

# A QUBIT REGULARIZATION OF THE $O(3)$ NON-LINEAR SIGMA MODEL

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# QFTs and quantum computers

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- Sign problem, non-equilibrium processes, real-time dynamics
- Quantum computing is a promising approach to overcome these bottlenecks
- Lots of recent work in this direction
  - (Jordan, Lee, and Preskill 2012; Casanova, Lamata, et al. 2011; Macridin et al. 2018; Roggero and Carlson 2018; Zohar, Cirac, and Reznik 2016; Pichler et al. 2016; Martinez et al. 2016; Bañuls et al. 2017; Klco et al. 2018; Kaplan and Stryker 2018; Frank, Huffman, and Chandrasekharan 2019).
  - See other talks in this session!

## Qubit regularization of QFT

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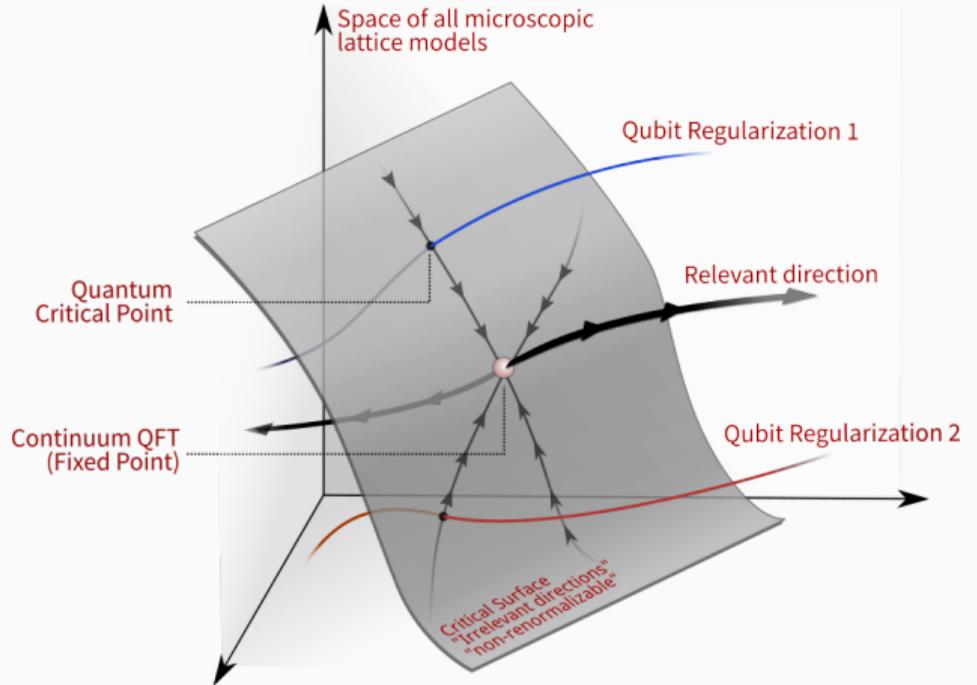
- QFT on a quantum computer  $\implies$  a local Hamiltonian with a finite dimensional Hilbert space at each lattice site.
- But traditional lattice formulations require an infinite dimensional Hilbert space at each site:

$$[\phi_x, \pi_y] = i\delta_{x,y} \quad (1)$$

- A **qubit regularization of a QFT** is defined as the construction of local quantum lattice Hamiltonian with a *finite dimensional* Hilbert space at each lattice site, which reproduces the continuum QFT at a quantum critical point.

