Phase transition

~~Critical behavior~~ of 4-dimensional Ising model with higher-order tensor renormalization group

Shinichiro Akiyama[1]

in collaboration with

Y. Kuramashi[2], T. Yamashita[3], and Y. Yoshimura[2]

[1]*Graduate School of Pure and Applied Sciences, Univ. of Tsukuba,*

[2]*CCS, Univ. of Tsukuba,*

[3]*Faculty of Engineering, Information and Systems, Univ. of Tsukuba*

Ref: arXiv:1906.06060

Lattice 2019 @Hilton Hotel Wuhan Riverside

June 18, 2019

Outline of Talk

1. Introduction

2. Tensor Network Scheme

3. Numerical Results

4. Summary and Outlook

ϕ^4 theory and the Ising model

Infinite-coupling limit

ϕ^4 theory → Ising model

the same universality class

In the 4-dimensional case,

Mean-field theory : $C \sim |t|^{-\alpha}$ with $\alpha = 0$

Perturbative RG : $C \sim |t|^{-\alpha} (\log |t|)^{1/3}$ with $\alpha = 0$

$(t = (T - T_{\rm c})/T_{\rm c})$

Kenna-Lang NPB393(1993)461, Kenna NPB691(2004)292

If the leading scaling behavior is the mean-field type and it is modified just by the multiplicative logarithmic factor, then the theory is trivial in the continuum limit.

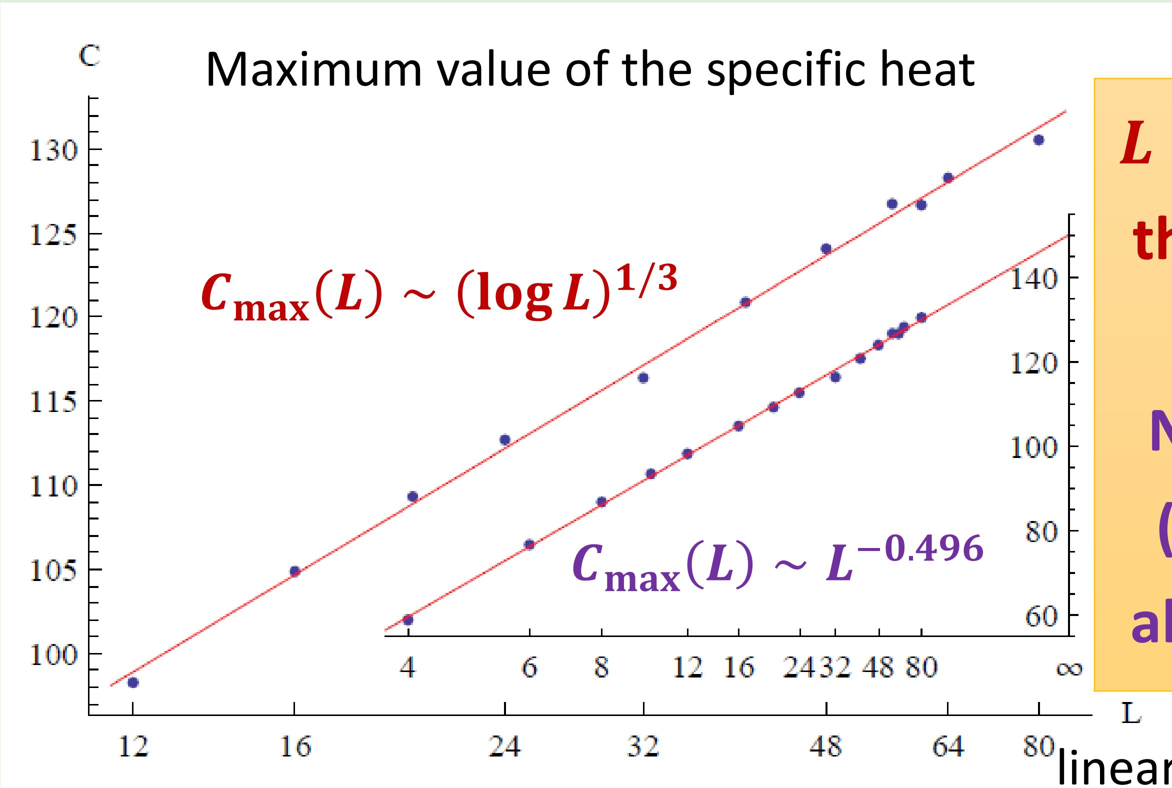
The latest Monte Carlo study

MC study of the 4-dimensional Ising model

⇨ a non-perturbative indirect test of the triviality

Lundow-Markstrom PRE80(2009)031104

Finite-size scaling analysis with linear system sizes $L \leq 80$



linear system size

$L = 80$ is too small to catch the logarithmic divergence

OR

No logarithmic correction (specific heat is bounded also in the infinite volume)

Tensor Network scheme

Hamiltonian approach

・Quantum many-body system

・Variational method (Ex. DMRG, MPS, PEPS, …)

Lagrangian approach

・Classical many-body system (path integral)

・Coarse-graining method (Ex. TRG, TNR, …)

Advantage of TN scheme

・No sign problem

・Direct treatment of Grassmann numbers

・Direct evaluation of thermodynamic limit

(simulation volume is increased just in one run)

Current status of TN scheme in higher dimensions (Lagrangian approach)

3-dimensional system

- Ising model Xie et al. PRB86(2012)045139, Wang et al. CPL31(2014)070503
- Potts model Wang et al. CPL31(2014)070503
- Free Wilson fermion Sakai et al. PTEP2017(2017)063B07
- $\mathbb{Z}_2$ gauge theory (finite temperature)

  Kuramashi-Yoshimura arXiv:1808.08025[hep-lat]

4-dimensional system

- Ising model with parallel computation (this work)

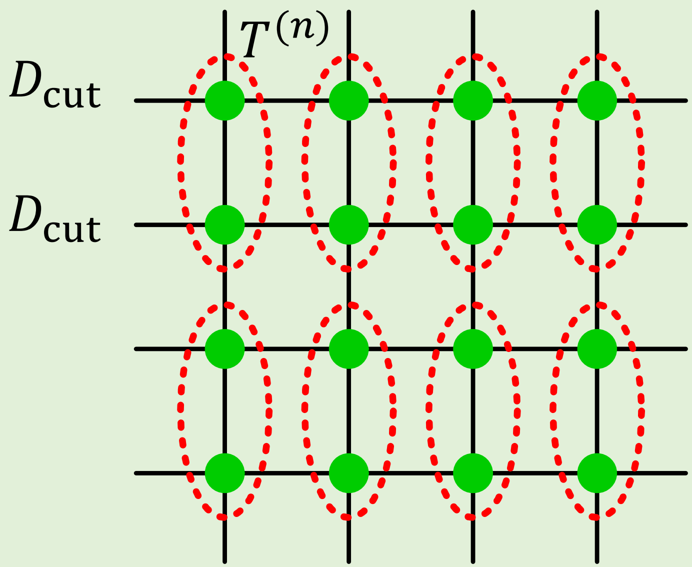
  For technical design details of the parallel computing,

  see Yamashita-Sakurai (in preparation)

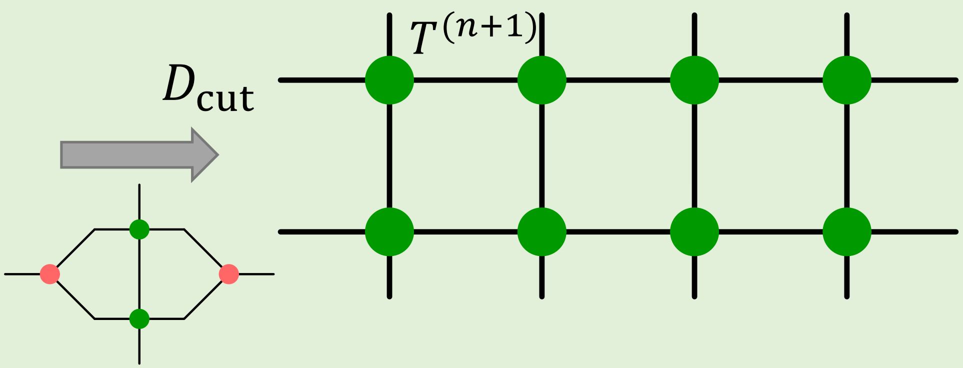
Higher-Order Tensor Renormalization Group

