Lattice 19
Wuhan • China | June 16-22

Prospects for large N gauge theories on the lattice

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Large N & Lattice - the standard approach

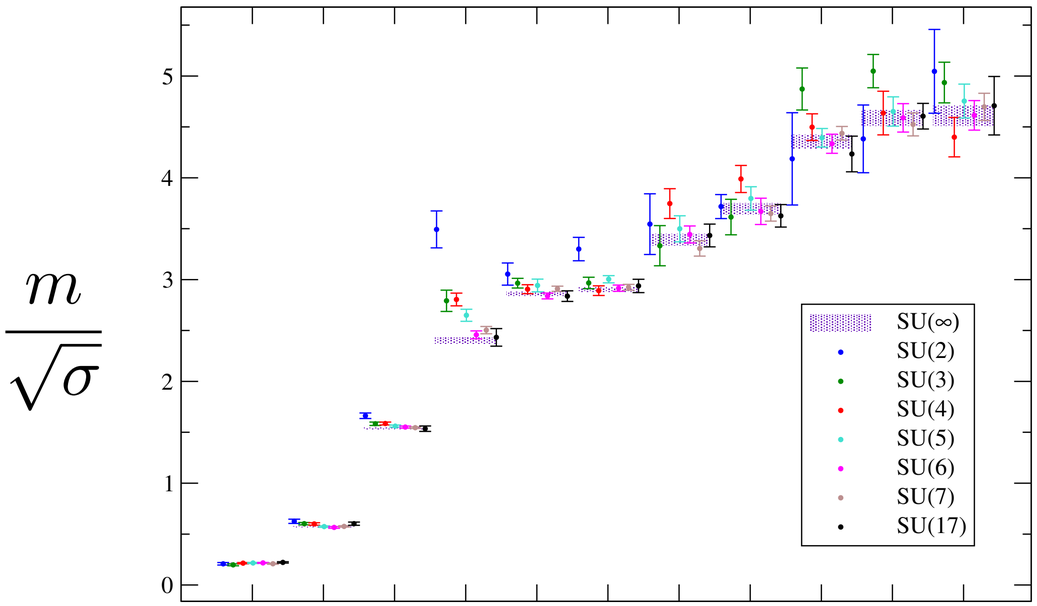
- Large N limit at fixed $a\sqrt{\sigma}$ + continuum limit at N = ∞

- Continuum limit at fixed N + large N extrapolation

Review - Lucini & Panero 2013

Meson spectrum, decay constans

[Bali et al. arXiv:1304.4437]



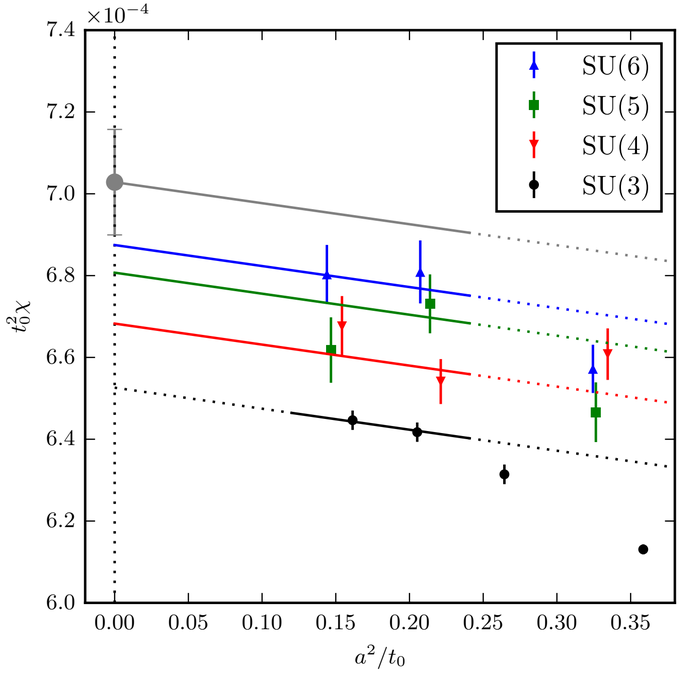
$\hat{F}_\pi$ $\hat{f}_\rho$ ρ a_0 a_1 b_1 π^* ρ^* a_0^* a_1^* b_1^*

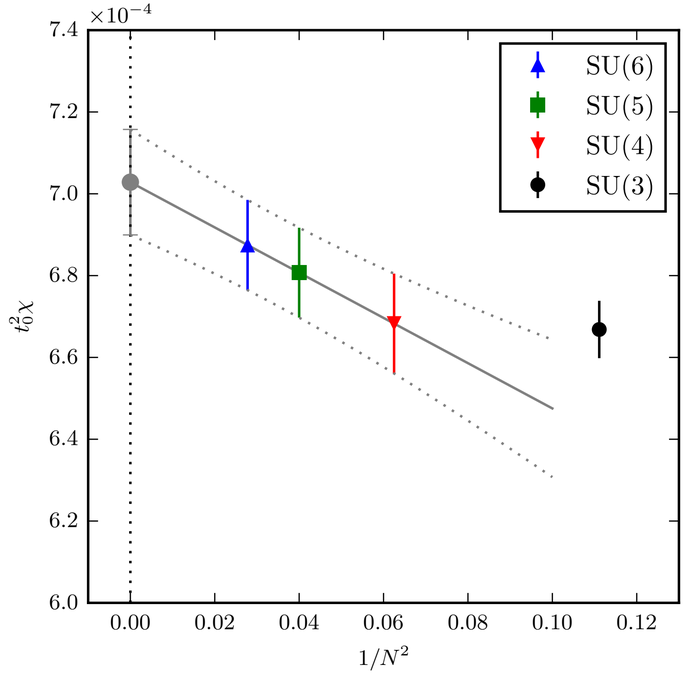
$a\sqrt{\sigma}$ constant

The standard approach

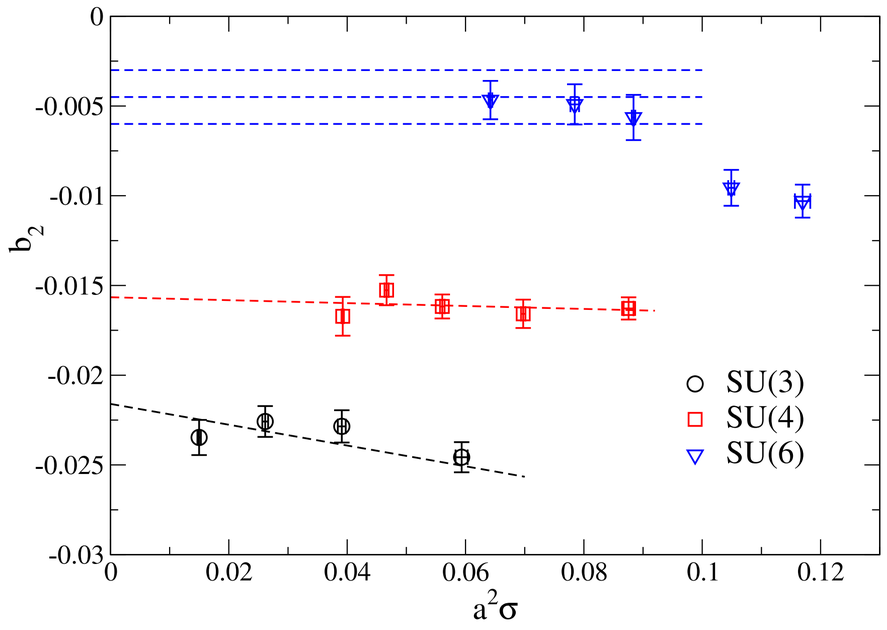
Limited values of N

[Cè et al. arXiv:1607.05039]





