Meson spectrum of large N gauge theories

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Large N gauge theories

SU(N) Gauge theories in the limit $N \longrightarrow \infty$ with re-scaled coupling $g^2 \longrightarrow \frac{\lambda}{N}$ and fermions in various representations

Important in their own right

- ♣ **NON-TRIVIAL, SIMPLER AND RICH**
- ♣ **Connection with Holography/String Theory**

Lattice Gauge Theories needed for non-perturbative study

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METHODOLOGY

- Extrapolation to Large N (Teper, Lucini, Panero, Bali, de Grand, Lopez-Romero, etc)
- With reduced models using Volume independence

QCD in the large N limit *'t Hooft limit*

N_f fermions in the fundamental with $N_f/N \longrightarrow 0$

$\Rightarrow$ Fermions are Quenched

Gauge Field Dynamics: TEK Matrix Model *(AGA, M. Okawa 1983)*

$$Z = \prod_{\mu=0}^{3} \left(\int dV_\mu \right) e^{-S_{\rm TEK}}$$

with

$$S_{\rm TEK} = -Nb \sum_{\mu,\nu} z_{\mu\nu} {\rm Tr}(V_\mu V_\nu V_\mu^\dagger V_\nu^\dagger)$$

and $z_{\mu\nu} = z^*_{\nu\mu} = e^{2\pi i n_{\mu\nu}/N}$ $\quad N = \hat{L}^2 \quad |n_{\mu\nu}| = k\hat{L}$ and $b = 1/\lambda_L$

QCD in the large N limit (continued)

Reduction for Wilson loops

$$\lim_{N\longrightarrow\infty} \prod_{P\in S(\mathcal{C})} (z(P))\langle \mathrm{Tr}(V(\mathcal{C}))\rangle_{\mathrm{TEK}} \longrightarrow W(\mathcal{C})\|_{V=\infty;\ N=\infty}$$

Validity tested

- Non-perturbative proof based on S-D equations
- Perturbative proof to all orders in PT
- Direct verification

	Extrapolated	Matrix Model N=841
Plaquette b=0.36	0.55800(2)	0.557998(5)
3×3 loop b=0.365	0.038592(59)	0.038554(8)
4×4 loop b=0.37	0.009966(45)	0.009926(3)
String tension $\Lambda_{\overline{\mathrm{MS}}}/\sqrt{\sigma}$	0.525(2)	0.523(5)

PROS and CONS of the method

PROS

- Volume reduction valid at the lattice level (artifacts included).
- Continuum limit works as usual $\lim_{N \longrightarrow \infty} a(b, N) = a_{\mathrm{T}EK}(b)$
- Finite N corrections understood:
  1. $\equiv$ to finite size effects in a $(\sqrt{N})^4$ lattice
  2. Non-Planar suppression which depends of k
- N values can be rather large $\Rightarrow$
  Good quenched approximation, no chiral logs, no chiral anomaly

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CONS

- ★ $1/N$ corrections different (Non-commutative field theory)
- ★ Some observables are hard (baryon and glueball masses)

Meson spectrum

Meson masses are interesting observables for which there are estimates/predictions from holography/string theory.
One can obtain a compact formula for $\vec{p} = 0$ time correlator of meson operators $\mathbf{O}_\Gamma(x) = \bar{\Psi}(x)\Gamma\Psi(x)$ for TEK:

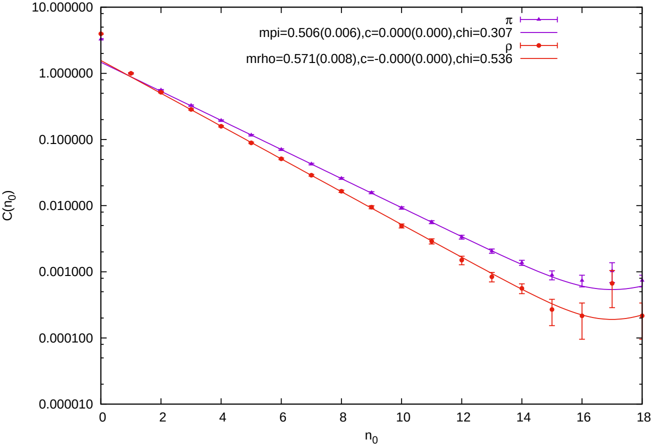
$$C_{\Gamma\Gamma'}(t) = \sum_{p_0} e^{i p_0 t} \, \mathrm{Tr}(\Gamma D^{-1}(p_0)\Gamma' D^{-1}(0))$$

