The twisted gradient flow running coupling in SU(3): a non-perturbative determination

37th Annual International Symposium on Lattice Field Theory
Wuhan, 2019

Eduardo I. Bribián

Instituto de Fisica Teorica UAM-CSIC

e.i.bribian@csic.es

Margarita García Pérez (IFT UAM-CSIC)
Eduardo I. Bribián (IFT UAM-CSIC)
Alberto Ramos (Trinity College Dublin)

June 17, 2019

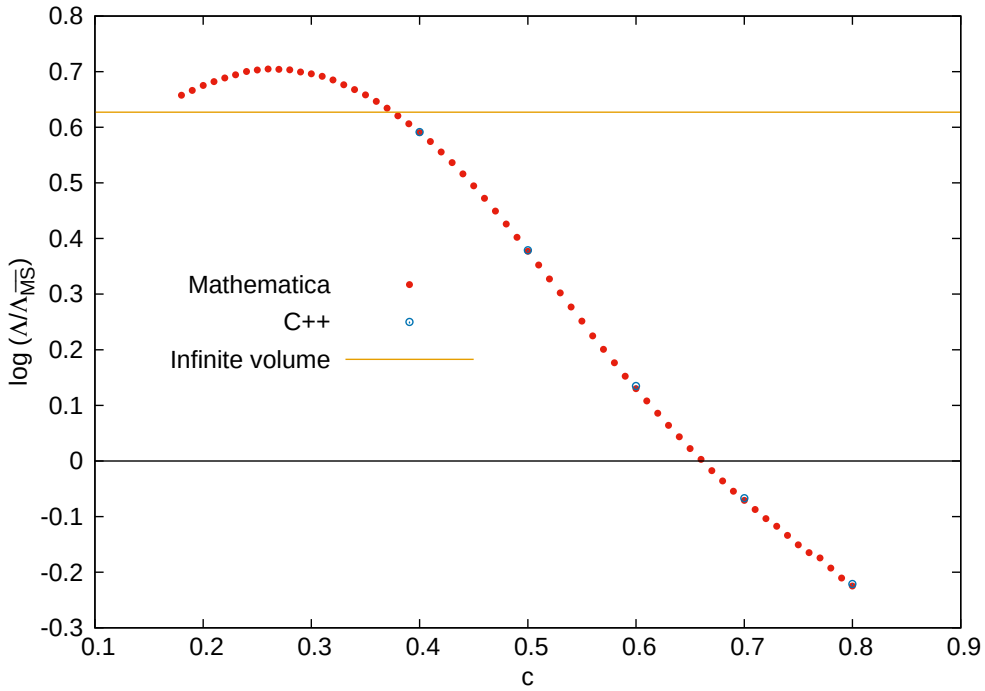
Trinity College Dublin
Coláiste na Tríonóide, Baile Átha Cliath
The University of Dublin

Outline

Outline

Introduction I

- The gradient flow has become a popular tool to study Yang-Mills theories, particularly when determining the scale dependence of the gauge coupling constant.
- We recently computed the one-loop running of the $SU(N)$ 't Hooft coupling in perturbation theory in a particular scheme. In the case of $SU(3)$ we obtained: Bribian, Garcia Perez '19



Introduction II

- For that computation, we used a finite volume gradient flow scheme implementing twisted boundary conditions on an asymmetrical torus.
- We parametrised the energy scale of the running in terms of an effective length $\tilde{L}$ combining the length of the torus and the rank of the group through $\mu = 1/c\tilde{L}$, c being a scheme-defining constant.
- The next step is to repeat the computation non-perturbatively, using step scaling techniques to compute the running coupling on the lattice on a similar twisted, asymmetrical torus.
- This computation is in progress. We are currently focusing on the case of $SU(3)$, but will extended the analysis to a wider range of $SU(N)$ groups.

