

The Anomaly Inflow of Domain-wall fermion in odd dimension

Yoshiyuki Matsuki (Osaka U.)

Collaboration w/ H.Fukaya, N.Kawai, M.Mori, T.Onogi, S.Yamaguchi

Work in progress



My talk

Revisiting Callan-Harvey mechanism

(Anomaly cancellation mechanism)

C.G. Callan Jr. and J.A. Harvey , Nucl.Phys. B250 (1985)

Outline

- Callan-Harvey mechanism
- The problem
- Our research

Set up

3-dim.

U(1) back ground
Continuum Theory

Euclidean signature

Outline

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Callan-Harvey mechanism

=Anomaly Cancellation Mechanism

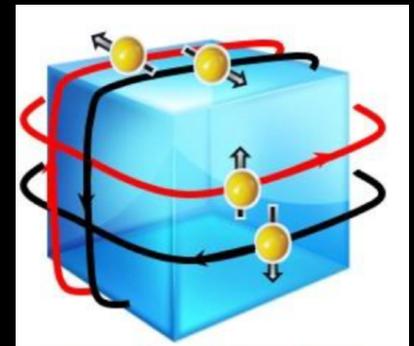
Anomaly from Bulk + Anomaly from Edge = 0

· Application

Condensed matter :

Bulk-Edge correspondence in topological matter

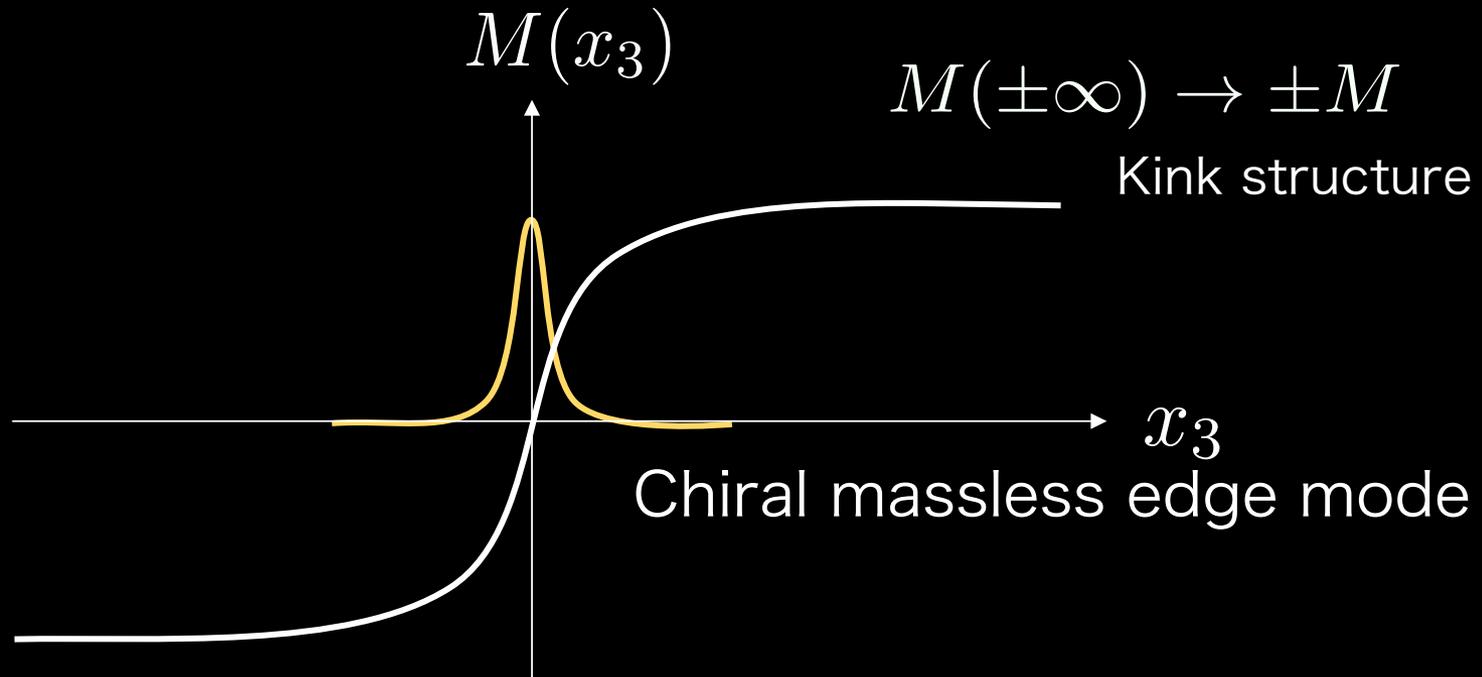
etc.



Lee, Seng Huat, "Dirac surface states of magnetic topological insulators" (2017).
Doctoral Dissertations. 2651.

Callan-Harvey mechanism

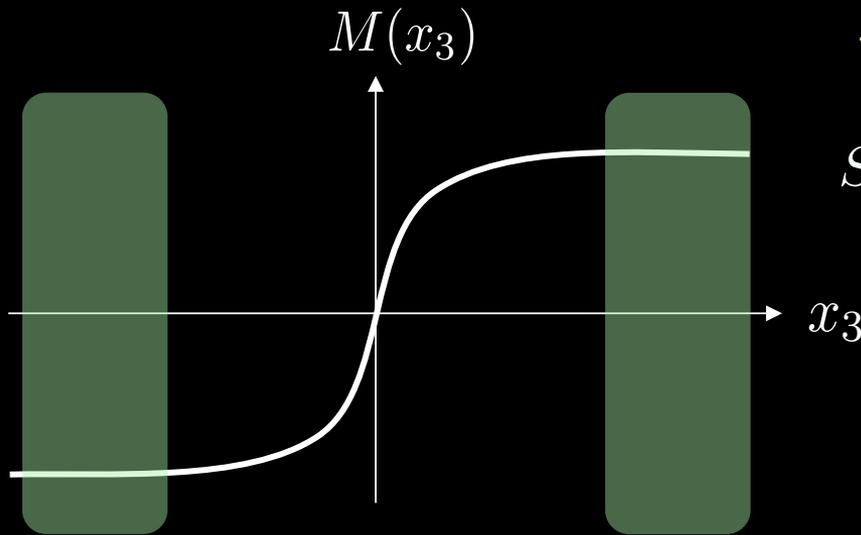
Set-up



Next, anomaly cancellation \rightarrow

Anomaly from Bulk + Anomaly from Edge = 0

Callan-Harvey mechanism



Deep in Bulk

$$S_{\text{Bulk}} \sim \int d^3x \bar{\psi} (\not{D} + M\epsilon(x_3)) \psi + PV$$

