Quark confinement in the Yang-Mills theory with a gauge-invariant gluon mass in view of the gauge-invariant BEH mechanism

SHIBATA, Akihiro (KEK);
KONDO, Kei-Ichi (Chiba University);
MATSUDO, Ryutaro (Chiba University);
NISHINO, Shogo (Chiba University)

The 37th international conference on lattice field theory (lattice2019) at Wuhan, China

Introduction

What is the mechanism of quark confinement?

- A promising scenario is the **dual superconductor picture** of the QCD vacuum. [Nambu,1974]['t Hooft,1975][Mandelstam,1976]
- One of the remarkable facts on this picture found in the preceding studies is *Infrared Abelian dominance* : The Abelian part (or diagonal component) of the gauge field becomes dominant for quark confinement in the low-energy or long-distance region [Ezawa & Iwazaki,1982].

  This hypothesis was confrmed by:
  Abelian dominance of the string tension: The string tension of the linear potential in the static quark-antiquark potential can be reproduced by the Abelian part alone [Suzuki & Yotsuyanagi,1990].
  Dynamical generation of the off-diagonal gluon mass: The off-diagonal gluon propagator exhibits the exponential fall-off in the distance [Amemiya & Suganuma,1999].
- However, these results were obtained only in the specific gauge called the maximal Abelian (MA) gauge based on the idea of Abelian projection method proposed by ['t Hooft,1981].
  The gauge invariance or independence was not clear in the Abelian projection method

Introduction (cont')

The decomposition method:

- We have succeeded to demonstrate the Abelian dominance of the string tension in the gauge-invariant way based on the novel reformulation of the Yang-Mills theory in terms of the new field variables obtained from the gauge covariant decomposition method and the non-Abelian Stokes theorem for the Wilson loop operator .

For more details, see the review: K.-I. Kondo, S. Kato, T. Shinohara and A. Shibata, Phys. Rept **579**, *1–226 (2015). arXiv:1409.1599 [hep-th]*

- How about the Abelian dominance of the diagonal propagator?

  The propagator can be obtained only after the gauge fixing. Therefore, Abelian dominance of the diagonal propagator cannot be extended in the gauge invariant way.
- Instead, however, we can give a gauge-invariant definition for the off-diagonal gluon mass.
- Therefore, we can study the mass generation of the off-diagonal gluon mass in the gauge-invariant way.

Introduction (cont')

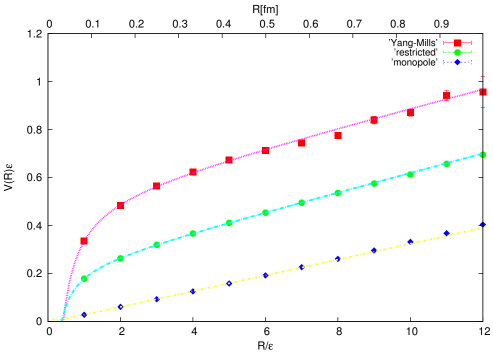
- This is based on the gauge-independent description of the Brout-Englert-Higgs (BEH) mechanism proposed recently by [Kondo, 2016, 2018], which needs
  neither the spontaneous breaking of gauge symmetry $G \to H$,
  nor the non-vanishing vacuum expectation value of the scalar field $\langle 0|\phi(x)|0\rangle := v \neq 0$.

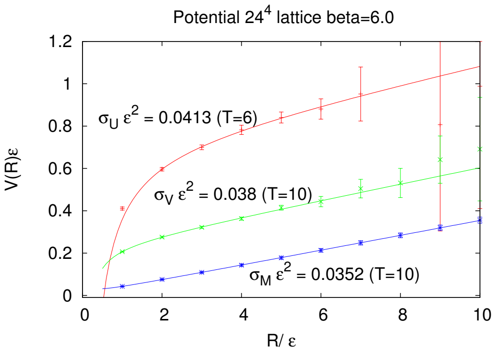
- To explain it, we need to introduce a specific gauge-scalar model (complementary gauge-scalar model) which reduces to the Yang-Mills theory with a gauge-invariant gluon mass term (massive Yang-Mills theory) .
- The gauge-invariant gluon mass term
  simulates the dynamically generated mass to be extracted in the low-energy effective theory of the Yang-Mills theory
  and plays the role of a new probe to study confinement mechanism through the phase structure (Confinement phase, Higgs phase, deconfinement phase) in the gauge-invariant way.

In this talk we give preliminary studies on the lattice in this direction.

The followings are the results by the decomposition method.