References

- A .Gonzalez-Arroyo, M .Okawa '83, *The Twisted Eguchi-Kawai Model: A Reduced Model for Large N Lattice Gauge Theory*, Phys.Rev.D27(1983)2397, and Phys.Lett.B120(1983)174
- M. Lüscher '10, *Properties and uses of the Wilson flow in lattice QCD*, arXiv: 1006.4518
- A. Ramos '14, *The gradient flow running coupling with twisted boundary conditions*, arXiv: 1409.1445
- M. Garcia Perez, A. Gonzalez-Arroyo, L. Keegan, M. Okawa '14, *The $SU(\infty)$ twisted gradient flow running coupling*, arXiv: 1412.0941
- K. Ishikawa *et al.* '17, *Non-perturbative determination of the Λ-parameter in the pure SU(3) gauge theory from the twisted gradient flow coupling*, arXiv: 1702.06289
- E.I. Bribian, M. Garcia Perez '19, *The twisted gradient flow coupling at one loop*, arXiv: 1903.08029

Outline

Twisted boundary conditions

- We defined a pure gauge $SU(3)$ theory on an asymmetrical four-dimensional torus, using twisted boundary conditions in one plane and periodic ones in the rest.
- The periodic directions were set to be thrice as long as the twisted ones, so that the effective length $\tilde{L} = NL$ is identical in all directions. We are thus working with a $L^2 x (3L)^2$ lattice.
- The twisted boundary conditions mean, for the gauge fields, that:

$$A_\mu(x + L_\nu \hat{\nu}) = \Gamma_\nu A_\mu(x) \Gamma_\nu^\dagger, \qquad \Gamma_\mu \Gamma_\nu = Z_{\mu\nu} \Gamma_\nu \Gamma_\mu,$$
$$\Gamma_\mu \in SU(3) \text{ if } \mu = 0, 1, \text{ and } \Gamma_\mu = \mathbb{I} \text{ otherwise.}$$
$$Z_{\mu\nu} = \exp\{2\pi i \epsilon_{\mu\nu}/3\}, \quad \epsilon_{01} = -\epsilon_{10} = 1, \epsilon_{\mu\nu} = 0 \text{ otherwise.}$$

- This choice of torus dimensions and boundary conditions forces momentum to be quantised in units of $2\pi/\tilde{L}$. Moreover, the use of TBC conveniently excludes zero modes.

't Hooft '79 ; Gonzalez-Arroyo *et al* '83

The gradient flow

- For the definition of the coupling we used the gradient flow, introducing a flow time parameter t and gauge fields $B_\mu(x,t)$ to smear down fluctuations towards the classical solutions of the e.o.m:

$$\partial_t B_\mu = -\partial S/\partial B_\mu = D_\nu G_{\nu\mu} + D_\mu \partial_\nu B_\nu, \qquad B_\mu(x,0) = A_\mu(x)$$
$$G_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu + i\left[B_\mu, B_\nu\right], \qquad D_\mu B_\nu = \partial_\mu B_\nu + i\left[B_\mu, B_\nu\right]$$

- As the expectation values of observables built from such fields can be shown to be renormalised quantities for $t > 0$, they can be used to define the coupling:

$$\lambda_{TGF}(\tilde{l}) = \frac{t^2 \mathcal{F}(c)}{2N} \left\langle \mathrm{Tr}\left(G_{\mu\nu}(x,t) G_{\mu\nu}(x,t)\right)\right\rangle \Big|_{t=\frac{1}{8}c^2\tilde{L}^2}$$

With $\mathcal{F}(c)$ such that $\lambda_{TGF} = \lambda_0 + \mathcal{O}(\lambda_0^2)$, and where the energy scale of the running is related to the size of the torus through $\mu^{-1} = \sqrt{8t} = c\tilde{L}$. Narayanan *et al* '06; Lüscher '10; Ramos '14

Lattice discretisation I

- Discretising the torus on a lattice of spacing a and sides $L = la$ and $\tilde{L} = \tilde{l}a$, there are three choices to make: one has to pick a lattice action to drive the simulations, an action to drive the r.h.s of the flow equations, and a discrete form of the observable used in the definition of the coupling:

$$t^2 E(t)/N \equiv t^2 \mathrm{Tr}(G_{\mu\nu}(x,t) G_{\mu\nu}(x,t))/N$$

- For both the action driving the lattice simulation and for the one driving the flow equations, we chose to simply use the standard (twisted) Wilson plaquette action:

$$S_w = bN \sum_{\mu\nu} \mathrm{Tr}\,[1 - Z^*_{\mu\nu}(n) P_{\mu\nu}(n,t)]$$

$$P_{\mu\nu}(n,t) = U_\mu(n) U_\nu(n+\hat{\mu}) U^\dagger_\mu(n+\hat{\nu}) U^\dagger_\nu(n)$$