# Qubit regularization and Wilson's RG

- Identifying quantum critical points can require non-perturbative methods



## $O(3)$ sigma model

- Goal: To show that sometimes we only need a *very small number* of qubits to accomplish this  $\implies$  Recover the continuum physics at the fixed points of  $O(3)$  sigma model in  $d + 1$  spacetime dimensions
- Traditional lattice regularized action

$$S = -\frac{1}{g} \sum_{\langle ij \rangle} \vec{\phi}_i \cdot \vec{\phi}_j \quad (2)$$

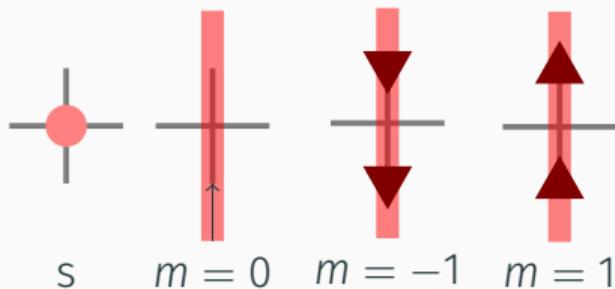
where  $\vec{\phi} \in R^3$  and  $\vec{\phi}^2 = 1$ .

- Fixed point:
  - $d = 1$ : Asymptotically free
  - $d = 2$ : Wilson-Fisher fixed point
  - $d = 3$ : Gaussian fixed point (free field theory)
- Q: Can we reproduce this using a qubit Hamiltonian? A: Yes! With 2 qubits.

1. Write down a qubit Hamiltonian for the  $O(3)$  nonlinear sigma model
2. Construct efficient Monte Carlo algorithm to locate the critical point
3. Perform a quantum simulation of the qubit Hamiltonian at the critical point

## $O(3)$ sigma model: A 2-qubit regularization

- 2 qubits per lattice site (4 dimensional Hilbert space):
  - singlets  $|s, r\rangle$ : Fock vacuum
  - triplets  $|m, r\rangle$  ( $m = 0, \pm 1$ ): particles carrying  $O(3)$  charge



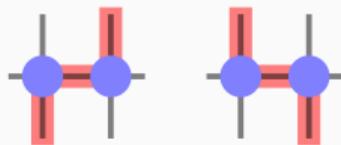
- Proposed Hamiltonian:

$$H = J_t \sum_{m,r} |m, r\rangle \langle m, r| - J \sum_{\langle r, r' \rangle} \left( H_{r,r'}^h + H_{r,r'}^p \right), \quad (3)$$

## $O(3)$ sigma model: A 2-qubit regularization

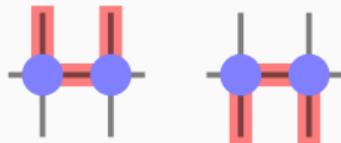
- $H_{r,r'}^h$  is a 'hopping' term

$$H_{r,r'}^h = \sum_m \left\{ |s, r\rangle |m, r'\rangle \langle m, r| \langle s, r'| + |m, r\rangle |s, r'\rangle \langle s, r| \langle m, r'| \right\} \quad (4)$$



- $H_{r,r'}^p$  is a pair creation/annihilation term

$$H_{r,r'}^p = \sum_m \left\{ |m, r\rangle |-m, r'\rangle \langle s, r| \langle s, r'| + |s, r\rangle |s, r'\rangle \langle m, r| \langle -m, r'| \right\}. \quad (5)$$



## $O(3)$ sigma model: A 2-qubit regularization

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### Symmetries

- Under an  $O(3)$  transformation  $\Lambda \in O(3)$

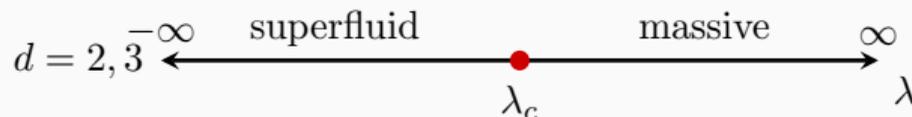
$$|m, \mathbf{r}\rangle \rightarrow D_{m,m'}^{(1)}(\Lambda)|m', \mathbf{r}\rangle \quad (6)$$

### Other models

- $O(2)$ : suppress the  $m = 0$  states
- $Z_2$ : suppress the  $m = \pm 1$  states