Xie et al. PRB86(2012)045139

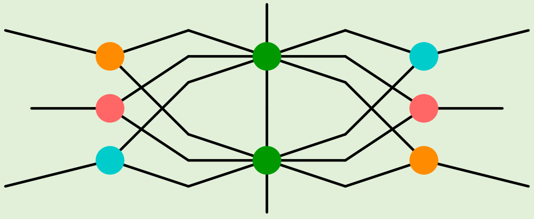
How to evaluate partition function



D_{cut} : bond dimension
(# of block-spin states)



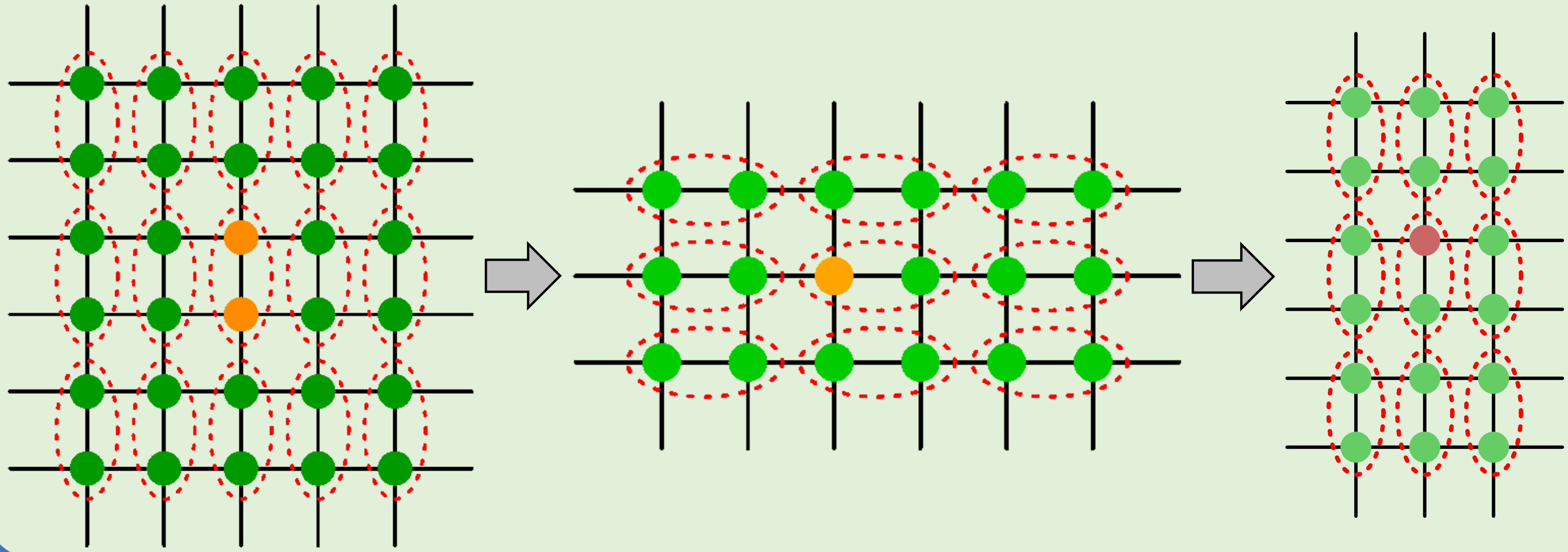
HOTRG in 4-dim. system



memory $\sim D_{\mathrm{cut}}^{8}$
computational time $\sim D_{\mathrm{cut}}^{15}$