Lattice discretisation II

- As for the observable in order to to estimate the importance of lattice artifacts we used both the plaquette (E_P) and clover (E_C) definitions of the energy density:

$$E_P(t) = \mathrm{Tr}\,[1 - Z_{\mu\nu}(n)P_{\mu\nu}(n,t)], \quad E_C(t) = \frac{1}{2}\mathrm{Tr}\,[\hat{G}_{\mu\nu}(n,t)\hat{G}_{\mu\nu}(n,t)]$$

- Where we defined:

$$\hat{G}_{\mu\nu}(n,t) = -\frac{i}{8}\{Z_{\mu\nu}(n)P_{\mu\nu}(n,t) + Z_{\mu\nu}(n-\hat{\nu})P_{-\nu\mu}(n,t) + Z_{\mu\nu}(n-\hat{\mu})P_{\nu-\mu}(n,t) + Z_{\mu\nu}(n-\hat{\mu}-\hat{\nu})P_{-\mu-\nu}(n,t) - c.c.\}$$

- A particular basis can be chosen to make $Z_{\mu\nu}(n) = 1$ in all plaquettes but the ones at the corners of the twisted directions.

Step scaling

- The strategy is then to use step scaling to compute the coupling, defining a step scaling function:

$$\Sigma(u, s, \lambda(\tilde{l})) = \lambda_{TGF}(s\tilde{l}, b)\Big|_{u=\lambda_{TGF}(\tilde{l},b)}$$

- We picked $s = 2$, and measured $\lambda(\tilde{l})$ at the same values of the bare coupling b for three pairs of lattices $l^2 x (3l)^2 \rightarrow (2l)^2 x (6l)^2$.
- We obtained a set of values of $\lambda_{TGF}(\tilde{l}, b)$ for each of the lattices, allowing us to compute a set of values of Σ that we fitted to a polynomial in order to interpolate Σ at any value of u.
- With those fits, one can extrapolate the continuum step scaling function σ by taking the $\tilde{l} \rightarrow \infty$ limit at constant u.