[Bonati et al. arXiv:1607.06360]



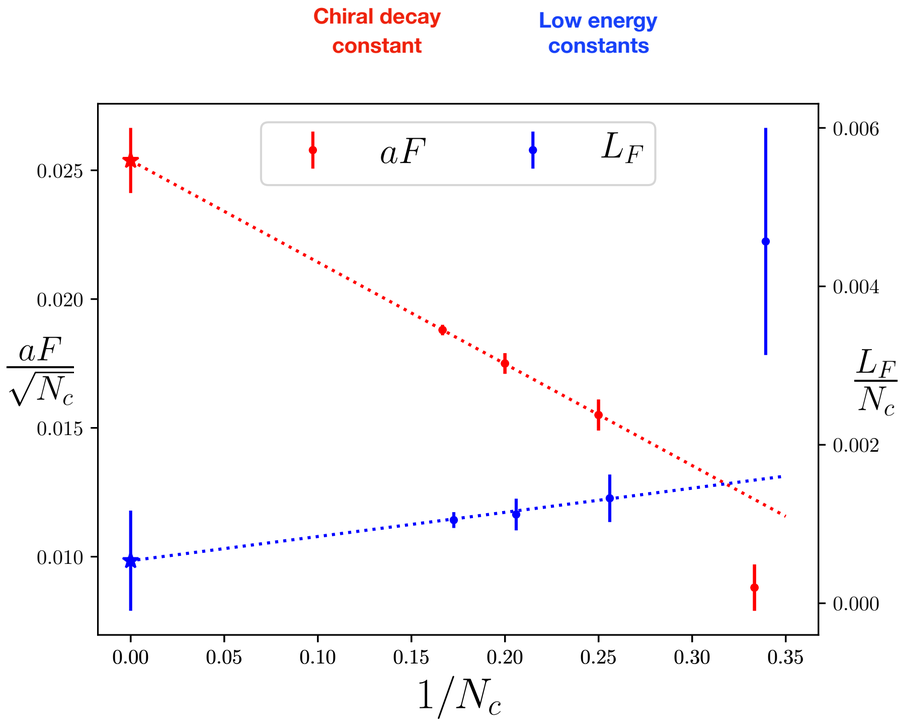
Meson interactions at Large N

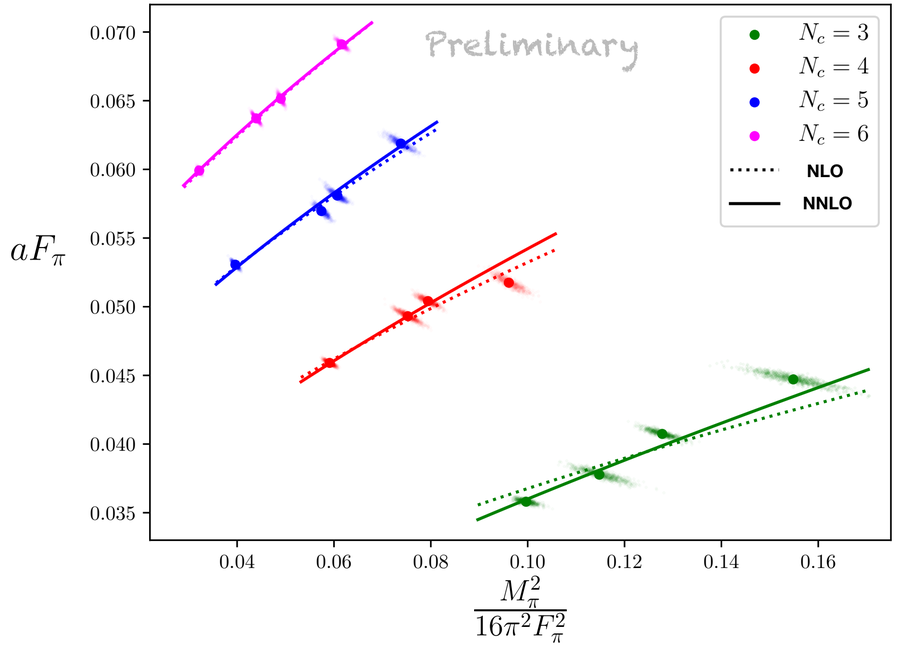
[F. Romero-López, this conference]

- N_c scaling of:
  1. Decay constant
  2. Meson masses
  3. Scattering length
  4. Flavour singlet
  5. Kaon decay amplitudes

Nf=4

N=3,4,5,6





also:
- arXiv:1607.03262
- arXiv:1810.06285
- arXiv:1907.xxxxx

[Donini, Hernández, Pena, Romero-López]

- Quenching effects see also [DeGrand & Liu 2016]

Related talks @ Lattice 19

- Emergent symmetry in baryon spectrum beyond large N [Kaplan]
- Cluster-size scaling in O(N) non-linear sigma model [Bietenholz]
- Thermal phase structure of a SUSY matrix model [Schaich]
- SU(N) models for a holographic description of cosmology [Juettner & Lee]
- SU(N) Interglueball potential [Yamanaka]
- Large N effective theory for heavy dense QCD [Scheunert]
- Sp(2N) Yang-Mills towards large N [Holligan]
- Partial deconfinement [Watanabe]

This talk - an alternative approach

Volume reduction [Eguchi & Kawai 1982]

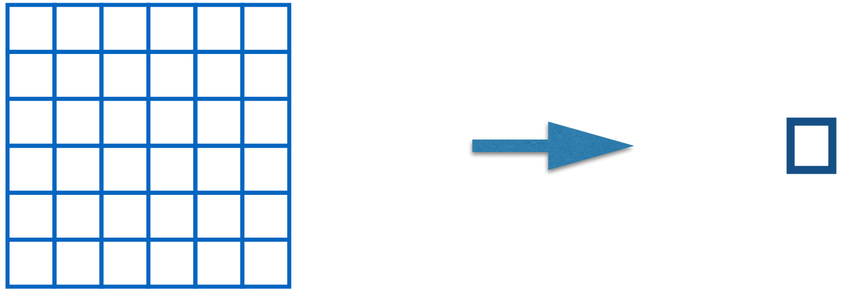
Revival - several working prescriptions

- Efficient tool for lattice large N studies
- Allows very large N (N=1369, Gonzalez-Arroyo & Okawa)
- Theoretically appealing - new avenues @ lattice

Eguchi-Kawai reduction

U(N) gauge theory at large N

is volume independent



- Exact result based on the equality of the loop equations **provided**

$$\boxed{\mathrm{Tr}\,(\longrightarrow) = 0}$$

Preserved center symmetry $\mathbf{Z_N^{\,d}}$

But with **periodic** boundary conditions & **d=4**

$\mathbf{Z_N^4}$ breaking by quantum fluctuations
[Bhanot, Heller & Neuberger]