Impure tensor method

Ex. How to specify the nearest-neighbor $\langle \sigma_i \sigma_j \rangle$



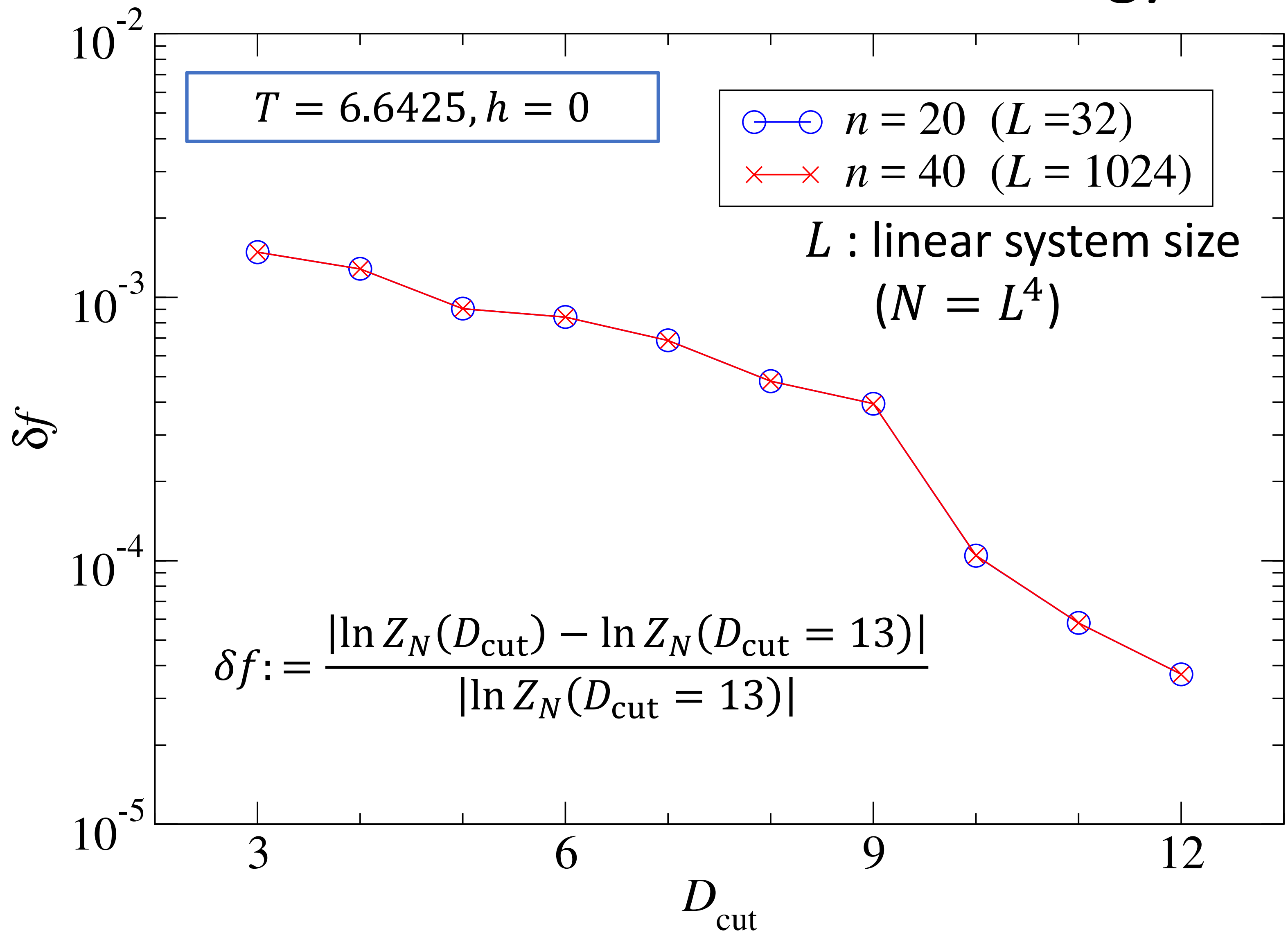
Coarse-graining of the tensor network

including local impure tensor(s) at the center of lattice

⇨ Evaluation of internal energy and magnetization

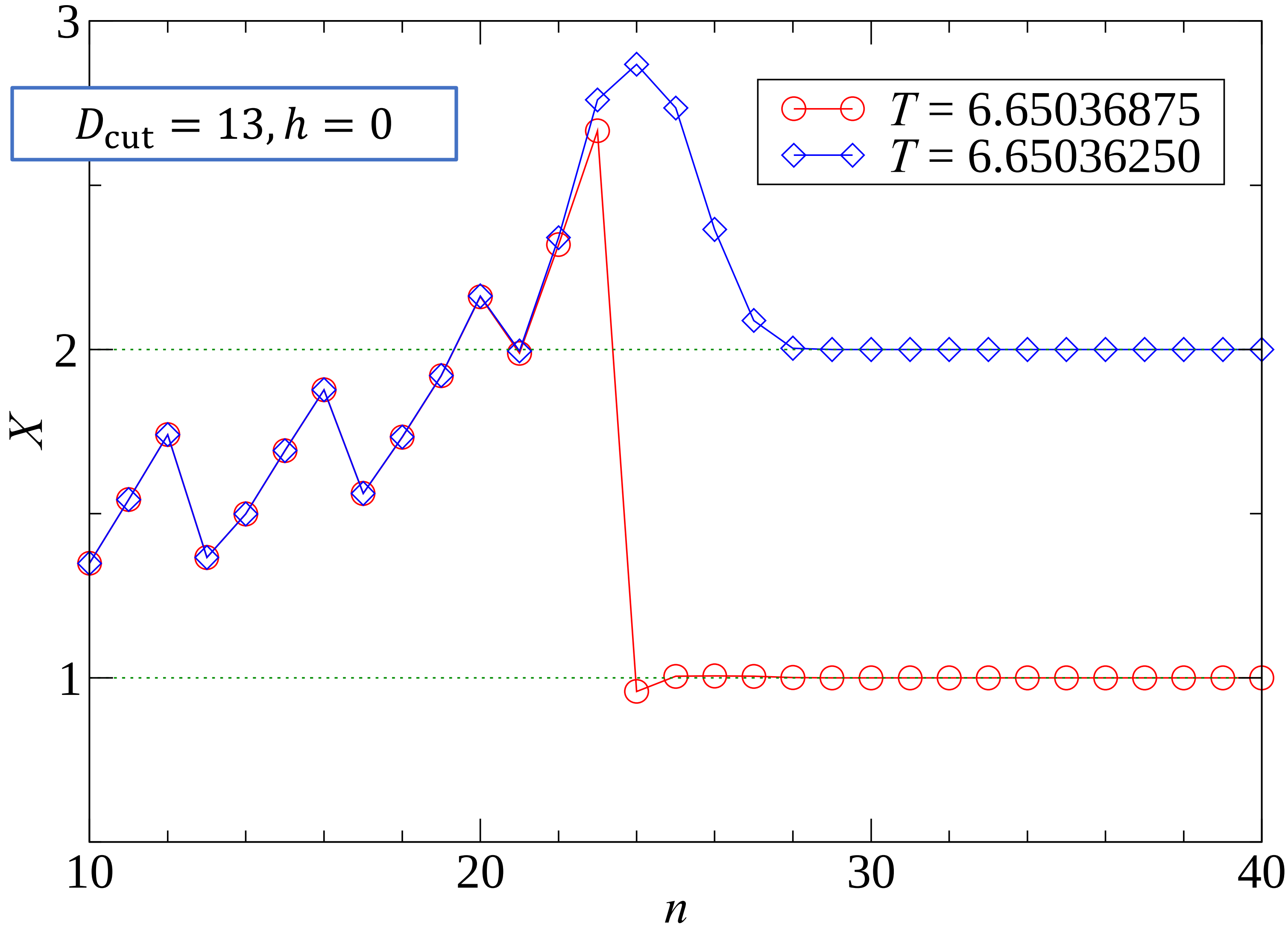
without numerical differentiation

Numerical Results: Free energy



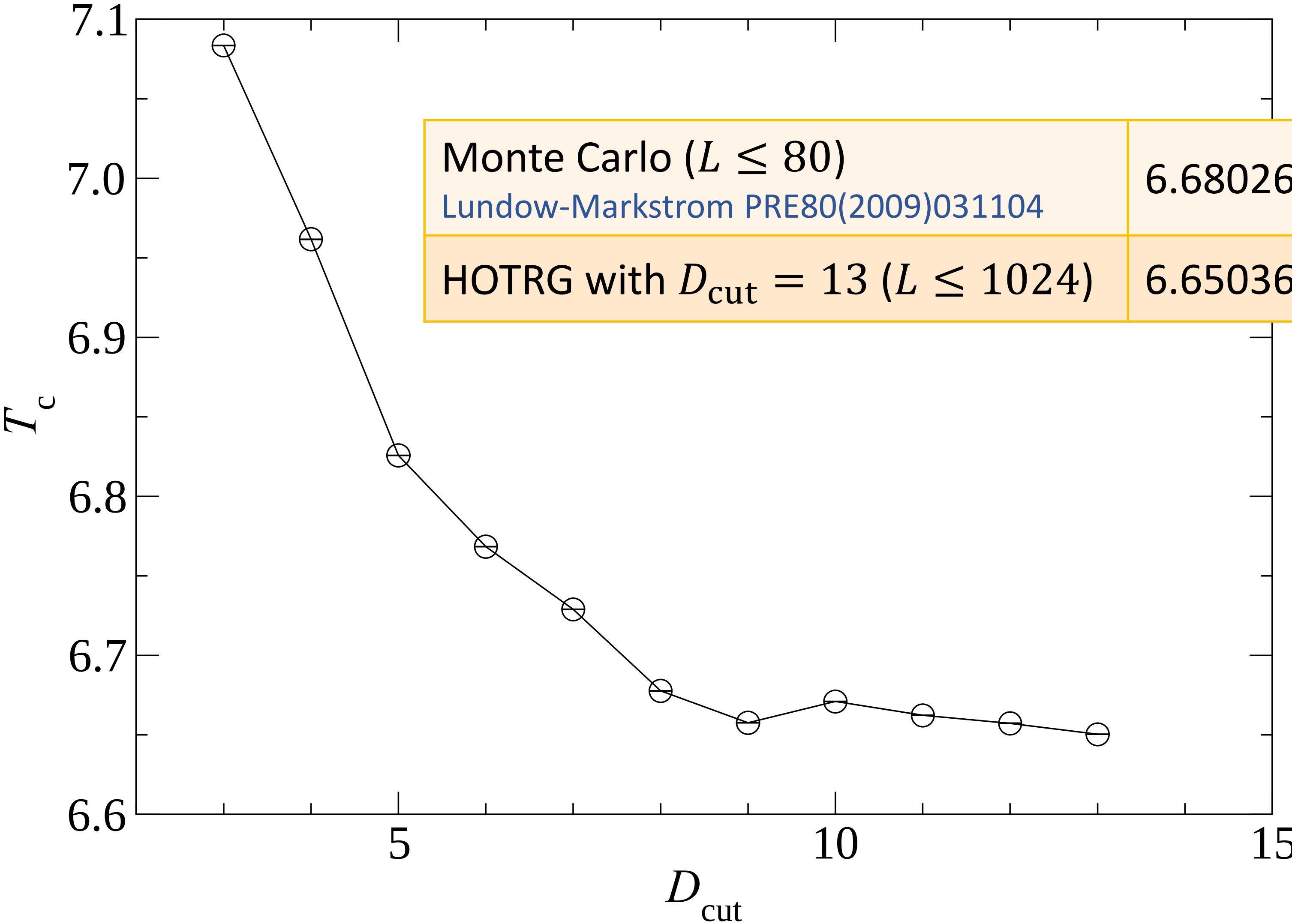
bond dimension (# of block-spin states)

Numerical Results: # of the largest eigenvalues



iteration number ($N = 2^n$)