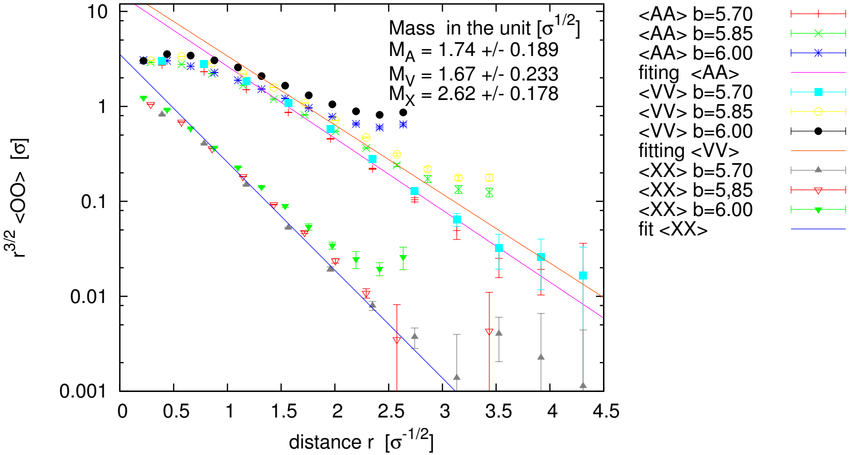
Abelian dominance of the string tension:





The static quark-antiquark potentials as functions of the quark-antiquark distance R:
(from above to below) full $V_{\rm full}(R)$, restricted part $V_{\rm rest}(R)$ and magnetic–monopole part $V_{\rm mono}(R)$.
(Left) $SU(2)$ at $\beta = 2.5$ on 24^4 lattice, [Kato, Kondo and Shibata, PRD**91**, 034506 (2015)]
(Right) $SU(3)$ at $\beta = 6.0$ on 24^4 lattice. [Kondo, Shibata, Shinohara & Kato, PRD**83**, 114016 (2011)]

Dynamical generation of the off-diagonal gluon mass:



The rescaled correlation functions $r^{3/2}\langle O(r)O(0)\rangle$ for $O = \mathbf{A}, \mathbf{V}, \mathbf{X}$ for 24^4 lattice with $\beta =$, 5.7, 5.85, 6.0. The physical scale is set in units of the string tension $\sigma_{\text{phys}}^{1/2}$.
[A. Shibata et al., PRD**87**, 054011 (2013)]

Contents

BEH mechanism for the gauge-scalar model

We consider $G = SU(2)$ gauge-scalar model with a single adjoint scalar field characterized by the gauge-invariant Lagrangian (no potential term):

$$\mathscr{L}_{\rm GS} = \mathscr{L}_{\rm YM} + \mathscr{L}_{\rm kin} = -\frac{1}{2}\mathrm{tr}\{\mathscr{F}^{\mu\nu}(x)\mathscr{F}_{\mu\nu}(x)\} + \mathrm{tr}\{(\mathscr{D}^{\mu}[\mathscr{A}]\boldsymbol{\phi}(x))(\mathscr{D}_{\mu}[\mathscr{A}]\boldsymbol{\phi}(x))\},$$

where the Lie algebra valued Yang-Mills field $\mathscr{A}_\mu(x) = \mathscr{A}_\mu^A(x)T_A$ $(A = 1, 2, 3)$ obey the gauge transformation:

$$\mathscr{A}_\mu(x) \to U(x)\mathscr{A}_\mu(x)U^{-1}(x) + ig^{-1}U(x)\partial_\mu U^{-1}(x), \quad U(x) \in G = SU(2)$$

and the Lie algebra valued scalar field $\boldsymbol{\phi}(x) = \phi^A(x)T_A$ $(A = 1, 2, 3)$ has the fixed radial length (modulus) $v > 0$:

$$\boldsymbol{\phi}(x) \cdot \boldsymbol{\phi}(x) \equiv 2\mathrm{tr}\{\boldsymbol{\phi}(x)\boldsymbol{\phi}(x)\} = \boldsymbol{\phi}^A(x)\boldsymbol{\phi}^A(x) = v^2.$$

and transforms according to the adjoint representation under the gauge transformation:

$$\boldsymbol{\phi}(x) \to U(x)\boldsymbol{\phi}(x)U^{-1}(x), \quad U(x) \in G = SU(2),$$

The covariant derivative $\mathscr{D}_\mu[\mathscr{A}] := \partial_\mu - ig[\mathscr{A}_\mu, \cdot]$ transforms according to the adjoint representation under the gauge transformation: $\mathscr{D}_\mu[\mathscr{A}] \to U(x)\mathscr{D}_\mu[\mathscr{A}]U^{-1}(x)$.

Conventional description for the BEH mechanism

- Suppose that the scalar field $\boldsymbol{\phi}(x)$ acquires a non-vanishing vacuum expectation value (VEV): $\langle \boldsymbol{\phi}(x) \rangle = \langle \boldsymbol{\phi} \rangle = \langle \boldsymbol{\phi}^A \rangle T_A$. Then the covariant derivative of the scalar field is

$$\mathscr{D}_\mu[\mathscr{A}]\boldsymbol{\phi}(x) := \partial_\mu \boldsymbol{\phi}(x) - ig[\mathscr{A}_\mu(x), \boldsymbol{\phi}(x)] \to -ig[\mathscr{A}_\mu(x), \langle \boldsymbol{\phi} \rangle] + \ldots.$$