Several working prescriptions

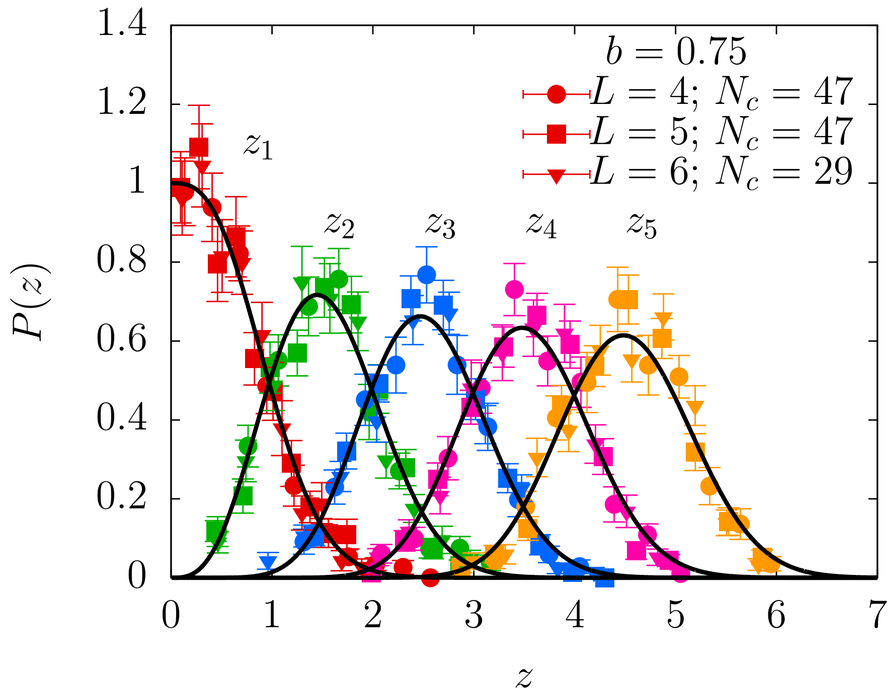
- TEK - twisted boundary conditions [Gonzalez-Arroyo & Okawa]
- QCD(Adj) - add massless adjoint fermions with PBC [Kotvun, Unsal &Yaffe]
- Continuum reduction PBC $La > 1/T_c$ [Kiskis, Narayanan & Neuberger]
- Trace deformations [Unsal & Yaffe]

Continuum reduction - QCD_3

[Karthik & Narayanan arXiv:1607.03905]