Outline

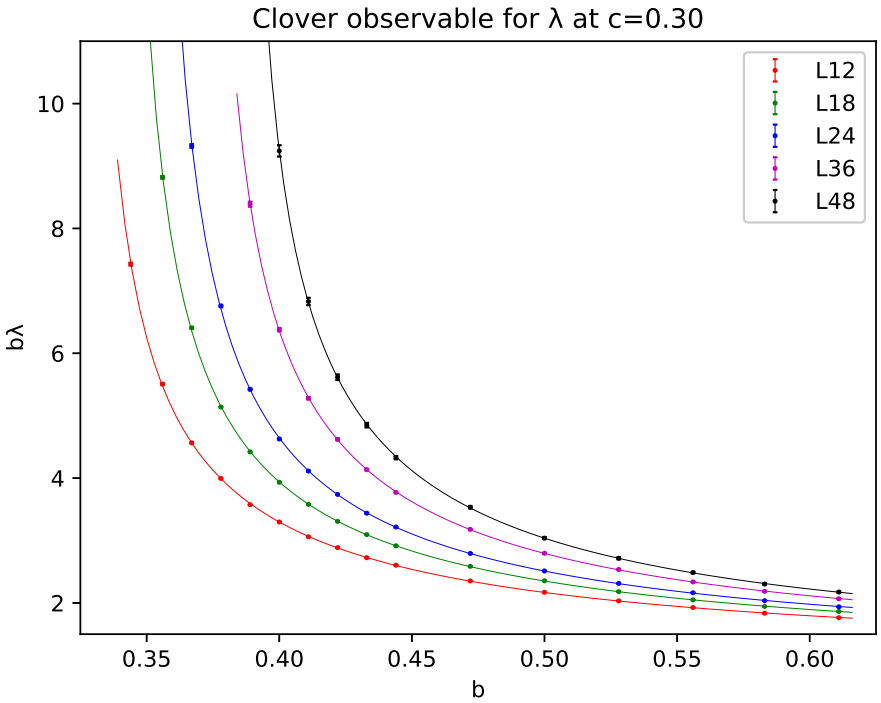
Simulations

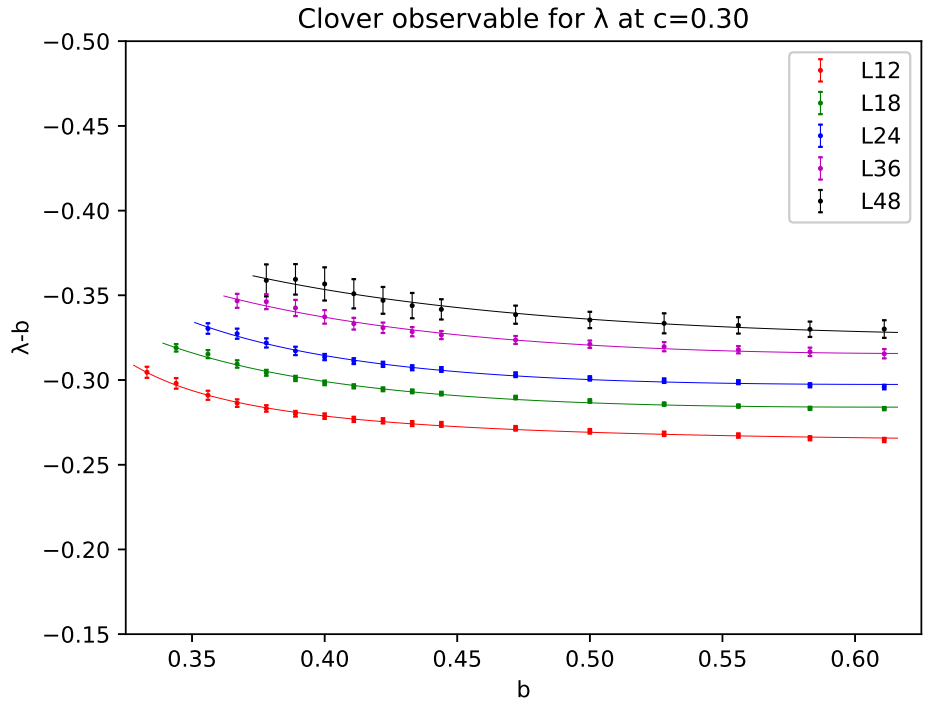
- We simulated lattices of five different sizes $l^2 x (3l)^2$, with $\tilde{l} = 3l = 12, 18, 24, 36, 48$, with the bare coupling b ranging from 0.35 to 0.61, and for c taking values between 0.1 and 0.8.
- We used a combination of heatbath and overrelaxation, performing a single heatbath step followed by $3l$ overrelaxations. This combination was repeated 2000 times to ensure thermalisation, and then was run $\tilde{l}$ times between measurements, as integrating the flow equations is more expensive than the MC updates. Between 500 and 10000 measurements were collected for each lattice, though we are currently running the simulations on the larger lattices to catch up.
- The integration of the flow equations was done using an adaptive step size integrator ensuring that the integration errors are below 10^{-6}.