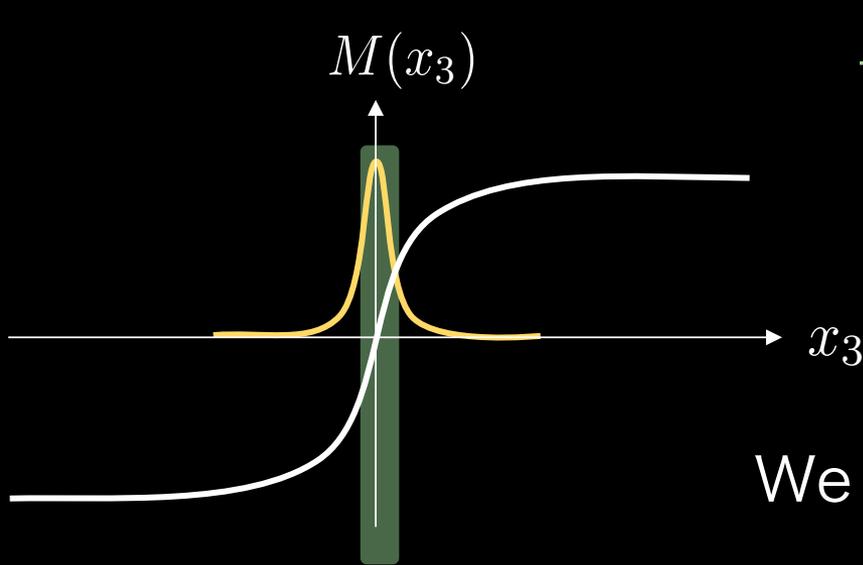
We extract
only the Bulk contribution

$$S_{\text{eff}} = \frac{i}{4\pi} \int d^3x \theta(x_3) \epsilon^{\mu\nu\rho} A_\mu \partial_\nu A_\rho$$

$$\partial_\mu J_{\text{bulk}}^\mu = \frac{i}{4\pi} \epsilon^{\mu\nu 3} \partial_\mu A_\nu \Big|_{x_3=0}$$

Anomalous

Callan-Harvey mechanism



Edge only

$$S_{\text{Edge}} = \int d^2x \bar{\psi}_R \not{D} \psi_R$$

We choose a certain regularization

$$\partial_i J_{\text{Edge}}^i = -\frac{i}{4\pi} \epsilon^{ij} \partial_i A_j$$

Gauge Anomaly

Callan-Harvey mechanism

Deep in Bulk + Edge only

$$\frac{i}{4\pi} \epsilon^{\mu\nu 3} \partial_\mu A_\nu \Big|_{x_3=0} + -\frac{i}{4\pi} \epsilon^{ij} \partial_i A_j = 0$$

Anomalous

Gauge Anomaly

Anomaly free

$$\partial_\mu J_{\text{Deep in Bulk}}^\mu + \partial_i J_{\text{Edge}}^i = 0$$

Outline

~~• Callan-Harvey mechanism~~

• The problem

• Our research

Set up

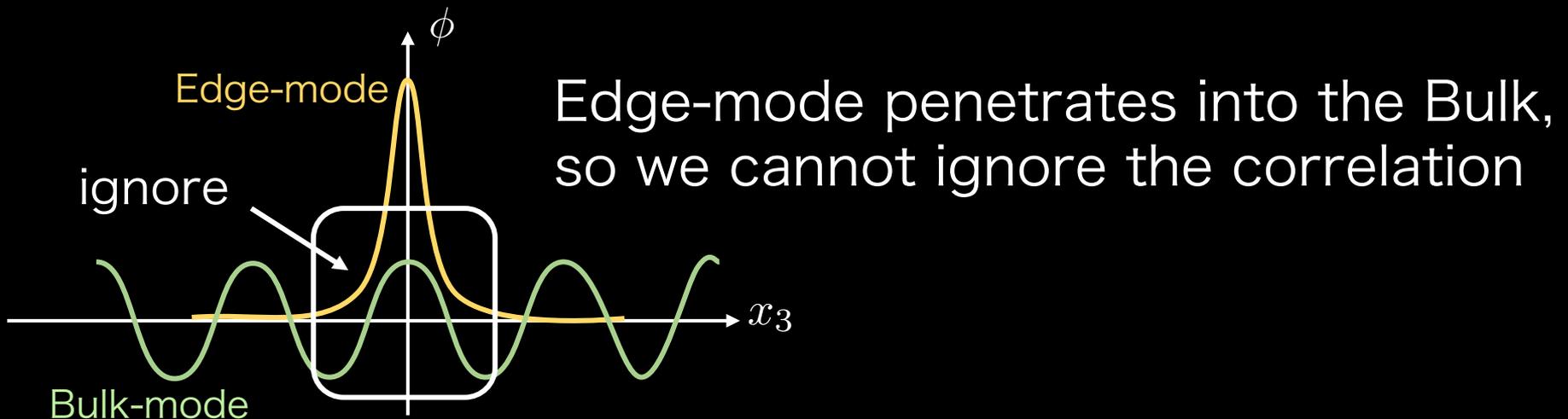
3-dim.

U(1) back ground
Continuum Theory

Euclidean signature

The problem

They evaluated Bulk contribution and Edge contribution separately, and simply **ignore** the **correlation** between **Bulk and Edge** mode



We will show that **the correlation** yields **the anomalous term**

Check

We have to check

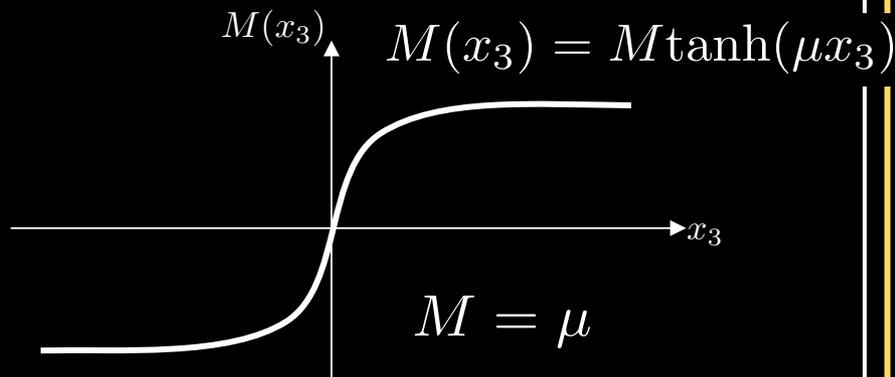
- Callan-Harvey mechanism is Correct? or not?
- If it is correct, how is the anomalous term canceled?

Previous research and Our research

S. Chandrasekharan, PRD49(1994)

He pointed out the same problem

His research

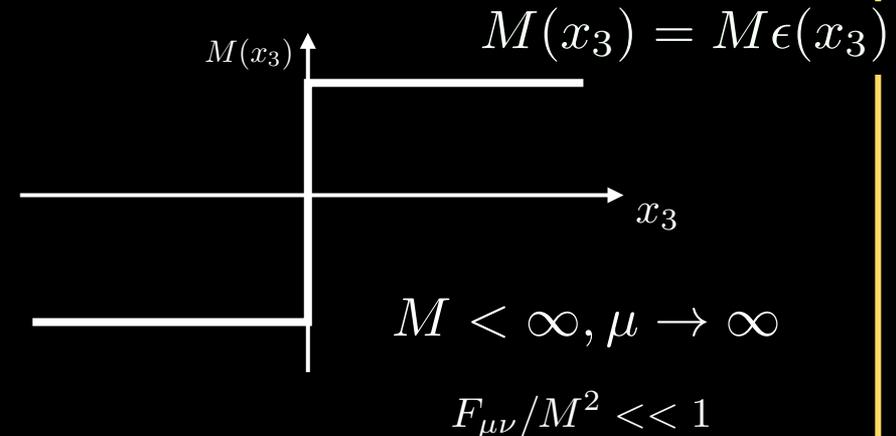


He focus on the anomaly
($\propto \epsilon$ tensor) $F_{\mu\nu}/M^2 \ll 1$

Calculation of higher order
terms are so complicated

$$\mathcal{O}((F_{\mu\nu}/M^2)^2)$$

Our research



We consider
the full contributions

We can evaluate
systematically up to higher
order terms, so it's easy to
extend to higher dim. case

Outline

~~• Callan-Harvey mechanism~~

~~• The problem~~

• Our research

Set up

3-dim.