Bilinear condensate in quenched large N QCD_3

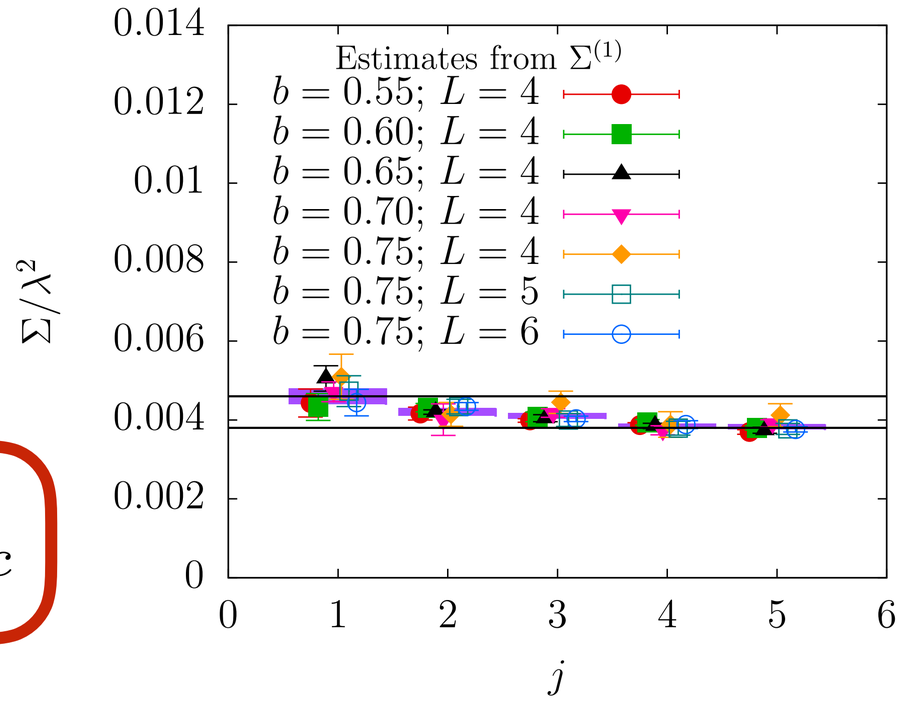
$$\Sigma/\lambda^2 = 0.0042(4)$$



L= 4 - 6

N= 7 - 47

$$La > 1/T_c$$

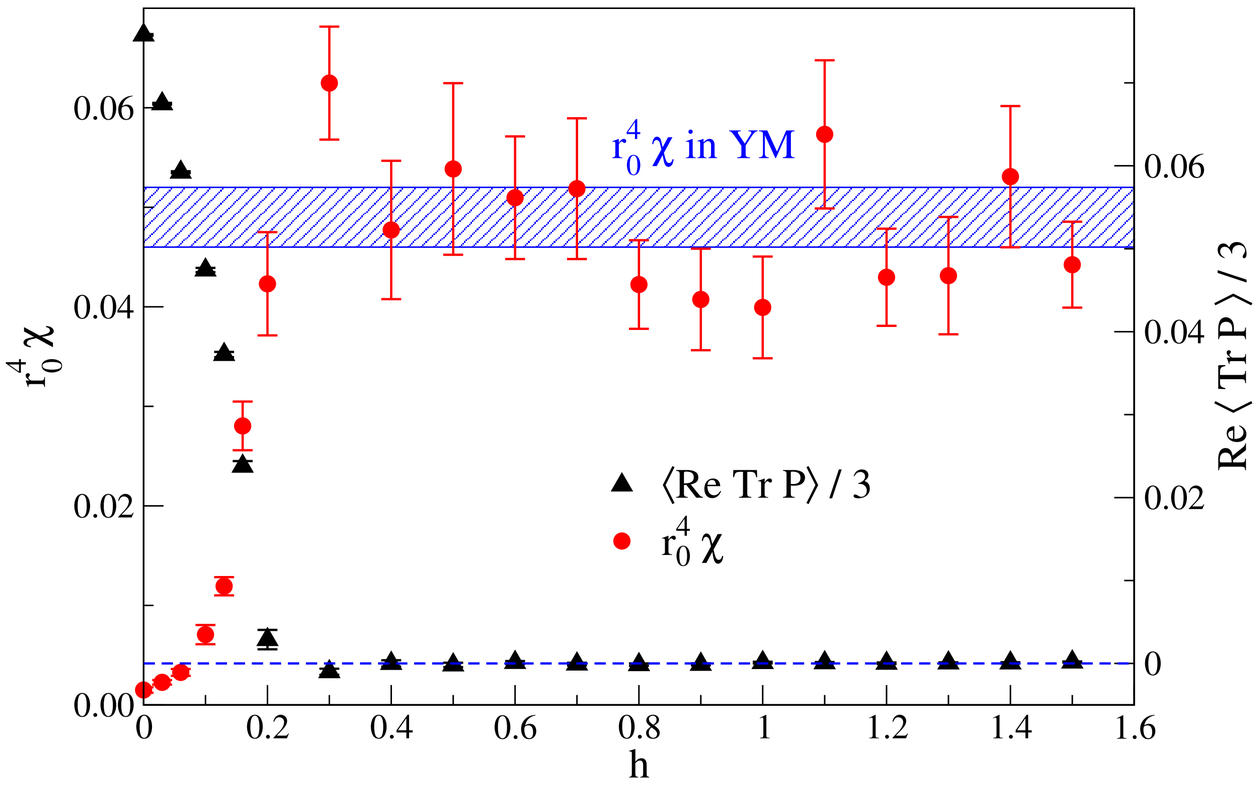


Overlap low lying eigenvalues vs RMM

Condensate from j-th eigenvalue

Trace deformed SU(3) on $R^3 \times S^1$

[Bonati, Cardinali & D'Elia arXiv:1807.06558]



8 x 32^3 lattice, $\beta = 6.4$

$$S_{TD} = h \int |\mathrm{Tr} P(\vec{x})|^2 d^3x$$

Topological susceptibility

Polyakov loop

Talk by Cardinali, this conference

Twisted bc

TBC

[Gonzalez-Arroyo & Okawa]

Adjoint QCD

QCD(Adj)

[Kotvun, Unsal &Yaffe]

Do they work? Why?

Does the PT vacuum respect the symmetry?

Hosotani mechanism

Yang-Mills SU(N) **$R^3 \times S^1$**

Polyakov loop

Minima have

$$\mathrm{Tr}\Omega \in Z_N$$

$$V_{\mathrm{eff}} = \quad - \quad \frac{2}{\pi^2 l^4} \sum_{n=1}^{\infty} \frac{1}{n^4} |\mathrm{Tr}\Omega^n|^2$$

S^1 period

Does the PT vacuum respect the symmetry?

Hosotani mechanism

QCD(Adj) **$R^3 \times S^1$**

Polyakov loop

Minima have

$$\mathrm{Tr}\Omega^n = 0$$

$$V_{\rm eff} = (2N_f - 1)\frac{2}{\pi^2 l^4}\sum_{n=1}^{\infty}\frac{1}{n^4}|\mathrm{Tr}\Omega^n|^2$$

with massless adjoint periodic fermions

Z_N preserved in QCD(Adj) with $2N_f > 1$

Center stabilization + EK reduction [Kotvun, Unsal &Yaffe]

$R^d \times T^n$ one-loop $V_{\rm eff}$ [Barbón & Hoyos based upon Lüscher, van Baal on $R \times T^3$]

Does the PT vacuum respect the symmetry?

[González-Arroyo, Okawa]

TEK

Pure Yang-Mills on **R^d x T^n**

with twisted boundary conditions

$$A_\mu(x + l\hat{\nu}) = \Gamma_\nu A_\mu(x) \Gamma_\nu^\dagger$$

$$\Gamma_\mu \Gamma_\nu = Z_{\mu\nu} \Gamma_\nu \Gamma_\mu$$

$$\boxed{Z_{\mu\nu} = \exp\left\{ i\epsilon_{\mu\nu} \frac{2\pi k}{\hat{N}} \right\}}$$

k & $\hat{N}$ coprime

$$\boxed{\hat{N} = N^{2/n}}$$

n even

For the vacuum configuration all loops winding less than $\hat{N}$ in each direction are traceless

$Z_{\hat{N}}$ preserved at zeroth PT order

Does the PT vacuum respect the symmetry?

[González-Arroyo, Okawa]

TEK

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k & $\hat{N}$ coprime

$$\hat{N} = N^{2/n}$$

n even

For the vacuum configuration all loops winding less than

$Z_{\hat{N}}$ preserved at zeroth

The choice of k is critical

k=1 does not work

(Ishikawa&Okawa, Teper&Vairinhos,Azeyanagi et al.)

k scaled with $\hat{N}$ (see later)

The result is an increased effective volume

Momentum quantization along compact cycles

$$p = \frac{2\pi n}{l_{\text{eff}}}$$

$$l_{\text{eff}} = \hat{N} l$$

$\hat{N} = N$	**R^3 x S^1**
$\hat{N} = N$	**T^2 x R**
$\hat{N} = \sqrt{N}$	**T^4**

The game

$$x = 1$$
$$l_{\text{eff}} = 1/M$$

$$x = M\hat{N}l$$

Large N limit

Eguchi Kawai reduction

- **TEK** $l_{\text{eff}} = a\hat{N}$

González-Arroyo & Okawa

- Continuum reduction $l > 1/M$

Narayanan & Neuberger

Thermodynamic limit

fixed volume

N to infinity

The game

$$x = 1$$
$$l_{\text{eff}} = 1/M$$

$$x = M\hat{N}l$$

Large N limits

TEK

Non-commutative gauge theory

[González-Arroyo & Korthals-Altes]

Finite x non-planar diagrams

Singular large N limit

[Alvarez-Gaumé & Barbón]

Singular large N limit

N to infinity

volume to zero

x fixed

Twisted Eguchi Kawai Reduction on $\mathbf{T^4}$

[González-Arroyo & Okawa]

’t Hooft limit -thermodynamic limit

From a practical point of view it can also be used at finite N to reduce volume effects

Implements a lattice with $\boxed{(L\hat{N})^4}$ → play the standard game

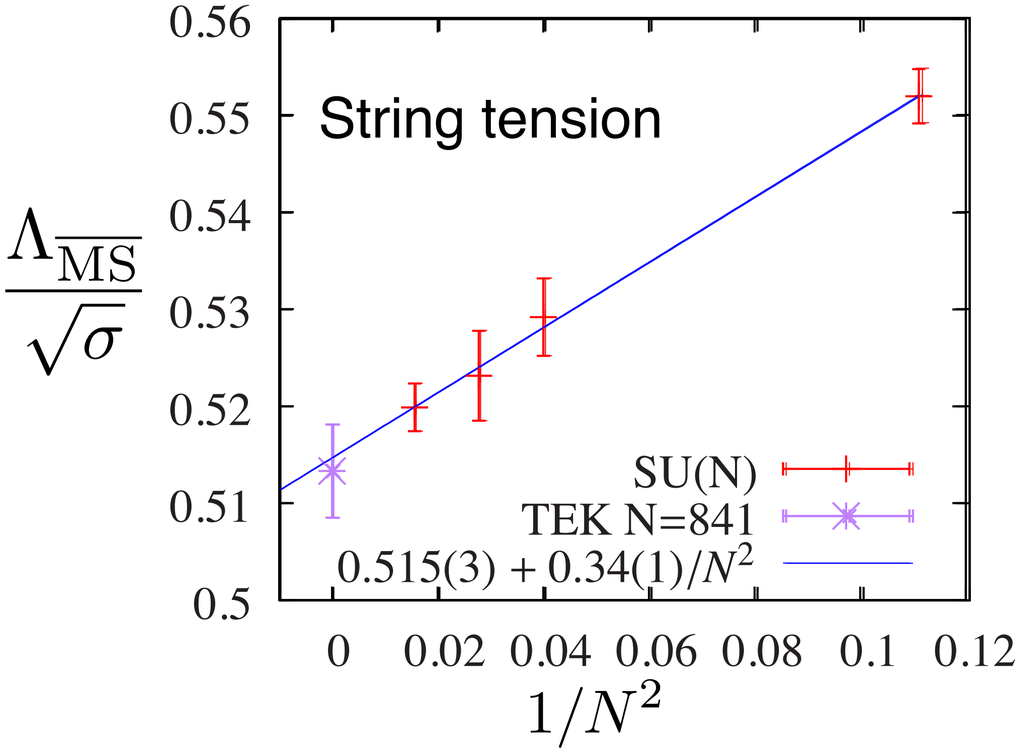
TEK $L = 1$

Twisted Eguchi Kawai Reduction on T^4

[González-Arroyo & Okawa]