$D(p_\mu) =$ lattice Dirac operator of a single-site with $U_\mu \longrightarrow V_\mu \otimes \Gamma^*_\mu e^{ip_\mu}$. D is an $N^2 \times N^2$ matrix.
The method applies for D being Wilson-Dirac, twisted mass, overlap, staggered, improved, . . .
Fermions living in an effective box of size $2\sqrt{N} \times (\sqrt{N})^3$

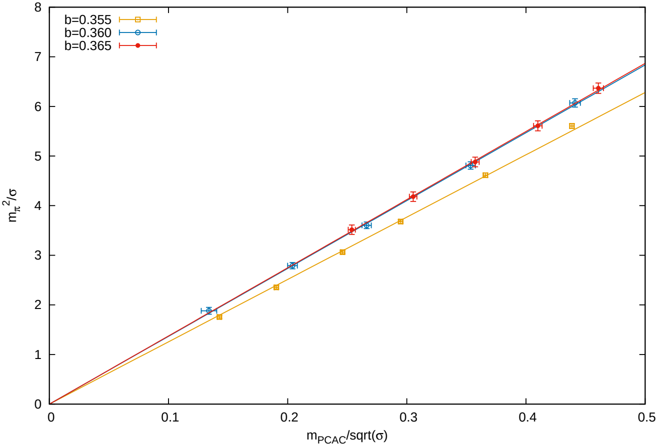
This work

1. 800 gauge configurations $N = 289 = 17^2$ Wilson fermions at 4 values of b (0.355,0.36, 0.365, 0.370) $\Rightarrow$ $a\sqrt{\sigma} = 0.241, 0.2058, 0.1783, 0.1573$
2. Wilson fermions for 5-7 kappa values
3. Local Meson operators (π ρ a_1 b_1) at 12 different spatial smearing levels
4. Pseudoscalar correlator for twisted mass and 4 values of μ (Fernando Romero-López et al)
5. Lowest masses extracted from variational method
6. Full analysis of systematic errors: Comparison to $N = 169 = 13^2$, extension to 3200 configurations, tests of inversion, mass determination method, etc

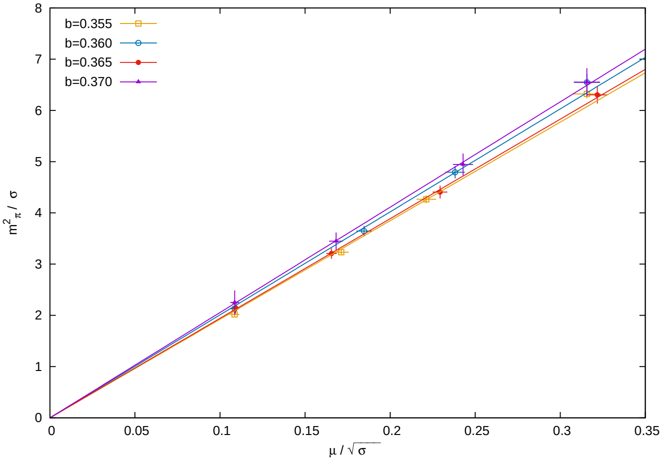
RESULTS: Sample Correlators of optimal operator