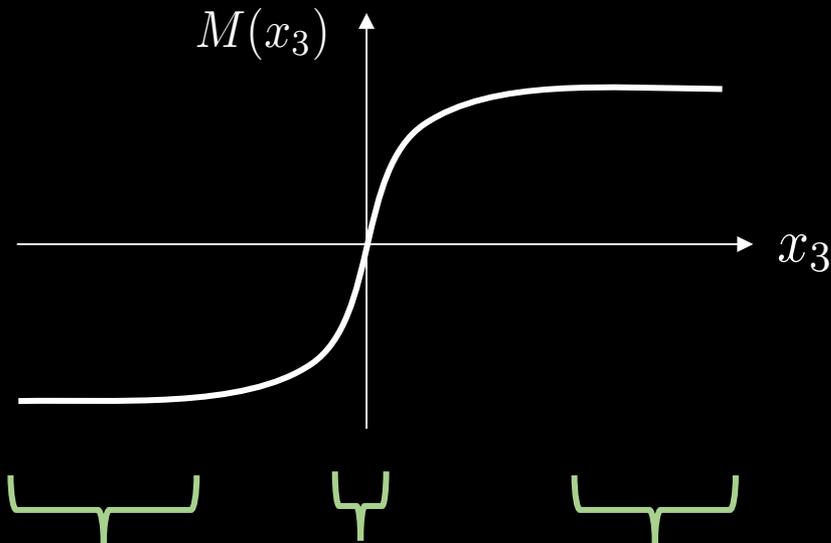
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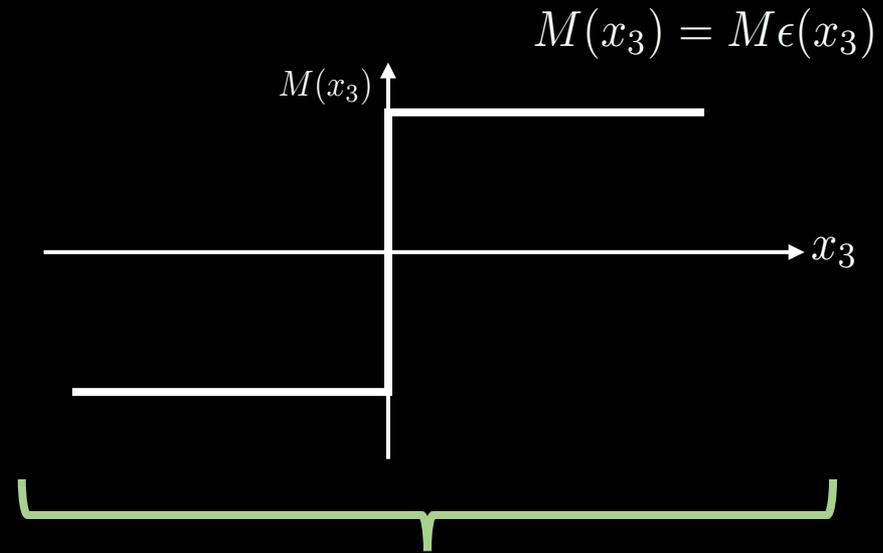
Our research

Callan, Harvey (1985)



evaluate
independently

Our research



$$S_{\text{Whole}} = \ln \det(\mathcal{D} + M\epsilon(x_3)) \cdot PV$$

Full contribution

Our research

Our goal is to show

$$\partial_{\mu} J_{\text{Whole}}^{\mu} = 0$$

$$J_{\text{Whole}}^{\mu} = - \frac{\delta S_{\text{Whole}}}{\delta A_{\mu}}$$

$$S_{\text{Whole}} = \ln \det(\not{D} + M \epsilon(x_3)) \cdot PV$$

Our research

Our goal is to show

$$\partial_{\mu} J_{\text{Whole}}^{\mu} = 0$$

$$J_{\text{Whole}}^{\mu} = - \frac{\delta S_{\text{Whole}}}{\delta A_{\mu}} \leftarrow \text{We evaluate}$$

$$S_{\text{Whole}} = \ln \det(\not{D} + M \epsilon(x_3)) \cdot PV$$

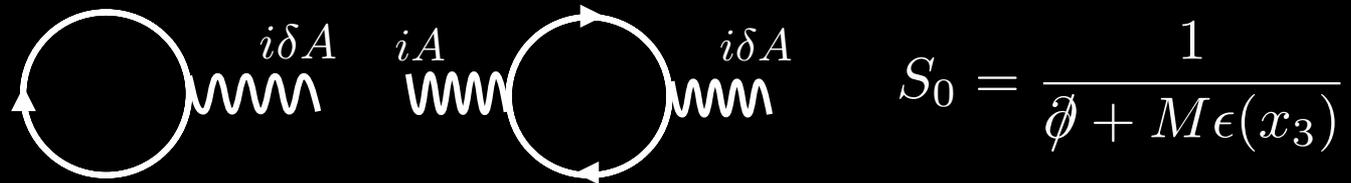
The variation of effective action

$$\delta S_{\text{Whole}} = \text{Tr} \left[\frac{i\delta A}{\not{D} + M\epsilon(x_3)} \right] + PV$$

$$S_{\text{Whole}} = \ln \det(\not{D} + M\epsilon(x_3)) \cdot PV$$

Expanding this in powers of gauge field A

$$\delta S_{\text{Whole}} = \text{Tr} [S_0 i\delta A - S_0 iA S_0 i\delta A + \dots] + PV$$



$$S_0 = \frac{1}{\not{D} + M\epsilon(x_3)}$$

Next, we find the expression of S_0

How should we evaluate $S_0 = \frac{1}{\phi + M\epsilon(x_3)}$
to see the correlation between Bulk and Edge?

How to evaluate S_0

First, we consider Hermitian operator

$$H \equiv (-\partial + M\epsilon(x_3))(\partial + M\epsilon(x_3)) = -\partial^2 + M^2 - 2M\gamma_3\delta(x_3)$$

The Eigen function for this operator

$$\begin{aligned} \phi_{\pm}^{\text{Bulk,e}}(x) &= \frac{1}{\sqrt{4\pi(k_3^2 + M^2)}} ((k_3 \mp iM)e^{-ik_3|x_3|} + (k_3 \pm iM)e^{ik_3|x_3|}) e^{i\vec{k}\cdot\vec{x}} u_{\pm} \\ \phi_{\pm}^{\text{Bulk,o}}(x) &= \frac{1}{\sqrt{4\pi}} (e^{-ik_3x_3} - e^{ik_3x_3}) e^{i\vec{k}\cdot\vec{x}} u_{\pm} \\ \phi_{+}^{\text{Edge,e}}(x) &= \sqrt{M} e^{-M|x_3|} e^{i\vec{k}\cdot\vec{x}} u_{+} \end{aligned} \quad \left. \begin{array}{l} \text{Bulk-mode} \\ |B_k\rangle \\ \text{Edge-mode} \\ |E_k\rangle \end{array} \right\}$$
$$u_{+} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad u_{-} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