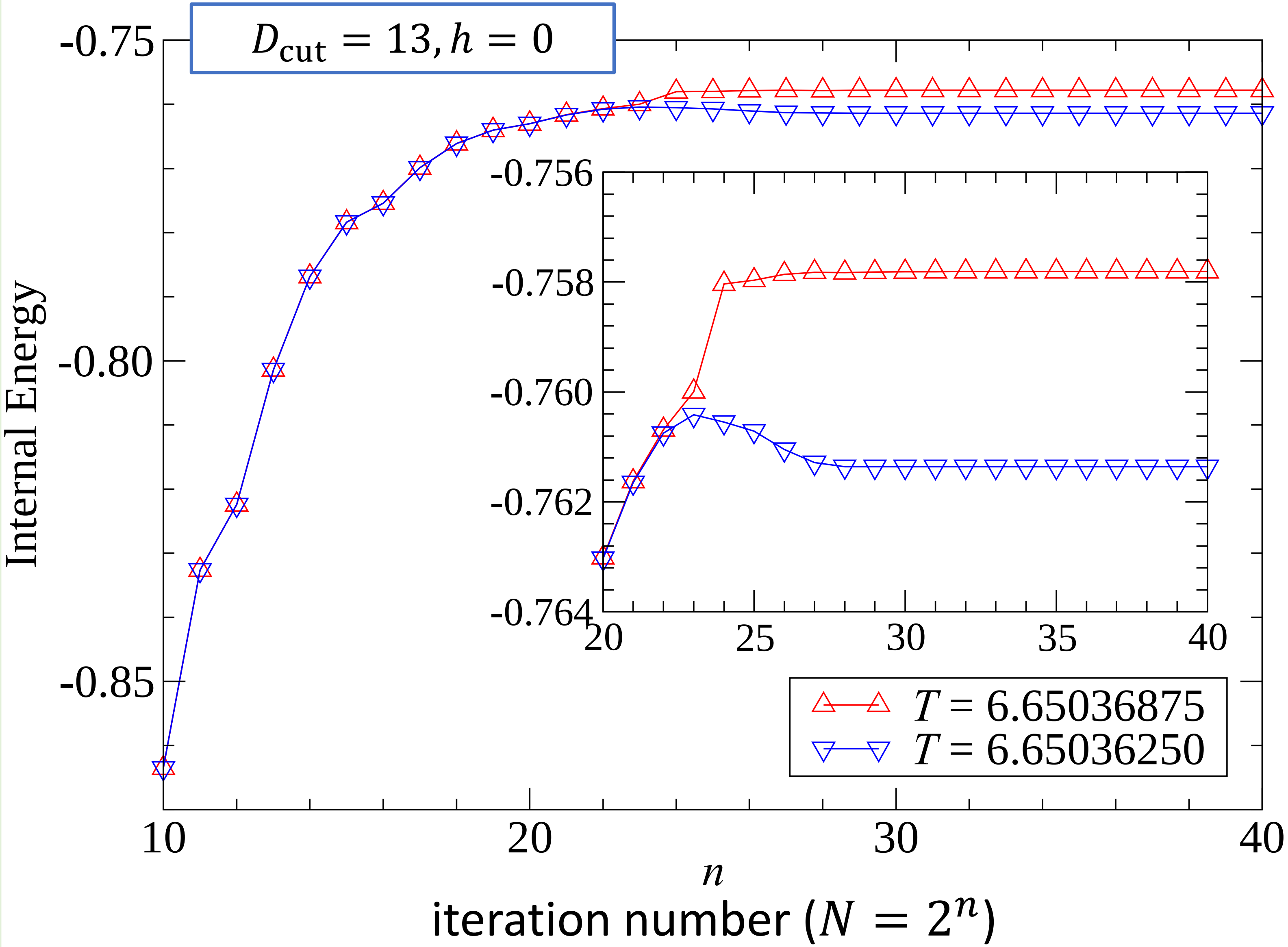
Numerical Results: Transition point



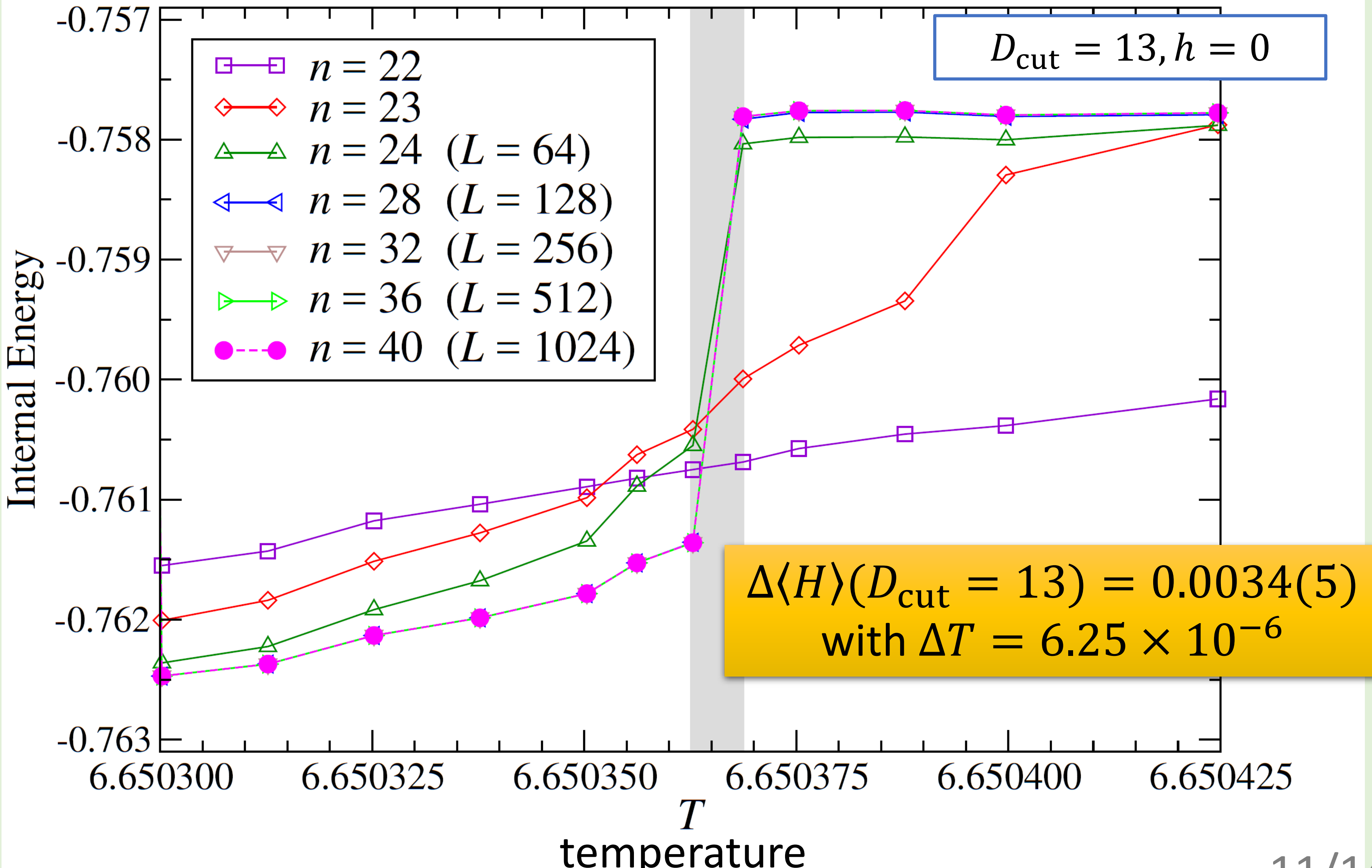
Monte Carlo ($L \leq 80$) Lundow-Markstrom PRE80(2009)031104	6.68026(2)
HOTRG with $D_{\mathrm{cut}} = 13$ ($L \leq 1024$)	6.650365(5)

bond dimension (# of block-spin states)

