Symmetry, Confinement, and the Higgs Phase

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What is "confinement"?

Suppose we have an SU(N) gauge theory with matter fields in the fundamental representation, e.g. QCD. Wilson loops have perimeter-law falloff asymptotically, Polyakov lines have a non-zero VEV, what does it mean to say such theories (QCD in particular) are confining?

Most people take it to mean “color confinement” or

C-confinement

There are only color neutral particles in the asymptotic spectrum.

The problem with C-confinement is that it also holds true for gauge-Higgs theories, deep in the Higgs regime, where there are

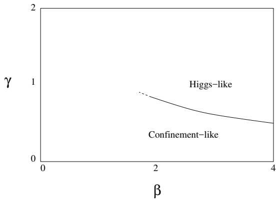
- only Yukawa forces,
- no linearly rising Regge trajectories,
- no color electric flux tubes.

If C-confinement is “confinement,” then the Higgs phase is also confining.

C-confinement in gauge-Higgs theories

How we know this:

1. **Elitzur's** Theorem: No such thing as spontaneous symmetry breaking of a local gauge symmetry.
2. The **Fradkin-Shenker-Osterwalder-Seiler** (FSOS) Theorem: There is no transition in coupling-constant space which isolates the Higgs phase from a confinement-like phase.
3. **Frölich-Morchio-Strocchi** (FMS) and also **'t Hooft** (1980): physical particles (e.g. W's) in the spectrum are created by gauge-invariant operators in the Higgs region.



FMS show how to recover the usual results of perturbation theory, starting from gauge-invariant composite operators.

Conclusion: If the confinement-like (QCD-like) region has a color neutral spectrum, then so does the Higgs-like region.

Higgs and Confinement: what's the difference?

The Higgs and confinement regions are both massive. Yet QCD and the weak interactions seem physically so different.

Questions:

- Can that difference be formulated precisely? Is there some variety of confinement other than C confinement?
- Are the confinement-like and Higgs-like regions of a gauge-Higgs theory differentiated by the breaking of a symmetry?
- If so, then what symmetry? And how is symmetry breaking related to a transition in the type of confinement?

Beyond C-confinement?

In a pure SU(N) gauge theory there is a different and stronger meaning that can be assigned to the word “confinement," which goes beyond C-confinement.

Of course the spectrum consists only of color neutral objects: glueballs.

But such theories *also* have the property that the static quark potential rises linearly or, equivalently, that large planar Wilson loops have an area-law falloff.

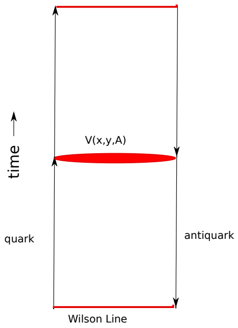
Is there any way to generalize this property to gauge theories with matter in the fundamental representation?

Separation-of-charge (“S_c”) confinement

The Wilson area-law criterion for pure gauge theories is equivalent to “S_c-confinement.”

A static $q\overline{q}$ pair, connected by a Wilson line, evolves in Euclidean time to some state

$$\Psi_V \equiv \overline{q}^a(\mathbf{x}) V^{ab}(\mathbf{x},\mathbf{y};A) q^b(\mathbf{y}) \Psi_0$$



where $V(\mathbf{x},\mathbf{y};A)$ is a gauge bi-covariant operator transforming as

$$V^{ab}(\mathbf{x},\mathbf{y};A) \rightarrow g^{ac}(\mathbf{x},t) V^{cd}(\mathbf{x},\mathbf{y};A) g^{\dagger db}(\mathbf{y},t)$$

The energy above the vacuum energy $\mathcal{E}_{vac}$ is

$$E_V(R) = \langle \Psi_V | H | \Psi_V \rangle - \mathcal{E}_{vac}$$

S_c-confinement

$$\lim_{R\to\infty} E_V(R) = \infty$$

for ***ANY*** choice of bi-covariant $V(\mathbf{x}, \mathbf{y}; A)$.