How to evaluate S_0

First, we consider Hermitian operator

$$H \equiv (-\not{\partial} + M\epsilon(x_3))(\not{\partial} + M\epsilon(x_3)) = -\partial^2 + M^2 - 2M\gamma_3\delta(x_3)$$

The Eigen function for this operator

$$\phi_{\pm}^{\text{Bulk,e}}(x) = \frac{1}{\sqrt{4\pi(k_3^2 + M^2)}} ((k_3 \mp iM)e^{-ik_3|x_3|} + (k_3 \pm iM)e^{ik_3|x_3|}) e^{i\vec{k}\cdot\vec{x}} u_{\pm}$$

$$\phi_{\pm}^{\text{Bulk,o}}(x) = \frac{1}{\sqrt{4\pi(k_3^2 + M^2)}} ((k_3 \mp iM)e^{-ik_3|x_3|} - (k_3 \pm iM)e^{ik_3|x_3|}) e^{i\vec{k}\cdot\vec{x}} u_{\pm}$$

the complete set

$$\phi_{+}^{\text{Edge,e}}(x) = \sqrt{M} e^{-M|x_3|} e^{i\vec{k}\cdot\vec{x}} u_{+} \quad u_{+} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad u_{-} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Bulk-mode

$|B_k\rangle$

Edge-mode

$|E_k\rangle$

How to evaluate S_0

$$S_0(x, y) = \langle x | \frac{1}{\not{\partial} + M\epsilon(x_3)} | y \rangle$$

Inserting the complete set $\sum_k |B_k\rangle \langle B_k| + |E_k\rangle \langle E_k| = 1$

$$\longrightarrow : S_0(x, y)$$

$$= \int \frac{d^3 k}{(2\pi)^3} \frac{-i\not{k} + M\epsilon(y_3)}{k^2 + M^2} e^{ik(x-y)} \equiv S_0^{\text{Bulk}}$$

$$- \int \frac{d^2 \vec{k}}{(2\pi)^2} \frac{M}{2E_k} \left(\epsilon(y_3) + (\gamma_3 E_k + M) \frac{i\vec{k}}{k^2} \right) e^{-E_k(|x_3|+|y_3|)+i\vec{k}\cdot(\vec{x}-\vec{y})} \equiv S_0^{\text{Edge}}$$

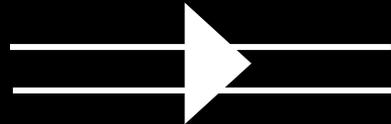
$$E_k = \sqrt{\vec{k}^2 + M^2}$$

How to evaluate S_0

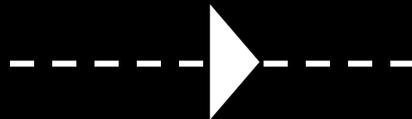
$$S_0(x, y) = \langle x | \frac{1}{\partial + M\epsilon(x_3)} | y \rangle$$

Inserting the complete set $\sum_k |B_k\rangle \langle B_k| + |E_k\rangle \langle E_k| = 1$

\longrightarrow : $S_0(x, y)$



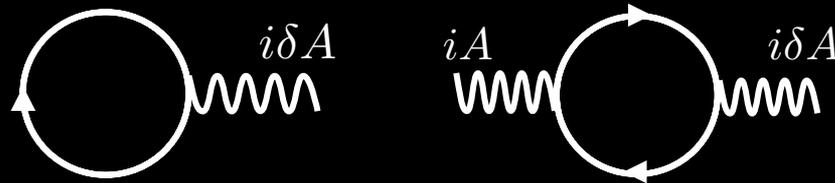
$$\equiv S_0^{\text{Bulk}}$$



$$\equiv S_0^{\text{Edge}}$$

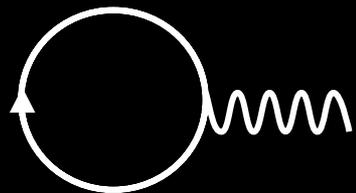
Let's evaluate

$$\delta S_{\text{Whole}} = \text{Tr} [S_0 i\delta A - S_0 iA S_0 i\delta A + \dots] + PV$$

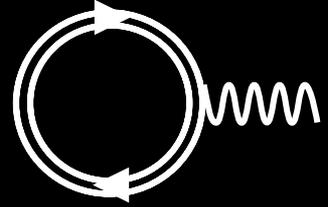


$$S_0 = \frac{1}{\not{\partial} + M\epsilon(x_3)}$$

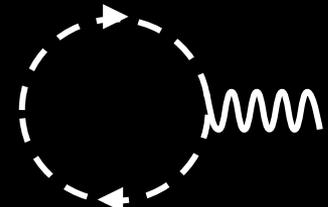
1 pt. function

$$\text{Tr} [S_0 i\delta\mathcal{A}]$$


$\text{Tr}[S_0^{\text{Bulk}} i\delta\mathcal{A}] = 0$



$\text{Tr}[S_0^{\text{Edge}} i\delta\mathcal{A}] = 0$



$\Rightarrow \Rightarrow : S_0^{\text{Bulk}}$

$- - \blacktriangleright - - : S_0^{\text{Edge}}$

$$\text{Tr} [S_0 i\delta\mathcal{A}] = 0$$

2pt. function

$-\text{Tr}[S_0 i\mathcal{A} S_0 i\delta\mathcal{A}]$

$-\text{Tr}[S_0^{\text{Bulk}} i\mathcal{A} S_0^{\text{Bulk}} i\delta\mathcal{A}]$

$-\text{Tr}[S_0^{\text{Bulk}(/Edge)} i\mathcal{A} S_0^{\text{Edge}(/Bulk)} i\delta\mathcal{A}]$

$-\text{Tr}[S_0^{\text{Edge}} i\mathcal{A} S_0^{\text{Edge}} i\delta\mathcal{A}]$

The correlation between Bulk and Edge

Result of 2pt. function

Under $F_{\mu\nu}/M^2 \ll 1$

$$\begin{aligned}
 \text{Diagram 1} &= -\frac{1}{8\pi} \int d^2x A_\mu \delta A^\mu \Big|_{x_3=0} - \frac{i}{4\pi} \int d^3x \theta(x_3) F_{\mu\nu} \delta A_\rho \epsilon^{\mu\nu\rho} \\
 &\quad - \frac{i}{4\pi} \int d^2x A_i \delta A_j \epsilon^{ij3} \Big|_{x_3=0}
 \end{aligned}$$

$$\text{Diagram 2} = \frac{1}{4\pi} \int d^2x A_3 \delta A^3 \Big|_{x_3=0} - \frac{1}{8\pi} \int d^2x A_i \delta A^i \Big|_{x_3=0}$$

$$\text{Diagram 3} = -\frac{1}{8\pi} \int d^2x A_3 \delta A^3 \Big|_{x_3=0} + \frac{1}{4\pi} \int d^2x A_i \delta A^i \Big|_{x_3=0}$$

$$\frac{1}{2\pi} \int d^2x \delta A^j \left(\frac{\partial_i \partial_j - \partial^2 \delta_{ij}}{\partial^2} A^i \right) \Big|_{x_3=0} + \frac{i}{4\pi} \int d^2x \epsilon^{jk3} \delta A_k \left(\frac{\partial_i \partial_j}{\partial^2} A^i \right) \Big|_{x_3=0}$$

$$+ \frac{i}{4\pi} \int d^2x \epsilon^{ik3} \delta A^j \left(\frac{\partial_i \partial_j}{\partial^2} A_k \right) \Big|_{x_3=0}$$