Consequently, the kinetic term of the scalar field is modified into

$$\begin{aligned}\mathrm{tr}\{(\mathscr{D}^\mu[\mathscr{A}]\boldsymbol{\phi}(x))(\mathscr{D}_\mu[\mathscr{A}]\boldsymbol{\phi}(x))\} \to & -g^2 \mathrm{tr}_G\{[\mathscr{A}^\mu(x), \langle \boldsymbol{\phi} \rangle][\mathscr{A}_\mu(x), \langle \boldsymbol{\phi} \rangle]\} + \ldots \\ = & -g^2 \mathrm{tr}_G\{[T_A, \langle \boldsymbol{\phi} \rangle][T_B, \langle \boldsymbol{\phi} \rangle]\}\mathscr{A}^{\mu A}(x)\mathscr{A}_\mu^B(x) + \ldots\end{aligned}$$

If the non-vanishing VEV $\langle \boldsymbol{\phi} \rangle = \langle \boldsymbol{\phi}^A \rangle T_A$ of the scalar field $\boldsymbol{\phi}$ is chosen to a specific direction, e.g., $\langle \boldsymbol{\phi} \rangle_\infty = \langle \boldsymbol{\phi}^3 \rangle T_3$, [unitary gauge] uniformly over the spacetime, then the original local continuous gauge symmetry $G = SU(2)$ is spontaneously broken to a subgroup $H = U(1)$.

- Thus the kinetic term of the scalar field generates the mass term of the gauge field:

$$-g^2 \mathrm{tr}_G\{[T_A, vT_3][T_B, vT_3]\}\mathscr{A}^{\mu A}\mathscr{A}_\mu^B = \frac{1}{2}(gv)^2(\mathscr{A}^{\mu 1}\mathscr{A}_\mu^1 + \mathscr{A}^{\mu 2}\mathscr{A}_\mu^2), \quad v := \langle \boldsymbol{\phi}^3 \rangle.$$

- The off-diagonal gluons $\mathscr{A}_\mu^1, \mathscr{A}_\mu^2$ acquire the same mass $M_W := gv = g\langle \boldsymbol{\phi} \rangle_\infty$,
- The diagonal gluon $\mathscr{A}_\mu^3$ remains massless.
  This description of the BEH mechanism depends on the specific gauge and is not gauge independent. Indeed, VEV $\langle \boldsymbol{\phi} \rangle_\infty$ is not a gauge invariant quantity.

Gauge-independent description for the BEH mechanism

We explain a gauge-independent description for the BEH mechanism, which does not rely on the SSB. [K.-I. Kondo, Phys. Lett. B**762**, 219–224 (2016). arXiv:1606.06194 [hep-th]].

- We construct a composite vector field $\mathscr{W}_\mu(x)$ which consists of $\mathscr{A}_\mu(x)$ and $\boldsymbol{\phi}(x)$:

$$\mathscr{W}_\mu(x) := -ig^{-1}[\hat{\boldsymbol{\phi}}(x), \mathscr{D}_\mu[\mathscr{A}]\hat{\boldsymbol{\phi}}(x)], \quad \hat{\boldsymbol{\phi}}(x) := \boldsymbol{\phi}(x)/v.$$

  We find that the kinetic term of the scalar field $\boldsymbol{\phi}$ is identical to the "mass term" of the vector field $\mathscr{W}_\mu$:

$$\mathscr{L}_{\rm kin} = \frac{1}{2}\mathscr{D}^\mu[\mathscr{A}]\boldsymbol{\phi}(x) \cdot \mathscr{D}_\mu[\mathscr{A}]\boldsymbol{\phi}(x) = \frac{1}{2}M_W^2 \mathscr{W}^\mu(x) \cdot \mathscr{W}_\mu(x), \quad M_W := gv,$$

  as far as the constraint $(\hat{\boldsymbol{\phi}}(x) \cdot \hat{\boldsymbol{\phi}}(x) = 1)$ is satisfied.

- This "mass term" of $\mathscr{W}_\mu$ is gauge invariant, since $\mathscr{W}_\mu$ obeys the adjoint gauge transformation:

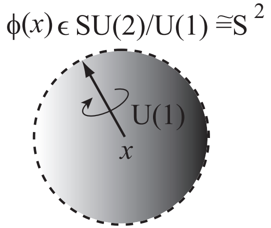
$$\mathscr{W}_\mu(x) \to U(x)\mathscr{W}_\mu(x)U^{-1}(x).$$

  The $\mathscr{W}_\mu$ gives a gauge-independent definition of the massive gluon mode in the operator level. The massive mode $\mathscr{W}_\mu$ can be described without breaking the original gauge symmetry. (We do not need to choose a specific vacuum from all possible degenerate ground states distinguished by the direction of $\boldsymbol{\phi}$.)

Gauge-independent description for the BEH mechanism(2)

- Despite its appearance of $\mathscr{W}_\mu$, the independent internal degrees of freedom in $\mathscr{W}_\mu = (\mathscr{W}_\mu^A)$ $(A = 1, 2, 3)$ is equal to $\dim(G/H) = 2$, since $\mathscr{W}_\mu(x) \cdot \hat{\boldsymbol{\phi}}(x) = 0$. Notice that this is a gauge-invariant statement.

- Thus, $\mathscr{W}_\mu(x)$ represent the massive modes corresponding to the coset space G/H components as expected.
  [We understand the **residual gauge symmetry** left in the partial SSB: $G = SU(2) \to H = U(1)$.]



