

# Quark masses and decay constants in $N_f = 2 + 1 + 1$ isoQCD with Wilson clover twisted mass fermions



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## outline

- Action details
- Ensembles available, 3 lattice spacing two of them with physical Pion mass
- Preliminary results on the decay constant  $f$  for the Pion, K , D and  $D_s$  mesons
- Preliminary result on  $m_{u/d}^{\overline{MS}}$

- Iwasaki glue+ clover twisted mass fermions [ETMC, Phys. Rev. D98 (2018) no.5, 054518]

$$S = S_g + S_{tm}^\ell + S_{tm}^h$$

$$S_g = \frac{\beta}{3} \sum_x \left( b_0 \sum_{\substack{\mu, \nu=1 \\ 1 \leq \mu < \nu}}^4 \{1 - \text{Re tr}(U_{x, \mu, \nu}^{1 \times 1})\} + b_1 \sum_{\substack{\mu, \nu=1 \\ \mu \neq \nu}}^4 \{1 - \text{Re tr}(U_{x, \mu, \nu}^{1 \times 2})\} \right)$$

$$\beta = 6/g_0^2, \quad b_1 = -0.331 \quad \text{and} \quad b_0 = 1 - 8b_1$$

- Light up and down quark doublet  $\chi_\ell = (u, d)^t$ ,

$$S_{tm}^\ell = \sum_x \bar{\chi}_\ell(x) \left[ D_W(U) + \frac{i}{4} c_{\text{SW}} \sigma^{\mu\nu} \mathcal{F}^{\mu\nu}(U) + m_\ell + i\mu_\ell \tau^3 \gamma^5 \right] \chi_\ell(x)$$

- Heavy quark pair  $\chi_h = (s, c)^t$  sea action

$$S_{tm}^h = \sum_x \bar{\chi}_h(x) \left[ D_W(U) + \frac{i}{4} c_{\text{SW}} \sigma^{\mu\nu} \mathcal{F}^{\mu\nu}(U) + m_h - \mu_\delta \tau_1 + i\mu_\sigma \tau^3 \gamma^5 \right] \chi_h(x)$$

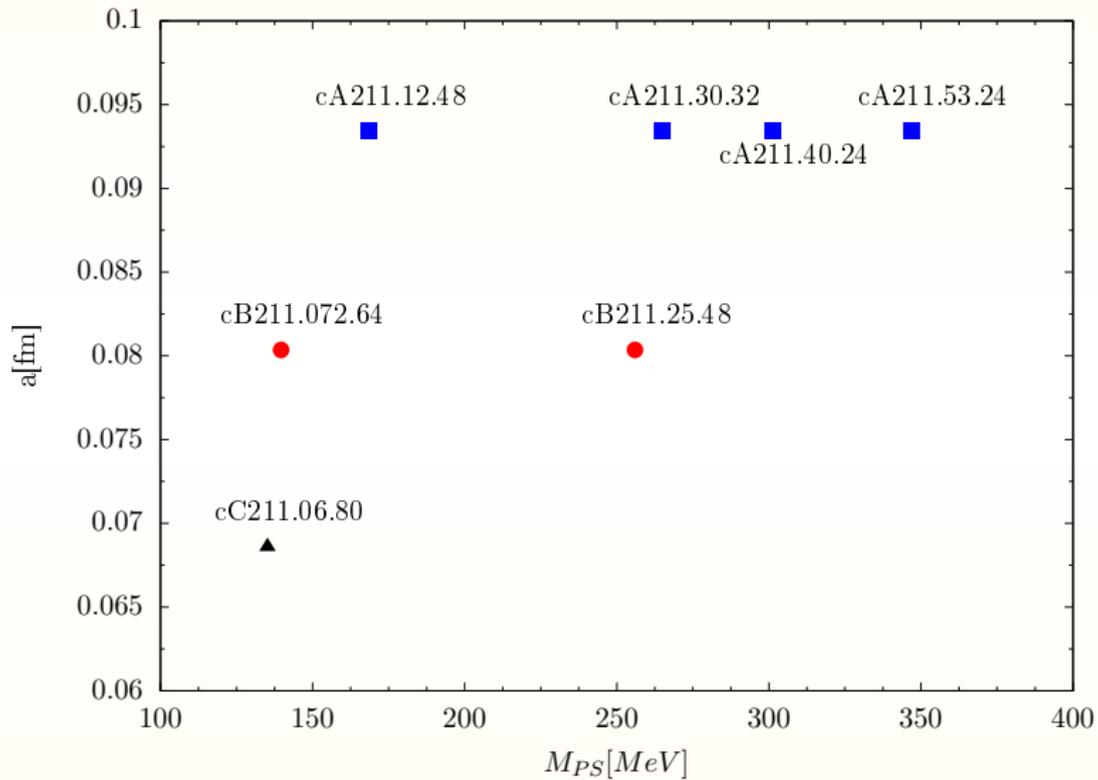
- Setting  $m_\ell = m_h = m_{crit} \implies$  physical observables  $\mathcal{O}(a)$ -improved
- One loop  $c_{SW} \cong 1 + 0.113(3) \frac{g_0^2}{P} \implies$  small  $\mathcal{O}(a^2)$  chiral breaking effects  
[S. Aoki, R. Frezzotti, P. Weisz, Nucl.Phys. B540 (1999) 501-519]
- Osterwalder Seiler fermions in the valence [Ann. Phys. (N.Y.) 110, 440 (1978)]  
[ R. Frezzotti and G. C. Rossi, JHEP 0410 (2004) 070]

