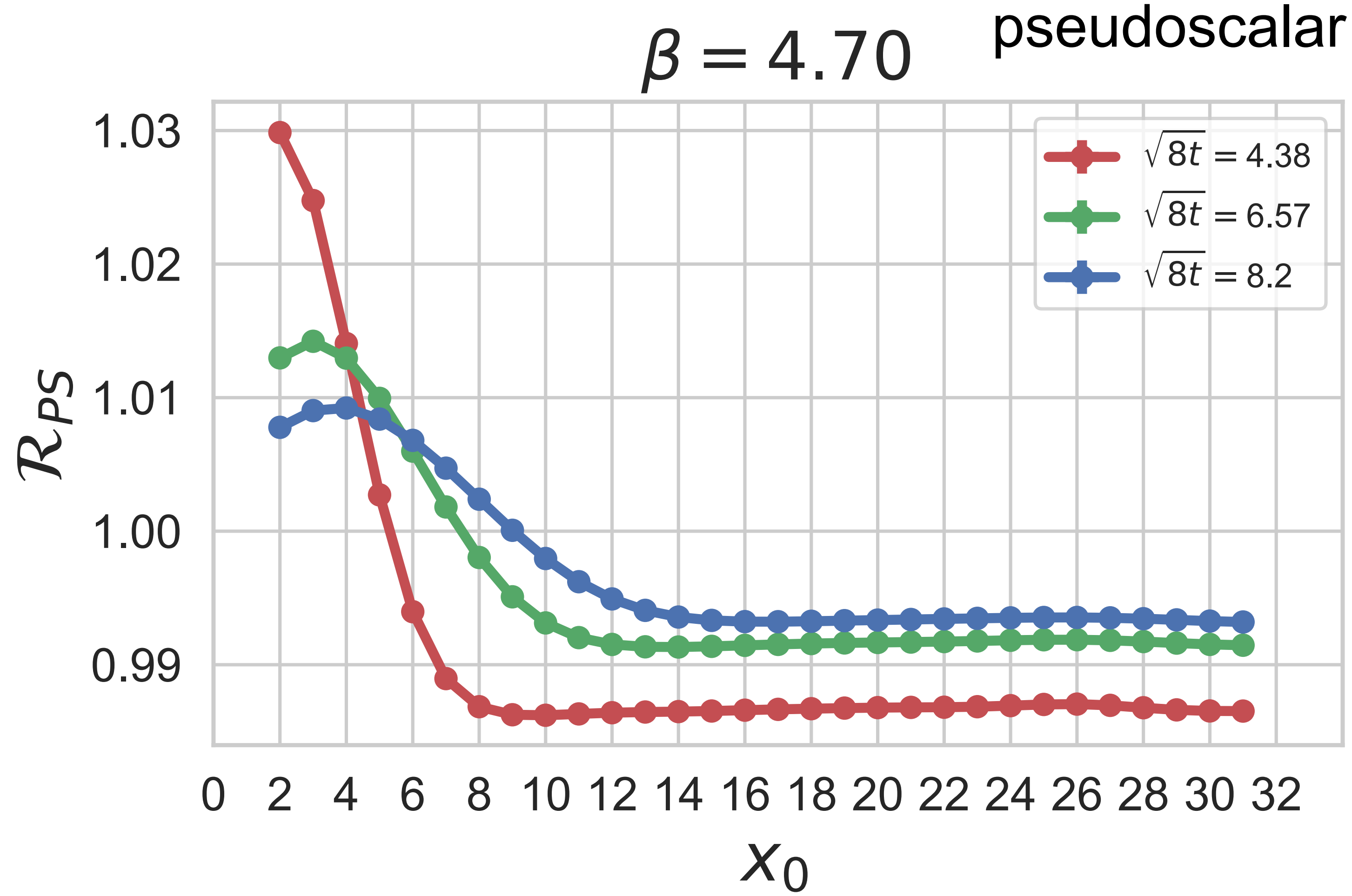
N_f=2 anomalous dimension

Super-ratio

$$\mathcal{R}_t^O(x_0) = \frac{\langle O(0)O_t(x_0)\rangle}{\langle O(0)O(x_0)\rangle}\left(\frac{\langle A(0)A(x_0)\rangle}{\langle A(0)A_t(x_0)\rangle}\right)^{n_O/n_A} = t^{\gamma_O}$$