Gauge-independent description v.s. conventional one

- In fact, by taking the unitary gauge $\hat{\boldsymbol{\phi}}^A(x) \to \hat{\boldsymbol{\phi}}^A_\infty$, $\mathscr{W}_\mu$ reduces to

$$\mathscr{W}_\mu(x) \to -ig^{-1}[\hat{\boldsymbol{\phi}}_\infty, \mathscr{D}_\mu[\mathscr{A}]\hat{\boldsymbol{\phi}}_\infty] = [\hat{\boldsymbol{\phi}}_\infty, [\hat{\boldsymbol{\phi}}_\infty, \mathscr{A}_\mu(x)]] = \mathscr{A}_\mu(x) - (\mathscr{A}_\mu(x)\cdot\hat{\boldsymbol{\phi}}_\infty)\hat{\boldsymbol{\phi}}_\infty.$$

Then $\mathscr{W}_\mu$ agrees with the off-diagonal components for the specific choice $\hat{\boldsymbol{\phi}}^A_\infty = \delta^{A3}$:

$$\mathscr{W}^A_\mu(x) \to \mathscr{A}^a_\mu(x) \text{ (for } A = a = 1,2), \quad 0 \text{ (for } A = 3)$$

This implies that the original gauge field $\mathscr{A}_\mu$ is separated into two pieces $\mathscr{V}_\mu$ and $\mathscr{W}_\mu$:

$$\mathscr{A}_\mu(x) = \mathscr{V}_\mu(x) + \mathscr{W}_\mu(x), \quad \mathscr{W}_\mu(x) := -ig^{-1}[\hat{\boldsymbol{\phi}}(x), \mathscr{D}_\mu[\mathscr{A}]\hat{\boldsymbol{\phi}}(x)].$$

- We find that $\mathscr{V}_\mu$ is constructed from $\mathscr{A}_\mu$ and $\boldsymbol{\phi}$ as

$$\mathscr{V}_\mu(x) = c_\mu(x)\hat{\boldsymbol{\phi}}(x) + ig^{-1}[\hat{\boldsymbol{\phi}}(x), \partial_\mu\hat{\boldsymbol{\phi}}(x)], \quad c_\mu(x) := \mathscr{A}_\mu(x)\cdot\hat{\boldsymbol{\phi}}(x),$$

and by definition transforms under the gauge transformation just like $\mathscr{A}_\mu$:

$$\mathscr{V}_\mu(x) \to U(x)\mathscr{V}_\mu(x)U^{-1}(x) + ig^{-1}U(x)\partial_\mu U^{-1}(x).$$

In the unitary gauge $\hat{\boldsymbol{\phi}}^A(x) \to \hat{\boldsymbol{\phi}}^A_\infty = \delta^{A3}$, $\mathscr{V}_\mu$ agrees with the diagonal component

$$\mathscr{V}_\mu(x) \to (\mathscr{A}_\mu(x)\cdot\hat{\boldsymbol{\phi}}_\infty)\hat{\boldsymbol{\phi}}_\infty \to 0 \ (A = a = 1,2), \quad \mathscr{A}^3_\mu(x) \ (A = 3)$$

So far, so good for a gauge-scalar model.

Complementary gauge-scalar model for the Yang-Mills theory

- In the gauge-scalar model, $\mathscr{A}_\mu(x)$ and $\boldsymbol{\phi}(x)$ are independent field variables.
- However, the Yang-Mills theory should be described by the Yang-Mills field $\mathscr{A}_\mu(x)$ alone and hence $\boldsymbol{\phi}$ must be supplied as a composite field made from the gauge field $\mathscr{A}_\mu(x)$ due to the strong interactions.
  [the scalar field $\boldsymbol{\phi}$ is to be given as a (complicated) functional of the gauge field $\mathscr{A}_\mu(x)$.]
- This is achieved by imposing the constraint which we call the **reduction condition**:

$$\chi(x) := [\hat{\boldsymbol{\phi}}(x), \mathscr{D}_\mu[\mathscr{A}]\mathscr{D}_\mu[\mathscr{A}]\hat{\boldsymbol{\phi}}(x)] = \mathbf{0} \Longleftrightarrow \mathscr{D}_\mu[\mathscr{V}]\mathscr{W}_\mu(x) = 0.$$

  This condition is gauge covariant, $\chi(x) \to U(x)\chi(x)U^{-1}(x)$.
- The reduction condition plays the role of eliminating the extra degrees of freedom introduced by the radially fixed adjoint scalar field into the Yang-Mills theory, since χ represents two conditions due to

$$\chi(x) \cdot \hat{\boldsymbol{\phi}}(x) = 0.$$

- The "complementary" gauge-scalar model is defined by taking into account the Faddeev-Popov determinant $\widetilde{\Delta}^{\text{red}}$ associated with the reduction condition $\chi = 0$ as

$$\tilde{Z}_{\text{RF}} = \int \mathcal{D}\mathscr{A}\mathcal{D}\hat{\boldsymbol{\phi}}\ \delta(\chi)\,\Delta^{\text{red}} e^{-S_{\text{YM}}[\mathscr{A}]-S_{\text{kin}}[\mathscr{A},v\hat{\boldsymbol{\phi}}]}.$$

