

Towards a holographic description of cosmology (I):

# Phase diagram of 3d $SU(N)$ matrix field theory

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# Non-perturbative IR regulation in 3d $SU(N)$ Matrix Field Theory

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# Collaboration

## **Edinburgh:**

Luigi Del Debbio

Guido Cosa

Joseph Lee (next talk)

Valentin Nourry

Antonin Portelli

## **Southampton:**

Masanori Hanada

Andreas Jüttner

Ben Kitching-Morley

Kostas Skenderis

## **LLNL:**

Pavlos Vranas

## **Waterloo:**

Elizabeth Gould

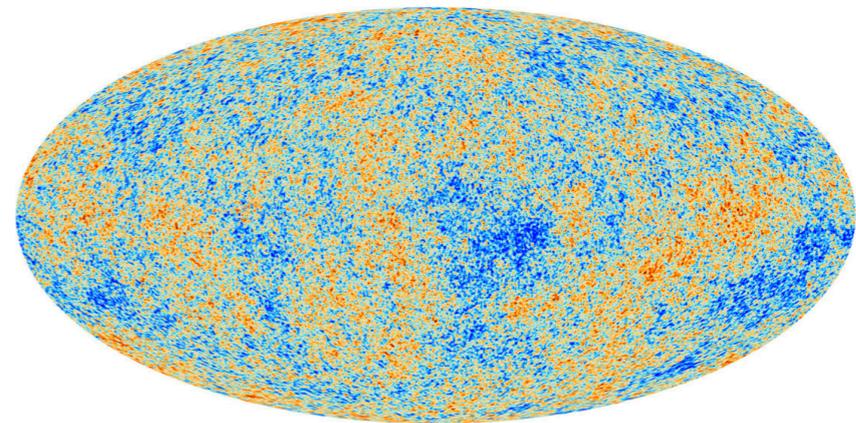
# Cosmology

## Problems in cosmology solved by inflation:

- Horizon problem
- Formation of structure
- Flatness problem

**Famous testable prediction in many models of inflation:**  
power-law primordial power spectrum

$$\mathcal{P}(q) = \Delta_0^2 \left( \frac{q}{q_*} \right)^{n_s - 1}$$



ESA and the Planck Collaboration

# Cosmology

**The particle physics interpretation of inflation seems ad hoc:**

- Fine tuning of inflation potential and inflaton
- Eternal inflation/initial conditions
- Effective theory — what about UV completion towards the very very early universe (initial singularity)?

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- Effective theory — what about UV completion towards the very very early universe (initial singularity)?

**The prospect of precision cosmological observations in the future motivates looking for more complete models or even first principles descriptions**

# Cosmology

**Idea:** Quantum gravity description of early universe  
in terms of Holographic dual QFT

Cosmological observables are mapped to correlation  
functions of the dual QFT

Maldacena JHEP 0305 (2003) 013

McFadden and Skenderis, PRD 81, 021301 (2010)

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**Prediction for power spectrum from general ansatz for QFT:**

$$S = \frac{1}{g_{\text{YM}}^2} \int d^3x \text{Tr} \left[ \frac{1}{2} F_{ij} F^{ij} + (D\phi)^2 + \bar{\psi} D_i \gamma^i \psi + \mu (\bar{\psi} \psi \phi) + \lambda \phi^4 \right]$$

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2-loop prediction for the power spectrum with 3 free parameters

McFadden and Skenderis, PRD 81, 021301 (2010)

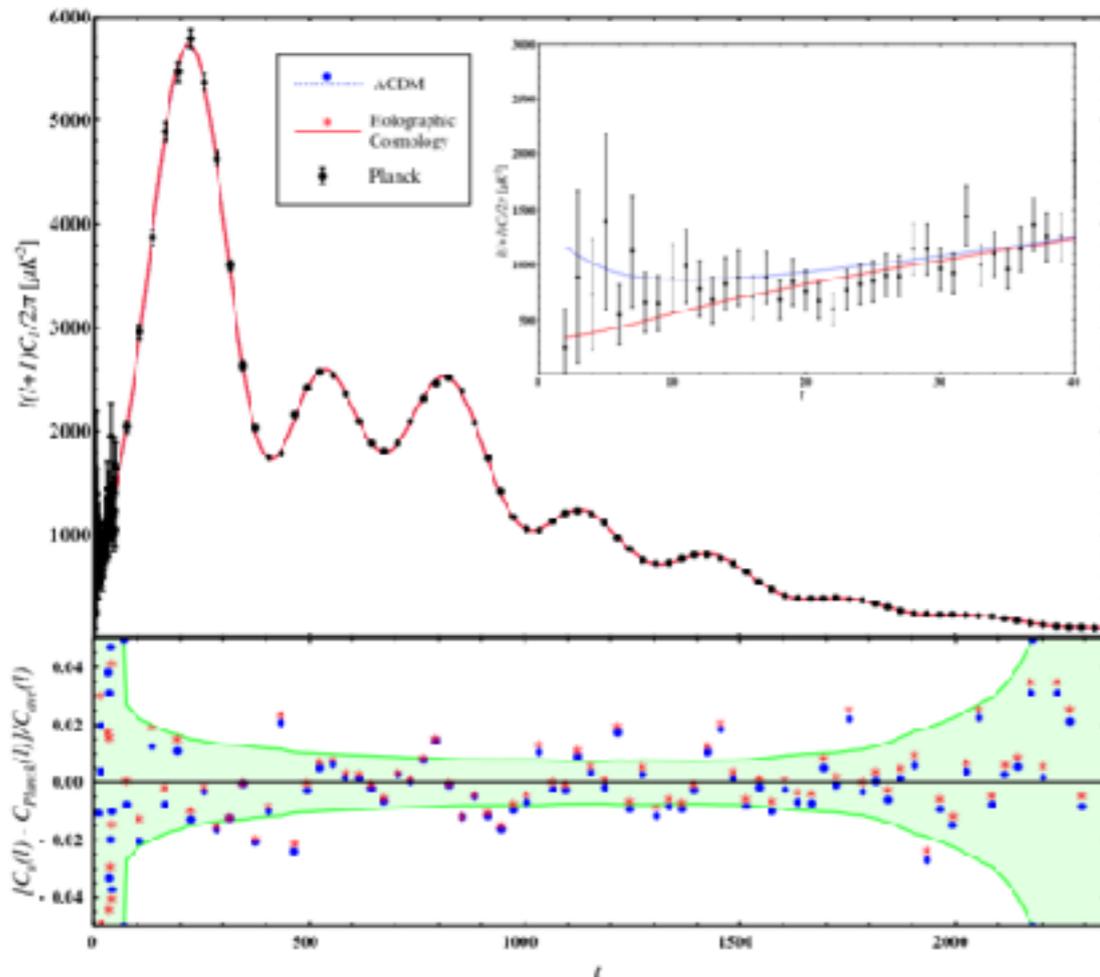
JPCS 222, 012007 (2010), JCAP 05 (2011) 013,06 (2011) 030

$$\Delta_{\mathcal{R}}^2(q) = \frac{\Delta_0^2}{1 + \frac{gq_*}{q} \log \left| \frac{q}{\beta g q_*} \right|}$$

# Holographic CMB spectrum

$$\Delta_{\mathcal{R}}^2(q) = \Delta_0^2 \left( \frac{q}{q_*} \right)^{n_s - 1}$$

$$\text{vs. } \Delta_{\mathcal{R}}^2(q) = \frac{\Delta_0^2}{1 + \frac{gq_*}{q} \log \left| \frac{q}{\beta g q_*} \right|}$$



- PT prediction of Holographic Cosmology competitive with  $\Lambda$ CDM
- low-multipole region corresponds to strong-coupling in dual QFT  
 → Lattice Holographic Cosmology

# Current study

Start with simplified model: **massless 3d scalar SU(N) matrix  $\phi^4$  theory**

$$\phi(x) = \phi^a(x)T^a$$

$$S = \frac{N}{g} \int d^3x \text{Tr} \left( (\partial_\mu \phi(x))^2 + (m^2 - m_c^2) \phi^2(x) + \phi^4(x) \right)$$

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- $[\phi] = 1, [g] = 1$  (t'Hooft coupling,  $g/N = g_{\text{YM}}^2$ )
- super-renormalisable
- log IR divergent in PT

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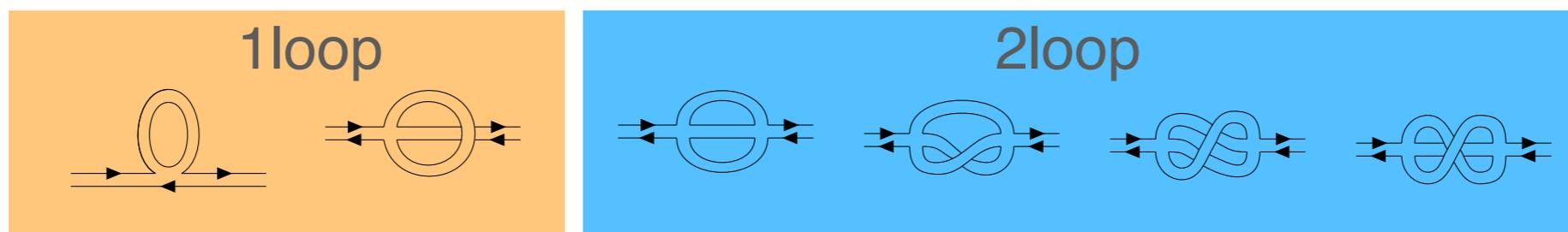
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## Programme:

1. non-perturbative study (is it IR divergent beyond PT?, determine critical properties, determine critical mass, ...)
2. compute  $n$ -pt. functions (e.g. energy-momentum tensor, next talk)
3. make a first post-diction for the power-spectrum of the CMB and test against Planck data

# Critical mass in NNLO lattice PT

$$S = \frac{N}{g} \int d^3x \text{Tr} \left( (\partial_\mu \phi(x))^2 + (m^2 - m_c^2) \phi^2(x) + \phi^4(x) \right)$$



$$m_c^2 = -g \frac{Z_0}{a} \left( 2 - \frac{3}{N^2} \right) + g^2 D_{\Lambda_{\text{IR}}} (0) \left( 1 - \frac{6}{N^2} + \frac{18}{N^4} \right)$$

- **2-loop integral** IR log-divergent
- use scalar mass  $\Lambda_{\text{IR}} = g/N$  as regulator
- evaluate momentum sums using Vegas

# Lattice study of scalar SU(N) matrix QFT

- Theory and observables implemented in Grid
- Ensemble generation on SKL cluster  
(STFC  Distributed Research utilizing Advanced Computing and University of Southampton Iridis5)
- O(100k) trajectories per ensemble

## Simulation parameters:

N	2,3,4,5
g	0.1,0.2,0.3
L	16,32,64,128
m	many masses in vicinity of 2-loop $m_c^2$

# Lattice study of scalar SU(N) matrix QFT

**Magnetisation:**  $M = \sum_x \phi(x)$

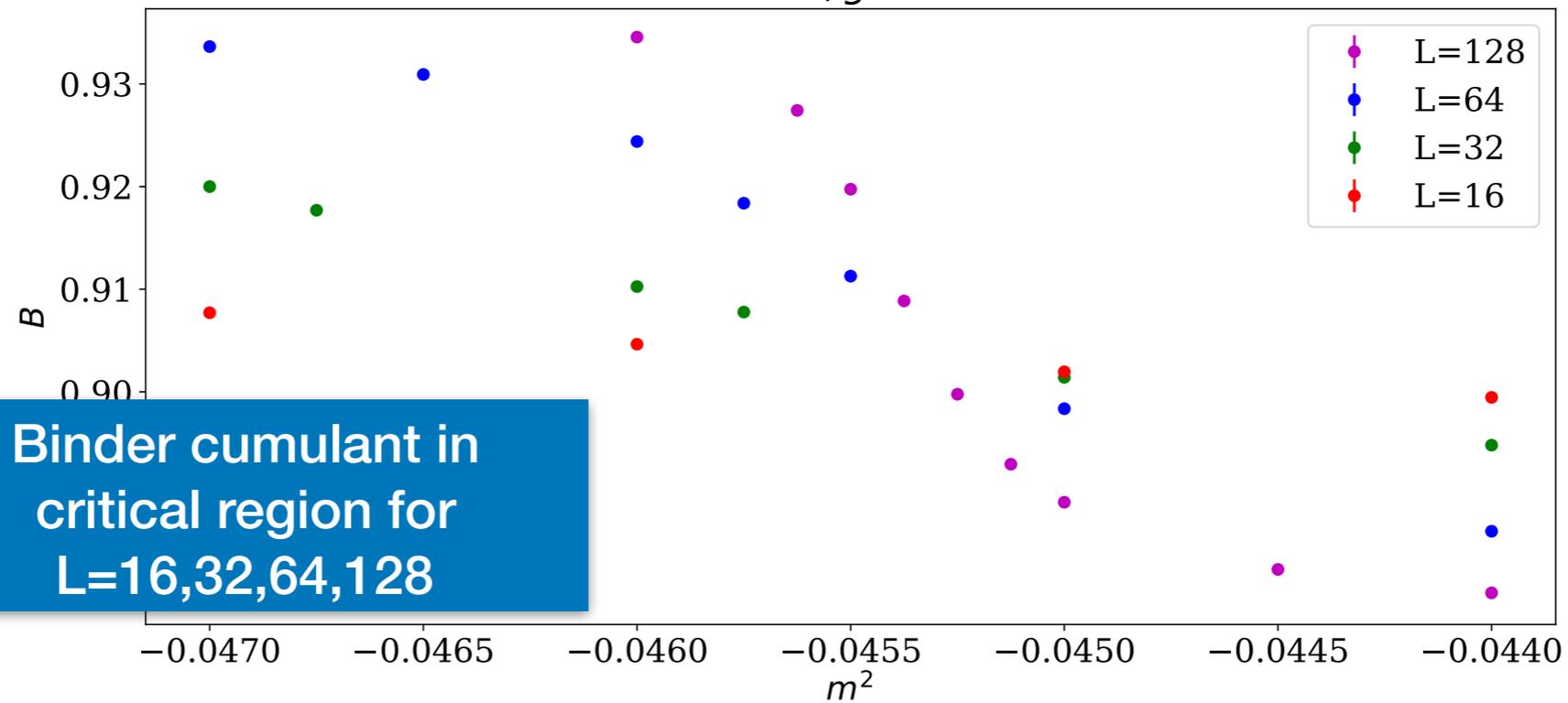
**Binder cumulant:**  $B = 1 - \frac{1}{N} \frac{\langle \text{Tr} [M^4] \rangle}{\langle \text{Tr} [M^2] \rangle^2}$

**Derivative of Binder cumulant:**

$$\frac{\partial B}{\partial m^2} = \frac{N^2}{g} (B - 1) \left( \langle \text{Tr} [\phi^2] \rangle - 2 \frac{\langle \text{Tr} [M^2] \text{Tr} [\phi^2] \rangle}{\langle \text{Tr} [M^2] \rangle} + \frac{\langle \text{Tr} [M^4] \text{Tr} [\phi^2] \rangle}{\langle \text{Tr} [M^4] \rangle} \right)$$

# A first look at the data

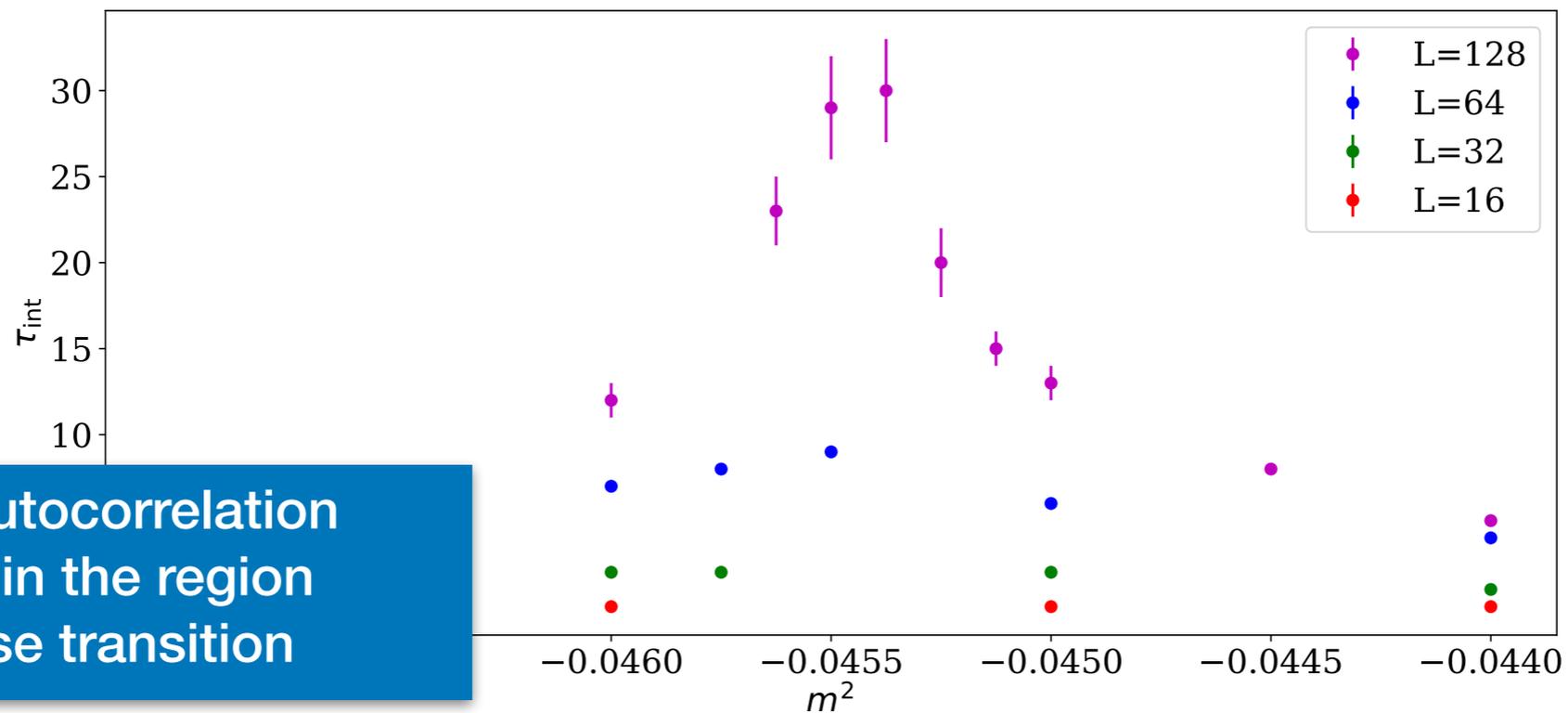
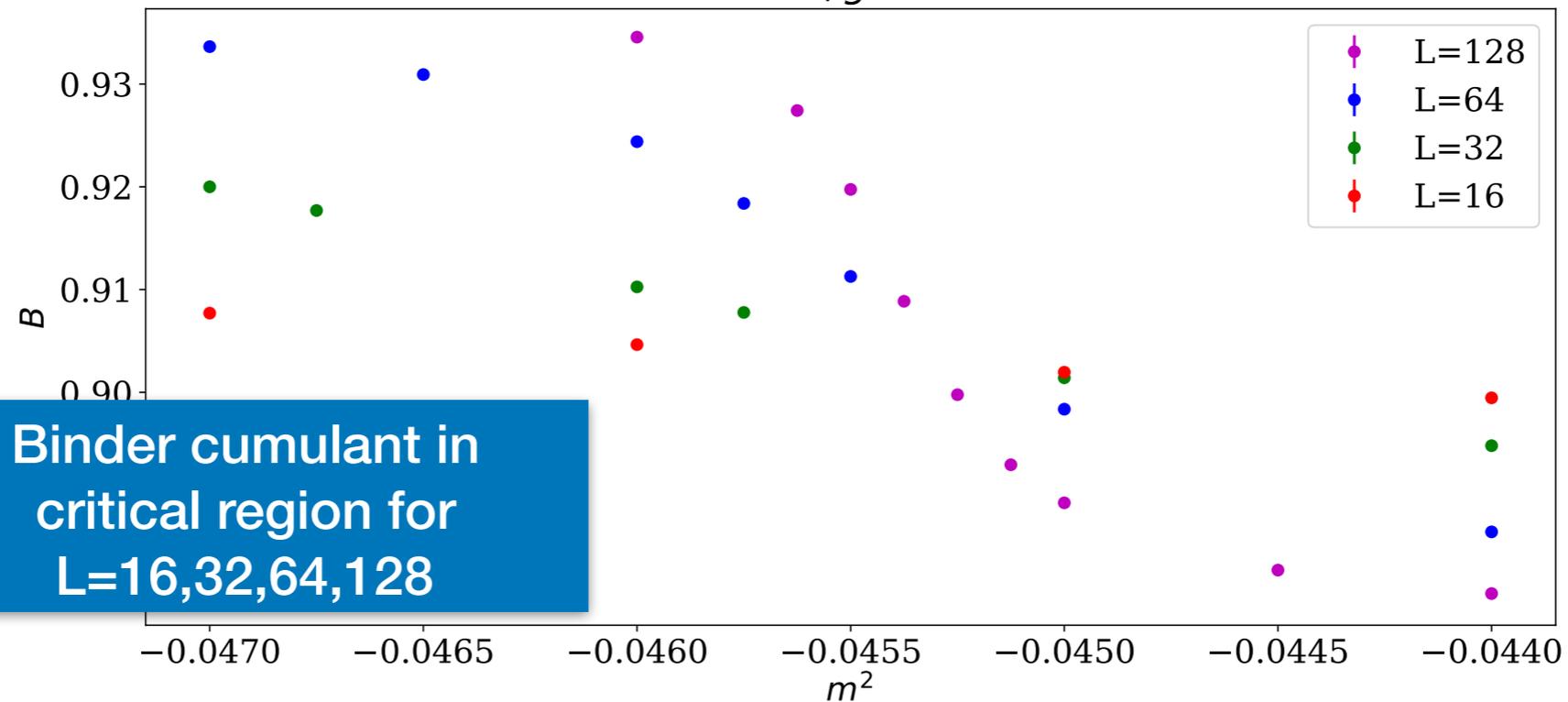
$N = 4, g = 0.1$