has no x_0 dependence if $x_0 >> b$

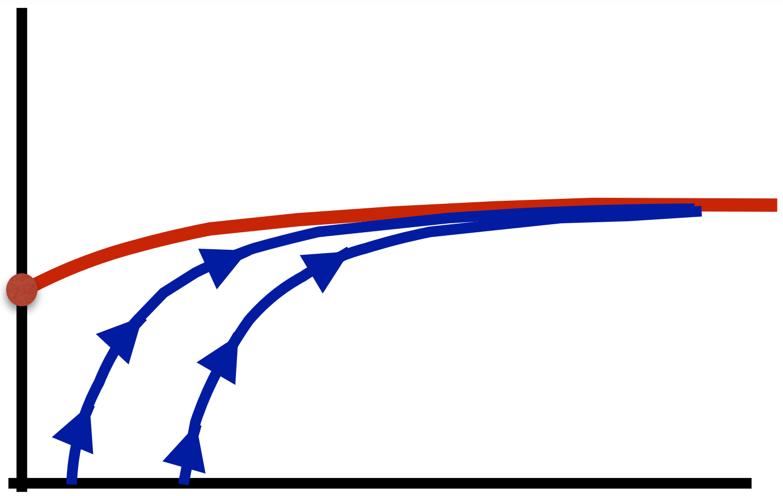
pseudoscalar



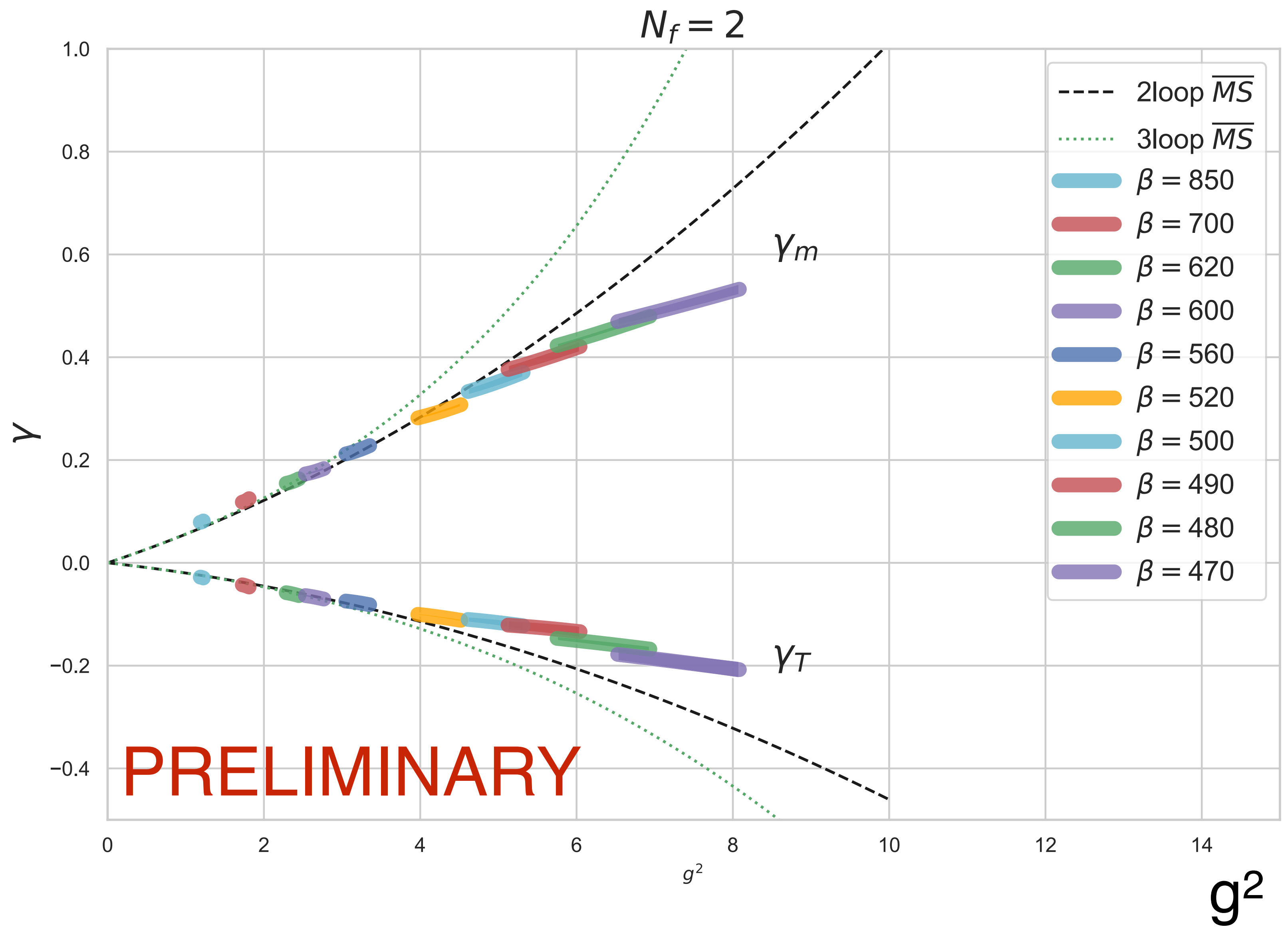
flow time dependence of the plateau gives anomalous dimension