Complementary gauge-scalar model for the Yang-Mills theory(2)

- We perform change of variables from the original variables to the new variables:

$$\{\mathscr{A}_\mu^A(x), \hat{\boldsymbol{\phi}}^a(x)\} \to \{c_\mu(x), \mathscr{W}_\nu^B(x), \hat{\boldsymbol{\phi}}^b(x)\}.$$

  Then we have

$$\tilde{Z}_{\mathrm{RF}} = \int \mathcal{D}c\mathcal{D}\mathscr{W}\mathcal{D}\hat{\boldsymbol{\phi}}\ J\delta\left(\widetilde{\boldsymbol{\chi}}\right)\widetilde{\Delta}^{\mathrm{red}} e^{-S_{\mathrm{YM}}[\mathscr{V}+\mathscr{W}]-iS_{\mathrm{m}}[\mathscr{W}]},$$

$$S_{\mathrm{m}}[\mathscr{W}] := \int d^D x \frac{1}{2} M_{\mathscr{W}}^2 \mathscr{W}_\mu \cdot \mathscr{W}_\mu,$$

- We can reproduce the well-known preceding cases by taking the special limit or choosing the gauge. For instance, by taking the unitary gauge,

$$\boldsymbol{\phi}^A(x) = v\hat{\boldsymbol{\phi}}^A(x),\ \hat{\boldsymbol{\phi}}^A(x) \to \delta^{A3},$$

  the new variables reduce to

$$c_\mu = \mathscr{A}_\mu \cdot \hat{\boldsymbol{\phi}} \to A_\mu^3, \quad \mathscr{V}_\mu^A \to A_\mu^3\delta^{A3}, \quad \mathscr{W}_\mu^A \to A_\mu^a\delta^{Aa},$$

  which means

$$\tilde{Z}_{\mathrm{RF}} \to \int \mathcal{D}A^3\mathcal{D}A^a\delta\left(\mathscr{D}^\mu[A^3]A_\mu^a\right)\Delta_{\mathrm{FP}} e^{-S_{\mathrm{YM}}[A^a+A^3]-S_{\mathrm{m}}[A^a]},$$

$$S_{\mathrm{m}}[A^a] := \int d^D x \frac{1}{2} M_{\mathscr{W}}^2 A_\mu^a A_\mu^a.,$$

Complementary gauge-scalar model for the Yang-Mills theory(3)

- $$\tilde{Z}_{\mathrm{RF}} = \int \mathcal{D}c\mathcal{D}\mathscr{W}\mathcal{D}\hat{\boldsymbol{\phi}}\ J\delta\left(\widetilde{\chi}\right)\widetilde{\Delta}^{\mathrm{red}} e^{-S_{\mathrm{YM}}[\mathscr{V}+\mathscr{W}]-iS_{\mathrm{m}}[\mathscr{W}]},$$
$$S_{\mathrm{m}}[\mathscr{W}] := \int d^Dx \frac{1}{2} M_{\mathscr{W}}^2 \mathscr{W}_\mu \cdot \mathscr{W}_\mu,$$

- In the limit, the gauge-scalar model with the radially fixed adjoint scalar field is reduced to the Yang-Mills theory with the gauge-fixing term of the Maximal Abelian gauge $\mathscr{D}^\mu[A^3]A_\mu^a = 0$ and the associated Faddeev-Popov determinant Δ_{FP}, plus a mass term $S_{\mathrm{m}}[A^a]$ for the off-diagonal gluons.
$$\tilde{Z}_{\mathrm{RF}} \to \int \mathcal{D}A^3\mathcal{D}A^a\delta\left(\mathscr{D}^\mu[A^3]A_\mu^a\right)\Delta_{\mathrm{FP}} e^{-S_{\mathrm{YM}}[A^a+A^3]-S_{\mathrm{m}}[A^a]},$$
$$S_{\mathrm{m}}[A^a] := \int d^Dx \frac{1}{2} M_{\mathscr{W}}^2 A_\mu^a A_\mu^a.,$$

- In other words, the pure Yang-Mills theory in the MA gauge with the off-diagonal gluon mass term has the gauge-invariant extension which is identical to the gauge-scalar model with the radially-fixed adjoint scalar field subject to the reduction condition, which we call the "complementary" gauge-scalar model.

Confined massive phase

The field strength $\mathscr{F}_{\mu\nu}[\mathscr{V}](x)$ of $\mathscr{V}_\mu(x)$ is shown to be proportional to $\hat{\boldsymbol{\phi}}(x)$:

$$\mathscr{F}_{\mu\nu}[\mathscr{V}](x) = \hat{\boldsymbol{\phi}}(x)\{\partial_\mu c_\nu(x) - \partial_\nu c_\mu(x) + H_{\mu\nu}(x)\},$$
$$H_{\mu\nu}(x) := ig^{-1}\hat{\boldsymbol{\phi}}(x)\cdot[\partial_\mu\hat{\boldsymbol{\phi}}(x), \partial_\nu\hat{\boldsymbol{\phi}}(x)].$$

