Does confinement imply CP invariance of the strong interactions?

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Undoubtedly, the two biggest unsolved problems of the Strong Interactions (and beyond) are

- Color confinement — CMI Millennium Problem
- CP invariance — Strong CP problem

$$S = S_{\mathrm{QCD}} + i\,\theta\,Q$$

$$|\theta| \lesssim 7.4 \times 10^{-11}$$

arXiv:1502.02295

Synopsis

Using the Gradient Flow, the strong coupling constant $\alpha_V(\mu)$ and the vacuum angle $\theta(\mu)$ of the Yang-Mills Theory are investigated in the infrared limit. While $\alpha_V(\mu)$ appears to diverge like $1/\mu^2$ at $\theta = 0$, which naturally leads to linear confinement, confinement is lost for $\theta \neq 0$

SU(3) Yang-Mills Theory

Effective Lagrangian

$$\mathcal{L}_{\rm YM}(\mu) = \frac{1}{4\,g^2(\mu)}\,F^a_{\mu\nu}F^a_{\mu\nu} + i\,\theta(\mu)\,\frac{1}{32\pi^2}\,F^a_{\mu\nu}\tilde{F}^a_{\mu\nu}$$

normalized at scale μ

$$\downarrow$$

$$\int d^4x\,\frac{1}{32\pi^2}\,F^a_{\mu\nu}\tilde{F}^a_{\mu\nu} = Q \in \mathbb{Z}$$

The static properties of the theory will be revealed by the coupling $g^2(\mu)$ and CP violating angle $\theta(\mu)$ in the infrared limit $\mu \to 0$ $\longrightarrow$ tree level

To obtain $\mathcal{L}_{\rm YM}(\mu' < \mu)$ we need to 'integrate' $\int_\mu^{\mu'} d\tau\,\mathcal{L}_{\rm YM}(\tau)$, which can be achieved by employing the Gradient Flow

Physics does not depend on the choice of scale μ $\leftarrow$ Renormalizability

For example: String Tension

$$a^2\sigma \simeq \chi(I,J)$$

$a = 1/\mu$ lattice spacing

This implies

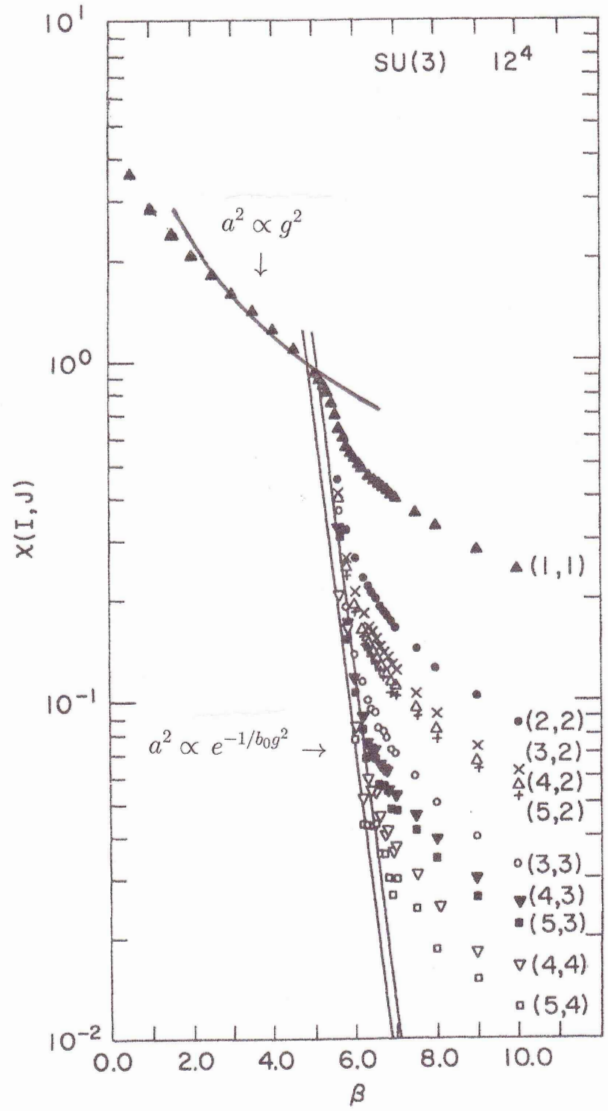
$g^2 \lesssim 1$: $a^2\Lambda^2 \simeq e^{-\frac{1}{b_0 g^2}}$ $\quad g^2 \simeq 1/b_0 \log(\mu^2/\Lambda^2)$