# Symmetries and Phase Diagram

Define coupling  $\lambda = J_t/J$



- $\lambda \rightarrow \infty$ 
  - Singlets dominate
  - Symmetric massive phase
- $\lambda \rightarrow -\infty$ 
  - Triplets dominate
  - Massless phase, SSB
- Second order phase transition at some  $\lambda_c \implies$  continuum QFT

1. Write down a qubit Hamiltonian for the  $O(3)$  nonlinear sigma model
2. **Construct efficient Monte Carlo algorithm to locate the critical point**  
     $\implies$  Worldline formulation and Worm algorithms
3. Perform a quantum simulation of the qubit Hamiltonian at the critical point

# Worldline Formulation

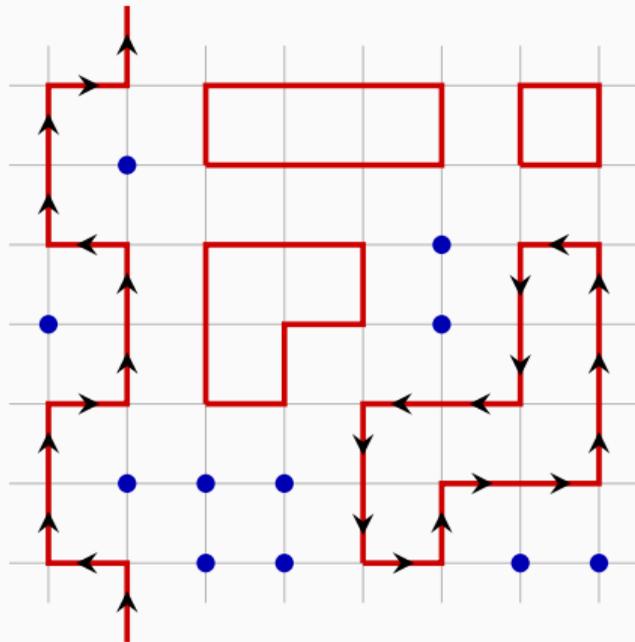
$$Z = \text{Tr} e^{-\beta H} = \sum_k \int [dt_k \dots dt_1] \text{Tr} \left( e^{-(\beta-t_k)H_1} (-H_2) e^{-(t_k-t_{k-1})H_1} \dots (-H_2) e^{-(t_1)H_1} \right),$$

$$= \sum_{\mathcal{C}} W[\mathcal{C}]$$

- Bond weights:

$$W_S = \varepsilon J, \quad W_t = e^{-\varepsilon J t}, \quad W_\mu = e^{\mu \varepsilon} \quad (7)$$

- Can construct a very efficient worm algorithm to sample configurations of this kind!



We consider two limits of the worldline model:

1. Relativistic limit

- Weights for temporal bonds = Weights for spatial bonds
- Manifest symmetry between space and time

2. Hamiltonian limit

- Relevant for quantum computers
- Temporal lattice spacing  $\varepsilon = 0.1$

# Worm Algorithm and the Observables

We focus on two observables:

- Susceptibility of the two-point correlation function  $\chi$

$$\chi = \frac{1}{ZL^d} \sum_{r,r'} \int_0^\beta dt \operatorname{Tr} \left( e^{-(\beta-t)H} a_{r,m} e^{-tH} a_{r',m}^\dagger \right). \quad (8)$$