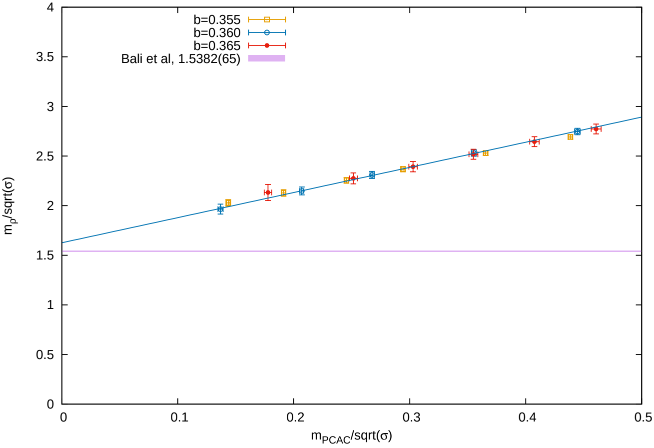
RESULTS: Pion mass and Chiral symmetry (wilson)



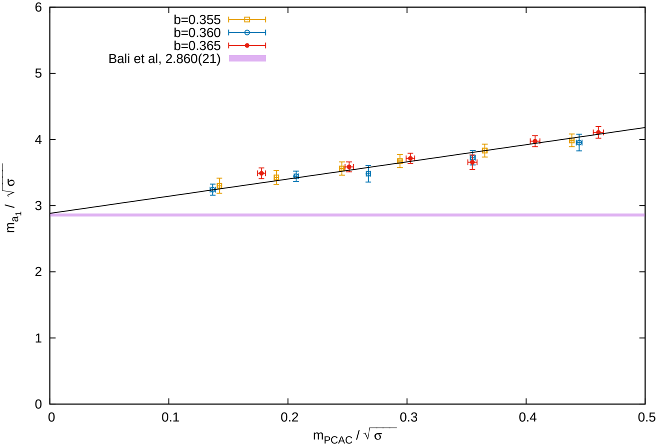
RESULTS: Twisted Mass



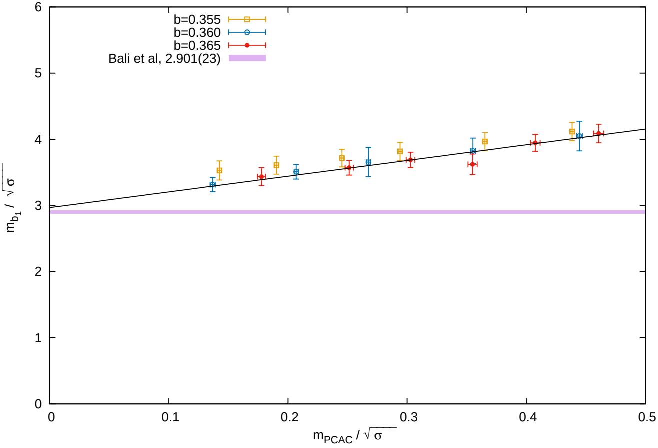
RESULTS: Vector mass



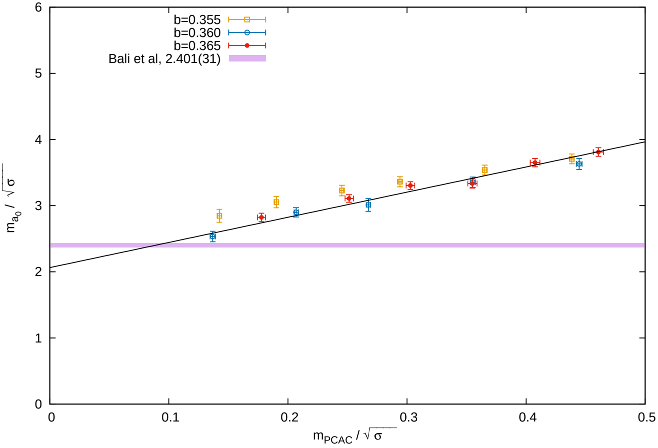
RESULTS: Axial vector a_1



RESULTS: Axial vector b_1



RESULTS: Scalar a_0



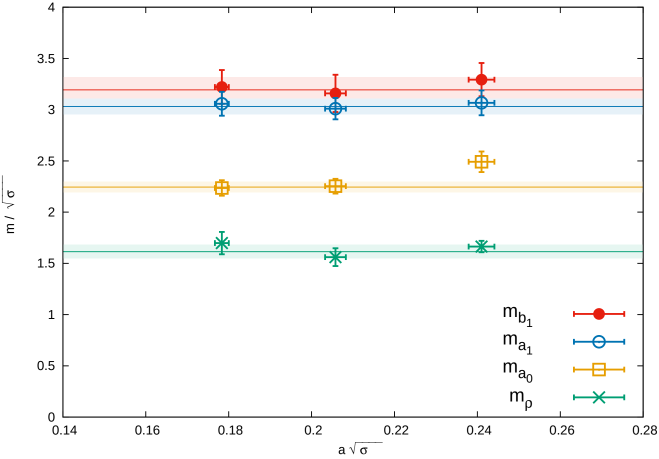
Results: Masses in the chiral and continuum limit

★ Masses linear in m_{PCAC} obtained by simultaneous fit to b=0.360 and 0.365 (Preliminary)

	slope	mass/$\sqrt{\sigma}$	Bali et al
ρ	2.42(12)	1.66(7)(5)	1.538(7)
a_0	3.53(22)	2.20(5)(4)	2.40(4)
a_1	2.32(15)	2.99(8)(2)	2.86(2)
b_1	2.22(17)	3.20(12)(18)	2.90(2)

♠ Renormalization constants and Decay constants also determined

RESULTS: Scaling and continuum limit



Large N theories with dynamical fermions

There are other interesting theories with dynamical quark fields that can be studied:

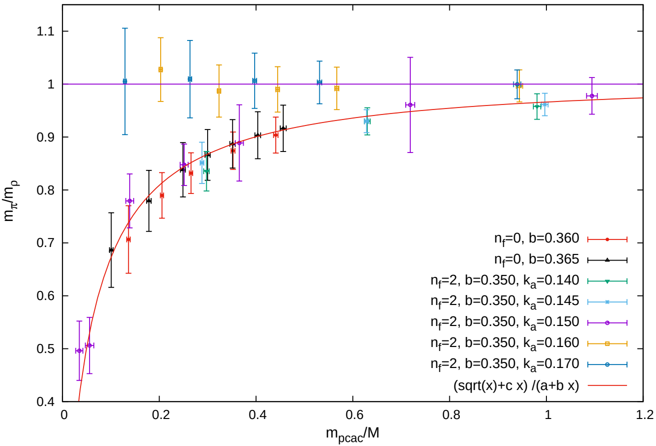
- With N_f/N finite: **Veneziano limit**
- With quarks in the two-index antisymmetric: a different large N limit of QCD *(Corrigan-Ramond)*.