For an SU(N) pure gauge theory, $E_V(R) \geq E_0(R)$, where $E_0(R) \sim \sigma R$ is the ground state energy of a static quark-antiquark pair.

Our proposal: S_c-confinement should also be regarded as the confinement criterion in gauge+matter theories. The crucial element is that the bi-covariant operators $V^{ab}(\mathbf{x}, \mathbf{y}; A)$ *must depend only on the gauge field A* at a fixed time, and not on the matter fields.

The idea is to study the energy $E_V(R)$ of physical states with large separations R of static color charges, *unscreened by matter fields*.

If $V^{ab}(\mathbf{x}, \mathbf{y}; A)$ would also depend on the matter field(s), then it is easy to violate the S_c-confinement criterion, e.g. let ϕ be a matter field in the fundamental representation, and

$$V^{ab}(\mathbf{x}, \mathbf{y}, \phi) = \phi^a(\mathbf{x})\phi^{\dagger b}(\mathbf{y})$$

Then

$$\Psi_V = \{\overline{q}^a(\mathbf{x})\phi^a(\mathbf{x})\} \times \{\phi^{\dagger b}(\mathbf{y})q^b(\mathbf{y})\}\Psi_0$$

corresponds to two color singlet (static quark + Higgs) states, only weakly interacting at large separations. Operators V of this kind, which depend on the matter fields, are excluded.

This also means that the lower bound $E_0(R)$, unlike in pure gauge theories, is *not* the lowest energy of a state containing a static quark-antiquark pair.

It is the lowest energy of such states when color screening by matter is excluded.

SU(2) gauge-Higgs theory

Most of our numerical work is one in this model, with a unimodular $|\phi| = 1$ Higgs field. In SU(2) the doublet can be mapped to an SU(2) group element

$$\vec{\phi} = \begin{bmatrix} \phi_1 \\ \phi_2 \end{bmatrix} \Longrightarrow \phi = \begin{bmatrix} \phi_2^* & \phi_1 \\ -\phi_1^* & \phi_2 \end{bmatrix}$$

and the corresponding action is

$$S = \beta \sum_{plaq} \frac{1}{2}\mathrm{Tr}[UUU^\dagger U^\dagger] + \gamma \sum_{x,\mu} \frac{1}{2}\mathrm{Tr}[\phi^\dagger(x)U_\mu(x)\phi(x+\widehat{\mu})]$$

Existence of S_c-confinement

1. Does S_c-confinement exist *anywhere* in the $\beta - \gamma$ phase diagram, apart from pure gauge theory ($\gamma = 0$)?

   Yes. We can show that gauge-Higgs theory is S_c-confining at least in the region

$$\boxed{\gamma \ll \beta \ll 1 \quad \text{and} \quad \gamma \ll \frac{1}{10}}$$

   This is based on strong-coupling expansions and a theorem (Gershgorim) in linear algebra.

2. Then does S_c-confinement hold *everywhere* in the $\beta - \gamma$ phase diagram?

   No. We can construct V operators which violate the S_c-confinement criterion when γ is large enough.

So there must exist a transition between S_c and C confinement.

Loss of S_c-confinement

Away from strong coupling, there is no guarantee of S_c-confinement.

If we can find *even one* V at some β, γ such that E_V does not grow linearly with R, then S_c-confinement is lost at that β, γ.

For $V =$ Wilson line, $E_V(R) \propto R$ even for non-confining theories. Not useful! Instead we consider

1. **The Dirac state**
   generalization of the lowest energy state with static charges in an abelian theory.
2. **Pseudomatter**
   Introduce fields built from the gauge field which transform like matter fields. See if these induce string-breaking.
3. **"Fat link" states**
   Wilson lines built from smoothed links.