$g^2 \gtrsim 1$: $a^2\Lambda^2 \simeq b_0 g^2$ $\quad g^2 \propto \Lambda^2/b_0\mu^2$

$$\downarrow$$

$$V(r) = \frac{\Lambda^2}{6\pi b_0} r \equiv \sigma\, r$$

Richardson



Barkai, Creutz and Moriarty

Lattice Matters

Action

$$S_{\mathrm{YM}} = \beta \sum_{x,\mu<\nu} \frac{1}{3} \mathrm{Tr}\left(1 - U_{\mu\nu}(x)\right) \ , \quad \beta = \frac{6}{g^2}$$

Wilson action

Lattices

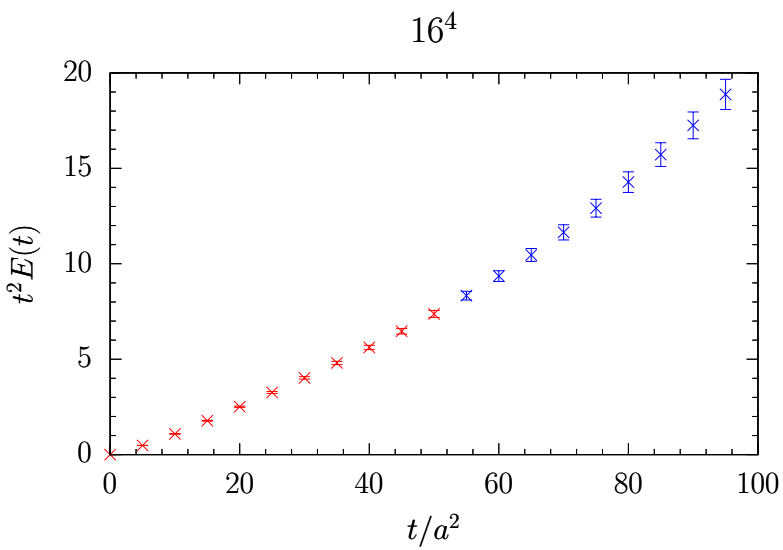
β	V	N_{conf}
6.0	16^4	4000
6.0	24^4	5000
6.0	32^4	WIP