Result of 2pt. function

Under $F_{\mu\nu}/M^2 \ll 1$

$$\begin{aligned}
 \text{Diagram 1} &= \boxed{-\frac{1}{8\pi} \int d^2x A_\mu \delta A^\mu \Big|_{x_3=0}} - \frac{i}{4\pi} \int d^3x \theta(x_3) F_{\mu\nu} \delta A_\rho \epsilon^{\mu\nu\rho} \\
 &\quad - \frac{i}{4\pi} \int d^2x A_i \delta A_j \epsilon^{ij3} \Big|_{x_3=0}
 \end{aligned}$$

$$\text{Diagram 2} = \boxed{\frac{1}{4\pi} \int d^2x A_3 \delta A^3 \Big|_{x_3=0} - \frac{1}{8\pi} \int d^2x A_i \delta A^i \Big|_{x_3=0}}$$

Mass term

$$\text{Diagram 3} = \boxed{-\frac{1}{8\pi} \int d^2x A_3 \delta A^3 \Big|_{x_3=0} + \frac{1}{4\pi} \int d^2x A_i \delta A^i \Big|_{x_3=0}}$$

$$\begin{aligned}
 &\frac{1}{2\pi} \int d^2x \delta A^j \left(\frac{\partial_i \partial_j - \partial^2 \delta_{ij}}{\partial^2} A^i \right) \Big|_{x_3=0} + \frac{i}{4\pi} \int d^2x \epsilon^{jk3} \delta A_k \left(\frac{\partial_i \partial_j}{\partial^2} A^i \right) \Big|_{x_3=0} \\
 &\quad + \frac{i}{4\pi} \int d^2x \epsilon^{ik3} \delta A^j \left(\frac{\partial_i \partial_j}{\partial^2} A_k \right) \Big|_{x_3=0}
 \end{aligned}$$

Result of 2pt. function

Under $F_{\mu\nu}/M^2 \ll 1$

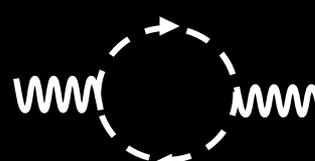


$$= \cancel{\frac{1}{8\pi} \int d^2x A_\mu \delta A^\mu \Big|_{x_3=0}} - \frac{i}{4\pi} \int d^3x \theta(x_3) F_{\mu\nu} \delta A_\rho \epsilon^{\mu\nu\rho} - \frac{i}{4\pi} \int d^2x A_i \delta A_j \epsilon^{ij3} \Big|_{x_3=0}$$



$$= \frac{1}{4\pi} \int d^2x A_3 \delta A^3 \Big|_{x_3=0} - \frac{1}{8\pi} \int d^2x A_i \delta A^i \Big|_{x_3=0}$$

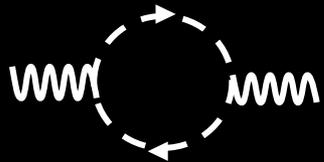
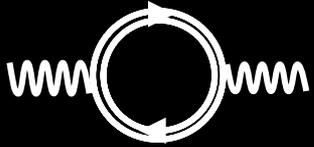
Cancel



$$= \cancel{\left(-\frac{1}{8\pi} \int d^2x A_3 \delta A^3 \Big|_{x_3=0} + \frac{1}{4\pi} \int d^2x A_i \delta A^i \Big|_{x_3=0} \right)}$$

$$\frac{1}{2\pi} \int d^2x \delta A^j \left(\frac{\partial_i \partial_j - \partial^2 \delta_{ij}}{\partial^2} A^i \right) \Big|_{x_3=0} + \frac{i}{4\pi} \int d^2x \epsilon^{jk3} \delta A_k \left(\frac{\partial_i \partial_j}{\partial^2} A^i \right) \Big|_{x_3=0} + \frac{i}{4\pi} \int d^2x \epsilon^{ik3} \delta A^j \left(\frac{\partial_i \partial_j}{\partial^2} A_k \right) \Big|_{x_3=0}$$

Result of 2pt. function



CS with Edge

$$-\frac{i}{4\pi} \int d^3x \theta(x_3) F_{\mu\nu} \delta A_\rho \epsilon^{\mu\nu\rho}$$

$$-\frac{i}{4\pi} \int d^2x A_i \delta A_j \epsilon^{ij3} \Big|_{x_3=0}$$

Anomaly from Bulk

Gauge-inv. term

$$\frac{1}{2\pi} \int d^2x \delta A^j \left(\frac{\partial_i \partial_j - \partial^2 \delta_{ij}}{\partial^2} A^i \right) \Big|_{x_3=0} + \frac{i}{4\pi} \int d^2x \epsilon^{jk3} \delta A_k \left(\frac{\partial_i \partial_j}{\partial^2} A^i \right) \Big|_{x_3=0}$$

Gauge-inv. non-local term

$$+\frac{i}{4\pi} \int d^2x \epsilon^{ik3} \delta A^j \left(\frac{\partial_i \partial_j}{\partial^2} A_k \right) \Big|_{x_3=0}$$

Anomaly from Edge

Result of 2pt. function

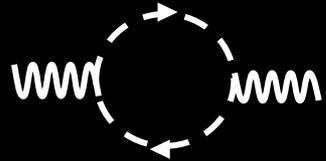


CS with Edge

$$\partial_\rho \frac{1}{\delta A_\rho} \left[-\frac{i}{4\pi} \int d^3x \theta(x_3) F_{\mu\nu} \delta A_\rho \epsilon^{\mu\nu\rho} - \frac{i}{4\pi} \int d^2x A_i \delta A_j \epsilon^{ij3} \Big|_{x_3=0} \right]$$



Anomaly from Bulk



Gauge-inv. term

$$\partial_\rho \frac{1}{\delta A_\rho} \left[\frac{1}{2\pi} \int d^2x \delta A^j \left(\frac{\partial_i \partial_j - \partial^2 \delta_{ij}}{\partial^2} A^i \right) \Big|_{x_3=0} + \frac{i}{4\pi} \int d^2x \epsilon^{jk3} \delta A_k \left(\frac{\partial_i \partial_j}{\partial^2} A^i \right) \Big|_{x_3=0} + \frac{i}{4\pi} \int d^2x \epsilon^{ik3} \delta A^j \left(\frac{\partial_i \partial_j}{\partial^2} A_k \right) \Big|_{x_3=0} \right]$$