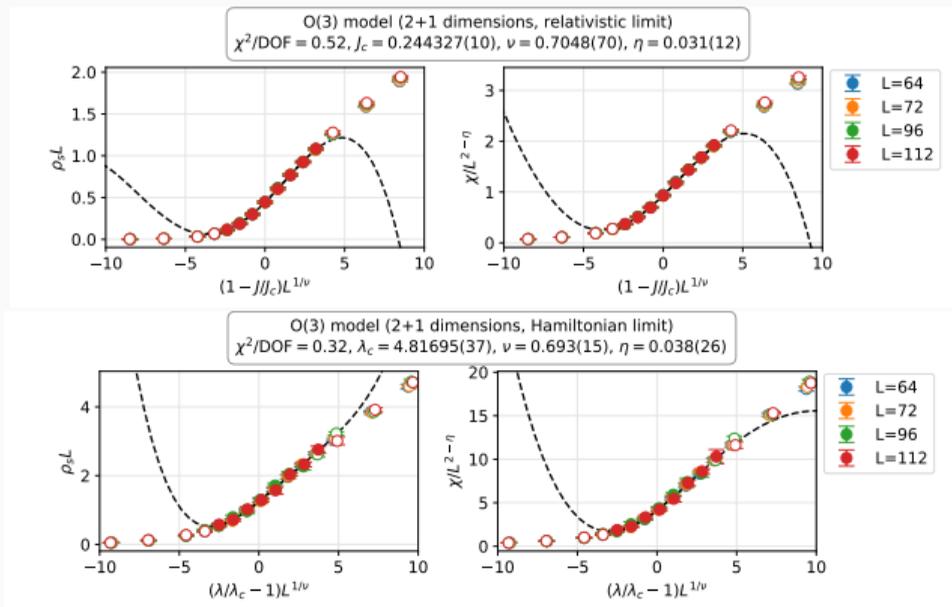
- Current-current susceptibility  $\rho_s$ : can be computed from the conserved  $O(3)$  charge  $Q_w$  along a spatial direction

$$\rho_s = \frac{1}{L^{d-2}\beta} \langle Q_w^2 \rangle. \quad (9)$$

Near the critical point  $J = J_c$ , we expect

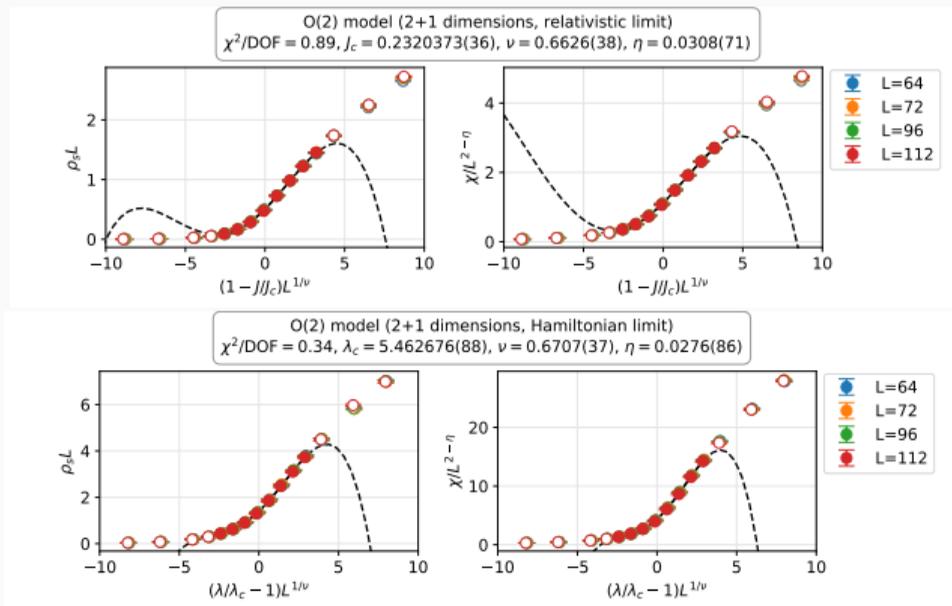
$$\begin{aligned} \rho_s L^{d-1} &= f((J - J_c)L^{1/\nu}), \\ \chi/L^{2-\eta} &= g((J - J_c)L^{1/\nu}). \end{aligned} \quad (10)$$