Domain wall

N_f=2 anomalous dimensions



Running anomalous dimension calculation works equally well :



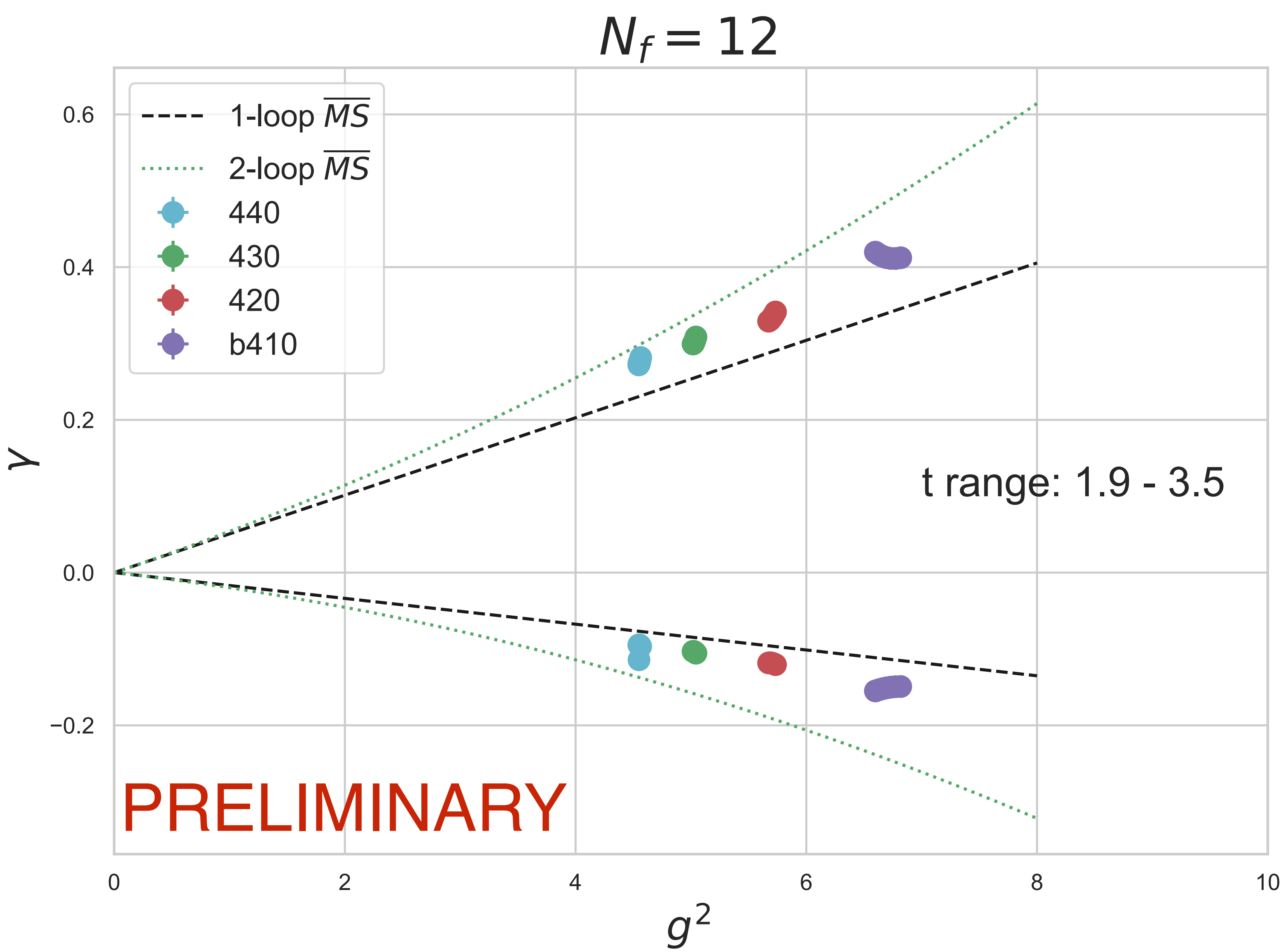
L=24 only
Seems to follow 1-loop PT

Domain wall

s

Running is very slow

g^2 vs γ is continuous, RG β fn is needed to find γ_{IRFP} (see poster)



Domain wall

Summary

- GF can be considered as real-space RG:
  - particularly helpful in conformal systems with IRFP