The Calculation

In general

$$E_V(R) = -\lim_{t\to 0} \frac{d}{dt} \log\left[\frac{\langle \Psi_V | e^{-Ht} | \Psi_V \rangle}{\langle \Psi_V | \Psi_V \rangle}\right] - \mathcal{E}_{vac}$$

on the lattice

$$\boxed{E_V(R) = -\log\left[\frac{\left\langle \mathrm{Tr}\left[U_0(x,t)V(x,y,t+1)U_0^\dagger(y,t)V(y,x,t)\right]\right\rangle}{\langle \mathrm{Tr}\left[V(x,y,t)V(y,x,t)\right]\rangle}\right]}$$

This is what we calculate numerically.

Results

Each of these states defines a V operator, and we can calculate $E_V(R)$ by lattice Monte Carlo.

- We find a line of $\mathrm{S_c}$ to C-confinement transition in the $\beta - \gamma$ plane for the V operator corresponding to the Dirac state.
- We find an $\mathrm{S_c}$ to C-confinement transition for the V operator constructed from pseudomatter fields. The transition line is close to (but a little below) the transition line for the Dirac state.
- The fat link state seems to be everywhere $\mathrm{S_c}$-confining. This doesn't mean that the gauge-Higgs theory is everywhere $\mathrm{S_c}$-confining. It means instead that not every operator can detect the transition to C-confinement.

See *arXiv:1708.08979* for details.

Gauge choice to V operator

Given an $F[U] = 0$ gauge, let $g_F(x;U)$ be the transformation to the gauge. Then define

$$V_F(\mathbf{x}, \mathbf{y}; U) = g_F(\mathbf{x}; U) g_F^\dagger(\mathbf{y}; U)$$

In an F-gauge, $V_F = \mathbb{1}$. Then in this gauge

$$E_V(R) = -\log \left[\frac{1}{N} \langle \mathrm{Tr} \left[U_0(\mathbf{x}, t) U_0^\dagger(\mathbf{x}_0, t) \right] \rangle_F \right]$$

gauge choice, continued

In any F gauge, there is a remnant global subgroup of the gauge symmetry of (at least) $g(\mathbf{x}, t) = z(t) \in Z_N$. Let $\langle\phi(x)\rangle_F$ be the VEV of ϕ in this gauge.

If $\langle\phi(x)\rangle_F \neq 0$ in the thermodynamic limit, then this remnant global symmetry is broken.

U_0 is sensitive to this symmetry, and will also pick up a finite VEV if the symmetry is broken. Then

$$\begin{aligned}\lim_{R\to\infty} E_V(R) &= \lim_{R\to\infty} -\log\left[\frac{1}{N}\mathrm{Tr}\left[\langle U_0(\mathbf{x},t)\rangle\langle U_0^\dagger(\mathbf{y},t)\rangle\right]\right] \\ &= \text{finite}\end{aligned}$$

So breaking remnant symmetry in an F gauge implies C confinement. But the breaking happens in different places in different gauges.

Where does C confinement begin? Is there a necessary condition of some kind?

Custodial Symmetry

Define a *custodial symmetry* to be a symmetry of the matter fields such that any operator which transforms under that symmetry also transforms under the gauge symmetry.

Example: SU(2) gauge-Higgs theory

$$S_H = \gamma \sum_{x,\mu} \frac{1}{2} \mathrm{Tr}[\phi^\dagger(x) U_\mu(x) \phi(x+\widehat{\mu})]$$

is invariant under SU(2)$_{gauge}\times$ SU(2)$_{global}$:

$$\begin{aligned} U_\mu(x) &\rightarrow L(x) U_\mu(x) L^\dagger(x+\hat{\mu}) \\ \phi(x) &\rightarrow L(x)\phi(x) R \end{aligned}$$

$R \in$ SU(2)$_{global}$ is a custodial symmetry transformation.