$$S_{tm}^h = \sum_{f=c,s} \sum_x \bar{\chi}_f(x) \left[ D_W(U) + \frac{i}{4} c_{SW} \sigma^{\mu\nu} \mathcal{F}^{\mu\nu}(U) + m_h + i\mu_f^{OS} \gamma^5 \right] \chi_f(x)$$

- Matching of the renormalised mass

$$\mu_{c,s}^{OS,ren} = \frac{1}{Z_P} \left( \mu_\sigma \pm \frac{Z_P}{Z_S} \mu_\delta \right)$$

name	Nconf	$\kappa$	$a\mu_\ell$	$\mu_\sigma$	$\mu_\delta$	$\beta$
cA211.53.24	4489	0.1400645	0.0053	0.1408	0.1521	1.726
cA211.40.24	4882	0.1400645	0.0040	0.1408	0.1521	1.726
cA211.30.32	4556	0.1400645	0.0030	0.1408	0.1521	1.726
cA211.12.48	2368	0.1400645	0.0012	0.1408	0.1521	1.726
cB211.25.48	4880	0.1394267	0.0025	0.1246864	0.1315052	1.778
cB211.072.64	3164	0.1394265	0.00072	0.1246864	0.1315052	1.778
cC211.06.80	1013	0.1387510	0.0006	0.1060	0.1135	1.836



- Ensemble cA211.12.48:  $m_{pcac}/\mu = -0.21(5)$
- Corrected for deviations from maximal twist

$$M_{PS}(m_{pcac} = 0) = M_{PS}(m_{pcac}) \left( 1 + \frac{m_{pcac}^2 Z_A^2}{\mu^2} \right)^{-1/4}$$

$$f_{PS}(m_{pcac} = 0) = F_{PS}(m_{pcac}) \left( 1 + \frac{m_{pcac}^2 Z_A^2}{\mu^2} \right)$$

- Correction for deviations from maximal twist can also be estimated through reweighting: preliminary estimates agree with the formula above