  - GF is continuous, making the RG efficient

- Continuum physics is along RT :
  - Take the $L/a \to \infty$ limit while keeping g^2 and t/a^2 fixed
  - Take the $a^2/t \to 0$ continuum limit

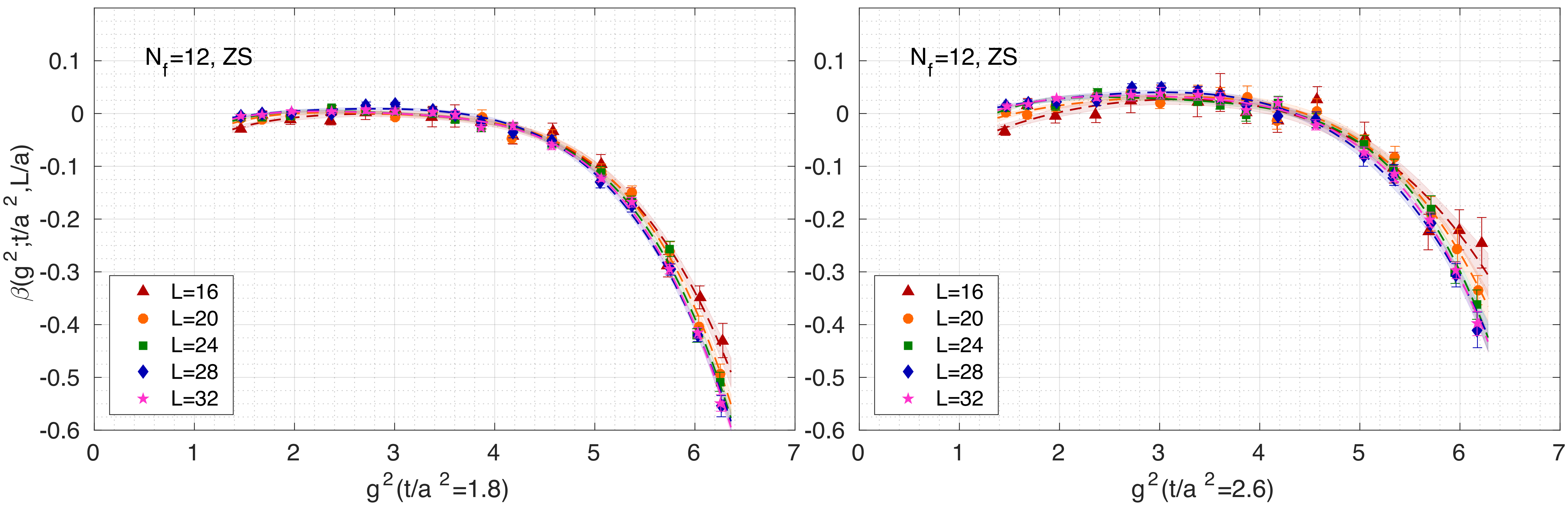
- Showed results for the continuous β function and anomalous dimensions in 2-flavor QCD
  - For a more difficult system see poster on N_f=12 flavors
  - O. Witzel's talk on Thursday on N_f=10 flavors