Binder cumulant in critical region for  $L=16,32,64,128$

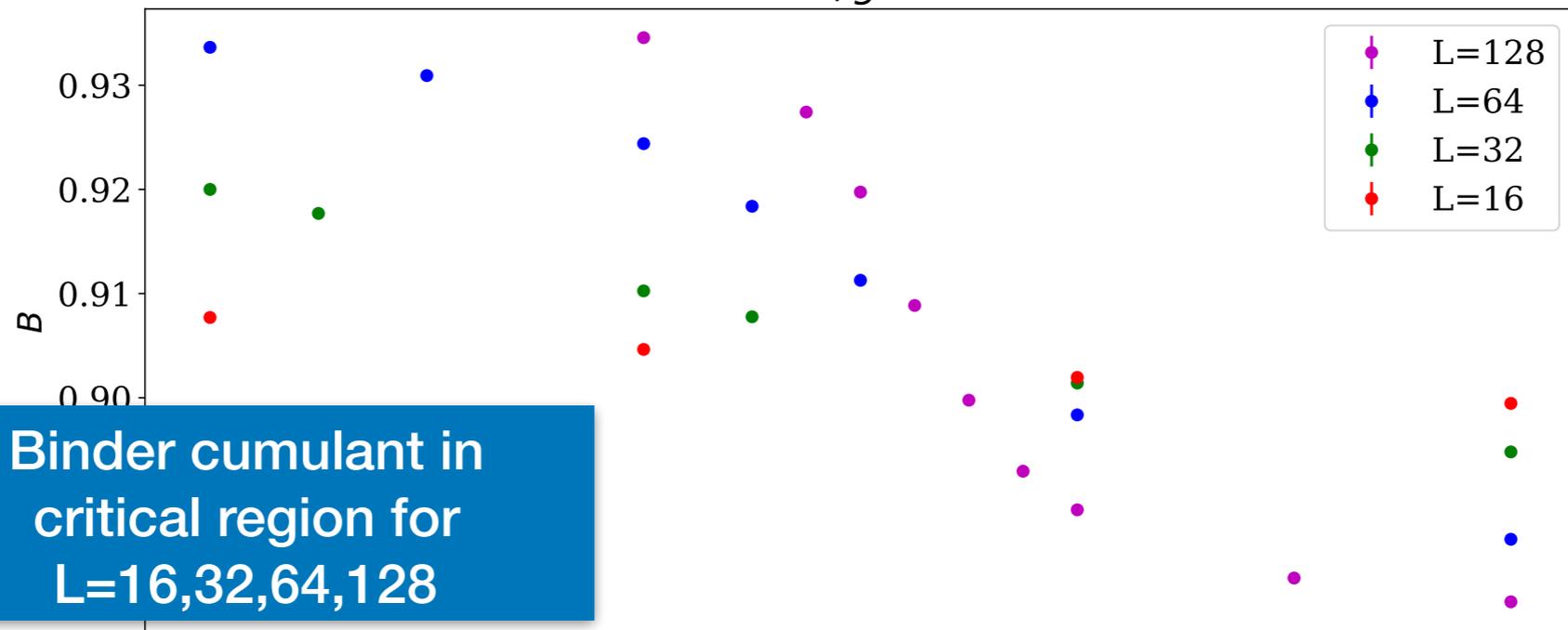
# A first look at the data

$N = 4, g = 0.1$

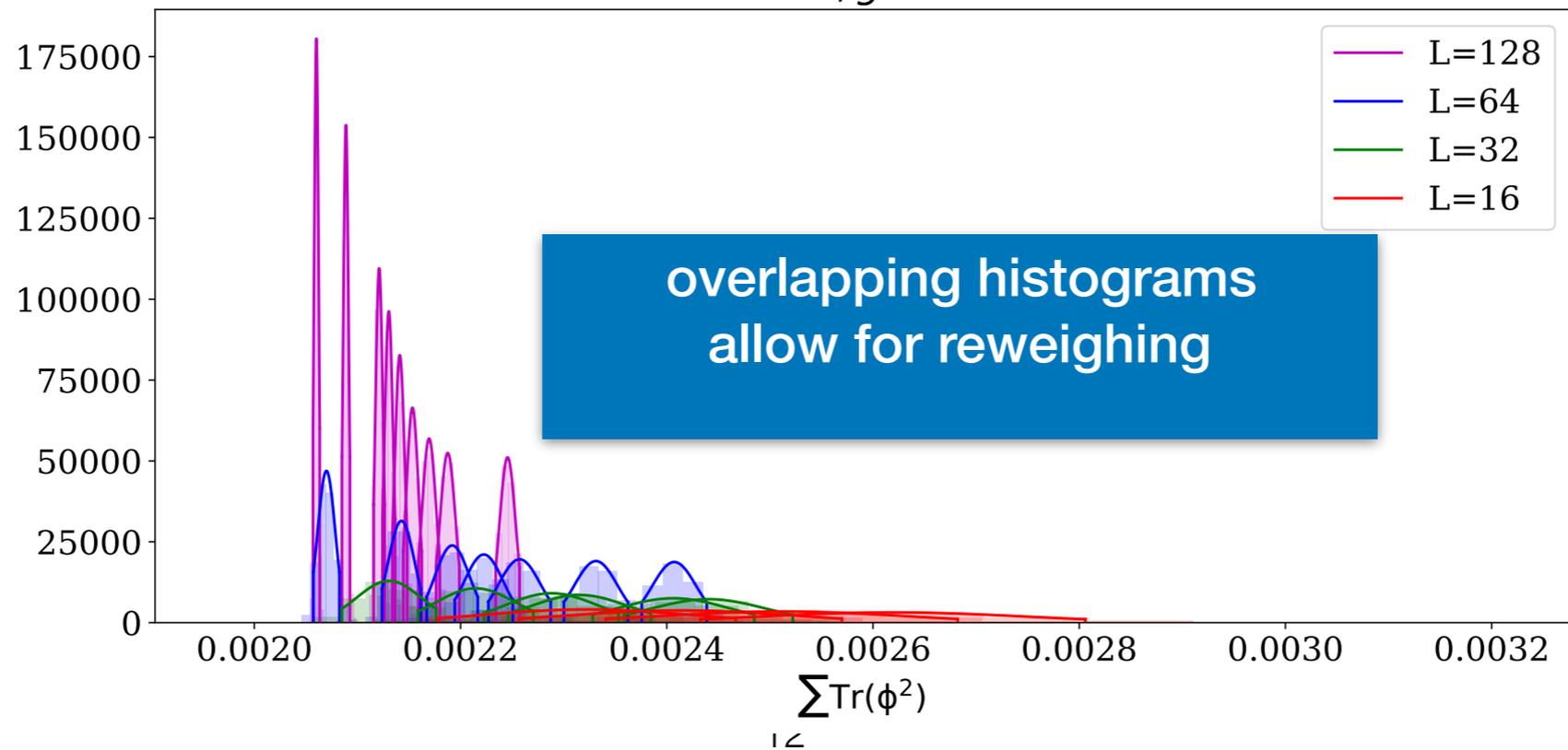


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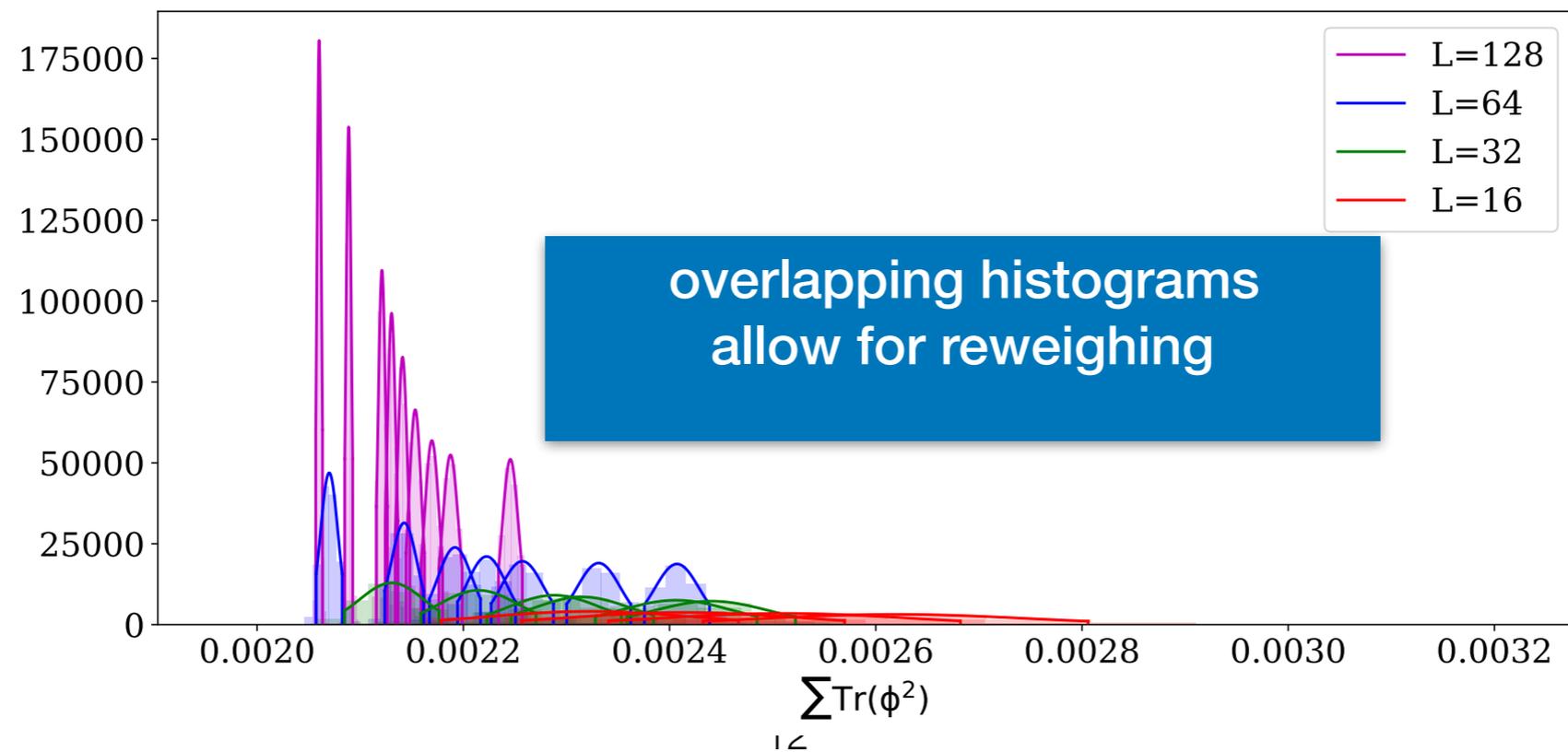
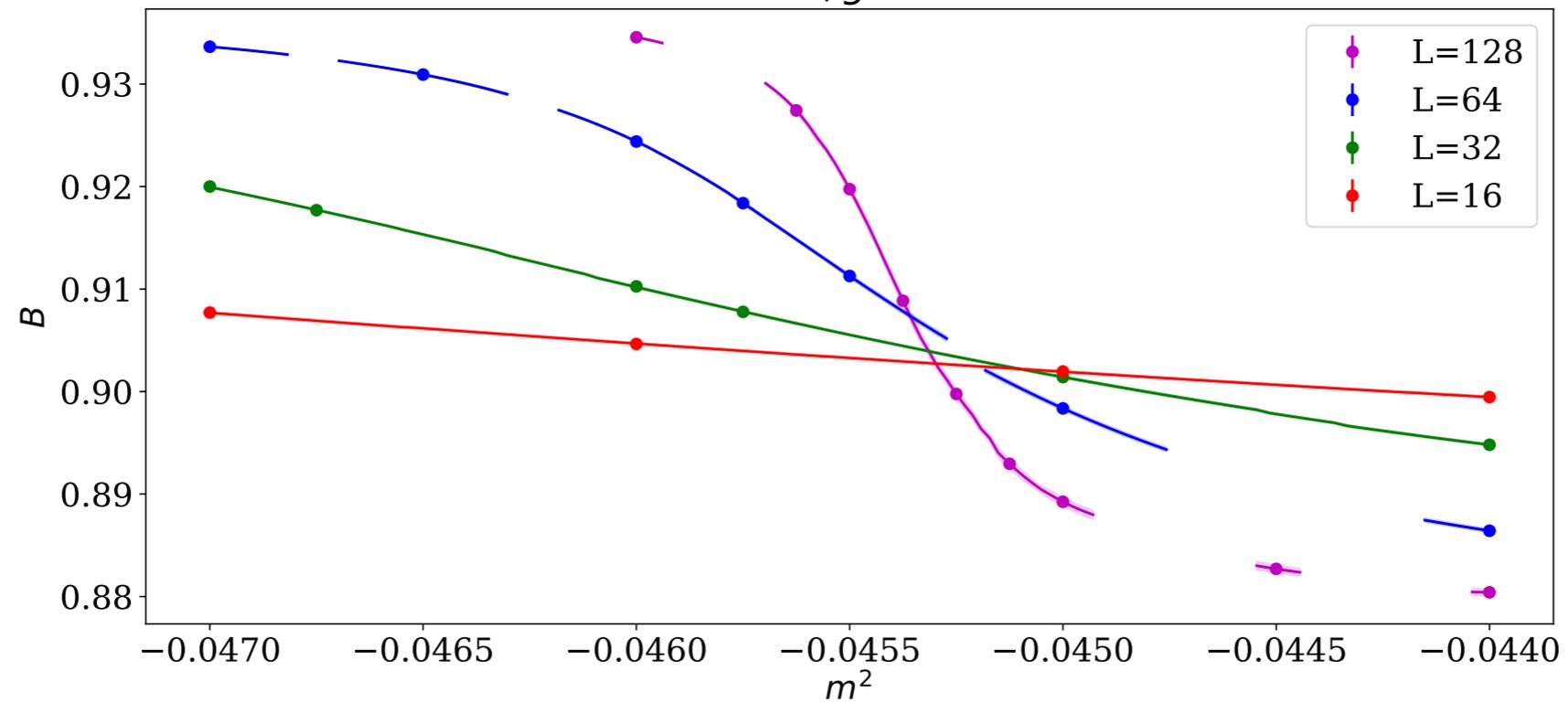