We can introduce the Abelian-like $SU(2)$ gauge-invariant field strength $f_{\mu\nu}$ by

$$f_{\mu\nu}(x) := \hat{\boldsymbol{\phi}}(x)\cdot\mathscr{F}_{\mu\nu}[\mathscr{V}](x) = \partial_\mu c_\nu(x) - \partial_\nu c_\mu(x) + H_{\mu\nu}(x).$$

In the low-energy $E \ll M_W$ or the long-distance $r \gg M_W^{-1}$ region, we can neglect $\mathscr{W}_\mu$. Then the dominant low-energy modes are described by the restricted Yang-Mills theory:

$$\mathscr{L}_{\rm YM}^{\rm rest} = -\frac{1}{4}\mathscr{F}^{\mu\nu}[\mathscr{V}]\cdot\mathscr{F}_{\mu\nu}[\mathscr{V}] = -\frac{1}{4}f^{\mu\nu}f_{\mu\nu}.$$

In the low-energy $E \ll M_W$ or the long-distance $r \gg M_W^{-1}$ region, the massive components $\mathscr{W}_\mu(x)$ become negligible and the other restricted (residual) fields become dominant. This is a phenomenon called the "Abelian" dominance in quark confinement. ['tHooft 81, Ezawa-Iwazaki 82]

The "Abelian" dominance in quark confinement is understood as a consequence of the BEH mechanism for the "complementary" gauge-scalar model in the gauge-invariant way.

From field equations to the reduction condition

If the fields $\mathscr{A}$ and $\boldsymbol{\phi}$ are a set of "solutions" of the field equations for the gauge-scalar model with a radially fixed scalar field, they are automatically field configurations satisfying the reduction condition for the pure Yang-Mills theory.

We introduce a Lagrange multiplier field $u(x)$ to incorporate the constraint

$$\mathscr{L}'_{\mathrm{RF}}(x) = \mathscr{L}_{\mathrm{GS}}(x) + u(x)\left(\boldsymbol{\phi}(x)\cdot\boldsymbol{\phi}(x) - v^2\right).$$

Then the field equations are obtained as

$$\frac{\delta S'_{\mathrm{RF}}}{\delta u(x)} = \boldsymbol{\phi}(x)\cdot\boldsymbol{\phi}(x) - v^2 = 0,$$

$$\frac{\delta S'_{\mathrm{RF}}}{\delta \mathscr{A}^{\mu}(x)} = \mathscr{D}^{\nu}[\mathscr{A}]\mathscr{F}_{\nu\mu}(x) - ig[\boldsymbol{\phi}(x), \mathscr{D}_{\mu}[\mathscr{A}]\boldsymbol{\phi}(x)] = 0, \tag{1}$$

$$\frac{\delta S'_{\mathrm{RF}}}{\delta \boldsymbol{\phi}(x)} = -\mathscr{D}^{\mu}[\mathscr{A}]\mathscr{D}_{\mu}[\mathscr{A}]\boldsymbol{\phi}(x) + 2u(x)\boldsymbol{\phi}(x) = 0. \tag{2}$$

The reduction condition is automatically satisfied:

- $\mathscr{D}^{\mu}(1) \Longrightarrow 0 = \mathscr{D}^{\mu}[\mathscr{A}]\mathscr{D}^{\nu}[\mathscr{A}]\mathscr{F}_{\nu\mu} = ig\mathscr{D}^{\mu}[\mathscr{A}][\boldsymbol{\phi}, \mathscr{D}_{\mu}[\mathscr{A}]\boldsymbol{\phi}] = ig[\boldsymbol{\phi}, \mathscr{D}^{\mu}[\mathscr{A}]\mathscr{D}_{\mu}[\mathscr{A}]\boldsymbol{\phi}]$
- $[\boldsymbol{\phi}, (2)] \Longrightarrow [\boldsymbol{\phi}, \mathscr{D}^{\mu}[\mathscr{A}]\mathscr{D}_{\mu}[\mathscr{A}]\boldsymbol{\phi}] = [\boldsymbol{\phi}, 2u\boldsymbol{\phi}] = 0$

Massive Yang-Mills theory on the lattice

Then, we discuss the numerical simulations for the proposed massive Yang-Mills theory on the lattice. By taking into account the reduction condition in the complementary gauge-scalar model, the gauge-invariant mass term is introduced:

$$Z_L = \int \mathcal{D}[U]\mathcal{D}[\mathbf{n}]\delta(\mathbf{n}-\hat{\mathbf{n}})e^{-\beta S_g - \gamma S_m}$$

$$S_g[U] := \sum_x \sum_{\mu>\nu} 2\,\mathrm{Re\,tr}\left(\mathbf{1} - U_{x,\mu}U_{x+\mu,\nu}U^\dagger_{x+\nu,\mu}U^\dagger_{x,\nu}\right)$$

$$S_m[U,\mathbf{n}] := \sum_{x,\mu} \mathrm{tr}\left((D^\epsilon_\mu[U]\mathbf{n}_x)^\dagger (D^\epsilon_\mu[U]\mathbf{n}_x)\right), \quad D^\epsilon_\mu[U]\mathbf{n}_x := U_{x,\mu}\mathbf{n}_{x+\mu} - \mathbf{n}_x U_{x,\mu}$$