- During production runs the two point correlators were computed for the light-light PS-masses

$$c(t) = \frac{1}{L^3} \sum_{\vec{x}} \langle \bar{Q} \gamma^5 Q(0) \bar{Q} \gamma^5 Q(x) \rangle$$

from which the masses and the decay constant can be evaluated

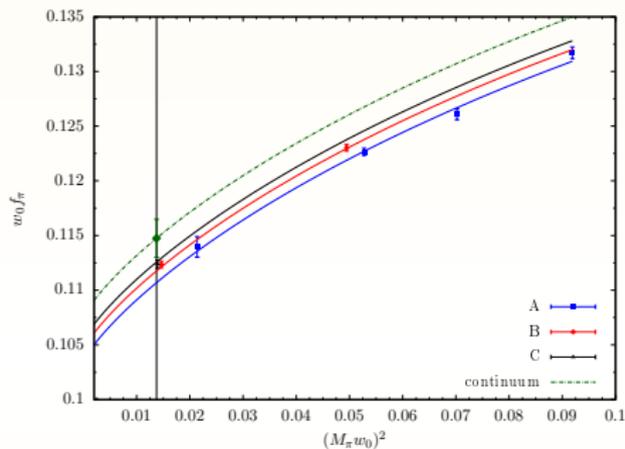
$$af_P(\mu_1, \mu_2) = a(\mu_1 + \mu_2) \frac{a^2 \langle 0 | P_5^{local} | P \rangle}{aM_P \sinh(aM_P)}.$$

- Analysis of full statistics ensembles measurements based on SU(2) ChPT:

$$(f_\pi w_0) = (fw_0) \left[ 1 - 2\xi_l^M \log \xi_l^M + P_3 \xi_l^M + \frac{a^2}{w_0^2} P_4 \right] K_f^{FSE},$$

$$\xi_l^M = \frac{M_\pi^2}{16\pi^2 f^2}$$

- does not rely on the knowledge of the quark mass RC ( Here  $Z_P$  )
- $K^{FSE}$  from [G. Colangelo, S. Dürr, C Haefeli, Nucl. Phys. B721: 136-174,2005]

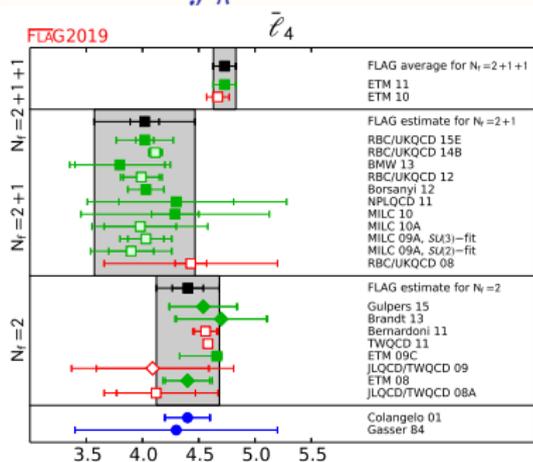
Preliminary Results  $f_\pi$ 

- Systematic error not included

$$f w_0 = (0.1076 \pm 0.0007) \quad \bar{\ell}_4 = (4.433 \pm 0.053) \quad P_4 = (-0.126 \pm 0.027)$$

We impose  $w_0 = 0.1714 \pm 0.0015$  fm [MILC, Phys. Rev. D 93, 094510] which was computed with the scale set through  $a f_\pi$ . We get:

$$f_\pi = (132.1 \pm 1.1) \text{MeV}$$



- Within the Prace call 17 project: "PLepNuGam - Radiative leptonic decays of light and heavy pseudoscalar mesons from Lattice QCD" we produce quark propagators with the following masses