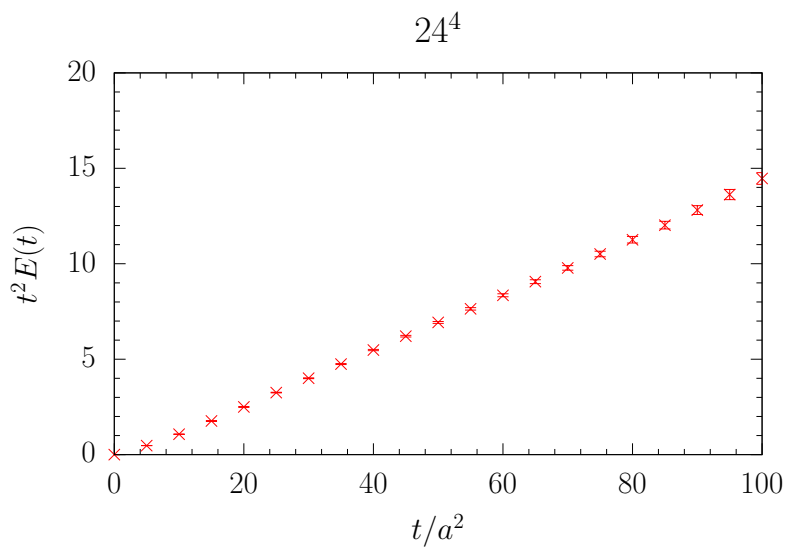
$$\sqrt{t_0}/a = 1.781(3)$$

$$\boxed{a = 0.082(2)\ \mathrm{fm}}$$

Gradient Flow

Observable E: clover





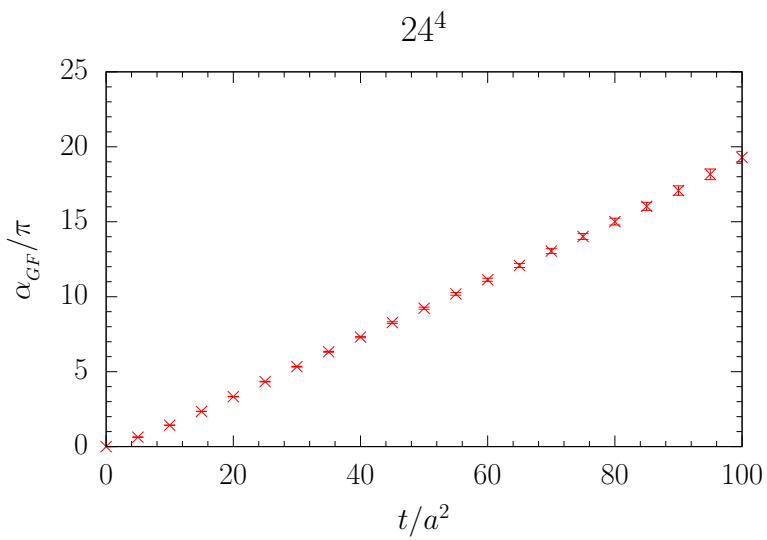
$\longrightarrow$

finite size effects

$\uparrow$

$\mu \approx 100\,\text{MeV}$

$$t^2 E(t) = \frac{3}{4\pi}\,\alpha_{GF}(\mu)\,, \quad \mu = \frac{1}{\sqrt{8\,t}}$$



$$\alpha_{GF} \propto t$$

$$\frac{\partial\,\alpha_{GF}}{\partial \ln\,\mu} = -2\,\frac{\partial\,\alpha_{GF}}{\partial \ln\,t} = \beta_{GF}(\alpha_{GF})$$

$$= -2\,\alpha_{GF}(\mu)$$

Change of Scheme $\quad GF \to V$

$$\frac{\Lambda_V}{\mu} = \exp\left\{-\int_1^{\alpha_V} d\alpha\,\frac{1}{\beta_V(\alpha)}\right\}$$

$$= \frac{\Lambda_V}{\Lambda_{GF}}\exp\left\{-\int_1^{\alpha_{GF}} d\alpha\,\frac{1}{\beta_{GF}(\alpha)}\right\}$$

$$= \frac{\Lambda_V}{\Lambda_{GF}}\sqrt{\alpha_{GF}}$$

Dalla Brida & Ramos

Solution

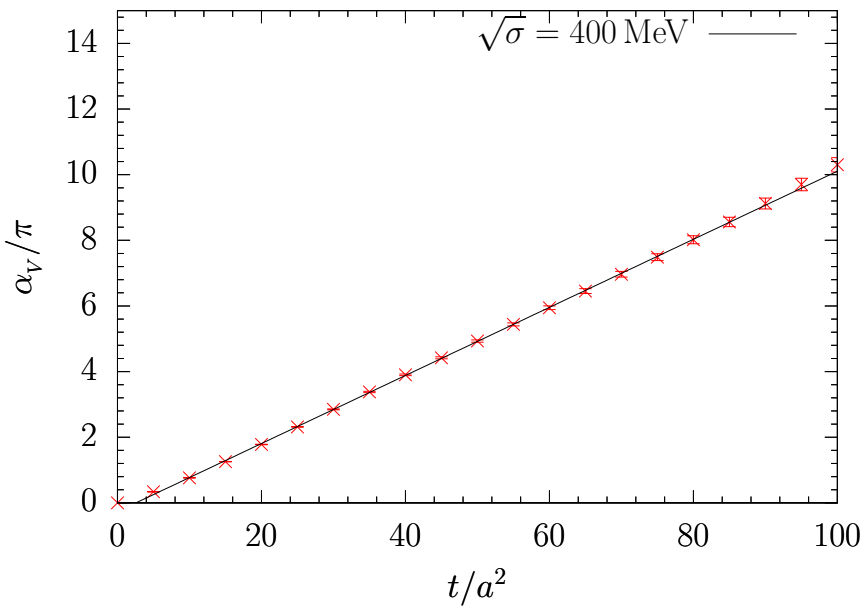
$$\beta_V = -2\,\alpha_V$$

$$\boxed{\alpha_V = \frac{\Lambda_V^2}{\Lambda_{GF}^2}\,\alpha_{GF}}$$

$\alpha_V(\mu) \quad [\theta = 0]$

$$\alpha_V = \frac{\Lambda_V^2}{\Lambda_{GF}^2}\,\alpha_{GF} = \frac{\Lambda_V^2}{\Lambda_{\overline{MS}}^2}\,\frac{\Lambda_{\overline{MS}}}{\Lambda_{GF}^2}\,\alpha_{GF} = 0.534\,\alpha_{GF}$$