where $U_{x,\mu} \in SU(2)$ is the link variable, $\mathbf{n} = \mathbf{n}_A T^A \in su(2)$ is the color field (scalar field $\boldsymbol{\phi}$) with $\mathbf{n}\cdot\mathbf{n} = 1$, and $D^\epsilon_\mu[U]\mathbf{n}_x$ is the covariant derivative.
$\delta(\mathbf{n}-\hat{\mathbf{n}})$ represents the reduction condition in the complementary gauge-scalar model, and $\hat{\mathbf{n}}$ is the solution of the reduction condition for given gauge configuration, which is obtaine by minimizing the functional:

$$F_{\mathrm{red}}[\mathbf{n};U] := \sum_{x,\mu} \mathrm{tr}\left((D^\epsilon_\mu[U]\mathbf{n}_x)^\dagger (D^\epsilon_\mu[U]\mathbf{n}_x)\right)$$

Nnmerical Simulation

Now, we perform the umerical simuration to genarate the gauge configuration:

$$\rho[U, \mathbf{n}] := \frac{\delta(\mathbf{n} - \hat{\mathbf{n}}) e^{-\beta S_g - \gamma S_m}}{Z_L}, \quad Z_L = \int \mathcal{D}[U] \mathcal{D}[\mathbf{n}] \delta(\mathbf{n} - \hat{\mathbf{n}}) e^{-\beta S_g - \gamma S_m}$$

- Without the reduction condition (or $\delta(\mathbf{n} - \hat{\mathbf{n}})$), this model is the usual gauge-scalar model with a radially fixed scalar field.
- If $\gamma = 0$, the model is reduced into the usual Yang-Mills theory with the standard Wilson action.
- In the massive Yang-Mills theory, $U_{x,\mu}$ and $\mathbf{n}$ are no more independent field variables.
- The theory should be described by the Yang-Mills gauge field $U_{x,\mu}$ alone, and hence the color field $\mathbf{n}$ must be supplied as a composite field made from the gauge field. This is achieved by imposing the reduction condition.
- Thus, the gauge configurations must be updated by solving the reduction condition simultaneously.

- As the first step, we investigate the reagion, $\gamma \sim 0$ by using the rewaiting technique.

lattice set up

- We perform the numerical simuration for 32^4 lattice for the standard Willson action ($\beta = 2.5$, $\gamma = 0$) with over-relaxation algorithm.
- After 80000 sweeps thurmalization we generate 4000 configurations every 400 sweeps.
- To obtain the color field (scalar field) configuration, we solve the reduction condition for each gauge configuration:

$$F_{\text{red}}[\mathbf{n};U] := \sum_{x,\mu} \mathrm{tr}\left((D^{\epsilon}_{\mu}[U]\mathbf{n}_x)^{\dagger} (D^{\epsilon}_{\mu}[U]\mathbf{n}_x) \right)$$

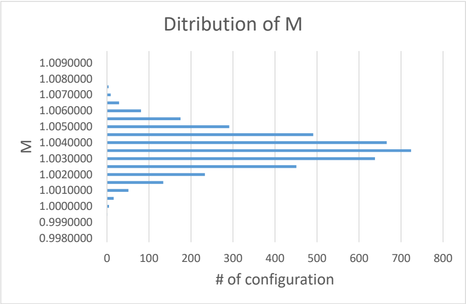
  The color field $\hat{\mathbf{n}}$ is obtained as function of the gauge configuration. $\hat{\mathbf{n}} = \hat{\mathbf{n}}[\mathbf{U}]$
- The observable $\mathcal{O}$ is measured by reweiting method.

$$\langle \mathcal{O} \rangle := \frac{\sum \mathcal{O}[U, \hat{\mathbf{n}}] e^{-\gamma S_m[U,\hat{\mathbf{n}}]}}{\sum e^{-\gamma S_m[U,\hat{\mathbf{n}}]}}$$

- We study the Wilson loops $W[R, T]$ of size $R \times T$ and the mass term M:

$$M := \frac{1}{N_{\text{site}}} \sum_{x,\mu} \mathrm{tr}\left((D^{\epsilon}_{\mu}[U]\mathbf{n}_x)^{\dagger} (D^{\epsilon}_{\mu}[U]\mathbf{n}_x) \right)$$

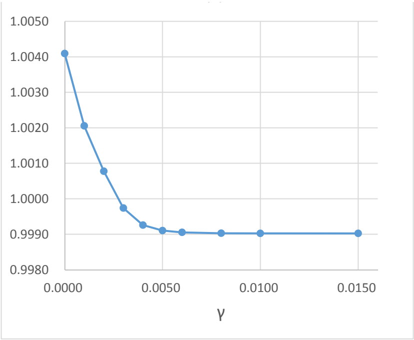
Numerical result (1)