name	Nconf	$a\mu_\ell$	$\sim L[\text{fm}]$
cA211.53.24	628	0.0053 0.0176 0.0220 0.0264 0.2077 0.2336 0.2596 0.2856 0.3115 0.3433 0.3948 0.4540 0.5221 0.6005 0.6905 0.7941	2.3
cA211.40.24	662	0.0040 0.0176 0.0220 0.0264 0.2077 0.2336 0.2596 0.2856 0.3115 0.3433 0.3948 0.4540 0.5221 0.6005 0.6905 0.7941	2.3
cA211.30.32	287	0.0030 0.0176 0.0220 0.0264 0.2077 0.2336 0.2596 0.2856 0.3115 0.3433 0.3948 0.4540 0.5221 0.6005 0.6905 0.7941	3.1
cA211.12.48	240	0.0012 0.0176 0.0220 0.0264 0.2077 0.2336 0.2596 0.2856 0.3115 0.3433 0.3948 0.4540 0.5221 0.6005 0.6905 0.7941	4.6
cB211.25.48	314	0.0025 0.0148 0.0185 0.0222 0.1745 0.1962 0.2181 0.2399 0.2617 0.2884 0.3316 0.3814 0.4386 0.5044 0.5800 0.6670	3.9
cB211.072.64	187	0.0072 0.0148 0.0185 0.0222 0.1745 0.1962 0.2181 0.2399 0.2617 0.2884 0.3316 0.3814 0.4386 0.5044 0.5800 0.6670	5.2
cC211.06.80	113	0.0006 0.0128 0.0161 0.0193 0.1526 0.1716 0.1907 0.2098 0.2288 0.2522 0.2901 0.3336 0.3837 0.4412 0.5075 0.5836	5.6

- four inversions per configuration (spin dilution)
- Lightest mass **unitary points**, heavier meson mixed action setup

- We compute all the bilinear correlators

$$c(t) = \frac{1}{L^3} \sum_{\vec{x}} \langle \bar{Q}^i \Gamma^A Q^i(0) \bar{Q}^j \Gamma^B Q^j(x) \rangle$$

with  $i, j$  = smeared, local Jacobi [UKQCD, Phys. Rev. D 47, 5128 (1993)]  
+APE [APE, Nucl. Phys. B251, 624 (1985)]  
 $A, B = 1, \dots, 16$  all Dirac

- Masses and decay constant from

$$c(t) = \frac{1}{L^3} \sum_{\vec{x}} \langle P_5^i(0) P_5^j(x) \rangle$$

$$af_P(\mu_1, \mu_2) = a(\mu_1 + \mu_2) \frac{a^2 \langle 0 | P_5^{local} | P \rangle}{aM_P \sinh(aM_P)}.$$

- Effective masses extracted using GEVP [B. Blossier *et al.*, JHEP 0904:094,2009]

## K meson

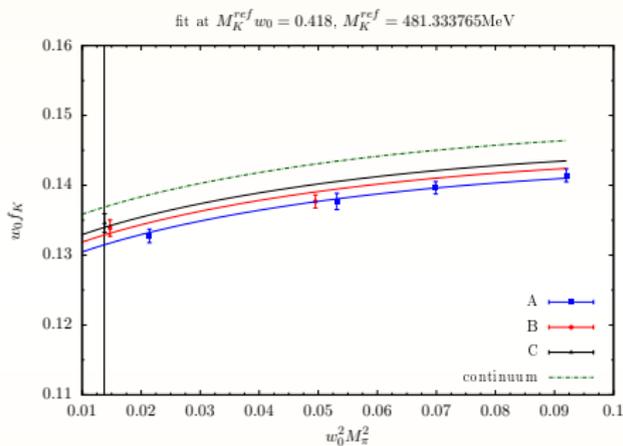
- Interpolation in the meson mass  $M_K^2$  of  $f_K = A + BM_K^2$  for each ensemble at  $(M_K^{ref} w_0)^2 = 0.1444, 0.1596, 0.1748$
- Chiral and continuum extrapolation for each  $(M_K^{ref} w_0)^2$

$$f_k = P'_1 \left[ 1 - \frac{3}{4} \xi_M \log \xi_M + P'_2 \xi_M + P'_4 a^2 \right] K_{f_K}^{FSE}$$

$$\xi_M = \frac{M_\pi^2}{16\pi^2 f^2}$$

- Interpolation in the meson mass  $M_K^2$  of  $f_K = A + BM_K^2$  at  $a = 0$ ,  $M_\pi = M_\pi^{exp}$