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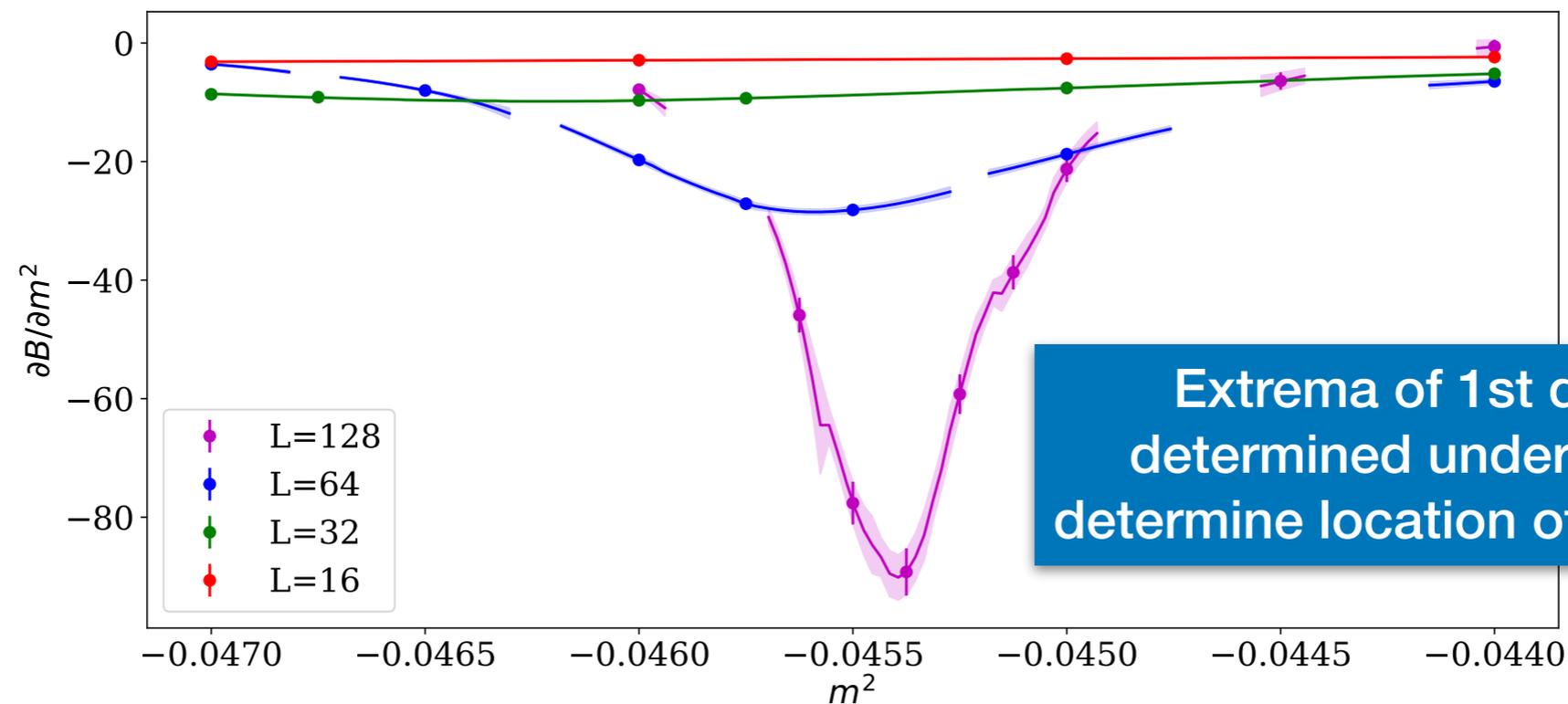
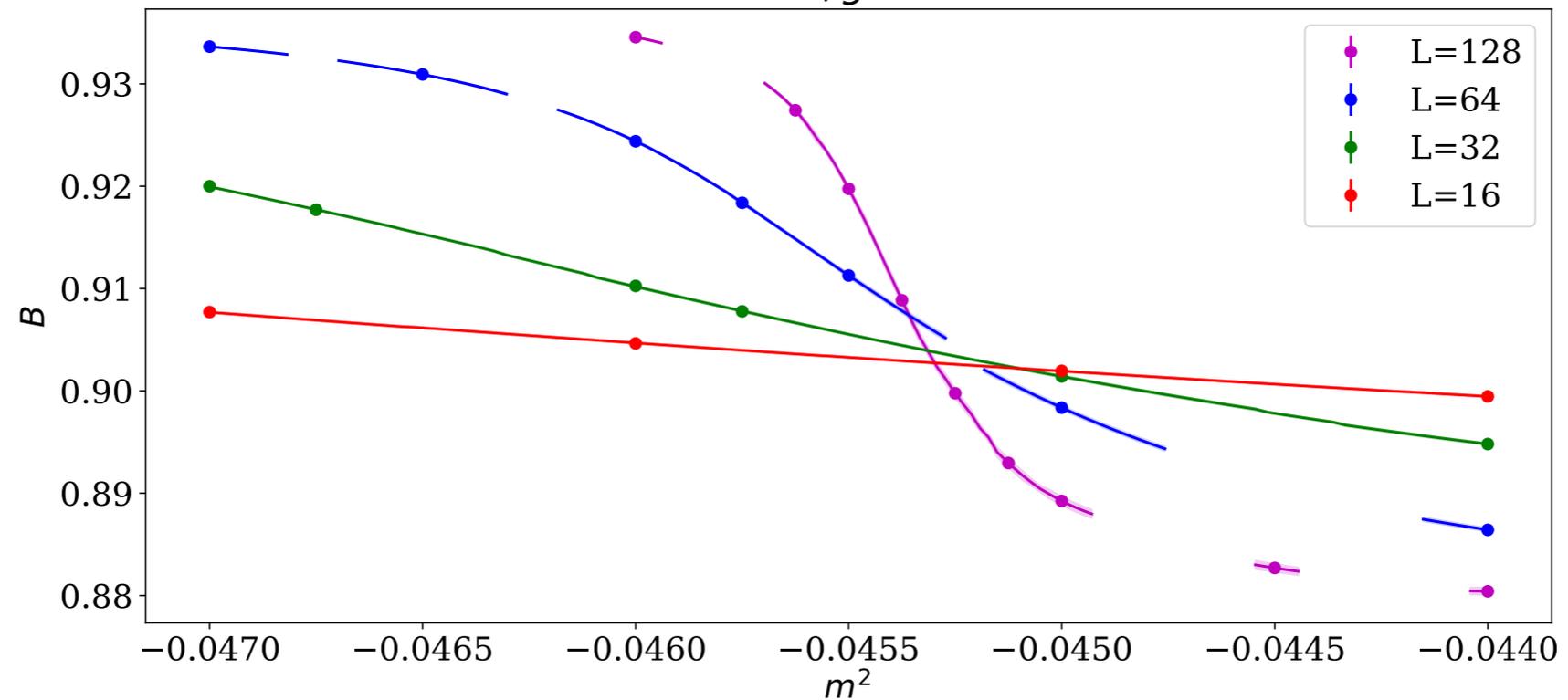
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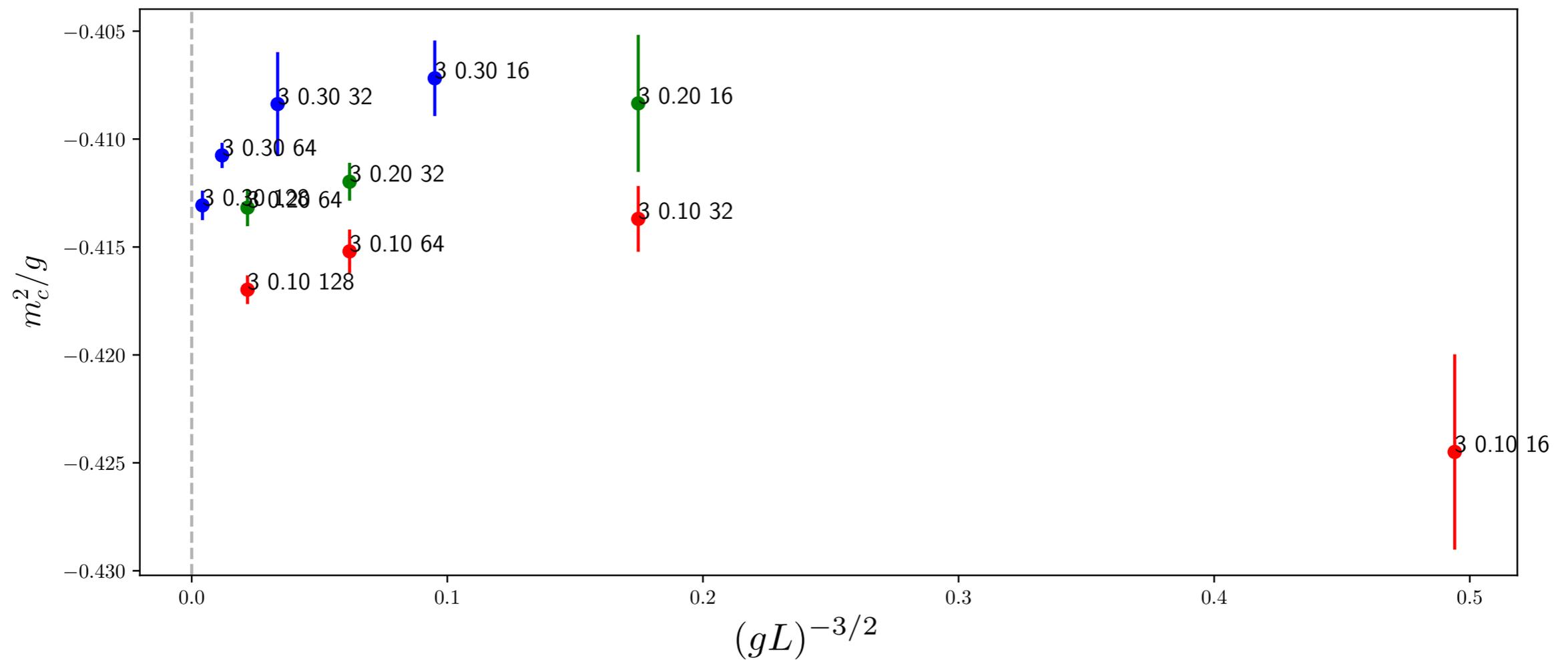
$N = 4, g = 0.1$



Extrema of 1st derivative  
determined under Bootstrap  
determine location of critical mass

# Global fit

Results for the critical mass as a function of  $g$  and  $L$  for  $N=3$ :



# Global fit

Volume dependence of critical mass in effective theory:

$$S_{\text{eff.}}[M] = \frac{L^3 N}{g} (m^2 \text{Tr} [M^2] + \text{Tr} [M^4])$$

$$\langle O[M] \rangle = \frac{1}{\mathcal{Z}_{\text{eff.}}} \int_{\text{su}(N)} dM O[M] e^{-S_{\text{eff.}}[M]}$$