Necco, Lüscher

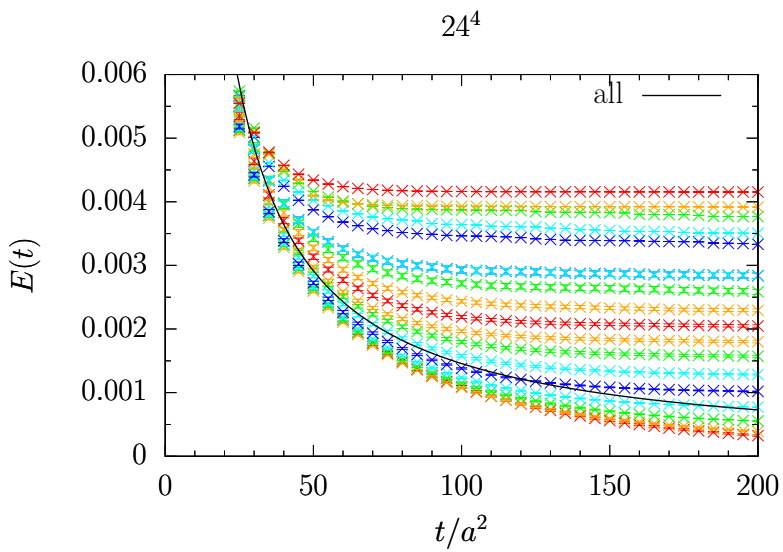


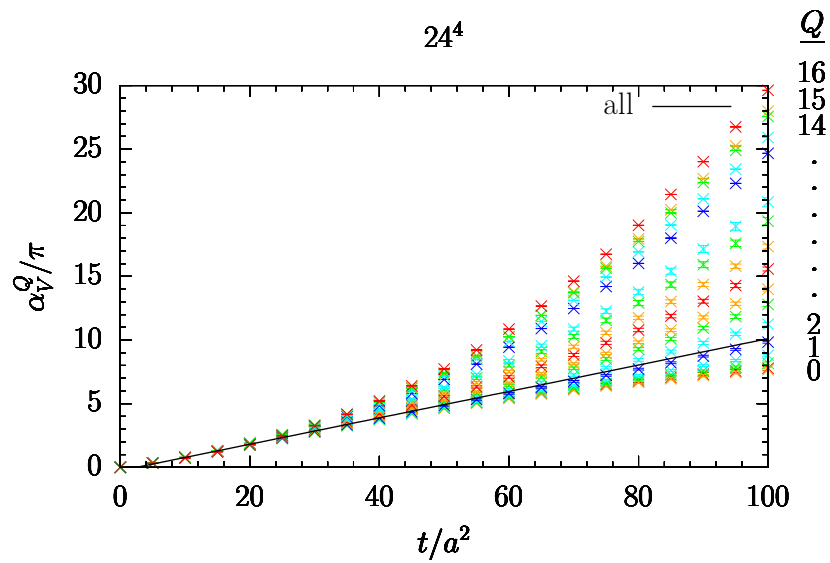
$$V(r) = \frac{1}{(2\pi)^3}\int d^3\mathbf{q}\,e^{i\mathbf{q}\mathbf{r}}\,\frac{4}{3}\,\frac{\alpha_V(q)}{\mathbf{q}^2 + i0} \equiv \sigma\,r \quad \Leftrightarrow \quad \alpha_V(\mu) = \frac{3\,\sigma}{2\,\mu^2}$$