Gauge-inv. non-local term

Anomaly from Edge

Conservation law

$$\partial_\rho J_{\text{Whole}}^\rho$$

$$= \frac{i}{4\pi} \delta(x_3) F_{ij} \epsilon^{ij3}$$

$$- \frac{i}{4\pi} \partial_i A_j \epsilon^{ij3} \delta(x_3)$$

$$- \frac{i}{4\pi} \epsilon^{ij3} \partial_i A_j \delta(x_3)$$

} Anomaly from Bulk

} Anomaly from Edge

Conservation law

$$\partial_\rho J_{\text{Whole}}^\rho$$

$$= \frac{i}{4\pi} \delta(x_3) F_{ij} \epsilon^{ij3}$$

$$- \frac{i}{4\pi} \partial_i A_j \epsilon^{ij3} \delta(x_3)$$

$$- \frac{i}{4\pi} \epsilon^{ij3} \partial_i A_j \delta(x_3)$$

$$= 0$$

Exactly
Cancel

We succeed in constructing

the EXACT anomaly cancellation mechanism

Effective action

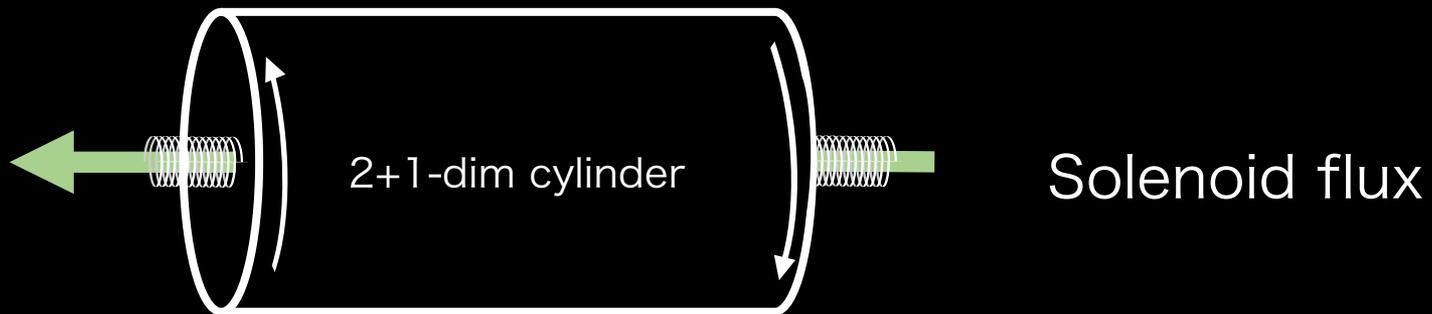
$$S_{\text{Whole}} = -\frac{i}{8\pi} \int d^3x \epsilon^{\mu\nu\rho} \theta(x_3) F_{\mu\nu} A_\rho \quad \text{CS with edge}$$
$$+\frac{1}{8\pi} \int d^2x F^{ij} \frac{1}{\partial^2} F_{ij} \quad \text{2D non-local term}$$
$$-\frac{i}{8\pi} \int d^2x \epsilon^{ij3} \partial_k A^k \frac{1}{\partial^2} F_{ij} \quad \text{Anomalous non-local term}$$

This gives us the viewpoint of Bulk-Edge correspondence as the FULL theory

Expected impact

This is the **future work**

- Laughlin's experiment



- S_{Whole} describes this system
- Chiral anomaly exists and gauge anomaly is cancelled in this system

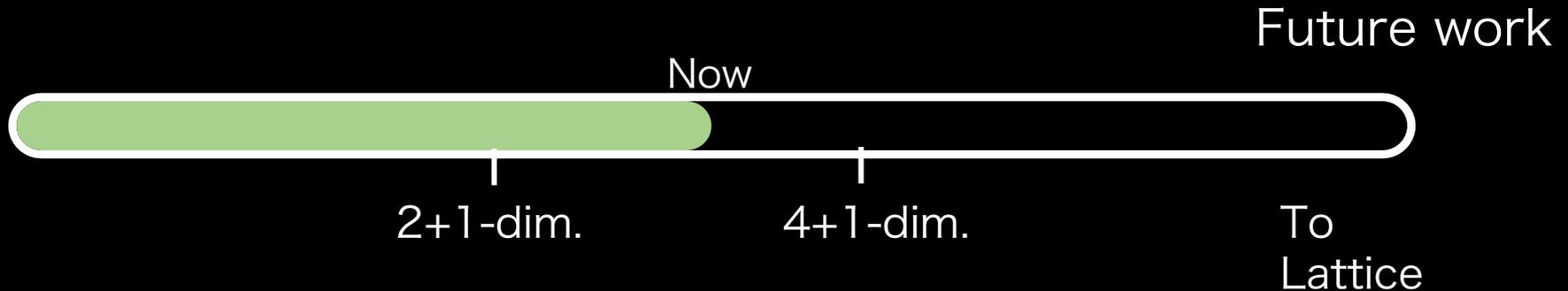
We expect that we can understand relationship between chiral anomaly and gauge anomaly by using S_{Whole}

Summary and Outlook

Callan-Harvey mechanism is important and useful, but they did not take the correlation into account

We checked that gauge-breaking mass term shows up from the correlation, but the total has gauge-sym.

Our method can evaluate systematically FULL effective action

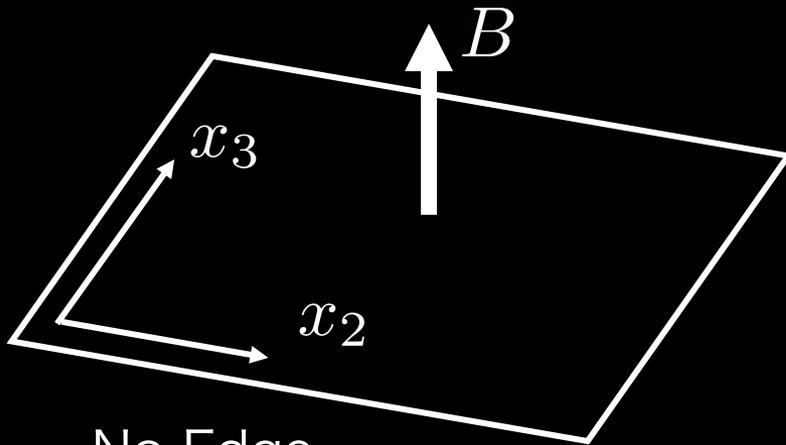


Back Up

Cond.-mat.

Quantum Hall System

Bulk current and charge density



No Edge

x_1 is time direction

$$J^\alpha = \sigma_{23} \epsilon^{\alpha\beta} E_\beta \quad (\alpha, \beta = 2, 3)$$

x_2 - x_3 sym.

$$\rho = \sigma_{23} B$$

$$\partial_1 \rho + \partial_\alpha J^\alpha = 0$$

Cond.-mat.

Bulk current and charge density

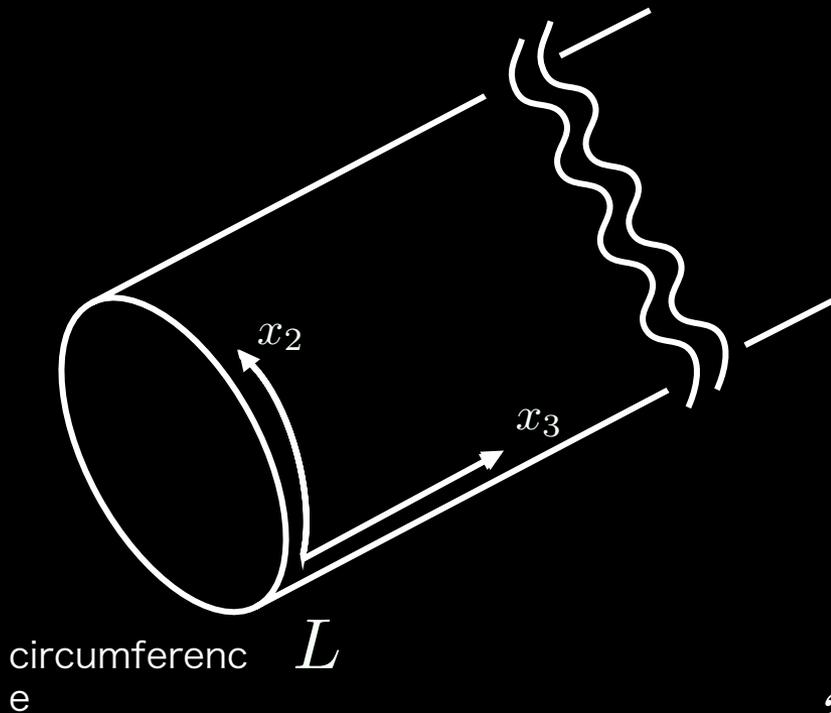
Intuitively (or Naively?)