Custodial symmetry breaking

Define a partition function for spacelike links and the scalar field at fixed time $t = 0$

$$\begin{aligned} \mathbf{Z}(U,\phi) &= \int DU_{i,t\neq 0} D\phi_{t\neq 0} DU_0 \; e^{-S} \\ \mathbf{Z}(U) &= \int D\phi_{t=0} \, \mathbf{Z}(U,\phi) \end{aligned}$$

and probability distribution

$$\langle Q \rangle = \int DU \; Q(U)P(U) \;\;, \;\; P(U) = \frac{\mathbf{Z}(U)}{Z}$$

At *fixed* U, $\mathbf{Z}(U,\phi)$ has no local gauge symmetry, but it does have custodial symmetry, which can break spontaneously:

$$\overline{\phi}(\mathbf{x};U) = \frac{1}{\mathbf{Z}(U)} \int D\phi \; \phi(\mathbf{x})\mathbf{Z}(U,\phi)$$

Of course this symmetry breaking depends on U, and $\overline{\phi}(\mathbf{x})$ depends on $\mathbf{x}$.

The question is whether the symmetry is broken for configurations U selected from probability distribution $P(U)$.

Gauge-invariant order parameter

The gauge-invariant order parameter for custodial symmetry breaking is

$$\langle \Omega \rangle = \int DU \frac{1}{V} \sum_{\mathbf{x}} |\overline{\phi}(\mathbf{x}; U)| \frac{\mathbf{Z}(U)}{Z}$$

which is non-zero in the thermodynamic limit if custodial symmetry is broken.

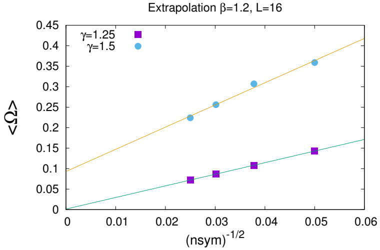
In practice compute $\langle \Omega \rangle$ by a Monte-Carlo-in-a-Monte-Carlo procedure. Update U, ϕ as usual, but at data-taking, keep spacelike links fixed on a $t = 0$ timeslice. Update everything else, and compute

$$\frac{1}{V_3} \sum_{\mathbf{x}} |\overline{\phi}(\mathbf{x}, t = 0; U)|$$

at $t = 0$. Average over data taking sweeps, and extrapolate to $V_3 \to \infty$.

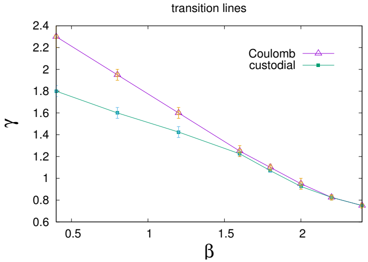
Note that there is no gauge-fixing of any sort in this computation.

Transition lines



Custodial symmetry breaking is determined from extrapolation of $\langle\Omega\rangle$ to $n_{sym} \to \infty$. In this case $\gamma_c \approx 1.4$

Remnant symmetry breaking in Coulomb gauge is determined from a peak in the susceptability of $\langle\phi\rangle$.



Transition lines for SU(2) gauge theory in the $\beta - \gamma$ plane.

Note that custodial symmetry breaking occurs before the breaking in Coulomb gauge.

Custodial and gauge symmetry breaking

Theorem

$$\langle \Omega \rangle \geq \langle \phi \rangle_F \tag{1}$$

for all physical gauges $F(U) = 0$. And there exists at least one physical gauge such that

$$\langle \Omega \rangle = \langle \phi \rangle_{F_\Omega} \tag{2}$$

Custodial symmetry breaking is therefore a

- necessary and sufficient, and
- gauge invariant

condition for the existence of spontaneous breaking of a global subgroup of the gauge symmetry, via $\langle \phi \rangle > 0$ in a physical F gauge.

Broken custodial symmetry

- implies spontaneous breaking of the subgroup of gauge transformations $g(\mathbf{x}, t) = z(t) \in Z_N$ in a physical gauge;
- which implies $\langle U_0 \rangle \neq 0$ in that gauge;
- which implies C confinement.