Fit: $\sqrt{\sigma} = 400\,\mathrm{MeV}$

Interplay of α_V and θ

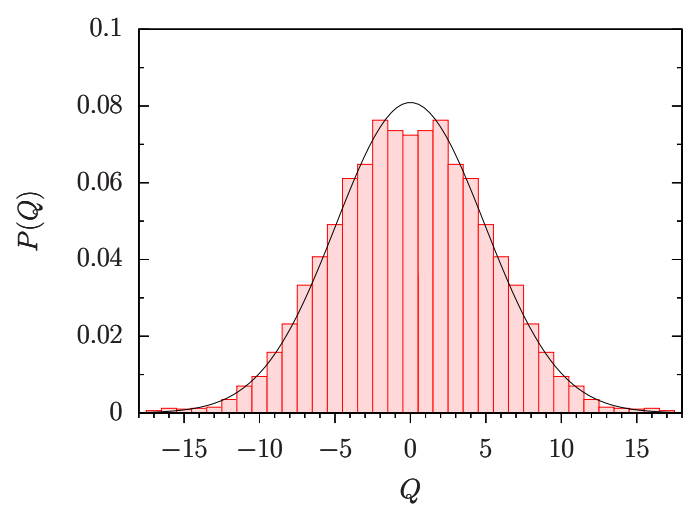
Break-down according to charge Q





Fourier transform to θ vacuum

$$\alpha_V^\theta(\mu) = \frac{1}{Z_\theta} \int dQ\, e^{i\theta Q} P(Q)\, \alpha_V^Q(\mu)$$