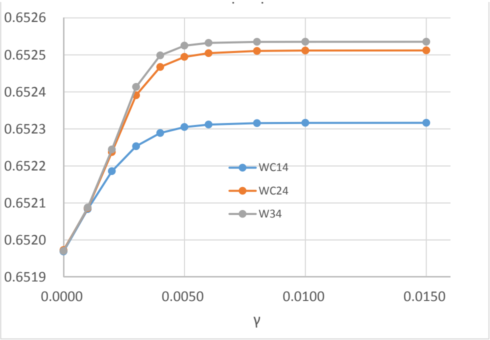


Preliminary

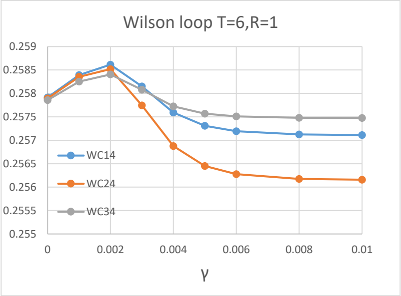
No error bars are ploted because they are very large for finite γ .

(Left) The histgram of the mass term M
(Lower left) The mesurement of the $\langle \mathcal{O} \rangle$
(Lower right) The mesurement of the 1×1 plaquet: for $X - T$, $Y - T$, $Z - T$.



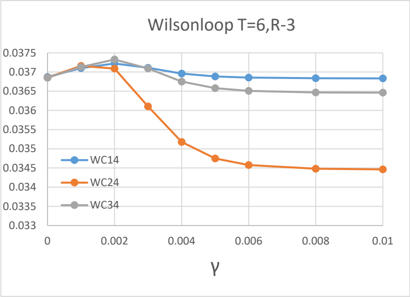


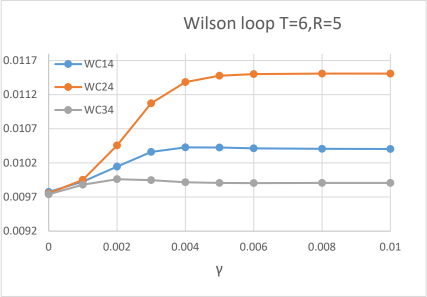
Numerical result (2)



Preliminary

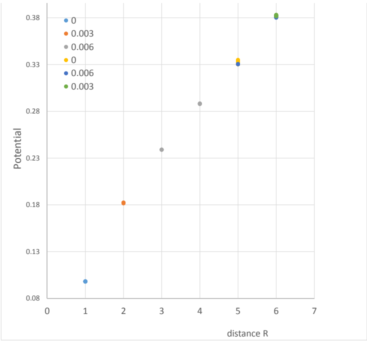
The measurment for the Wilson loop for $X-T$, $Y-T$, $Z-T$ plane.
(Left) $R=1,\ T=6$
(Lower Left) $R=3,\ T=6$
(Lower Right) $R=5,\ T=6$

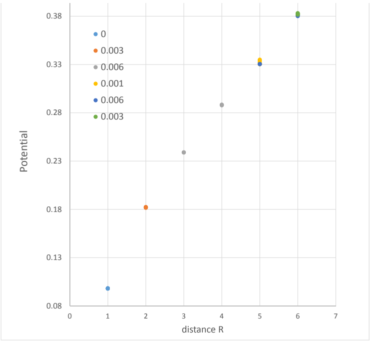




Preliminary

We finally show resulta of the static potential.
(left pannel) for the X–T plane (right panel) for the Z-T plane





Summary

- In order to clarify the mechanism of quark confinement in the Yang-Mills theory with mass gap, we propose to investigate the massive Yang-Mills model, namely, Yang-Mills theory with a gauge-invariant gluon mass term to be deduced from a specific gauge-scalar model with a single radially-fixed scalar field under a suitable constraint called the reduction condition.
- The gluon mass term simulates the dynamically generated mass to be extracted in the low-energy effective theory of the Yang-Mills theory and plays the role of a new probe to study the phase structure and confinement mechanism.
- We first explain why such a gauge-scalar model is constructed without breaking the gauge symmetry through the gauge-independent description of the Brout-Englert-Higgs mechanism which does not rely on the spontaneous breaking of gauge symmetry.
- Then we discuss how the numerical simulations for the proposed massive Yang-Mills theory can be performed by taking into account the reduction condition in the complementary gauge-scalar model on a lattice.
- By using the reweiying method, we investigate the effect of the gluon mass term to Wilson loops (the static potential) and the dynamically generated mass.
  These are very preliminary results.
- This gives an alternative understanding for the physical meaning of the gauge-covariant decomposition for the Yang-Mills field known as the Cho-Duan-Ge-Faddeev-Niemi decomposition.

Outlook

- In this study, we give the first lattice calcuration of Yang-Mills theory with a gauge-invariant gluon mass term for small mass parater γ by using the reweithing technique.
- It has been found the full simulation with gluon mass term to investigate the whole parameter space of the gauge coupling β and the massterm γ.
- We should take care of the fact that massive Yang-Mills models of distinct type are obtained depending on representations of the scalar field.
- For the fundamental representation, the massive Yang-Mills model is expected to have a single confining phase with continuously connecting confining and Higgs regions as suggested by the Fradkin-Shenker continuity.
- For the adjoint representation, the two regions will be separated by the phase transition and become two different phases showing confinement and deconfinement even at zero temperature.