- $M_K^{exp} = \sqrt{\frac{M_{K^+}^2 + M_{K^0}^2}{2} - \frac{(1+\epsilon+2\epsilon_{K^0} - \epsilon_m)}{2} (M_{\pi^+}^2 - M_{\pi^0}^2)} = 494.2(4)\text{MeV}$   
[FLAG 2013, arXiv:1310.8555]

Preliminary results  $f_K$ 

- Systematic error not included

$$f_K = (158.2 \pm 2.9)\text{MeV}$$

$$f_K/f_\pi = (1.197 \pm 0.023)$$

Collaboration	Ref.	$N_f$	publication status	chiral extrapolation	continuum extrapolation	finite-volume errors	$f_K/f_\pi$	$f_{K^\pm}/f_{\pi^\pm}$
FNAL/MILC 17	[5]	2+1+1	A	★	★	★	1.1980(12)( $^{+5}_{-15}$ )	1.1950(15)( $^{+6}_{-18}$ )
ETM 14E	[35]	2+1+1	A	○	★	○	1.188(11)(11)	1.184(12)(11)
FNAL/MILC 14A	[18]	2+1+1	A	★	★	★	1.1956(10)( $^{+26}_{-18}$ )	1.1956(10)( $^{+26}_{-18}$ )
ETM 13F	[253]	2+1+1	C	○	★	○	1.193(13)(10)	1.183(14)(10)
HPQCD 13A	[34]	2+1+1	A	★	○	★	1.1948(15)(18)	1.1916(15)(16)
MILC 13A	[254]	2+1+1	A	★	★	★	1.1947(26)(37)	1.1947(26)(37)
MILC 11	[255]	2+1+1	C	○	○	○	1.1872(42) $^{\dagger}_{stat.}$	1.1872(42) $^{\dagger}_{stat.}$
ETM 10E	[256]	2+1+1	C	○	○	○	1.224(13) $^{\dagger}_{stat.}$	1.224(13) $^{\dagger}_{stat.}$
QCDSF/UKQCD 16	[40]	2+1	A	○	★	○	1.192(10)(13)	1.190(10)(13)
BMW 16	[39, 257]	2+1	A	★	★	★	1.182(10)(26)	1.178(10)(26)
RBC/UKQCD 14B	[10]	2+1	A	★	★	★	1.1945(45)	1.1945(45)
RBC/UKQCD 12	[154]	2+1	A	★	○	★	1.199(12)(14)	1.199(12)(14)
Laiho 11	[98]	2+1	C	○	★	○		1.202(11)(9)(2)(5) $^{\dagger\dagger}$
MILC 10	[37]	2+1	C	○	★	★		1.197(2)( $^{+2}_{-2}$ )
JLQCD/TWQCD 10	[258]	2+1	C	○	■	○	1.230(19)	
RBC/UKQCD 10A	[157]	2+1	A	○	○	★	1.204(7)(25)	
BMW 10	[38]	2+1	A	★	★	★	1.192(7)(6)	
MILC 09A	[17]	2+1	C	○	★	★		1.198(2)( $^{+6}_{-6}$ )
MILC 09	[128]	2+1	A	○	★	★		1.197(3)( $^{+6}_{-13}$ )
Aubin 08	[259]	2+1	C	○	○	○		1.191(16)(17)
RBC/UKQCD 08	[160]	2+1	A	○	■	★	1.205(18)(62)	
HPQCD/UKQCD 07	[36]	2+1	A	○	○	○	1.189(2)(7)	
MILC 04	[163]	2+1	A	○	○	○		1.210(4)(13)
ETM 14D	[260]	2	C	★	■	○	1.203(5) $^{\dagger}_{stat.}$	
ALPHA 13A	[261]	2	C	★	★	★	1.1874(57)(30)	
ETM 10D	[237]	2	C	○	★	○	1.190(8) $^{\dagger}_{stat.}$	
ETM 09	[41]	2	A	○	★	○	1.210(6)(15)(9)	
QCDSF/UKQCD 07	[262]	2	C	○	○	★	1.21(3)	

$^{\dagger}$  Result with statistical error only from polynomial interpolation to the physical point.