- Existing step scaling GF data are easy to reanalyze

EXTRA SLIDES

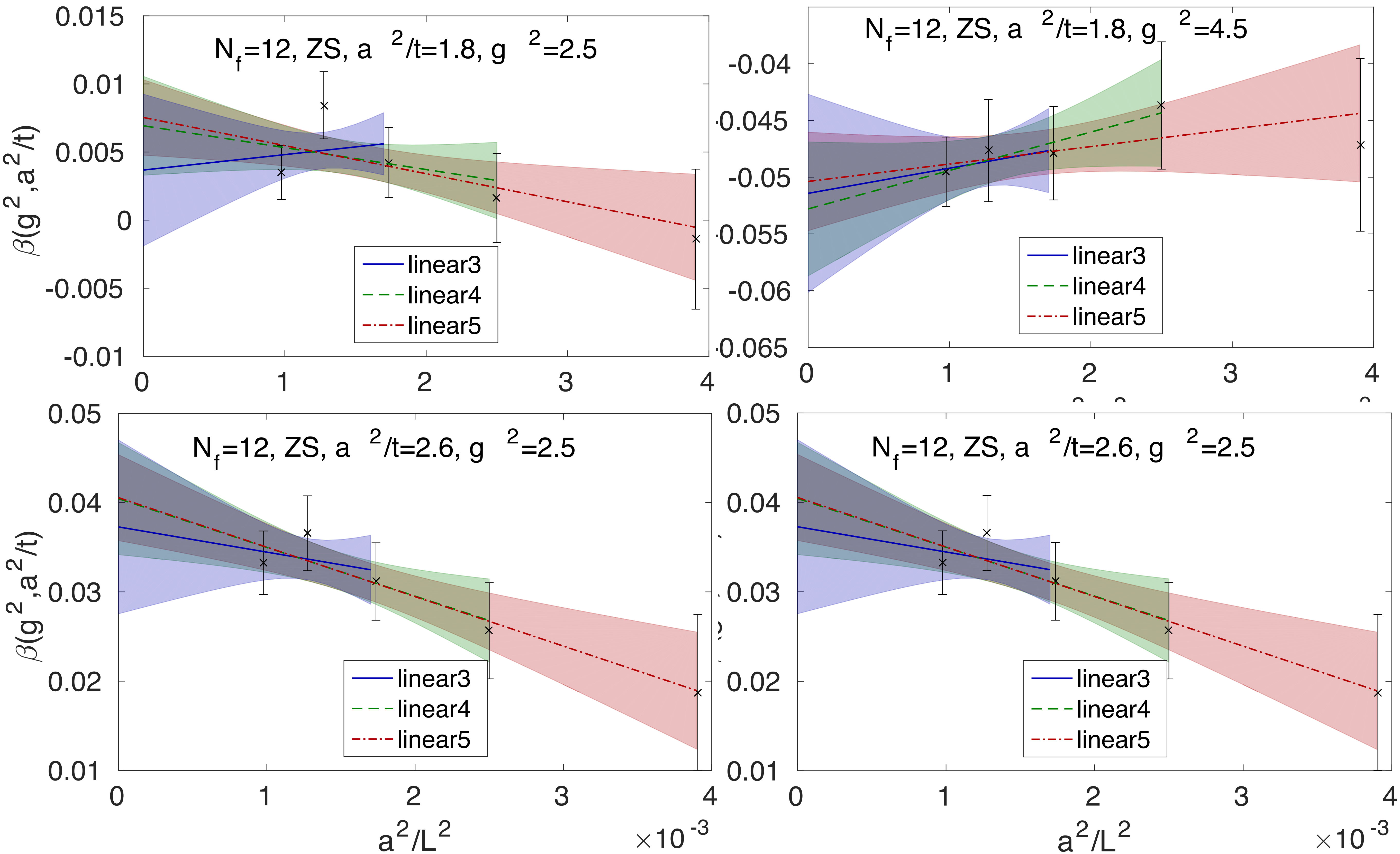
N_f=12 continuous β function

Using existing configurations on L/a = 20,24,28,32 volumes
Domain wall fermions, APBC, Zeuthen flow