$L = 1$ **Pure Yang-Mills**

N=841

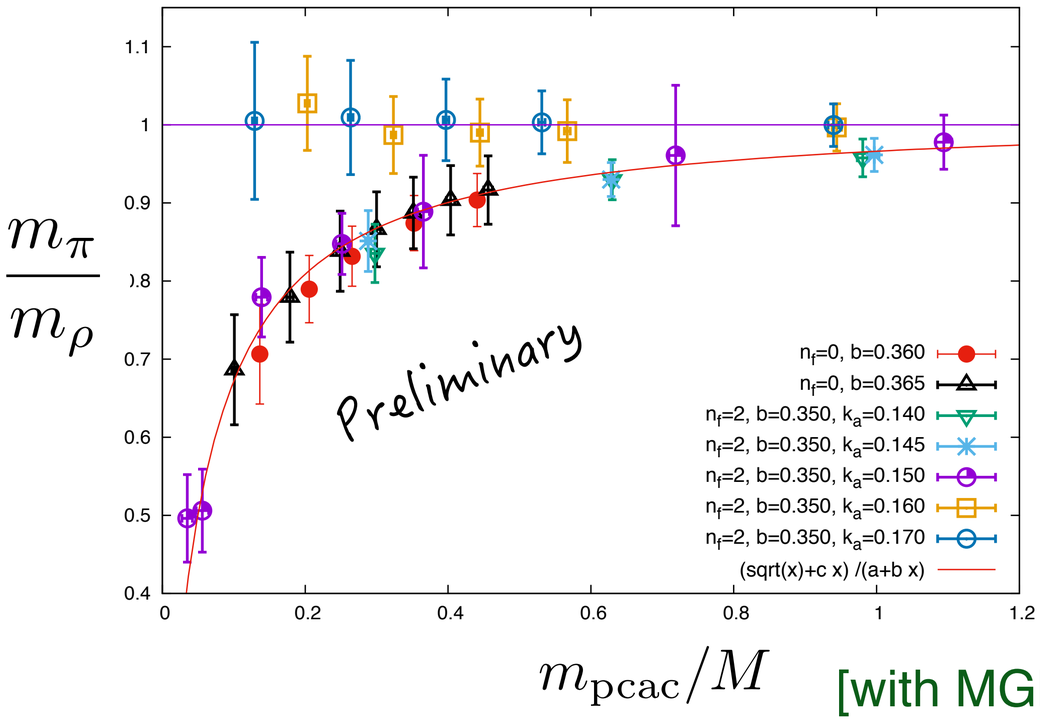


With adjoint fermions N_f=2

N=289

$$\gamma_* = 0.269 \pm 0.002 \pm 0.05$$

[with MGP, Keegan]



[with MGP]

Twisted Eguchi Kawai Reduction on $\mathbf{T^4}$

[González-Arroyo & Okawa]

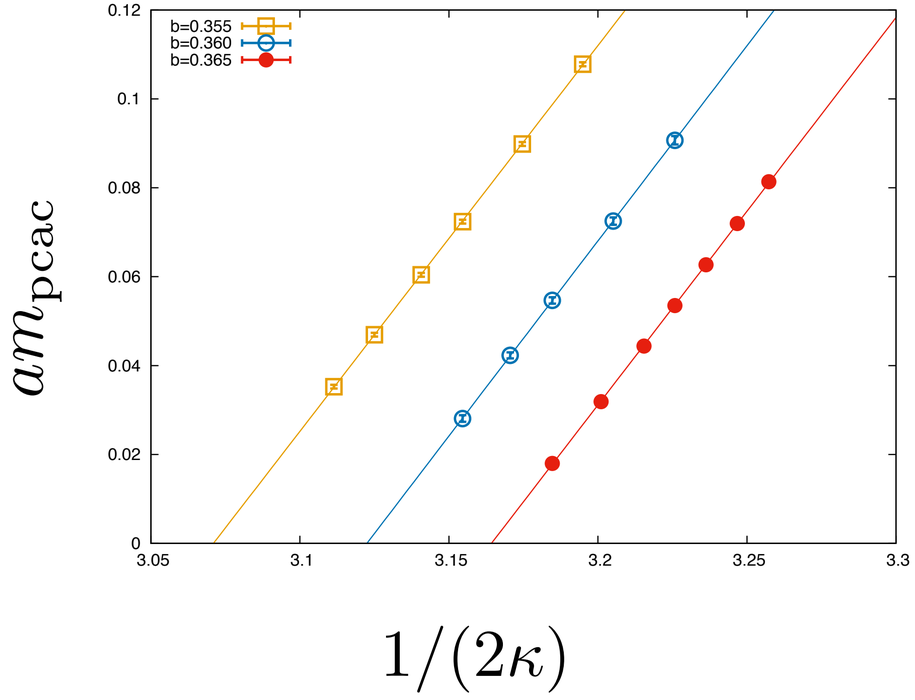
[this conference talk by A. González-Arroyo]