$^{\dagger\dagger}$  This work is the continuation of Aubin 08.

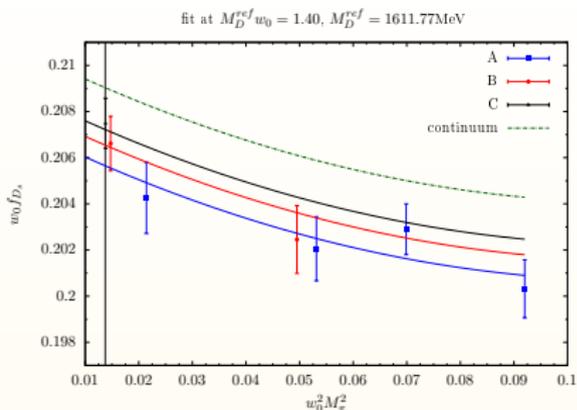
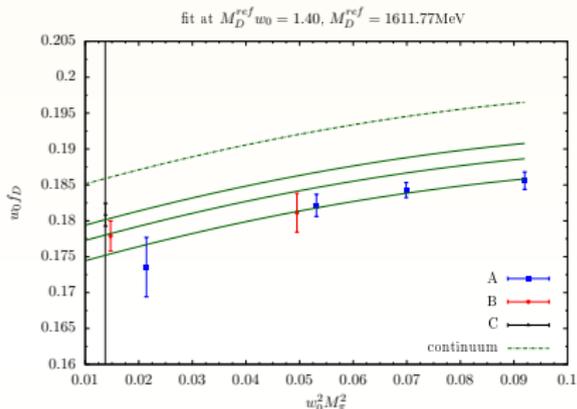
## D and $D_s$ meson

- Interpolation in the meson mass  $M_D$  of  $f_{D/D_s} = A + BM_D$  for each ensemble at three reference values  $(M_D^{ref} w_0) = 0.1444, 0.1596, 0.1748$
- Chiral and continuum extrapolation a

$$f_{D/D_s} = P_1(1 + P_2 M_\pi^2 + P_3 M_\pi^4 + P_4 a^2)$$

- Interpolation in the meson mass  $M_D$  of  $f_{D/D_s} = A + BM_D$  at  $a = 0$ ,  $M_\pi = M_\pi^{exp}$
- $M_D^{exp} = \frac{M_{D^\pm} + M_{D^0}}{2} = 1.867\text{GeV}$
- In the  $D_s$  meson an extra interpolation in  $M_K^2$  of  $f_{D_s} = A + BM_K^2$  to  $M_K^{exp} = 494.2(4)\text{MeV}$  is required

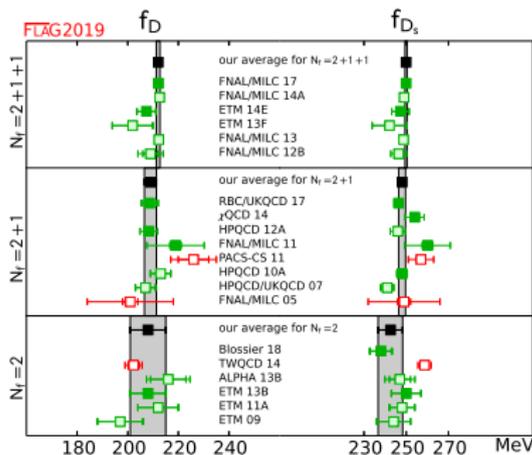
# Preliminary results $f_D$ and $f_{D_s}$



- Systematic error not included

$$f_D = (219 \pm 6) \text{ MeV}$$

$$f_{D_s} = (246 \pm 3) \text{ MeV}$$



## Light quark mass

- Chiral fits in SU(2)ChPT in terms of  $m_\ell = \mu/Z_P$ :