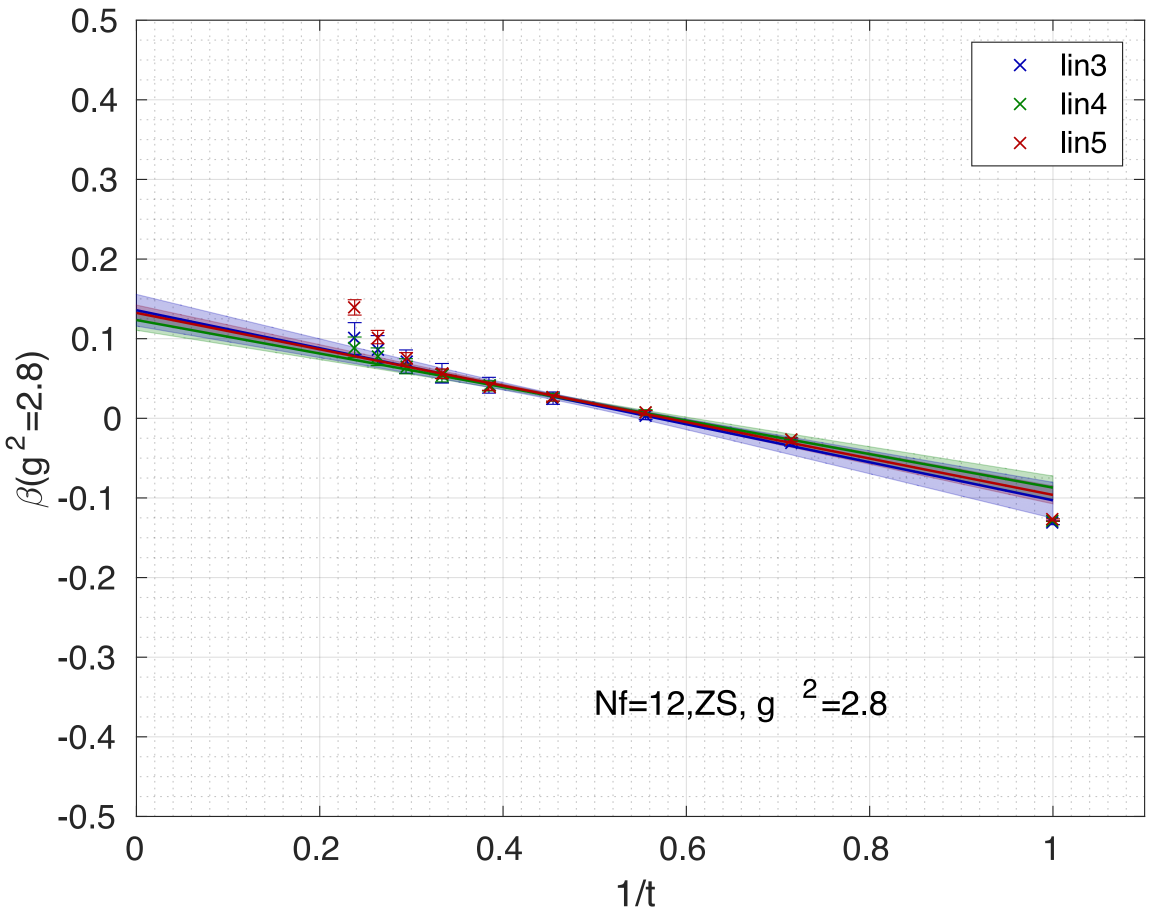
N_f=12 continuous β function