Numerical Results: Internal energy

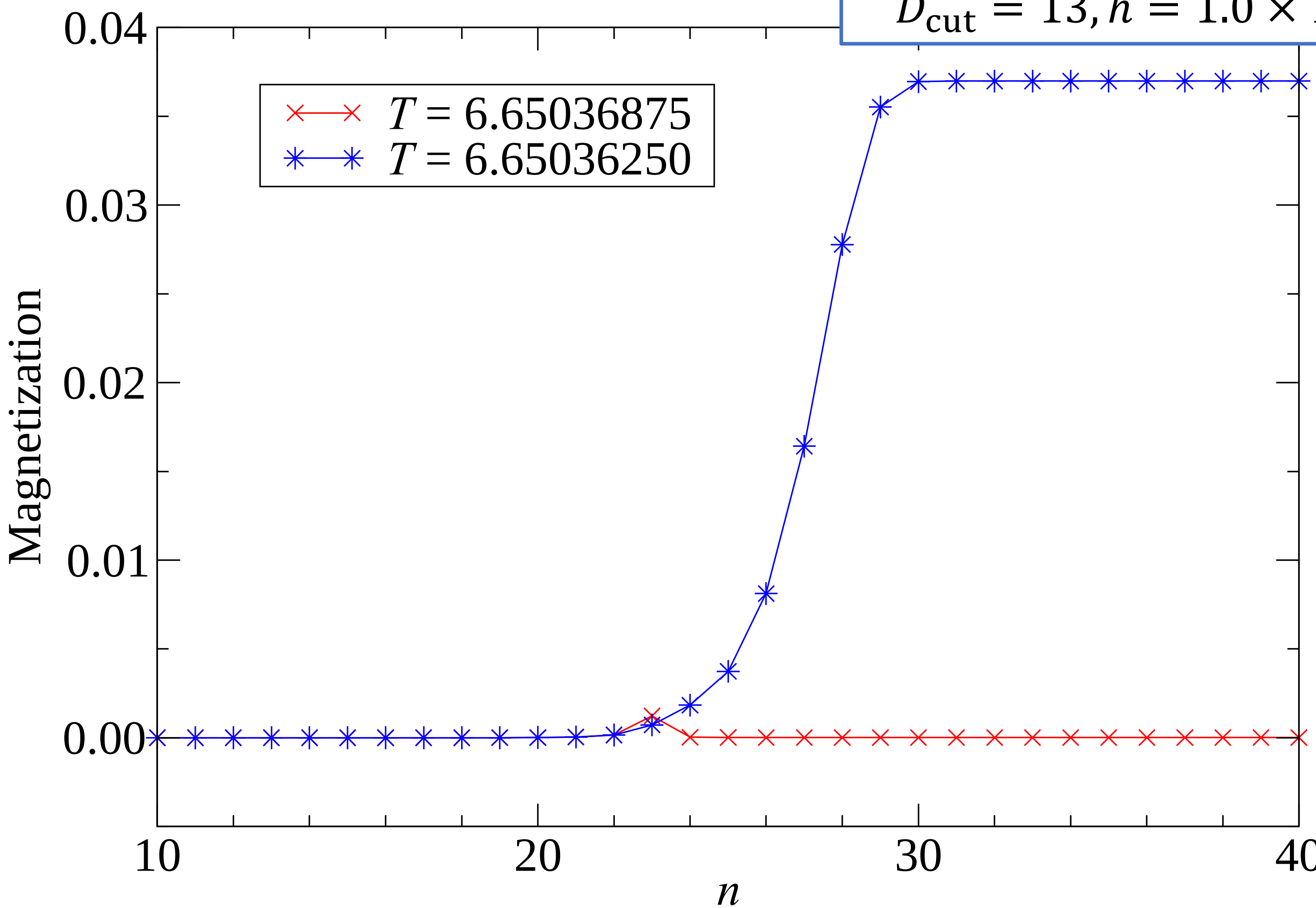


Numerical Results: Internal energy



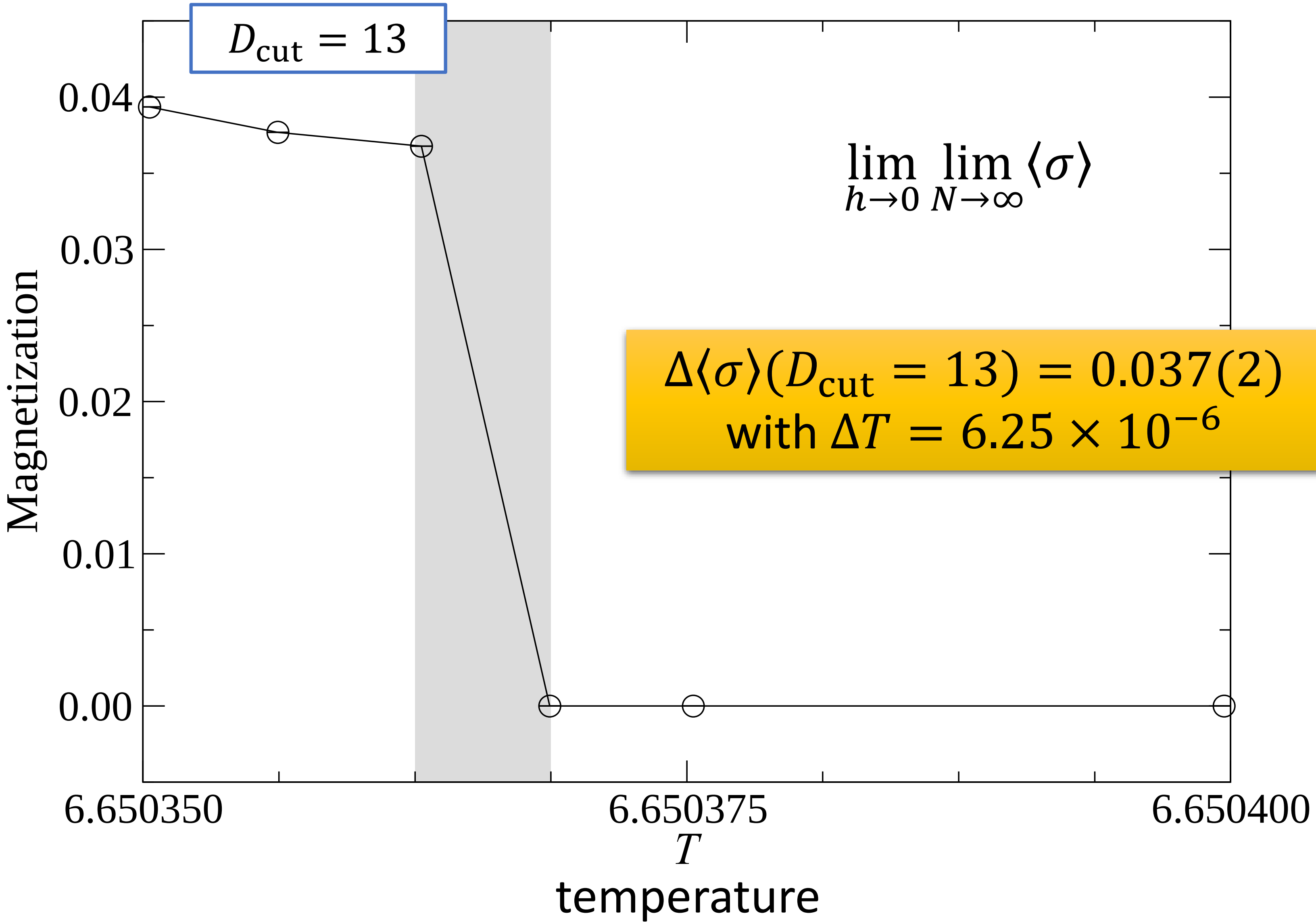
Numerical Results: Magnetization

$D_{\text{cut}} = 13, h = 1.0 \times 10^{-9}$



iteration number ($N = 2^n$)

Numerical Results: Spontaneous magnetization



$\Delta\langle\sigma\rangle(D_{\mathrm{cut}} = 13) = 0.037(2)$
with $\Delta T = 6.25 \times 10^{-6}$

Summary and outlook

	T_c	Note
Monte Carlo ($L \leq 80$) Lundow-Markstrom PRE80(2009)031104	6.68026(2)	logarithmic corrections may not exist
HOTRG with $D_{\mathrm{cut}} = 13$ ($L \leq 1024$)	6.650365(5)	seems like the weakly 1^{st} - order phase transition