Ramos '14

Preliminary results I - Running coupling

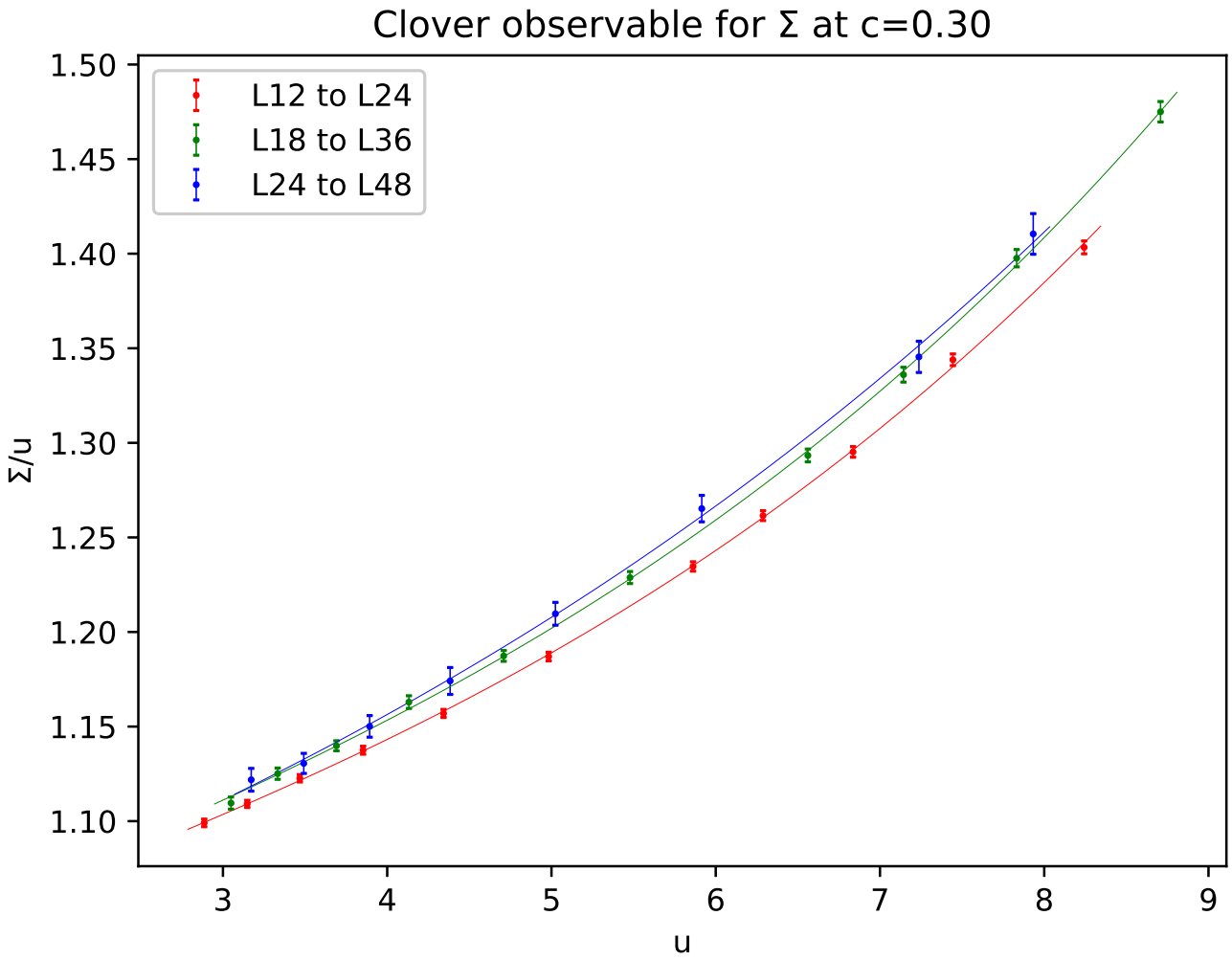
- With that, we computed the renormalised coupling λ_{TGF} and then fitted its b dependence to a Padé ansatz $\lambda_{TGF}(\tilde{l}, b) = \frac{1}{b}\frac{a_0+a_1 b+b^2}{a_2+a_3 b+b^2}$.
- For instance, for the clover observable at $c = 0.3$, and **preliminarily**:





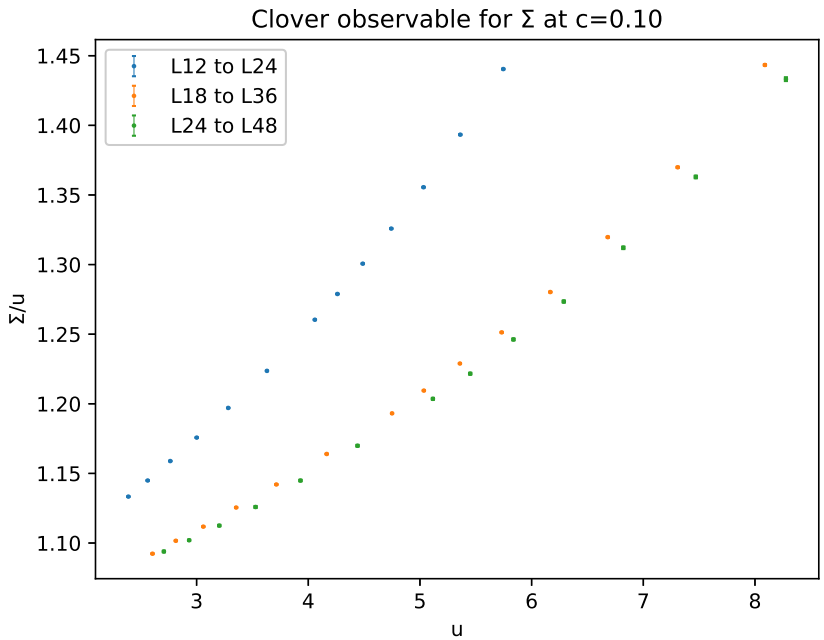
Preliminary results II - Sigma functions