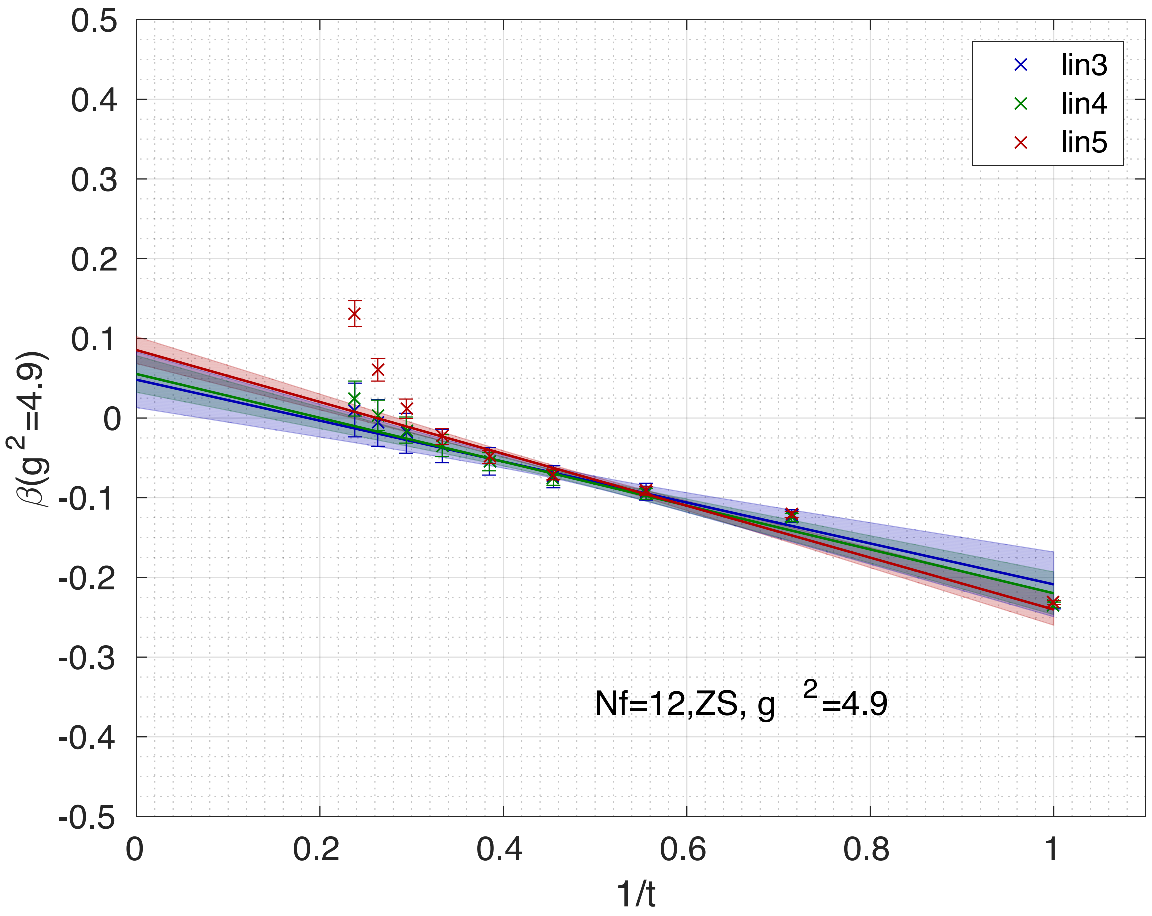
1/L^2 → 0 extrapolation:



N_f=12 continuous β function

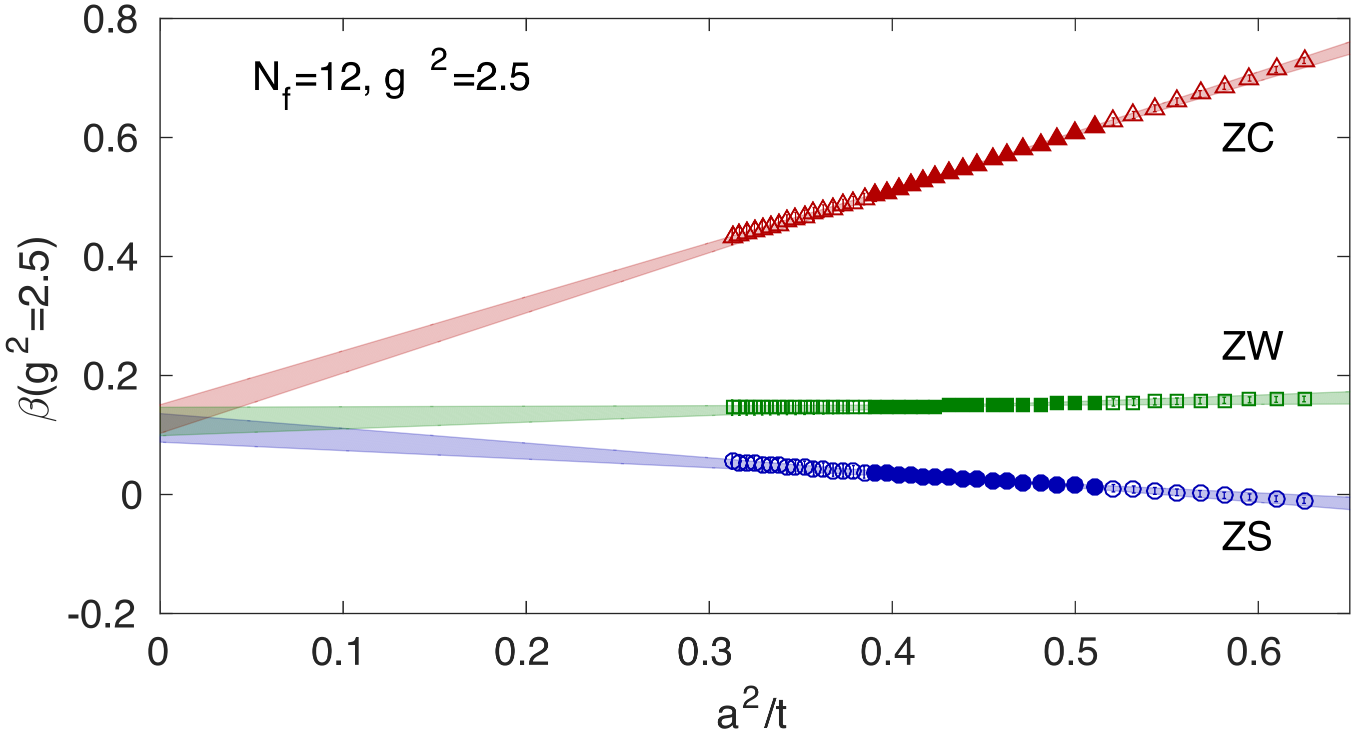
1/t → 0 extrapolation

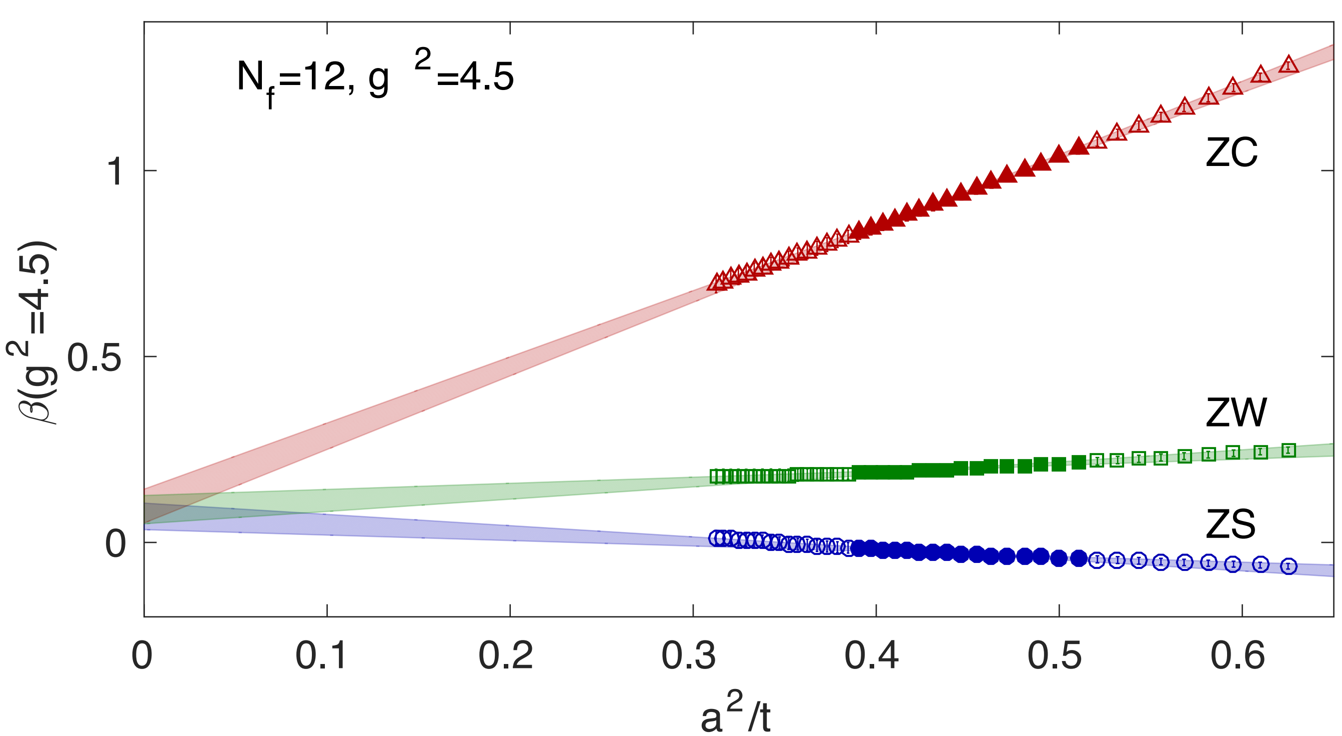




N_f=12 continuous β function

1/t → 0 continuum extrapolation, 3 operators independently





Continuum limit: N_f=12 continuous β function

Various 1/L, 1/t , operator extrapolations