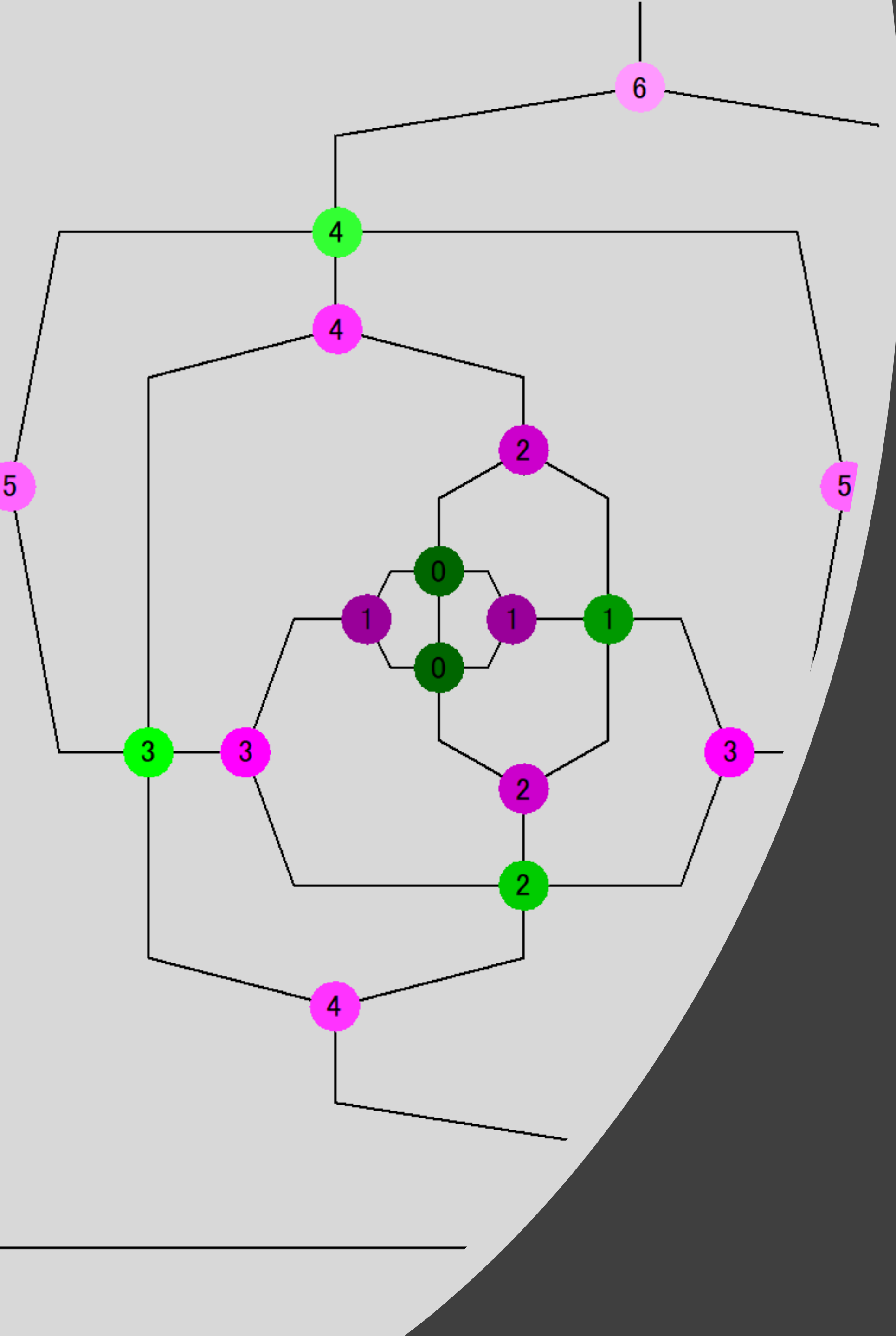
・A finite jump emerges with mutual crossings of curves for different volume in the internal energy. A jump has also been observed in the spontaneous magnetization.

Future work

★ Improvement of the impure tensor method

・patterns of coarse-graining for the network including impurity

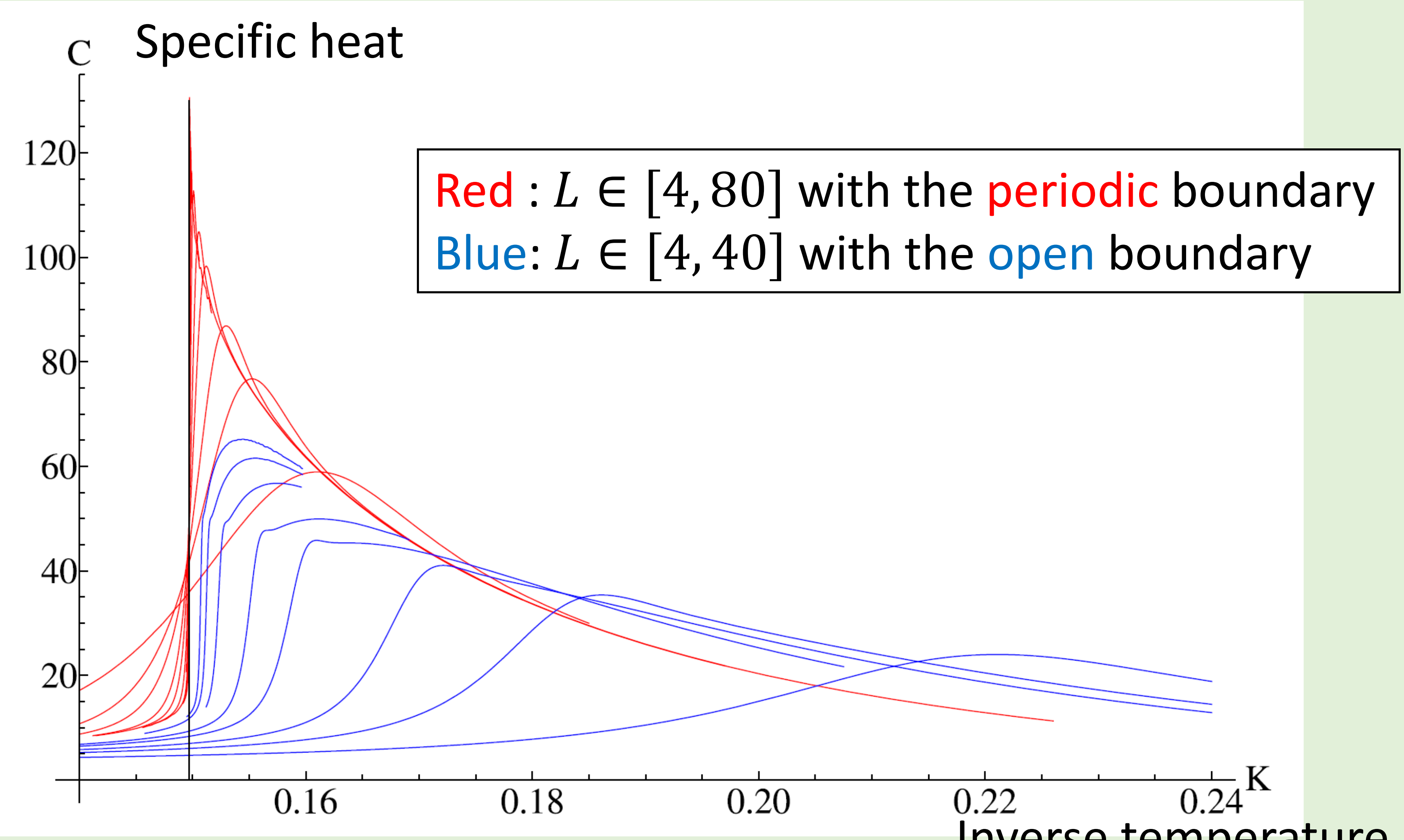
・Best optimization of the Frobenius norm of impure tensor