- With N_f fermions in the adjoint representation (*GA Okawa 2013*):
  1. $N_f = 1/2$: $\mathcal{N} = 1$ **SUSY Yang-Mills**
  2. $N_f = 1$: Orientifold planar equivalence to Corrigan-Ramond *(Armoni-Shifman-Veneziano)*
  3. $N_f = 2$: Within the conformal window (infrared fixed point)

Mass anomalous dimension for $N_f = 2$:

$\gamma_* = \mathbf{0.269 \pm 0.002 \pm 0.05}$ (*Garcia Perez, GA, Keegan, Okawa 2015*

Mesons of “additional” fundamental quarks

Mesons of fundamental quarks in $N_f = 2$ Adjoint QCD



CONCLUSIONS

- Use of reduced models is a competitive method to determine observables for large N gauge theories. This is **complementary** to the extrapolation method.
- The meson spectrum can be obtained with quite high precision
- Twisted mass correlators are very well behaved
- Spectrum results for large N QCD in the continuum limit are obtained.
- Theories with dynamical fermions are accesible with this method. Results for $N_f = 2$ adjoint QCD will follow soon.
- Other interesting developments:
  NSPT *(AGA, I. Kanamori, K-I Ishikawa, K. Miyahana, M. Okawa, R. Ueno)*