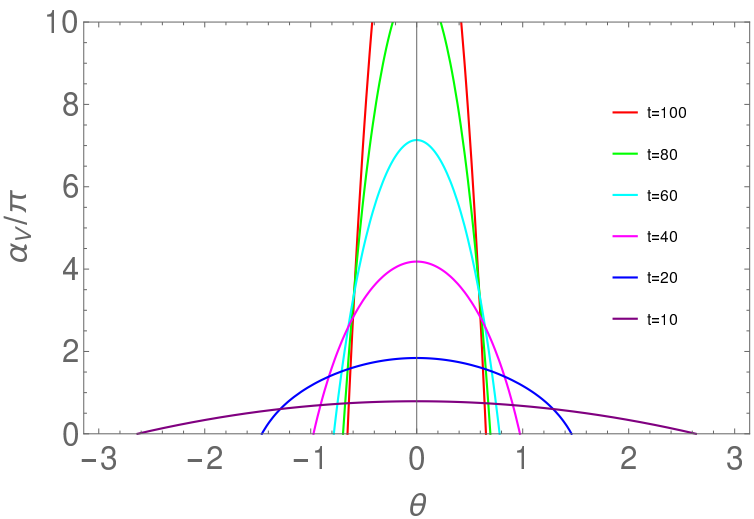


$\alpha_V^\theta(\mu)$

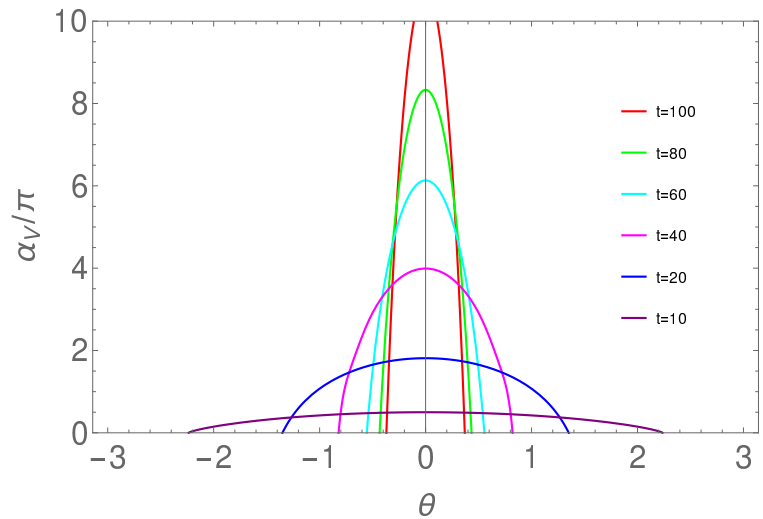
$L \to \infty$

$\to \quad \leftarrow$

16^4



24^3



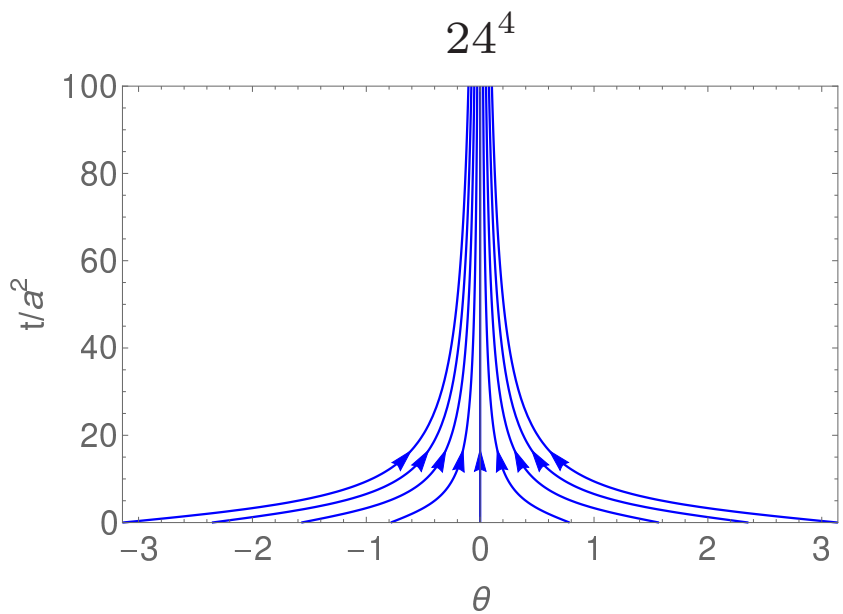
$$t = \frac{1}{8\mu^2}$$

$\theta(\mu)$

$\alpha_V =$ constant

$$\frac{\partial\,\theta}{\partial \ln\, t} \simeq \sin(\theta)$$

Solution : $\theta = 2\,\mathrm{arccot}(t + c)$



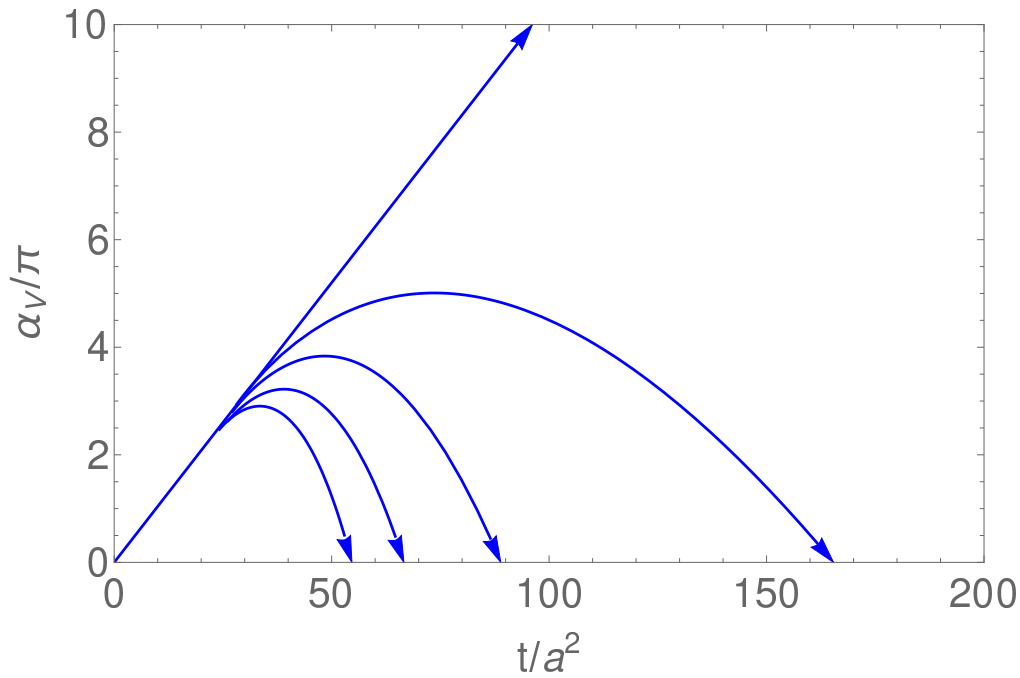
Confinement?