Motivates model for global fit:

$$m_c^2(N, g, L) = m_{c,2\text{-loop}}^2(N, g) + \alpha_0 g^2 + \alpha_1 \frac{\sqrt{g}}{L^{3/2}} + \text{power suppressed FVE}$$

# Global fit

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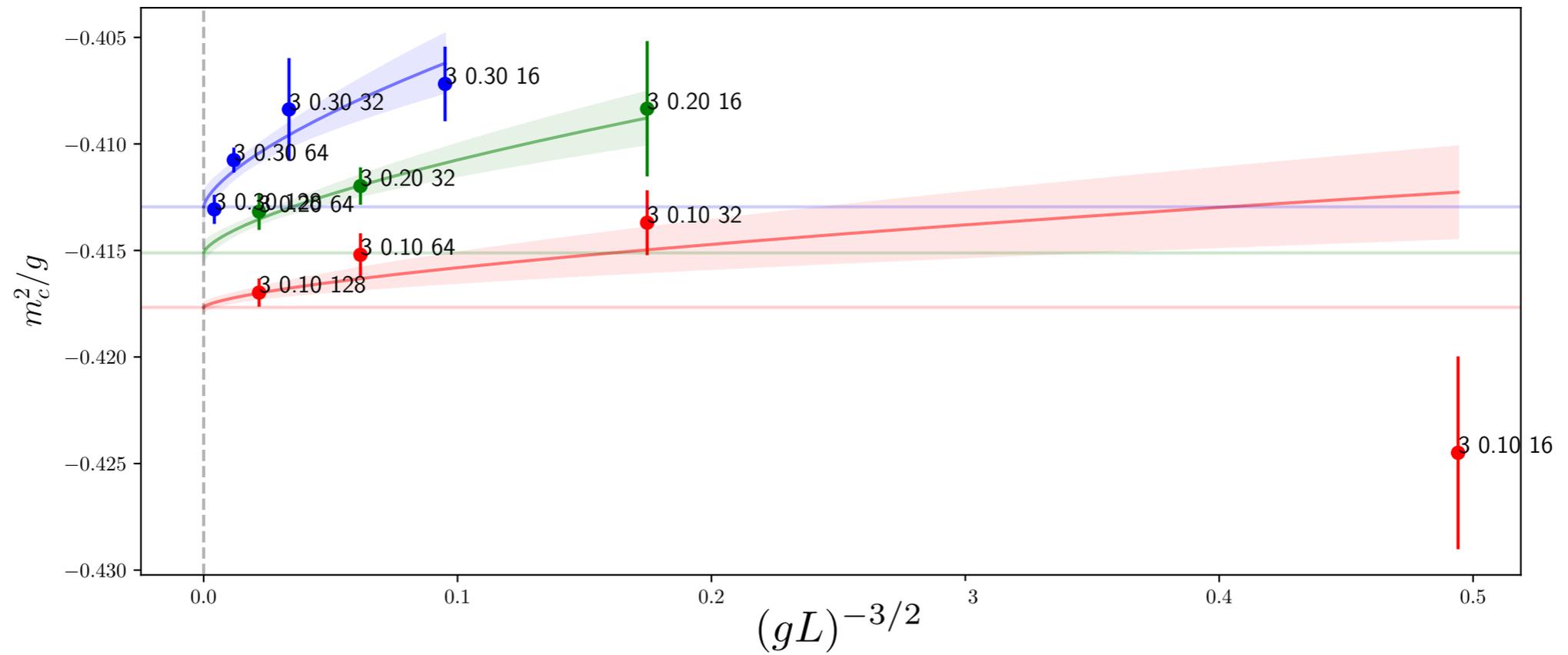
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Use this ansatz and variations to extrapolate  
lattice data to infinite volume.  
Determine critical mass and tune further simulations