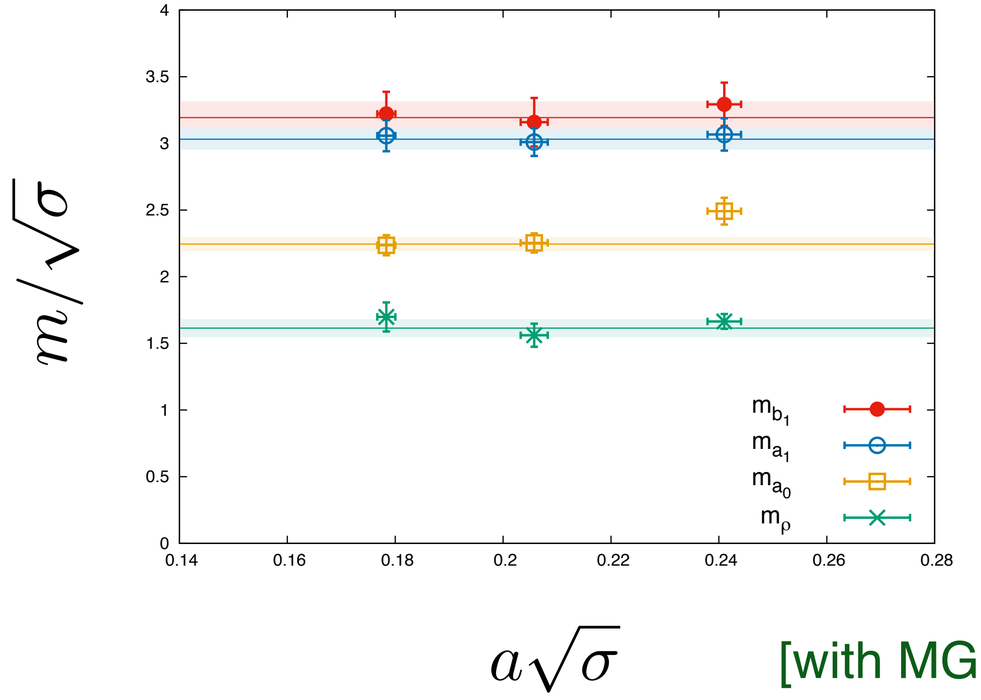
Meson spectrum $N_f/N \to 0$

Fundamental fermions live on a lattice $\hat{N}^3 \times l_0 \hat{N}$

N=289

Wilson & Twisted mass fermions





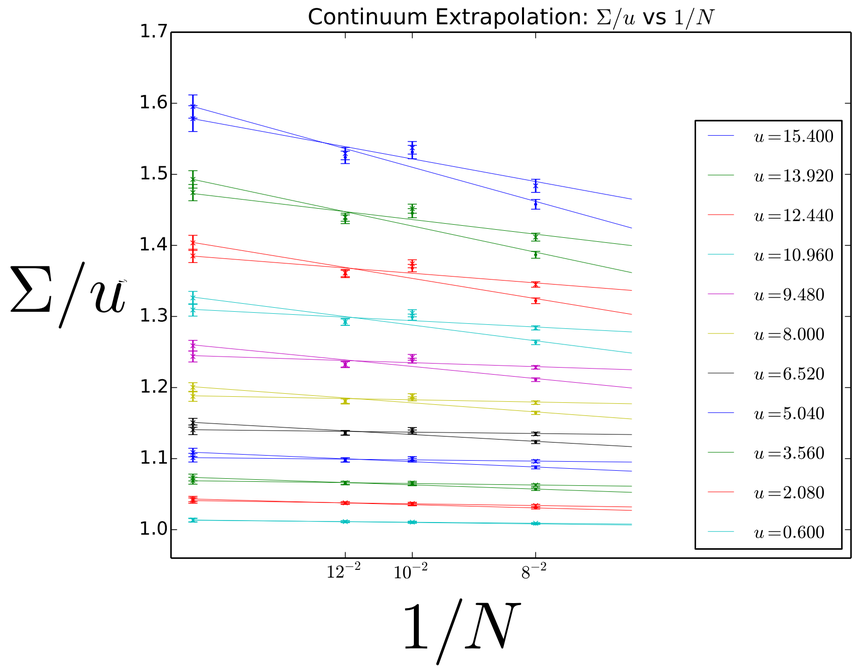
[with MGP]

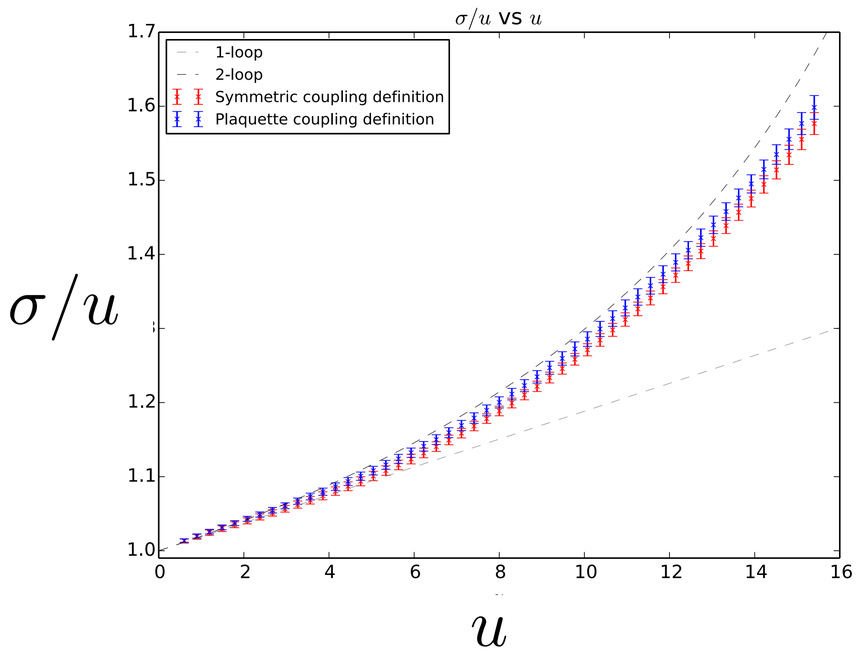
Twisted Eguchi Kawai Reduction on T^4

[MGP, Keegan, González-Arroyo & Okawa]

Gradient flow coupling

Step scaling with the rank of the group $l_{\text{eff}} = a\hat{N}$ → $\lambda(l_{\text{eff}})$





One loop PT [Bribián & MGP]

NSPT [Ishikawa, this conference]

The game

$$x = 1$$
$$l_{\text{eff}} = 1/M$$

$$x = M\hat{N}l$$

Analytic + Semi-classical

Thermodynamic limit

Lattice

The standard game

fixed N

varying volume

80s-90s Gonzalez-Arroyo, Lüscher, van Baal, …

$$z = Ml$$

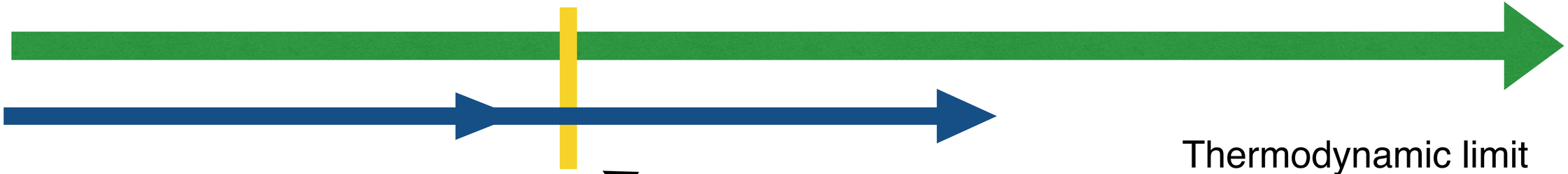
Adiabatic continuity

Shifman, Unsal, Yaffe, …

The game

$$x = 1$$
$$l_{\text{eff}} = 1/M$$

$$x = M\hat{N}l$$



Analytic + Semi-classical

Thermodynamic limit

Lattice

The standard game

fixed N

varying volume

80s-90s Gonzalez-Arroyo, Lüscher, van Baal, …

$$z = Ml$$

Adiabatic continuity

Shifman, Unsal, Yaffe, …

The standard game

Analytic + Semi-classical + Lattice at finite N

80s-90s $T^3 \times R$ SU(2)

- **PBC** mass gap - tunnelling through vacuum induced barrier [van Baal et al.]
- **TBC** mass gap - tunnelling by fractional instantons [González-Arroyo et al.]