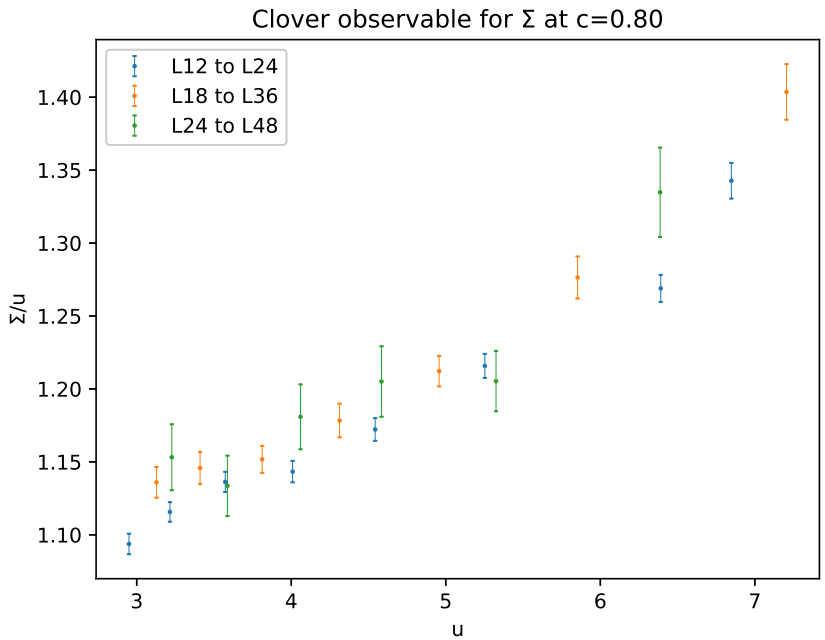
- For the Σ functions, we fitted their dependence in u to a simple three-parameter polynomial $\Sigma(u, \tilde{l}) = u + a_0 u^2 + a_1 u^3 + a_2 u^4$.
- For instance, for the clover observable at $c = 0.3$, and **preliminarily**:



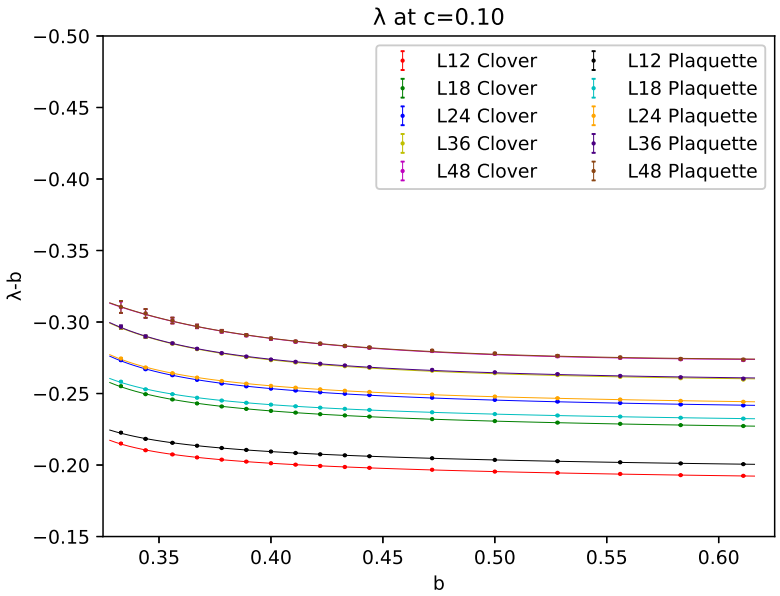
Issues I - Effect of c

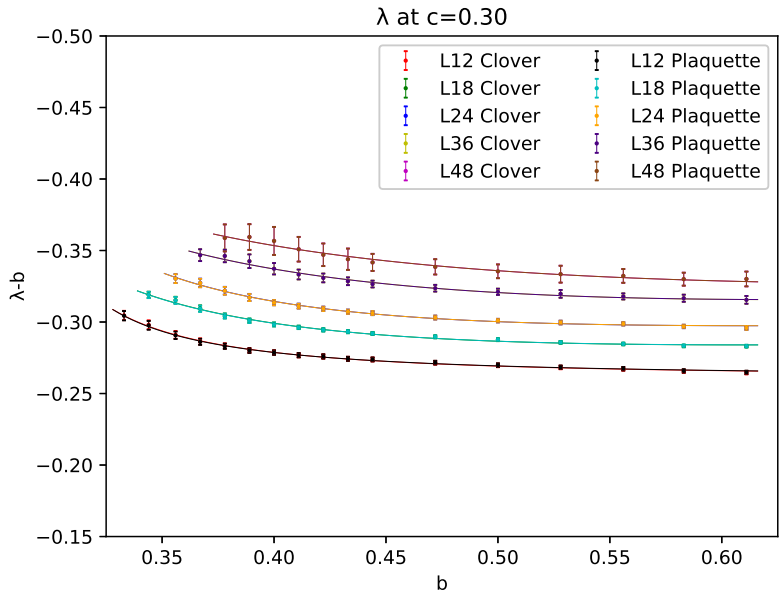
- There are a few issues with the simulation that had to be addressed.
- First of all is the effect of the flow, controlled by the parameter c. Short flow times (low c) leads to larger lattice artifacts, whereas larger values of c yield worse statistics.
- The effect of choosing the plaquette or the clover observable quickly becomes negligible compared to the effect of c.