$$J^\alpha = \theta(x_3) \sigma_{23} \epsilon^{\alpha\beta} E_\beta$$

We assume that the system has x_2 - x_3 sym. on the Edge

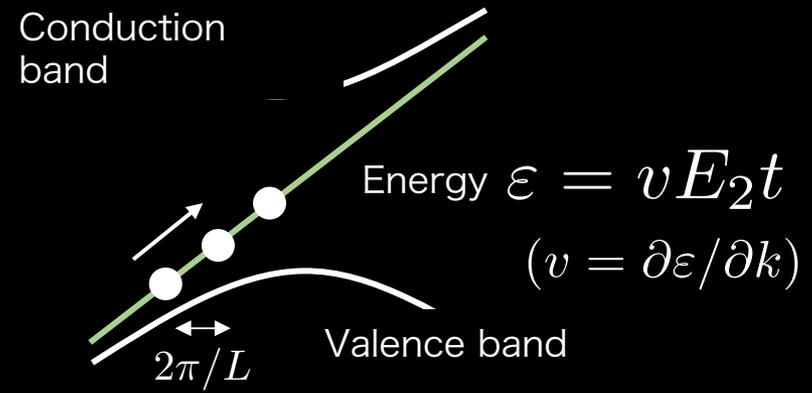
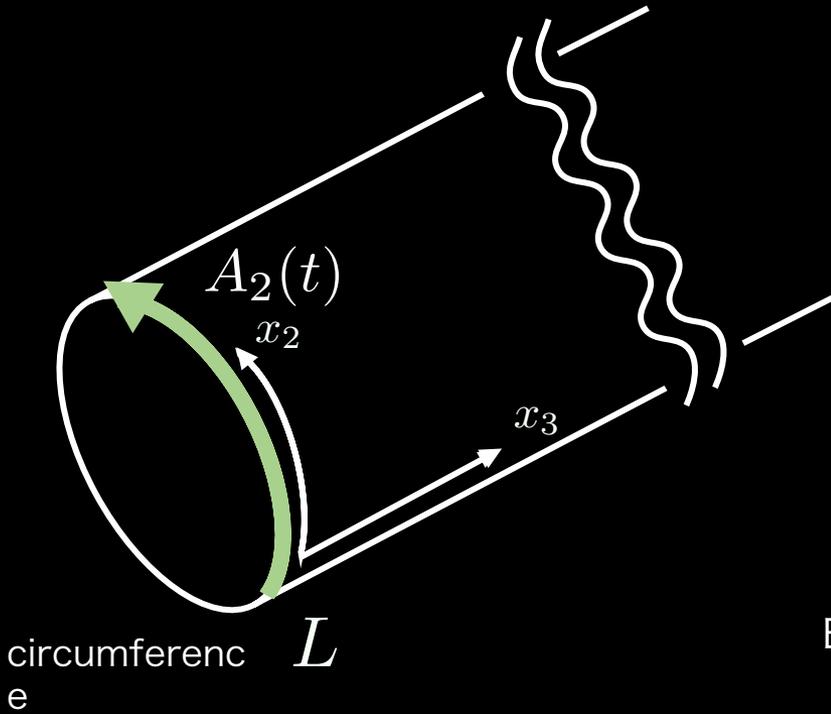
$$\rho = \theta(x_3) \sigma_{23} B$$

$$\partial_1 \rho + \partial_\alpha J^\alpha = \delta(x_3) \frac{i}{2\pi} E_2$$



Cond.-mat.

Edge has a chiral fermion

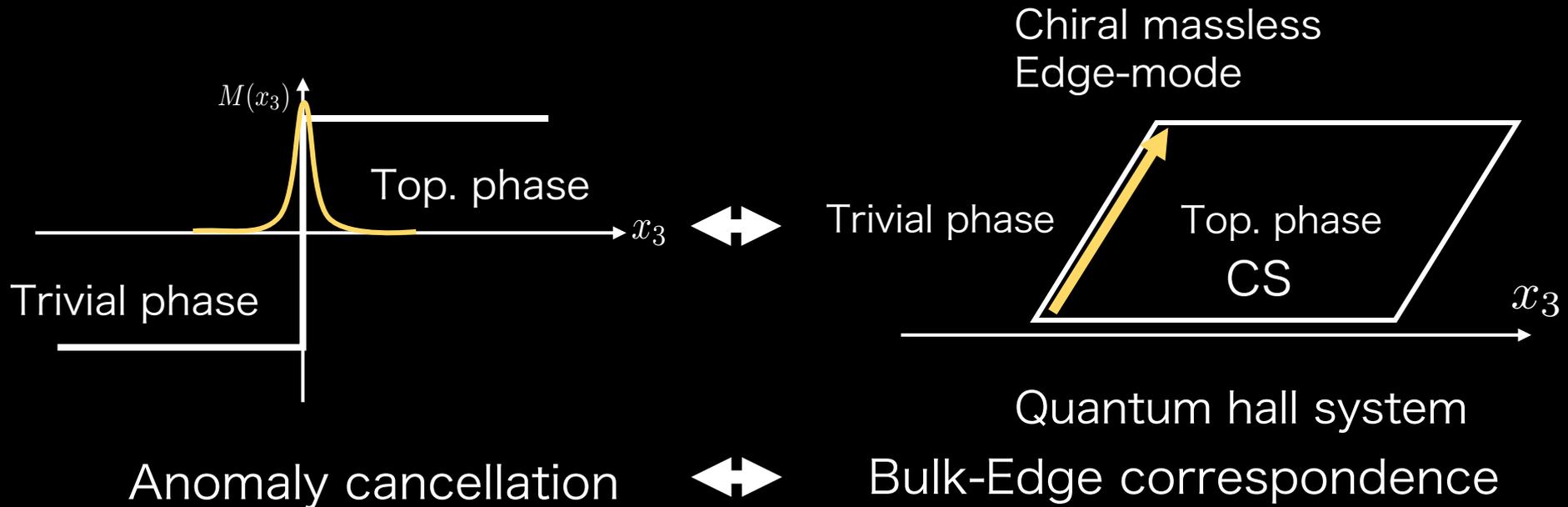


$$\Delta\rho = \frac{1}{L} \frac{E_2 \Delta t}{2\pi/L}$$

Euclid

$$\partial_1 \rho + \partial_\alpha J^\alpha = -\delta(x_3) \frac{i}{2\pi} E_2$$

Callan-Harvey mechanism



This mechanism is **very important and useful** for understanding Topological matter