$T^2 \times R$ SU(N)

- Electric flux and glueball spectrum - one loop PT and lattice

[García Pérez, Koren, González-Arroyo & Okawa]

The standard game

Analytic + Semi-classical + Lattice at finite N

A large amount of analytic work on **QCD(Adj)** Review 2016 [Dunne & Unsal]

- $R^3 \times S^1$ mass gap - induced by bions [Unsal]
- Analytic glueball, meson, string tension [Aitken&Cherman&Poppitz&Yaffe]
- CP^{N-1} phase structure, resurgence [i.e. this conference talk by T. Misumi]
- $\mathcal{N}$=1 SUSY with mass deformation [Poppitz & Schaefer & Unsal] [Bergner et al. - lattice SU(2)]

The standard game

- $\mathcal{N}$=1 SUSY - mass deformation on $R^3 \times S^1$

[Poppitz, Schäfer & Unsal]

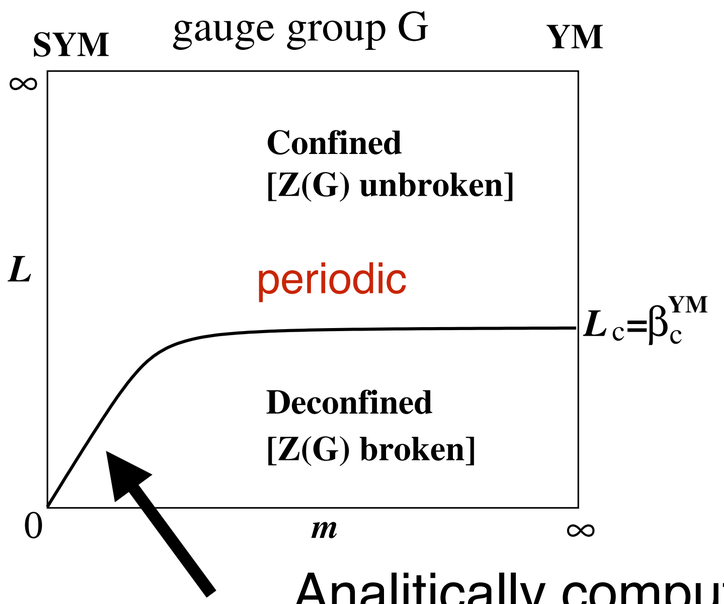
- $S^3 \times S^1$

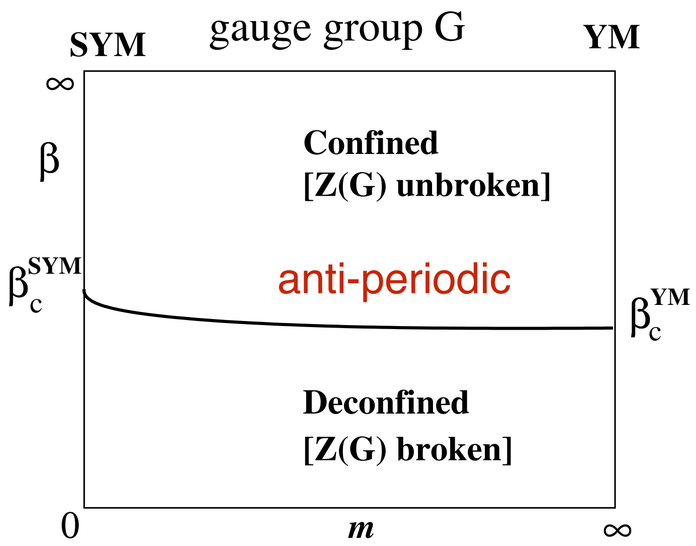
[Hollowood & Myers]

- Lattice tests

[Bergner et al.]

[Bergner, this conference]





Analitically computable

$N_f > 1/2$ [Cossu&D'Elia, Cossu et al.]

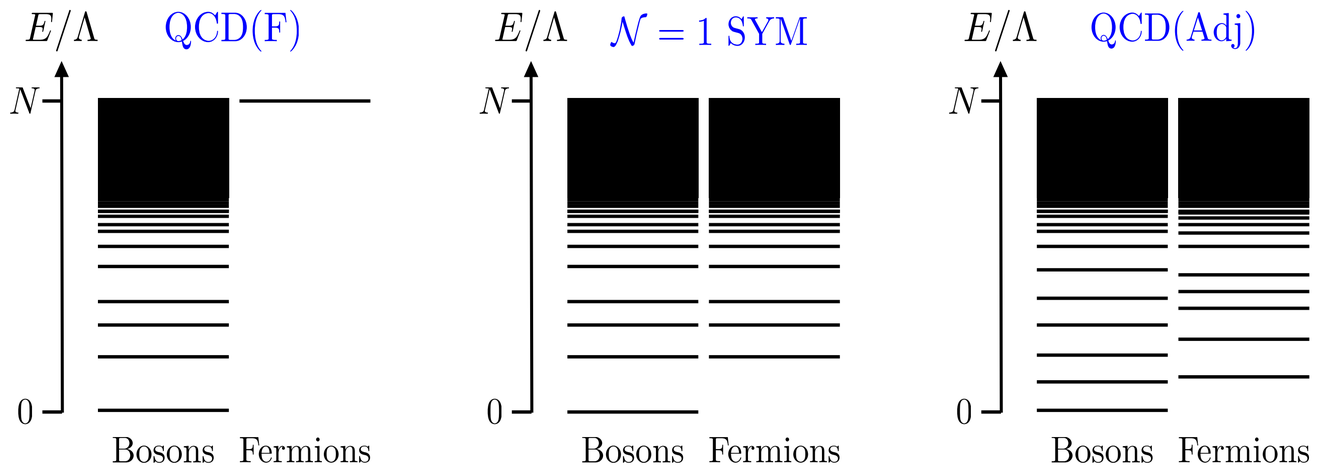
T^4 [Bringoltz, Koren & Sharpe, Hietanen & Narayanan, Lohmayer & Narayanan, Cunningham & Giedt]

Going to large N - Emergent fermionic symmetry

QCD(ADJ)

[Basar, Cherman , Dorigoni & Unsal]

Volume reduction has implications for the spectrum



From [Cherman, Shifman & Unsal, arXiv:1812.04642]

(Basar, Cherman , Dienes, Dorigoni, McGady,……)

- Cancelation of Hagedorn growth

$$I(l) = \mathrm{Tr}\left[(-1)^F e^{-lH}\right]$$

$$I(l) = \int dE\,[\Omega_B(E) - \Omega_F(E)]e^{-lE}$$

- Discussed also in the context of planar equivalences

  i.e. Barbón & Hoyos

- Missaligned SUSY Dienes

Going to large N - Singular large N limits

T^2 x R with twist SU(N)

$$x = \lambda N l/(4\pi)$$

$$\tilde{\theta} = 2\pi \bar{k}/N$$

Morita duality

Non-commutative U(1)

$$l_{\text{eff}} = N l$$

$$\Theta = \frac{l_{\text{eff}}^2}{(2\pi)^2}\,\tilde{\theta}$$

- Possible non-perturbative regulator of NC

[Ambjorn, Makeenko, Nishimura & Szabo]