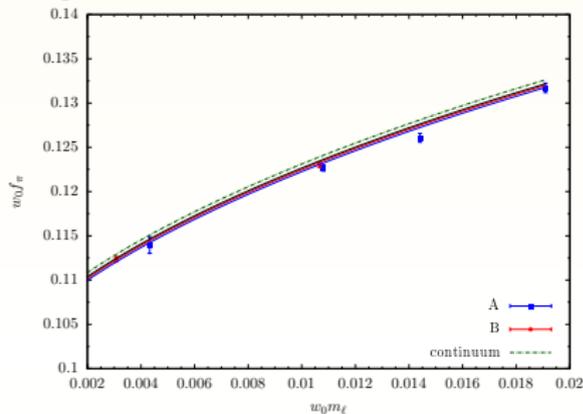
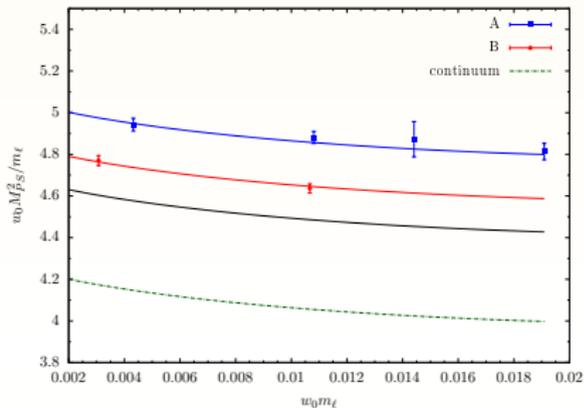
$$(M_\pi w_0)^2 = 2(Bw_0)(m_\ell w_0) \left[ 1 + \xi_\ell \log \xi_\ell + P_1 \xi_\ell + \frac{a^2}{w_0^2} P_2 \right] K_{M^2}^{FSE},$$

$$(f_\pi w_0) = (fw_0) \left[ 1 - 2\xi_\ell \log \xi_\ell + P_3 \xi_\ell + \frac{a^2}{w_0^2} P_4 \right] K_f^{FSE},$$

$$\xi_\ell = 2B_0 m_\ell / (16\pi f^2), \quad P_1 = -\bar{\ell}_3 - 2 \log \left( \frac{M_\pi^{phys}}{4\pi f} \right), \quad P_3 = 2\bar{\ell}_4 - 4 \log \left( \frac{M_\pi^{phys}}{4\pi f} \right)$$

- $Z_P$  computed with sub-percent accuracy employing RI'-MOM techniques on  $N_f = 4$  gauge ensembles generated for the (mass independent) RCs computation
- only preliminary result at the two coarser lattice spacings for  $Z_P(\overline{MS}2GeV)$

## Very preliminary results



- Systematic errors not included

$$m_{ud}^{\overline{MS}}(2\text{GeV}) = (3.84 \pm 0.05)\text{MeV}$$

$$f_\pi = (131 \pm 1)\text{MeV}$$

- Lattice artefacts in the relation  $M_{PS}^2$  to  $m_\ell$  not small (as usual)
- Can be reduced assuming  $M_{SS'}$  scaling variable [ETMC, arXiv:1403.4504v3]
- Without a third lattice spacing systematic error  $\sim 5 - 10\%$

## Conclusions

- Simulations at the physical point
- Three lattice spacings at (0.096, 0.08, 0.07) fm.
- $O(a^2)$  lattice artefacts generally small
- The results presented are very preliminary:
  - Full statistics not included yet
  - Systematic uncertainties not included yet
  - Check of the FVE: ensembles cB211.25.24 and cB211.25.32 in production
  - Quark mass RC ( $Z_P$ ) at the finest lattice spacing not available yet

## Outlook

- Scale setting from  $m_\Omega$
- Accurate calculation of  $m_{u/d}$ ,  $m_s$ ,  $m_c$  and  $m_b$
- Accurate calculation of  $f_K$ ,  $f_D$ ,  $f_{D_s}$  and  $f_{B(s)}$  (ration method)