How to evaluate S_0

$$S_0(x, y) = \langle x | \frac{1}{\partial + M\epsilon(x_3)} | y \rangle$$

$$H = (-\partial + M\epsilon(x_3))(\partial + M\epsilon(x_3))$$

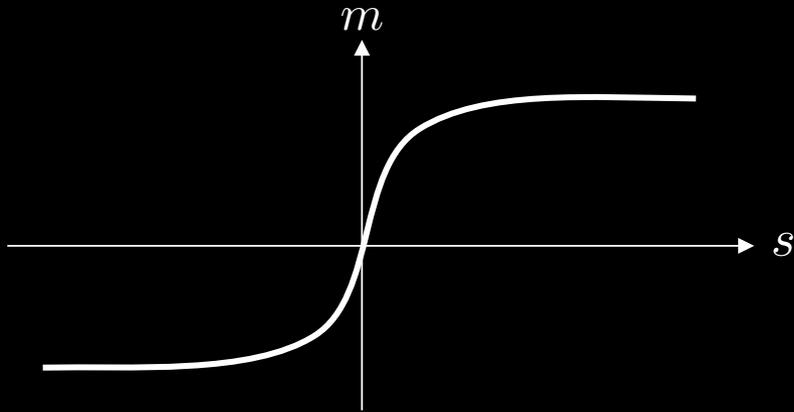
$$= \langle x | \frac{1}{H} (-\partial + M\epsilon(x_3)) | y \rangle$$

Inserting the complete set

$$\sum_k |B_k\rangle \langle B_k| + |E_k\rangle \langle E_k| = 1$$

$$= S_0^{\text{Bulk}} + S_0^{\text{Edge}}$$

Shailesh Chandrasekharan's Work



$$m(s) = m_0 \tanh m_0 s$$

$$\underbrace{\gamma^0 \{i\gamma^\mu \partial_\mu + m(s)\}}_S \psi_\lambda = \lambda \psi_\lambda$$

Eigenfunction of S

$$\psi_{Bulk,o} \propto \begin{pmatrix} k_2 \cos k_2 s - m(s) \sin k_2 s \\ -(\pm\omega + k_1) \sin k_2 s \end{pmatrix} e^{-ik_0 t - ik_1 x}$$

$$\psi_{Bulk,e} \propto \begin{pmatrix} k_2 \sin k_2 s + m(s) \cos k_2 s \\ (\pm\omega + k_1) \cos k_2 s \end{pmatrix} e^{-ik_0 t - ik_1 x}$$

$$\psi_{Edge} \propto \begin{pmatrix} \operatorname{sech} m_0 s \\ 0 \end{pmatrix} e^{-ik_0 t - ik_1 x}$$

This point is different from ours

Shailesh Chandrasekharan's Work

$$S(z, z') = \sum_{\lambda} \frac{\psi_{\lambda}(z) \bar{\psi}_{\lambda}(z')}{\lambda} \quad z = (t, x, s)$$

$$\left\{ \begin{array}{l} S_{\text{Bulk}} = \int \frac{d^3 k}{(2\pi)^3} \frac{\not{k} + M}{k^2 - m_0^2} e^{-ik(z-z')} \quad M = \begin{pmatrix} -m(s) & \frac{k_0+k_1}{k_2^2+m_0^2} [m(s)m(s') + ik_2(m(s) - m(s')) - m_0^2] \\ 0 & -m(s') \end{pmatrix} \\ S_{\text{Edge}}(z, z') = \frac{1}{2}(1 + i\gamma^2) \frac{m_0}{2} \text{sech}(m_0 s) \text{sech}(m_0 s') \int \frac{d^2 k}{(2\pi)^2} \frac{\gamma_i k_i}{k_0^2 - k_1^2} e^{-ik_0(t-t') - ik_1(x-x')} \end{array} \right.$$

Shailesh Chandrasekharan's Work

He evaluated

$$S_{eff} = \frac{1}{2} \int d^3 z d^3 z' A_\mu(z) S(z, z') A_\nu(z') S(z, z')$$

$\frac{1}{2}$ is the coefficient of expansion of log

$$S_{eff} = -\frac{i}{8\pi} \int d^3 z \operatorname{sgn}(s) \epsilon^{\mu\nu\rho} A_\mu \partial_\nu A_\rho - \frac{1}{16\pi^2} \int d^3 z d^3 z' A_i \frac{\epsilon^i \epsilon^{*j} + \epsilon^j \epsilon^{*i}}{2\epsilon^4} A_j(z') \\ \times m_0^2 [1 - f]^2 \operatorname{sech}^2(m_0 s) \operatorname{sech}^2(m_0 s')$$

$$f = \frac{e^{-m_0 r}}{r} [r \cosh m_0 (s - s') + (s - s') \sinh m_0 (s - s')]$$

$$r^\mu = z - z', \epsilon^i = (t - t', x - x')$$

$$\epsilon^{*i} = i \epsilon^{ji} \epsilon_j$$

Shailesh Chandrasekharan's Work

He evaluated

$$S_{eff} = \frac{1}{2} \int d^3z d^3z' A_\mu(z) S(z, z') A_\nu(z') S(z, z')$$

$$S_{eff} = - \text{Complicated} \frac{-\epsilon^j \epsilon^{*i}}{4} A_j(z') \times m_0^2 [1 - f]^2 \text{sech}^2(m_0 s) \text{sech}^2(m_0 s')$$

$$f = \frac{e^{-m_0 r}}{r} [r \cosh m_0 (s - s') + (s - s') \sinh m_0 (s - s')]$$

$$r^\mu = z - z', \epsilon^i = (t - t', x - x')$$

$$\epsilon^{*i} = i \epsilon^{ji} \epsilon_j$$

Consistent anomaly

$$\text{Diagram: wavy line} \circlearrowleft \text{ wavy line} = -\frac{i}{4\pi} \delta(x_3) F_{ij} \epsilon^{ij3} + \frac{i}{4\pi} \partial_i A_j \epsilon^{ij3} \delta(x_3)$$

$$\text{Diagram: wavy line} \circlearrowright \text{ wavy line} = +\frac{i}{4\pi} \epsilon^{ij3} \partial_i A_j \delta(x_3) \quad \text{Consistent anomaly}$$

Wess - Zumino condition demand the anomaly, which come from the Edge, is proportional to consistent anomaly

But, in U(1) case the covariant anomaly is the twice of consistent anomaly.

In our next work, we will reveal why the mode expansion choose the consistent anomaly

Laughlin's experiment



$$J_{\text{Bulk}} = \sigma_{xy} E_y$$

The chiral mode on the edge moves to the other edge through the bulk

$$\partial_\mu \langle J_5^\mu \rangle = -\frac{i}{\pi} \epsilon^{\mu\nu} \partial_\mu A_\nu$$

$$J_5^\mu = J_{R,\text{chiral}} - J_{L,\text{chiral}}$$

We combine the two edges and regard this as the vector-like theory

Gauge Anomaly

$$\partial_\mu \langle J_{R,\text{gauge}}^\mu \rangle = -\frac{i}{4\pi} \epsilon^{\mu\nu} \partial_\mu A_\nu$$

$$\partial_\mu \langle J_{L,\text{gauge}}^\mu \rangle = \frac{i}{4\pi} \epsilon^{\mu\nu} \partial_\mu A_\nu$$

Why systematic?

Shailesh Chandrasekharan's Method

The form of eigenfunction in higher odd-dim is different from 2+1-dim. case

Our Method

The form of eigenfunction in higher odd-dim. is almost same in 2+1-dim. case

We can get the propagator if we change naively,

$$\gamma_3 \rightarrow \gamma_5$$