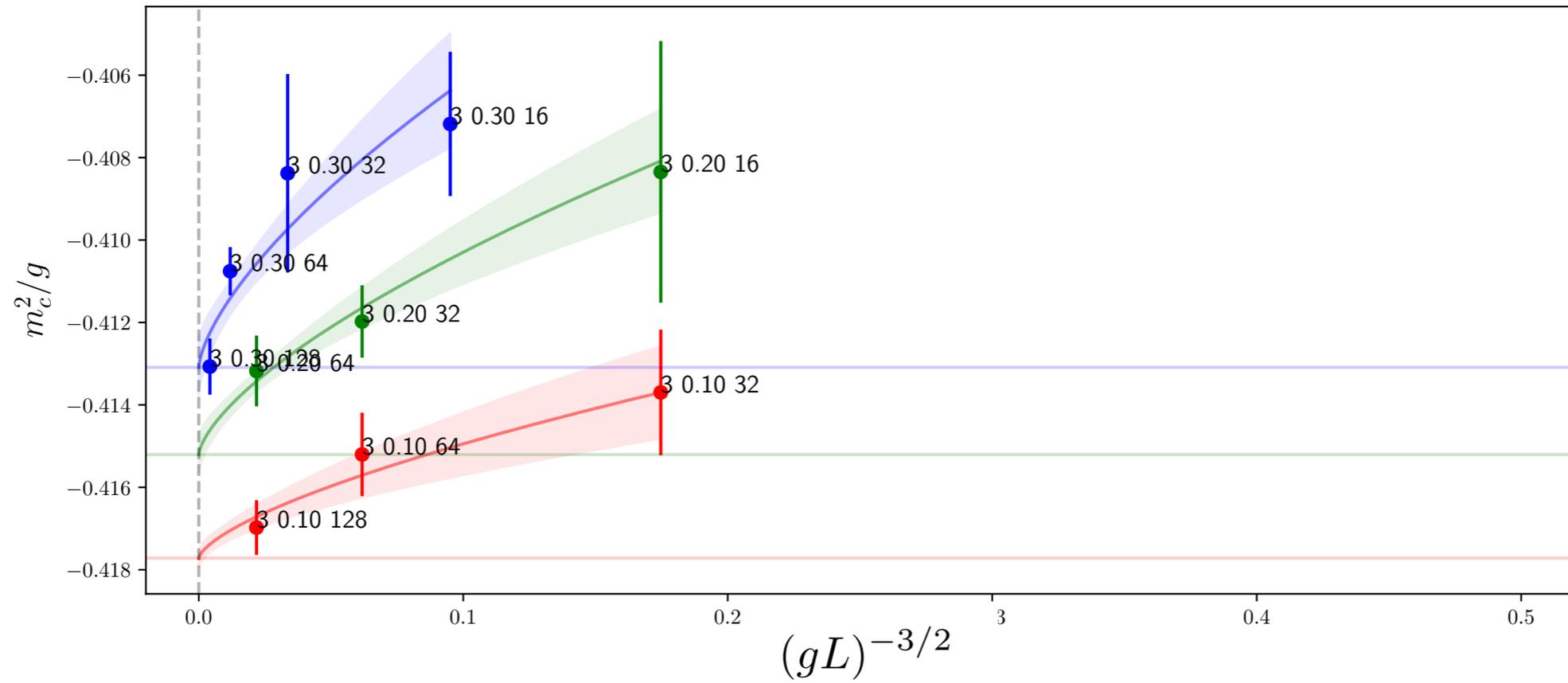
# Global fit

N fix NLO N=3 Lgmin=1.0 p=0.18



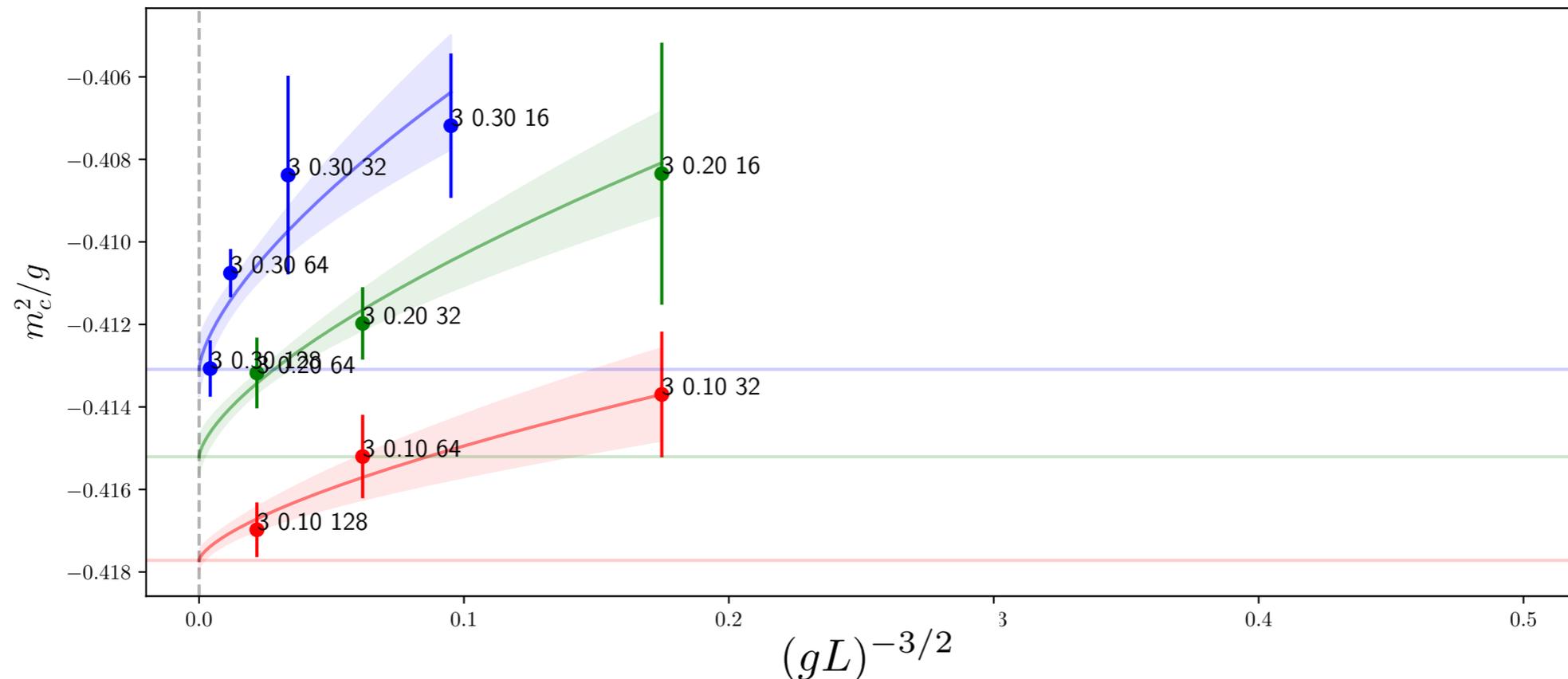
# Global fit

N fix NLO N=3 Lgmin=2.0 p=0.87



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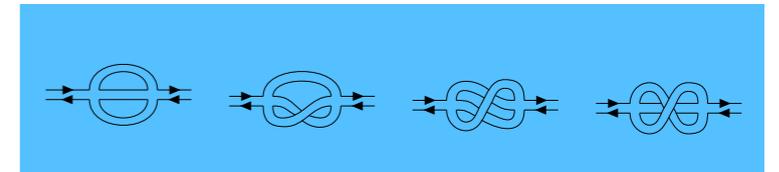


Example for global fit with  $N=3$

- fit describes data well and is stable under variation of ansatz
- we obtain precise predictions for critical mass needed to simulate massless theory
- data not good enough to determine critical exponent