Backup

Non-vanishing boundary effect

Lundow-Markstrom NPB845(2011)120



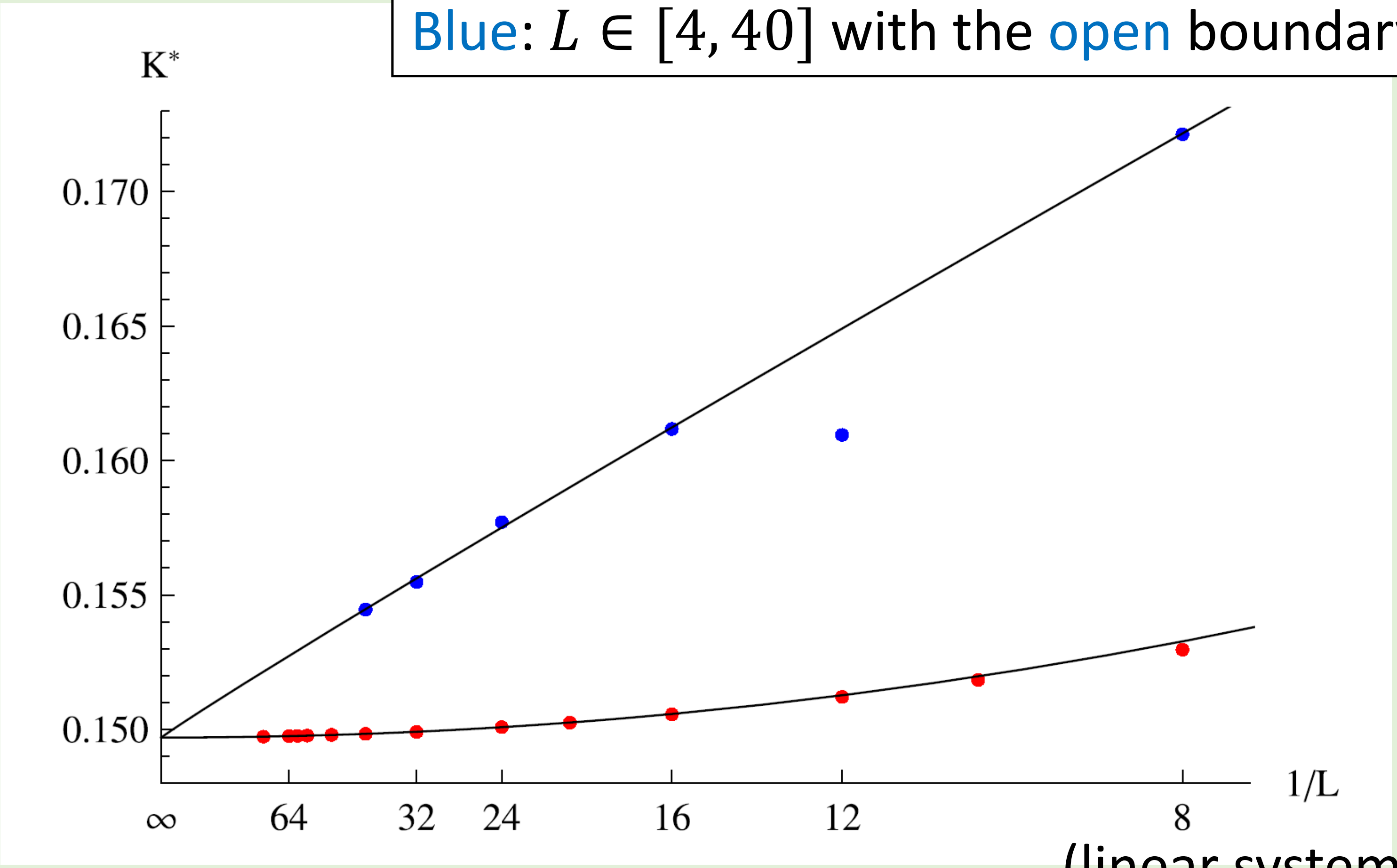
Inverse temperature

Non-vanishing boundary effect

Lundow-Markstrom NPB845(2011)120

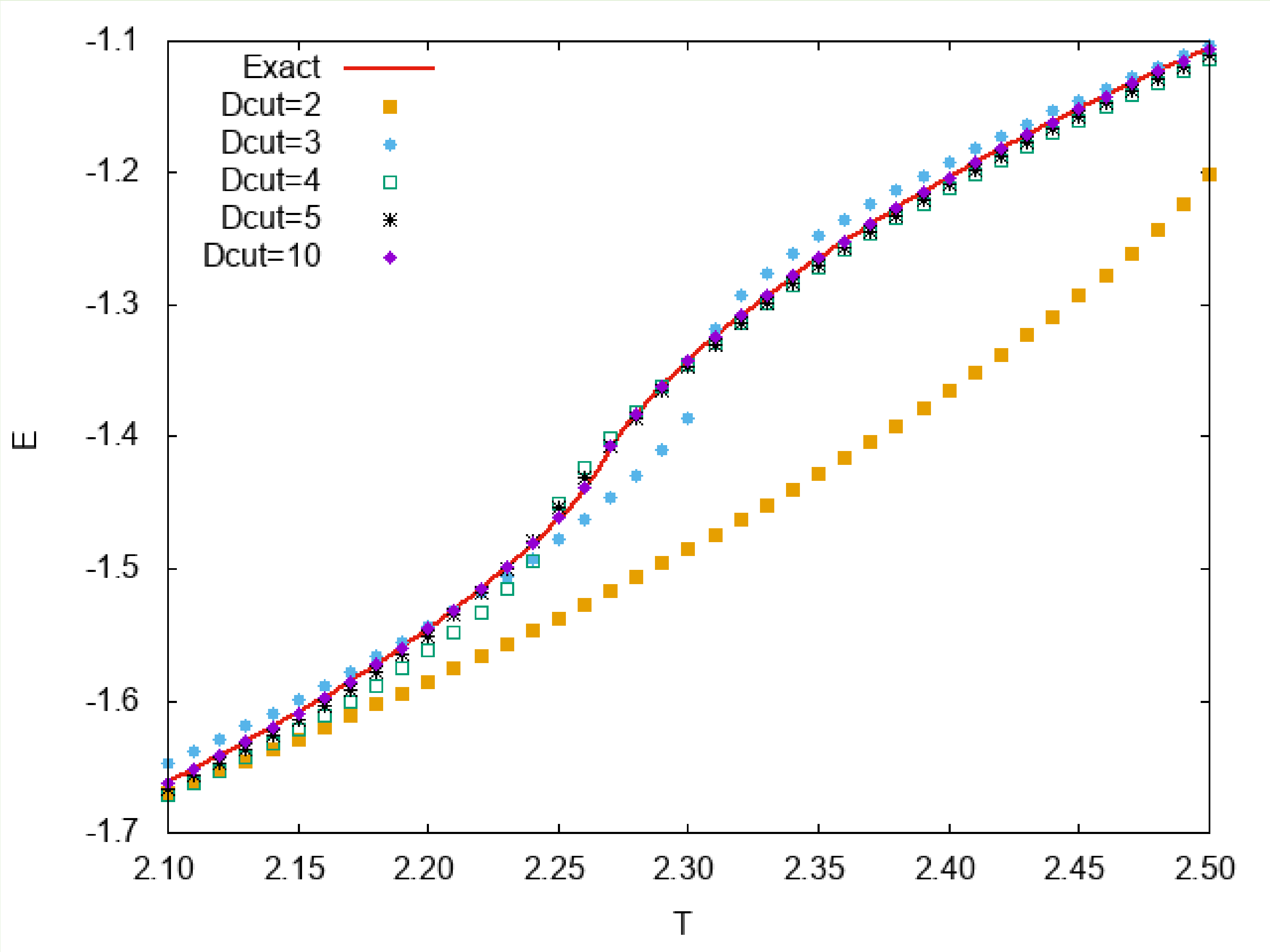
Transition point

Red : $L \in [4, 80]$ with the periodic boundary
Blue: $L \in [4, 40]$ with the open boundary