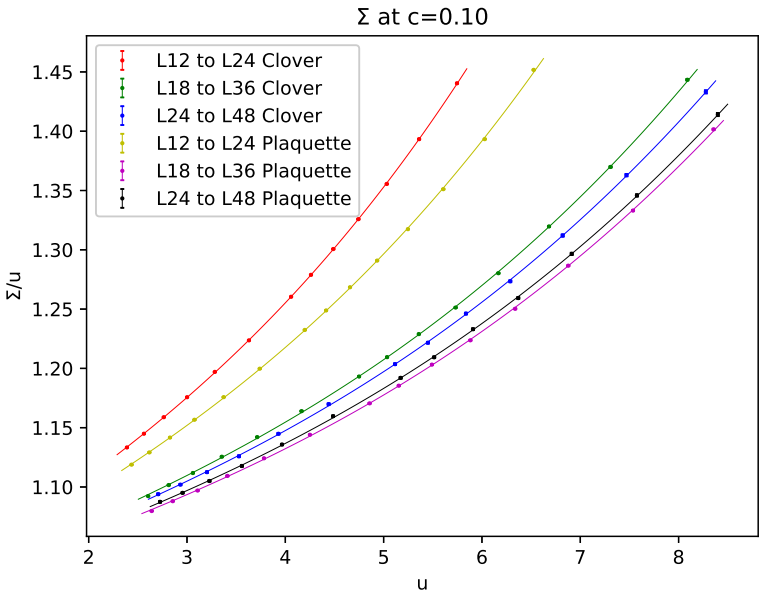


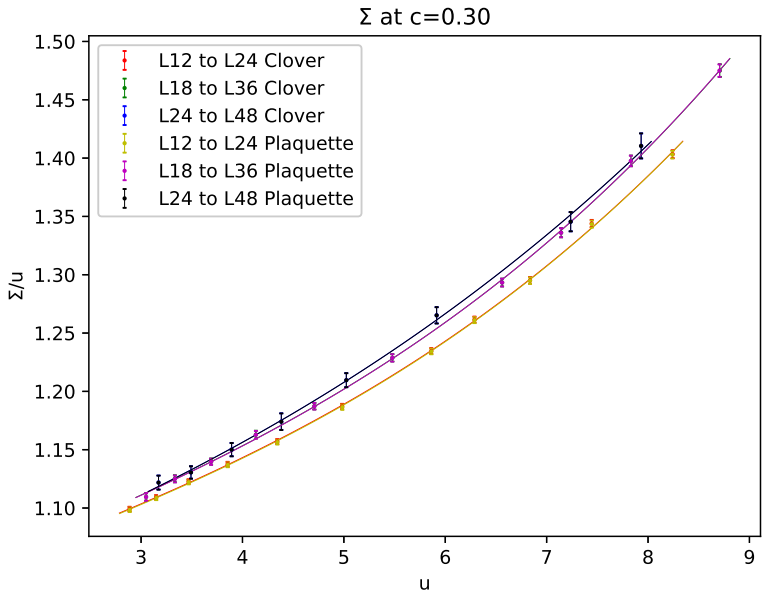


Issues II - Choice of observable



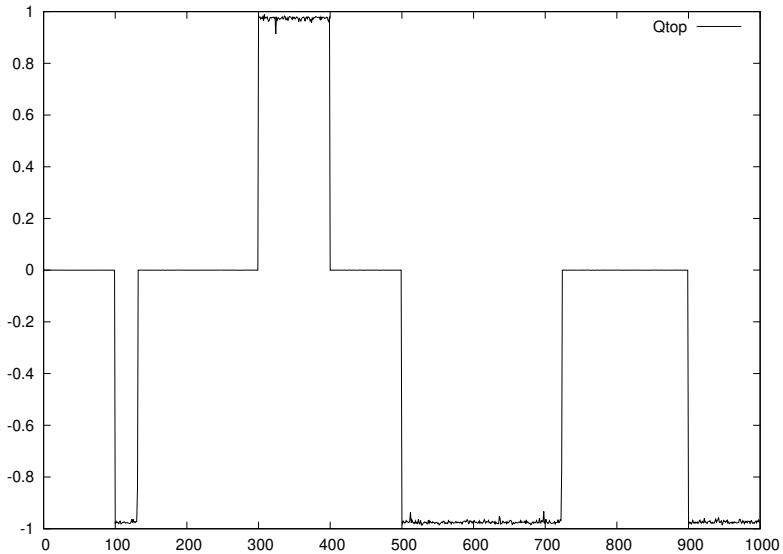


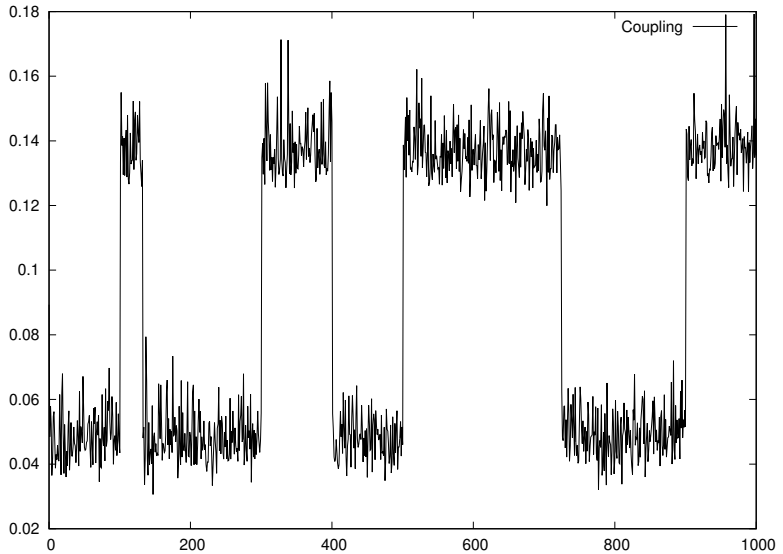




Issues III - Topological Freezing

- Topological freezing is also problematic, more so with the larger lattices and at small values of the coupling. Our observable is particularly sensitive to this, as it is essentially the plaquette.
- We chose to only use the subset of data with $Q_{top} = 0$ as part of our scheme definition, to avoid weighting errors, and chose to start from a cold (zero action) configuration, but this makes computations more expensive, as measurements with nonzero topological charge have to be discarded.





Summary and conclusions

- We considered a $SU(3)$ pure gauge theory, defined on an asymmetrical four-dimensional torus endowed with twisted boundary conditions in one plane and periodical ones in the rest.
- We defined a renormalised 't Hooft coupling through the gradient flow, using an effective length scale combining the physical size of the torus and the rank of the gauge group as the running scale.
- We discretised the theory on the lattice and are in the process of computing the running coupling through a step scaling procedure, using lattices with $\tilde{l} = 12, 18, 24, 36, 48$ and values of c ranging from 0.1 to 0.8. We will extrapolate the results to the continuum as soon as the full simulations are finished.
- Once the analysis of $SU(3)$ is complete, we will extend this work to other $SU(N)$ groups to explore the extent of finite N effects in the context of volume reduction.

Thank you.