# Results: $O(3)$ model in 2+1 dimensions



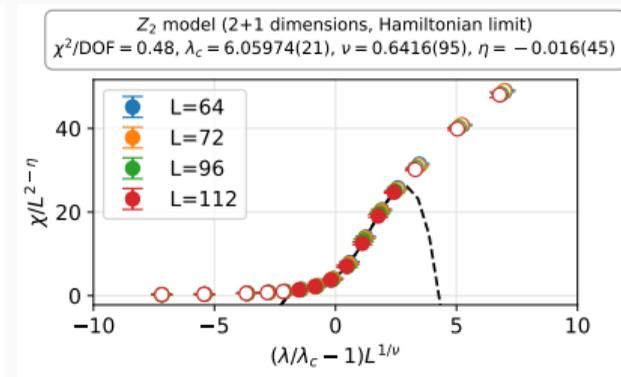
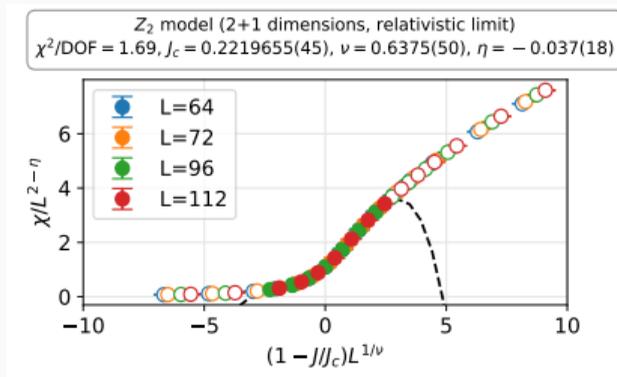
- Known critical exponents:  $\nu = 0.7113(13)$  and  $\eta = 0.0378(6)$  (Pelissetto and Vicari 2002)

# Results: $O(2)$ model in 2+1 dimensions



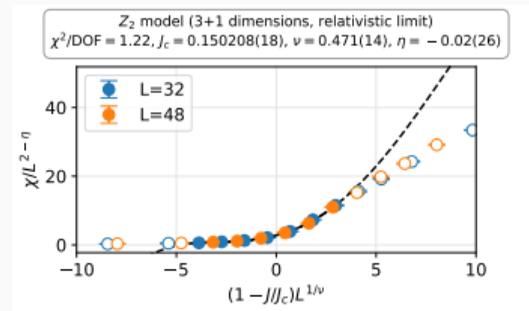
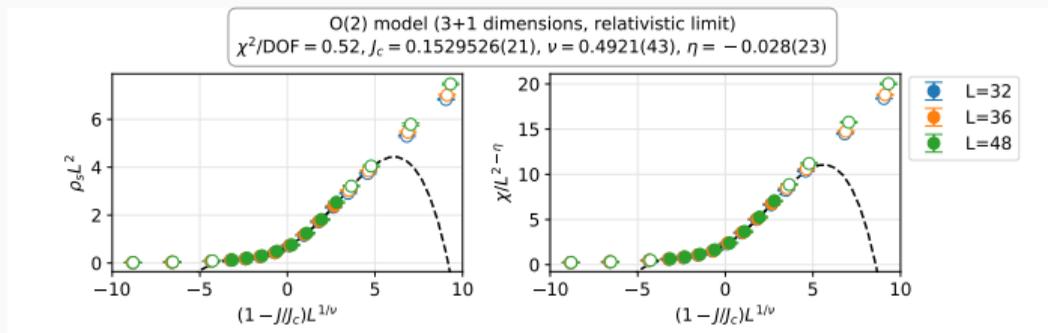
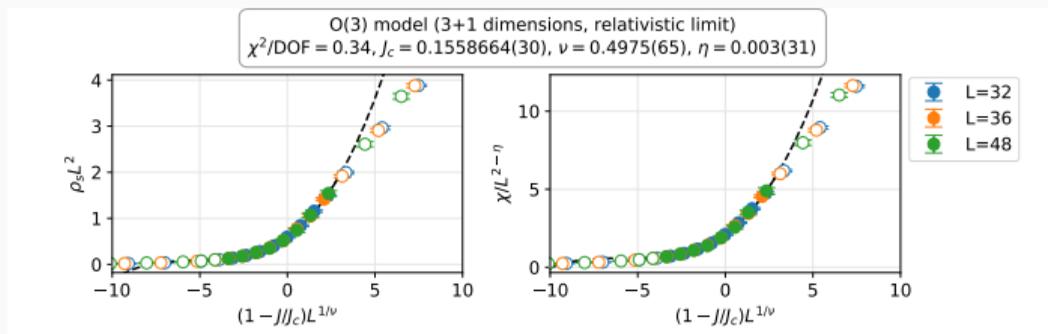
- Known critical exponents:  $\nu = 0.6717(1)$  and  $\eta = 0.0381(2)$  (Pelissetto and Vicari 2002)