BUT

Tachyonic instabilities at one-loop [Guralnik, Landsteiner, López]

Also non-perturbatively i.e. [Bietenholz, Nishimura, Susaki & Volkholz]

T^2 x R

$$\mathcal{E}^2_{\vec{n}}(x,\tilde{\theta}) = \frac{|\vec{n}|^2}{4x^2} - G\left(\frac{\tilde{\theta}\vec{n}}{2\pi}\right)\frac{1}{x}$$

Energy of electric flux (in units of λ)

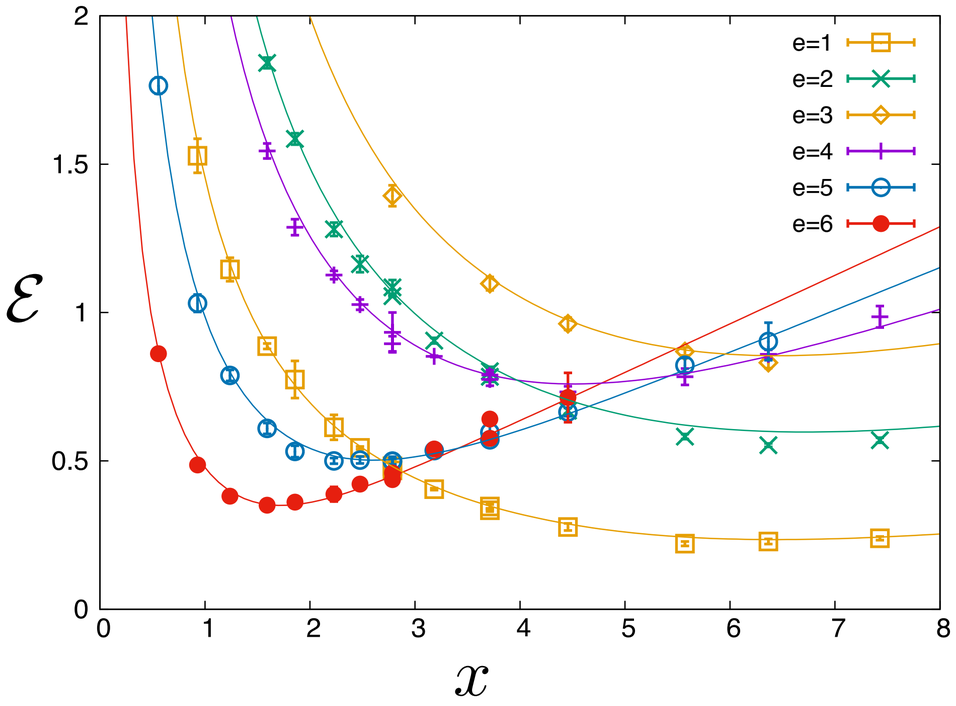
[García Pérez, Koren, González-Arroyo & Okawa]

Going to large N - Singular large N limits

[García Pérez, Koren, González-Arroyo & Okawa]

Combined analytic and numerical analysis for the electric flux spectrum at various N

$$\mathcal{E}^2 = \mathcal{E}^2_{1-\mathrm{loop}} + \mathcal{E}^2_{\mathrm{Nambu-Goto}}$$



The absence of tachyonic behaviour in the electric flux spectrum requires

$$Z_{\min}(N,k) \equiv \min_{e\perp N} e \left|\left|\frac{ke}{N}\right|\right| \gtrsim 0.1$$

[González-Arroyo & Chamizo]

Is it possible for any N to choose an appropriate k?

Unproven Zaremba's conjecture

It holds for almost all values of N [Huang]

The Golden Ratio and Non-Commutative gauge theories

[García Pérez, Koren, González-Arroyo & Okawa]

A different question - singular large N limits

Can we reach any value of the NC parameter at large N?

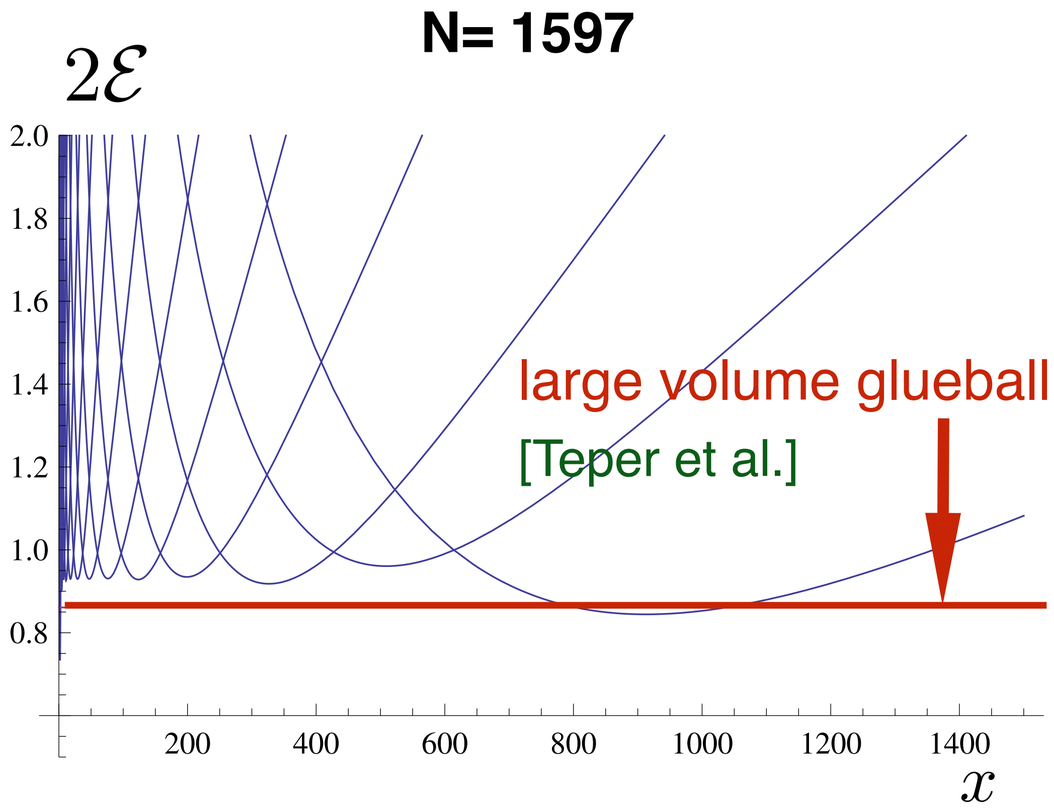
$$\lim_{i \longrightarrow \infty} \frac{\bar{k}_i}{N_i} = \frac{\tilde{\theta}}{2\pi}$$

NO, only for an uncountable zero-measure set

$$\frac{\bar{k}_i}{N_i} = \frac{F_{i-2}}{F_i} \longrightarrow \frac{3-\sqrt{5}}{2}$$

optimal Z_{min}

Fibonacci numbers



Lattice Large N from the perspective of volume independence

Efficient tool for large N extrapolations

Mesons, string tension, running coupling …

Conformality

Theta dependence

Veneziano limit

Several conjectures in the game

Continuity in the crossover region

Phase structure

SUSY - mass deformations

Non - commutativity

Resurgence

many games to play