Let us define

$$\begin{aligned} |\text{charged}\rangle &= \overline{q}^a(\mathbf{x}) V^{ab}(\mathbf{x}, \mathbf{y}; U) q^b(\mathbf{y}) |\text{vac}\rangle \\ |\text{neutral}\rangle &= (\overline{q}^a \phi^a)_{\mathbf{x}} (\phi^{\dagger b} q^b)_{\mathbf{y}} |\text{vac}\rangle \end{aligned}$$

Take $\mathbf{x} \to \infty$. This defines charged and uncharged quark states at site $\mathbf{y}$, distinguished by a long-range color electric field of the charged quark.

Charged and uncharged states

Custodial broken phase: Evaluate overlap with $V(\mathbf{x},\mathbf{y};U) = g_F^\dagger(\mathbf{x};U)g_F(\mathbf{y};U)$, $\langle\phi\rangle_F > 0$. Then in the $R = |\mathbf{x}-\mathbf{y}| \to \infty$ limit

$$\boxed{\langle \text{charged}|\text{neutral}\rangle \propto \langle \phi^{\dagger a}(\mathbf{x})\phi^a(\mathbf{y})\rangle \neq 0}$$

Custodial unbroken phase: $\langle\phi\rangle_F = 0$, and there is no obvious Higgs mechanism in any physical $F(U) = 0$ gauge. Moreover, at $R \to \infty$,

$$\boxed{\langle \text{charged}|\text{neutral}\rangle = 0}$$

for all charged states.

- If finite energy electric field $\Longrightarrow$ massless phase.
  This has been ruled out numerically in SU(2) gauge-Higgs.
- The alternative is an infinite energy electric field $\Longrightarrow$ S_c confinement.

Conclusions

We have suggested two gauge-invariant criteria for distinguishing between the confinement and Higgs sectors of a gauge-Higgs theory:

1. S_c vs. C confinement
2. unbroken vs. spontaneously broken custodial symmetry

Broken custodial symmetry $\Longrightarrow \langle\phi\rangle_F > 0$ in a physical gauge $\Longrightarrow$ C confinement.
In the unbroken phase there is no obvious route to the Higgs mechanism, and we have argued that S_c confinement is implied.

According to our arguments...

the custodial symmetry transition line and the S_c to C transition line coincide.

EXTRA

SLIDES

Introduce a breaking term

$$S_\eta = -J\sum_{\mathbf{x}} \mathrm{Tr}[\eta^\dagger(\mathbf{x})\phi(\mathbf{x})]$$

and

$$\begin{aligned}
\overline{\phi}_{JV}(\mathbf{x};\eta) &= \frac{1}{\mathbf{Z}(U)}\int D\phi\,\phi(\mathbf{x})\mathbf{Z}(U,\phi)e^{-S_\eta} \\
\Omega_{JV}(U) &= \max_\eta \frac{1}{V}\sum_{\mathbf{x}} |\overline{\phi}_{JV}(\mathbf{x};\eta)|
\end{aligned}$$

Then

$$\begin{aligned}
|\langle\phi\rangle_F| &= \lim_{J\to 0}\lim_{V\to\infty}\frac{1}{Z}\left|\int DU\delta[F(U)]\Delta_F[U]\right. \\
&\quad \left.\times\frac{1}{V}\sum_{\mathbf{x}}\int D\phi\,\phi(x)\mathbf{Z}(U,\phi)e^{J\sum_{\mathbf{x}}\mathrm{Tr}\phi(\mathbf{x})}\right| \\
&\le \lim_{J\to 0}\lim_{V\to\infty}\frac{1}{Z}\int DU\delta[F(U)]\Delta_F[U] \\
&\quad \times\frac{1}{V}\sum_{\mathbf{x}}\left|\int D\phi\,\phi(x)\mathbf{Z}(U,\phi)e^{J\sum_{\mathbf{x}}\mathrm{Tr}\phi(\mathbf{x})}\right|
\end{aligned}$$