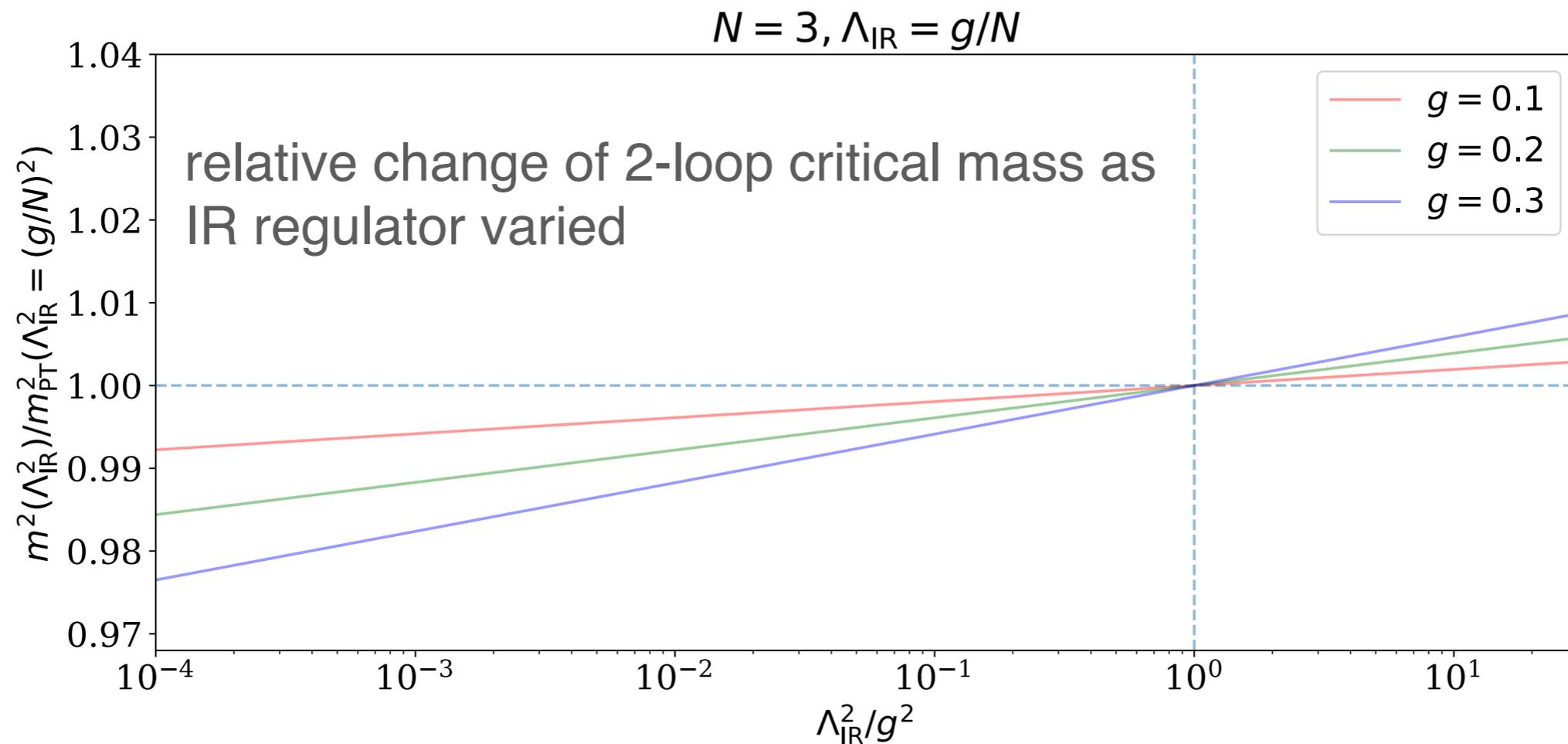
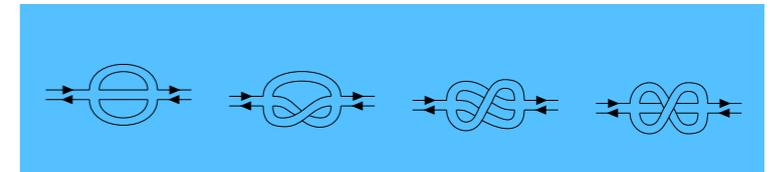
# What about IR divergence?

$$D_{\Lambda_{\text{IR}}}(p) = \int_{-\pi/a}^{\pi/a} \frac{d^3 k}{(2\pi)^3} \frac{d^3 q}{(2\pi)^3} \frac{1}{(\hat{k}^2 + \Lambda_{\text{IR}}^2)(\hat{q}^2 + \Lambda_{\text{IR}}^2)(\hat{r}^2 + \Lambda_{\text{IR}}^2)}$$



# What about IR divergence?

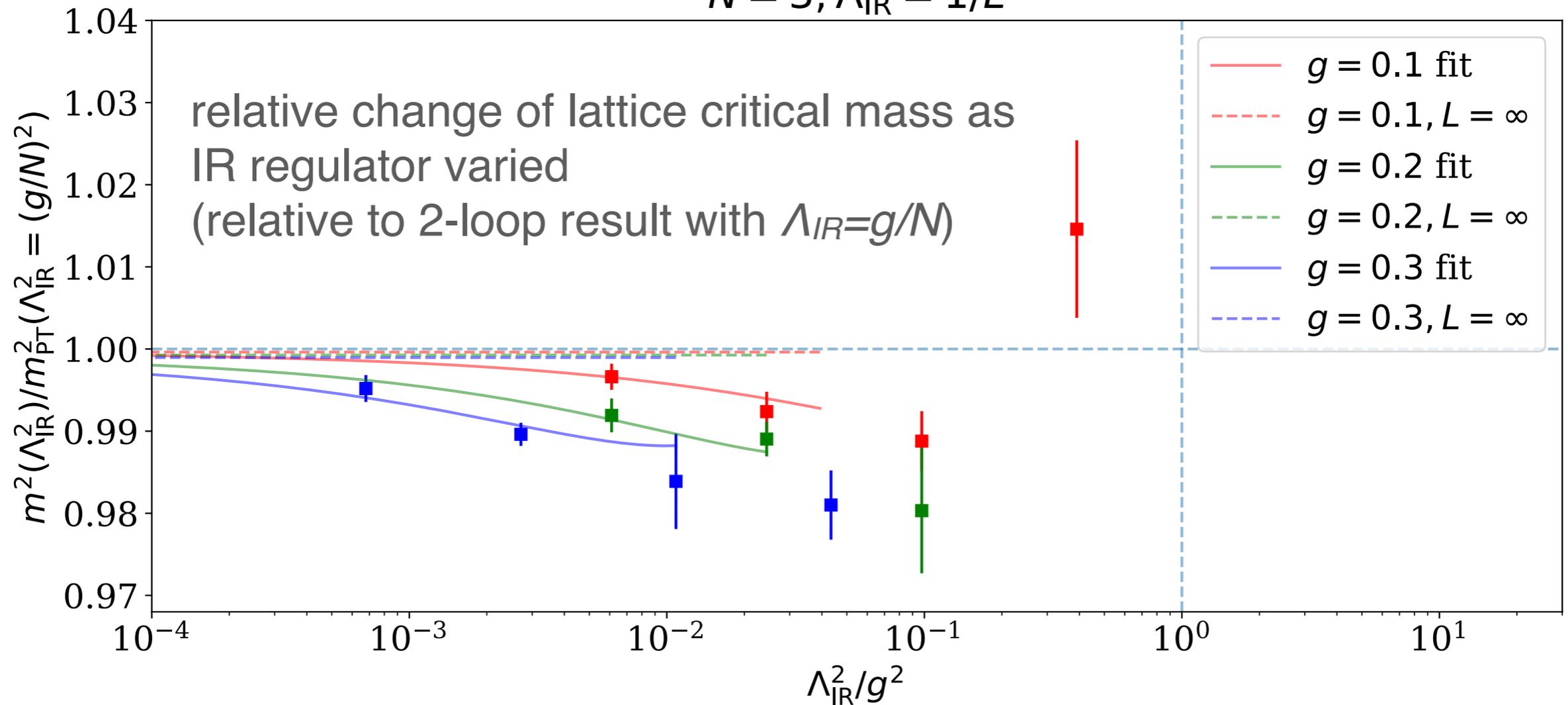
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plot shows logarithmic divergence as IR-cutoff removed