$\theta \;=\;$ constant

Confining: $V(r) = \sigma\, r$

$\theta = 0$

$\downarrow$



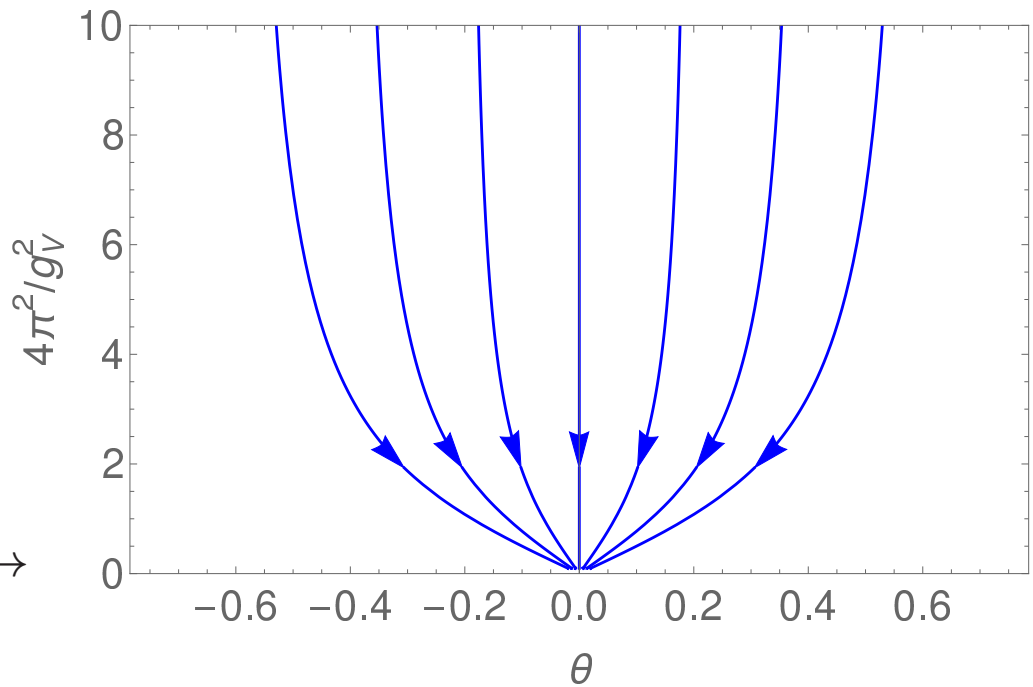
$\uparrow$ $\uparrow$ $\uparrow$ $\uparrow$

$\theta = 0.6$ 0.5 0.4 0.3

Coulomb phase: $V(r) \propto \dfrac{1}{r}$

Flow: $(1/g_V^2, \theta)$ – plane

$t/a^2 = 100 \rightarrow$

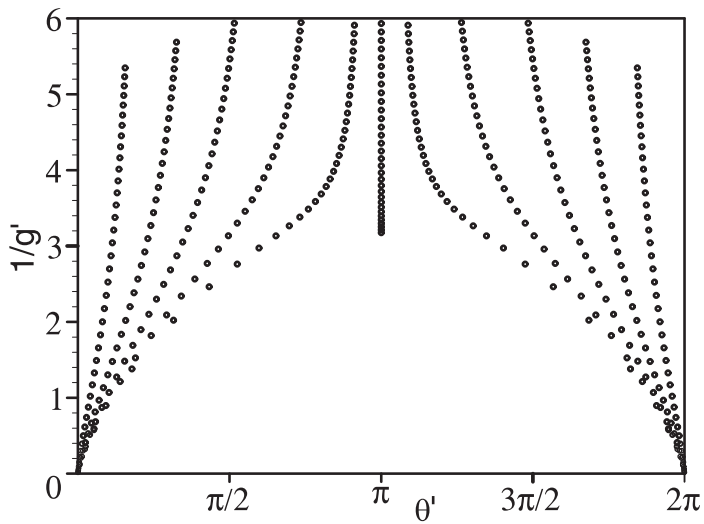


Similar to

Quantum Hall Effect

Dilute Instanton Gas

Quantum Hall Effect



θ: magnetic flux

Apenko

Knizhnik & Morozov

Levine & Libby

Pruisken

Conclusions

Answer: YES!

Scaling behavior of the strong coupling constant $\alpha_V(\mu) = g_V^2(\mu)/4\pi$ and the CP violating vacuum angle $\theta(\mu)$ at long distances (i.e. the hadronization scale) suggests

- Confinement for $\theta = 0$ $\qquad \alpha_V \propto t \qquad \Rightarrow \; V(r) = \sigma\, r$

  $\uparrow$

  $\sqrt{\sigma} = 400$ MeV

- Coulomb phase for $\theta \neq 0$ $\qquad \alpha_V \propto t\,(t_0 - t)/t_0 \qquad \Rightarrow \; V(r) \propto 1/r$

  cf. dual superconductor: $e_{\rm mon} = e\,\theta/2\pi$

  Witten

- $1/g^2(\mu)\,,\; \theta(\mu) \;\longrightarrow\; 0$ $\qquad$ IR fixed point

$\boxed{\text{Axion} \rightarrow \text{Coulomb phase}}$