Thank you for your attention

Action  
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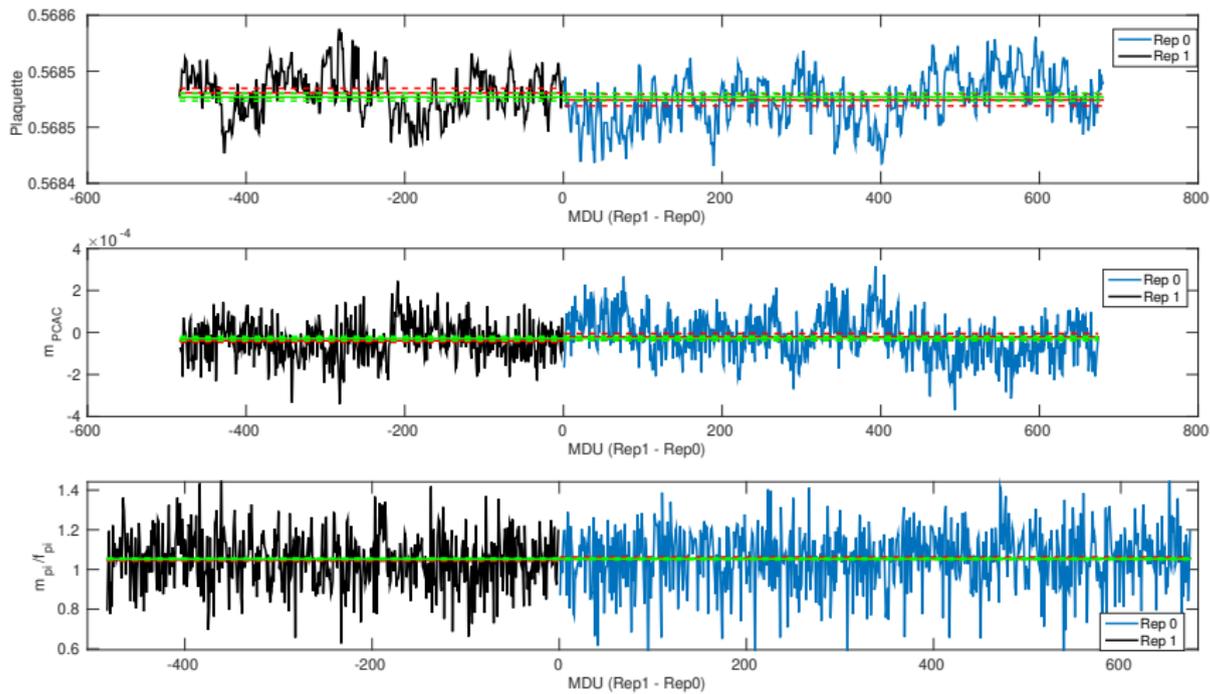
Pion  
oo

K,D,Ds  
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quark masses  
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# Backup slides

## cC211.06.80



$Z_P$  in RI'-MOM

- Quark renormalization constant

$$Z_q^{-1} \frac{-i}{12} \text{Tr} \left[ \frac{\not{p} S_\chi^{-1}(p)}{p^2} \right] = 1$$

where

$$S_\chi(p) = a^4 \sum_x e^{-ipx} \langle \chi_f^{\text{val}}(x) \bar{\chi}_f^{\text{val}}(0) \rangle_{\mathcal{M}, \omega}^{\text{latt}}$$

$$\frac{Z_q}{Z_P} = \text{Tr} \left[ S_{\chi_1}^{-1}(p) \left( a^8 \sum_{x,y} e^{-ip(x-y)} \langle \chi_1^{\text{val}}(x) (\bar{\chi}_1^{\text{val}} \gamma_5 \chi_2^{\text{val}})(0) \bar{\chi}_2^{\text{val}}(y) \rangle_{p^2=\mu^2}^{\text{latt}} \right) S_{\chi_2}^{-1}(p) \gamma_5 \right]$$