# What about IR divergence?

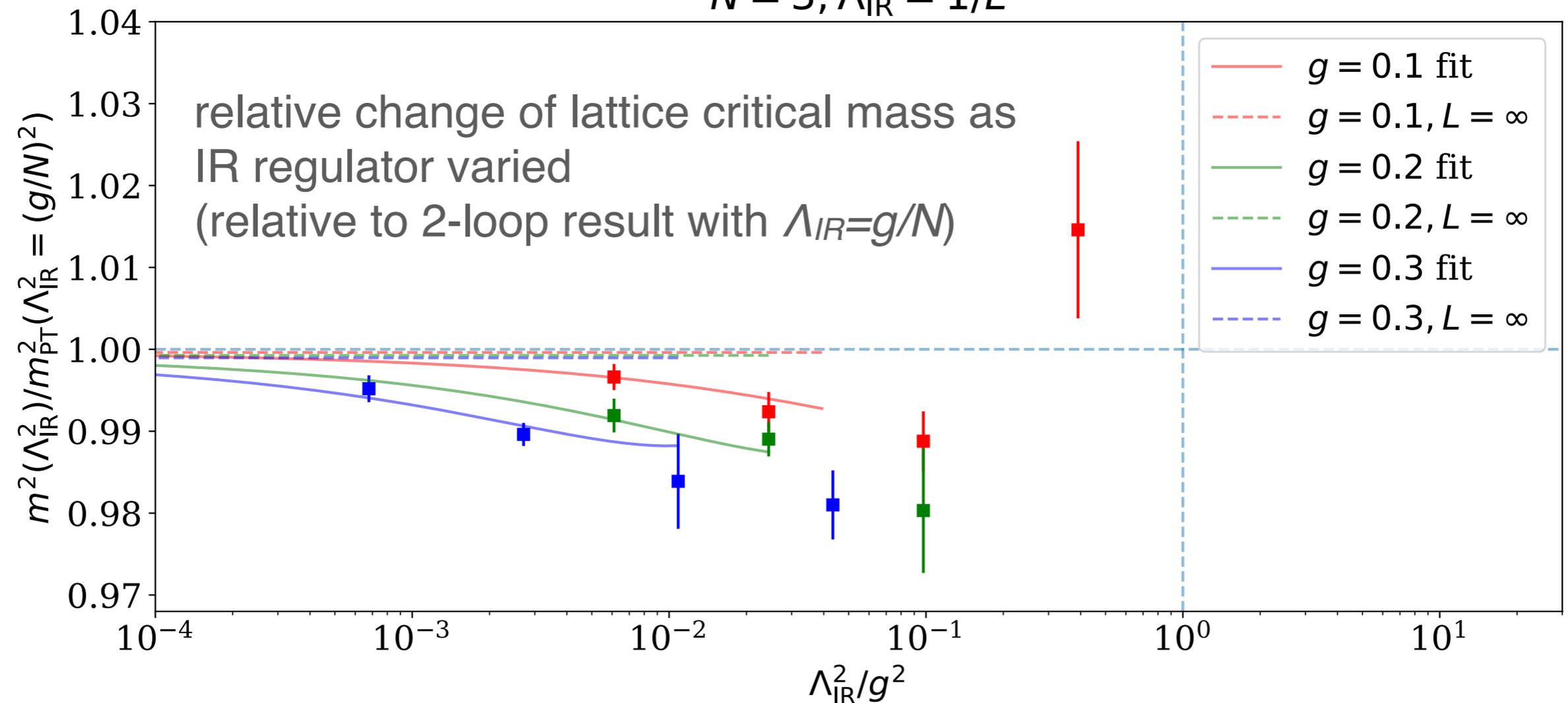
$$N = 3, \Lambda_{\text{IR}} = 1/L$$



- 2-loop PT agrees with lattice results for critical mass at below-per-cent level when YM coupling used as IR regulator  $\Lambda_{\text{IR}}=g/N$
- For  $L \rightarrow \infty$  lattice data converges near  $L=\infty$  PT — it does not seem to diverge

# What about IR divergence?

$$N = 3, \Lambda_{\text{IR}} = 1/L$$



Accumulating evidence for anticipated nonperturbative IR regularisation in superrenormalisable QFT

see Jackiw, Templeton PRD 23 1981  
Appelquist, Pisarski PRD 23 1981

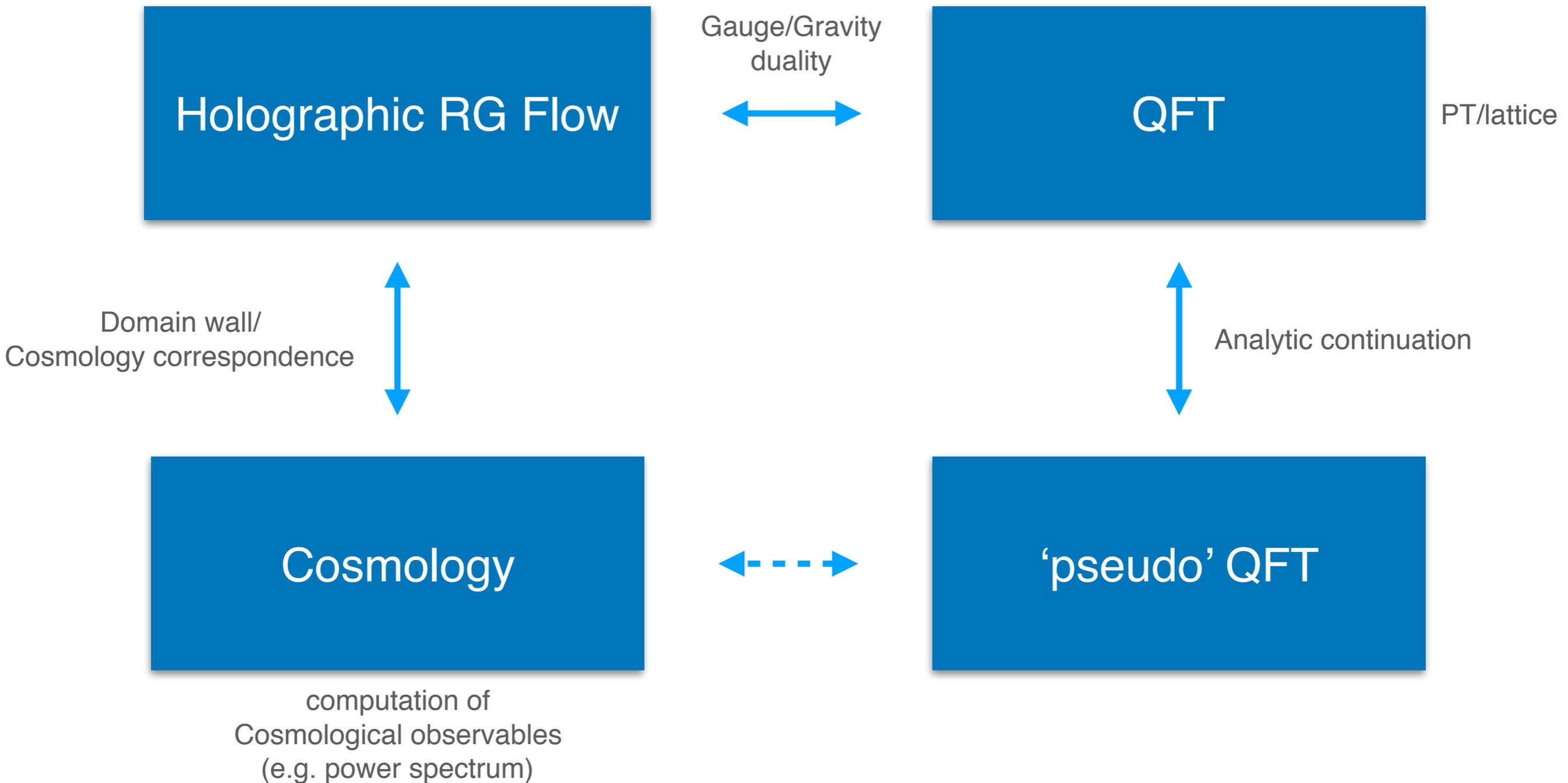
# Conclusions

- Exciting project that covers the entire range of lattice QFT, Holography, theoretical and observational Cosmology
- Infinite volume critical masses determined in PT and non-perturbatively do agree very well if YM coupling used as PT IR regulator
- We see evidence accumulating for absence of IR divergence in 3d superrenormalisable QFT beyond PT
- **Next steps:**
  - computation of correlation functions (next talk by Joseph Lee)
  - test predictions for Cosmology against Planck data
  - study theories with different matter content

# Supplementary slides

# Holographic Cosmology

McFadden and Skenderis, PRD 81, 021301 (2010)



# N-scaling

L=128, g=0.3