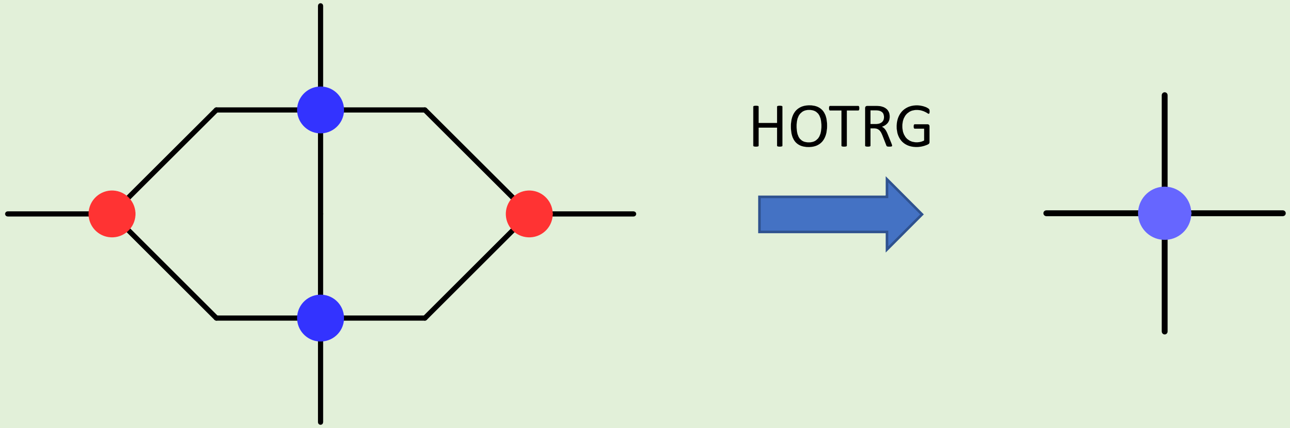
(linear system size)$^{-1}$

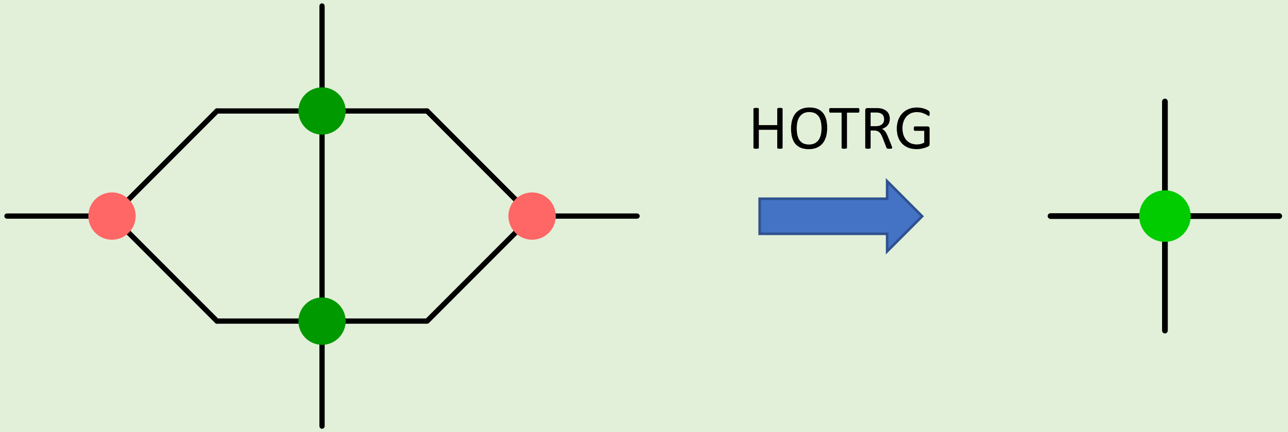
Internal energy of 2-dim. Ising model with impurity tensor method



Impurity Tensor Method

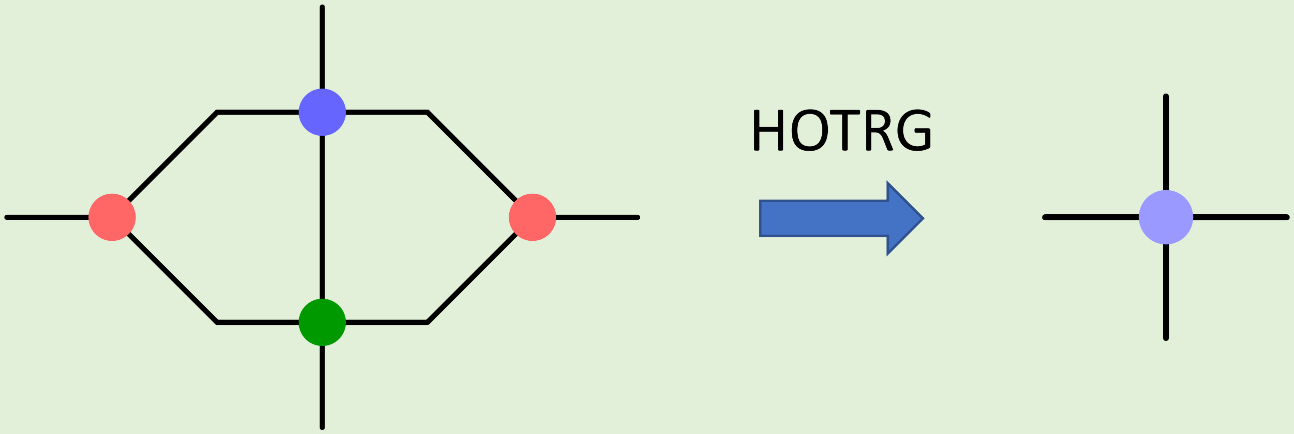
$n = 1$ (internal energy)

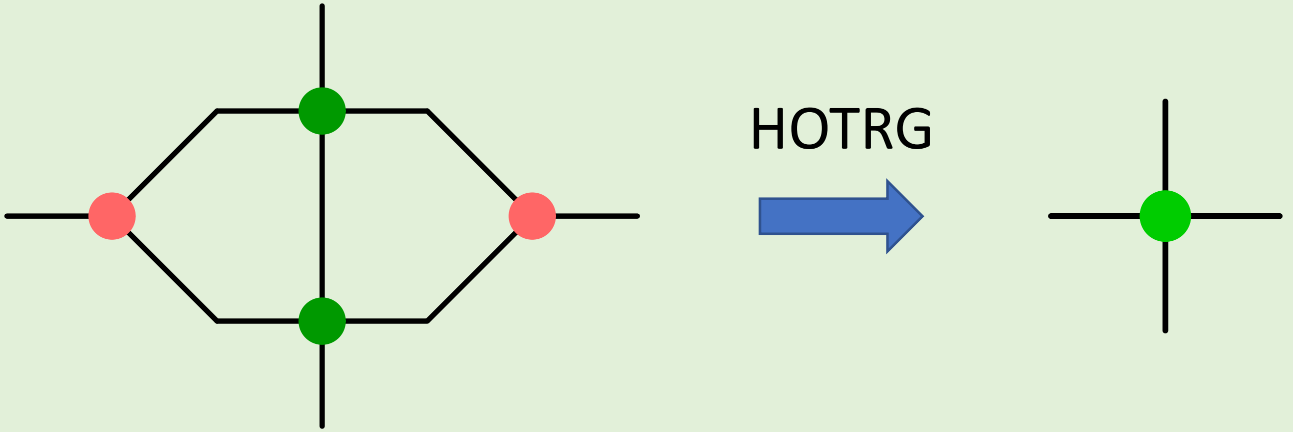




Impurity Tensor Method

$n \geq 2$ (internal energy), $n \geq 1$ (magnetization)





How to specify $T_{\mathrm{c}}(D_{\mathrm{cut}})$

$X := \frac{(\mathrm{Tr}[A])^2}{\mathrm{Tr}[A^2]}$, where $A_{tt'} := \sum_{xyz} T^{(n)}_{xxyyzztt'}$

$h = 0$

Ordered phase ⇒ $\mathbb{Z}_2$ symmetry is broken spontaneously

The largest eigenvalue of A is 2-fold degenerated, $X = 2$

Disordered phase ⇒ $\mathbb{Z}_2$ symmetry is preserved

The largest eigenvalue of A is unique, $X = 1$