## Results: $Z_2$ model in 2+1 dimensions



- Known critical exponents:  $\nu = 0.629971(4)$  and  $\eta = 0.036298(2)$  (Kos et al. 2016)

# Results: 3+1 dimensions



- Mean-field theory critical exponents:  $\nu = 0.5$  and  $\eta = 0.0$ .

1. Write down a qubit Hamiltonian for the  $O(3)$  nonlinear sigma model
2. Construct efficient Monte Carlo algorithm to locate the critical point
3. Perform a quantum simulation of the qubit Hamiltonian at the critical point  
 $\implies$  Future!

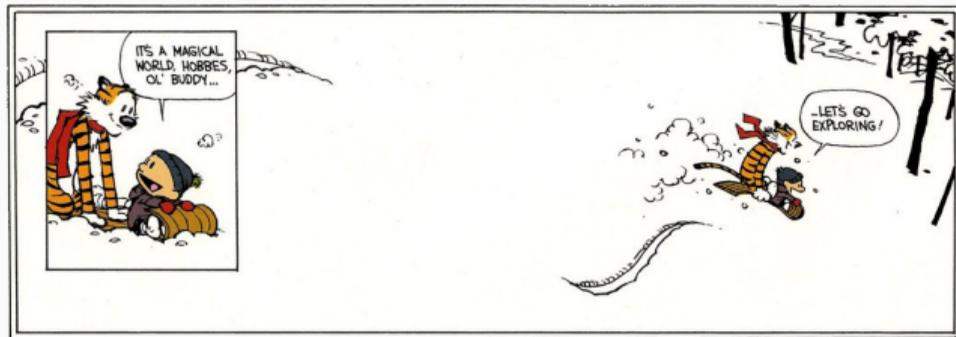
## Conclusions - I

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- Quantum simulations of QFTs might happen in two steps:
  1. A 'qubit regularized' Hamiltonian for a QFT of interest:
    - requires a non-perturbative classical method to identify the critical point
  2. Simulate the Hamiltonian on a quantum computer at the critical point
- Implemented this idea for the  $O(3)$  non-linear sigma model
  - Qubit Hamiltonian for the  $O(3)$  non-linear sigma model
  - Worldline formulation and an efficient worm algorithm precisely identify the critical point,
    - Wilson-Fisher critical exponents in  $d = 2$ ,
    - mean-field theory critical exponents in  $d = 3$  (relativistic and Hamiltonian limits)
- Bonus:  $O(2)$  and  $Z_2$  field theories ( $d = 2, 3$ ; relativistic and Hamiltonian limits)

## Conclusions - II

- Further work: qubit regularizations of  $O(N)$  models, gauge theories (quantum link models); proof-of-concept quantum simulations
- Questions: What kind of QFTs have qubit regularizations? Do all of them have it? What is the smallest number of qubits we need? What about asymptotically free QFTs?
- Lots of exciting directions!



Thank you for listening!