proof, part 2

$$
\begin{aligned}
|\langle\phi\rangle_F| &\leq \lim_{J\to 0}\lim_{V\to\infty}\frac{1}{Z}\int DU\delta[F(U)]\Delta_F[U] \\
&\quad \times\frac{1}{V}\max_{\eta}\sum_{\mathbf{x}}\left|\int D\phi\phi(x)\mathbf{Z}(U,\phi)e^{-S_\eta}\right| \\
&\leq \langle\Omega\rangle
\end{aligned}
$$

which is the first part of the the theorem. Next, define the gauge

$$
\widehat{F}(U) = \frac{\overline{\phi}(\mathbf{x};U)}{|\overline{\phi}(\mathbf{x};U)|} - \mathbb{1} = 0
$$

Since $\Omega(U)$ is gauge invariant, it can be evaluated in this $\widehat{F}(U) = 0$ gauge in particular.

Then

$$\begin{aligned}
\langle\Omega\rangle &= \lim_{J\to 0}\lim_{V\to\infty}\frac{1}{Z}\int DU\delta[\widehat{F}(U)]\Delta_{\widehat{F}}[U] \\
&\quad \times\frac{1}{V}\max_{\eta}\sum_{\mathbf{x}}\left|\int D\phi\phi(x)\mathbf{Z}(U,\phi)e^{-S_\eta}\right| \\
&= \lim_{J\to 0}\lim_{V\to\infty}\frac{1}{Z}\left|\int DU\delta[\widehat{F}(U)]\Delta_{\widehat{F}}[U]\right. \\
&\quad \left.\times\frac{1}{V}\max_{\eta}\sum_{\mathbf{x}}\int D\phi\phi(x)\mathbf{Z}(U,\phi)e^{-S_\eta}\right| \\
&= \langle\phi\rangle_{\widehat{F}}
\end{aligned}$$

which establishes the second part of the theorem.

Orthogonality

Integrating out the static quark fields, it is easy to see that

$$\langle \text{charged}_{\mathbf{xy}} | \text{neutral}_{\mathbf{xy}} \rangle \quad \propto \quad \langle \phi^{\dagger a}(\mathbf{x}) V^{ab}(\mathbf{x}, \mathbf{y}; U) \phi^{b}(\mathbf{y}) \rangle$$

Then we integrate out the scalar field to obtain

$$\begin{aligned} &\lim_{R\to\infty} \langle \text{charged}_{\mathbf{xy}} | \text{neutral}_{\mathbf{xy}} \rangle \\ &\qquad = \lim_{R\to\infty} \int DU \, \mathrm{Tr}[V(\mathbf{x}, \mathbf{y}; U) G(\mathbf{y}, \mathbf{x}; U)] \frac{\mathbf{Z}(U)}{Z} \end{aligned}$$

where

$$G(\mathbf{y}, \mathbf{x}; U) \quad = \quad \frac{1}{\mathbf{Z}(U)} \int D\phi \phi^{\dagger}(\mathbf{x}) \phi(\mathbf{y}) e^{-S}$$

Unbroken custodial symmetry in $\mathbf{Z}(U, \phi)$, for fixed U configurations drawn from the probability distribution $\mathbf{Z}(U)/Z$, implies $G(\mathbf{x}, \mathbf{y}, U) \to 0$ as $R \to \infty$, even if the background U field is fixed to some F-gauge. Then, since $V(\mathbf{x}, \mathbf{y}; U)$ is normalized and therefore bounded in the $R \to \infty$ limit, we conclude that

$$\lim_{R\to\infty} \langle \text{charged}_{\mathbf{xy}} | \text{neutral}_{\mathbf{xy}} \rangle \quad = \quad 0$$