Towards a holographic description of cosmology (II):

Renormalisation of the 3D SU(N) scalar energy-momentum tensor

Joseph Lee

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Lattice 2019 Wuhan

Collaboration

Edinburgh

Luigi Del Debbio

Joseph Lee

Antonin Portelli

Guido Cossu

Valentin Nourry

Southampton

Masanori Hanada

Andreas Jüttner

Ben Kitching-Morley

Kostas Skenderis

Queen's

Elizabeth Gould

LLNL

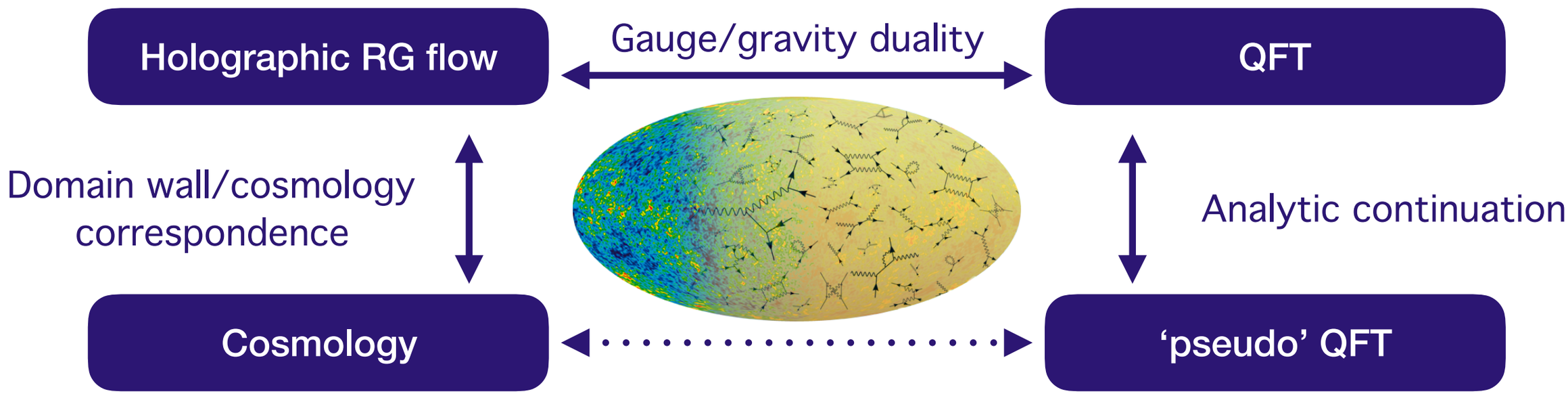
Pavlos Vranas

Outline

- Introduction: Holographic CMB spectrum
- A first step: scalar SU(N) matrix model
- Non-perturbative lattice simulations
- Energy Momentum Tensor (EMT) renormalisation
- Conclusion & outlook

Holographic Cosmology

- Very early Universe - strong gravity: use holographic principle



P.L McFadden, K. Skenderis
[PRD 81(2) 2010]
[JPCS 222(1) 2010]
[JCAP 05 2011]

- Dual theory ansatz: 3D SU(N) gauge theory with arbitrary content of massless scalars and fermions

Holographic CMB Spectrum

- CMB primordial scalar power spectrum:

Energy momentum tensor

$$\Delta^2_{\mathcal{R}}(q) = -\frac{q^3}{4\pi^2}\frac{1}{\langle T(q)T(-q)\rangle} \qquad (T = T_{\mu\mu})$$

[Afshordi et al., PRL 118(4)

& PRD 95(1), 2017]

- Holographic Cosmology 2-loop PT

$$\Delta^2_{\mathcal{R}-\mathrm{HC}}(q) = \frac{\Delta_0^2}{1 + \frac{g q_*}{q}\log\left|\frac{q}{\beta g q_*}\right|}$$

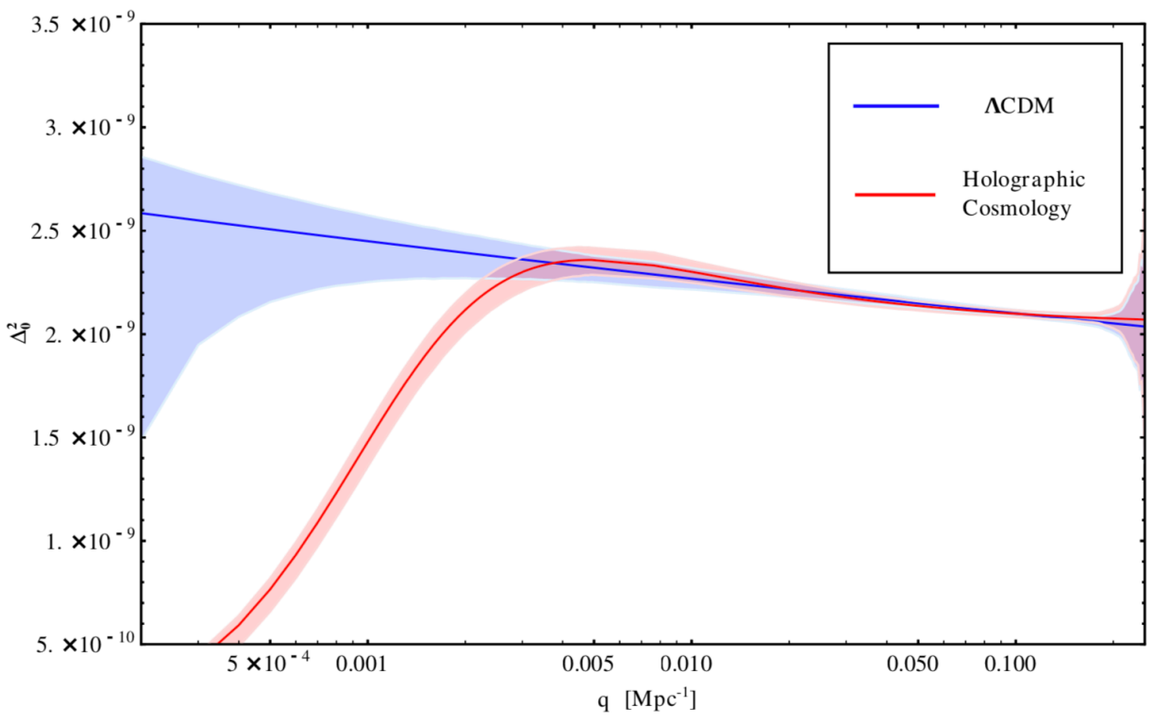
Free parameters: depend on content of theory

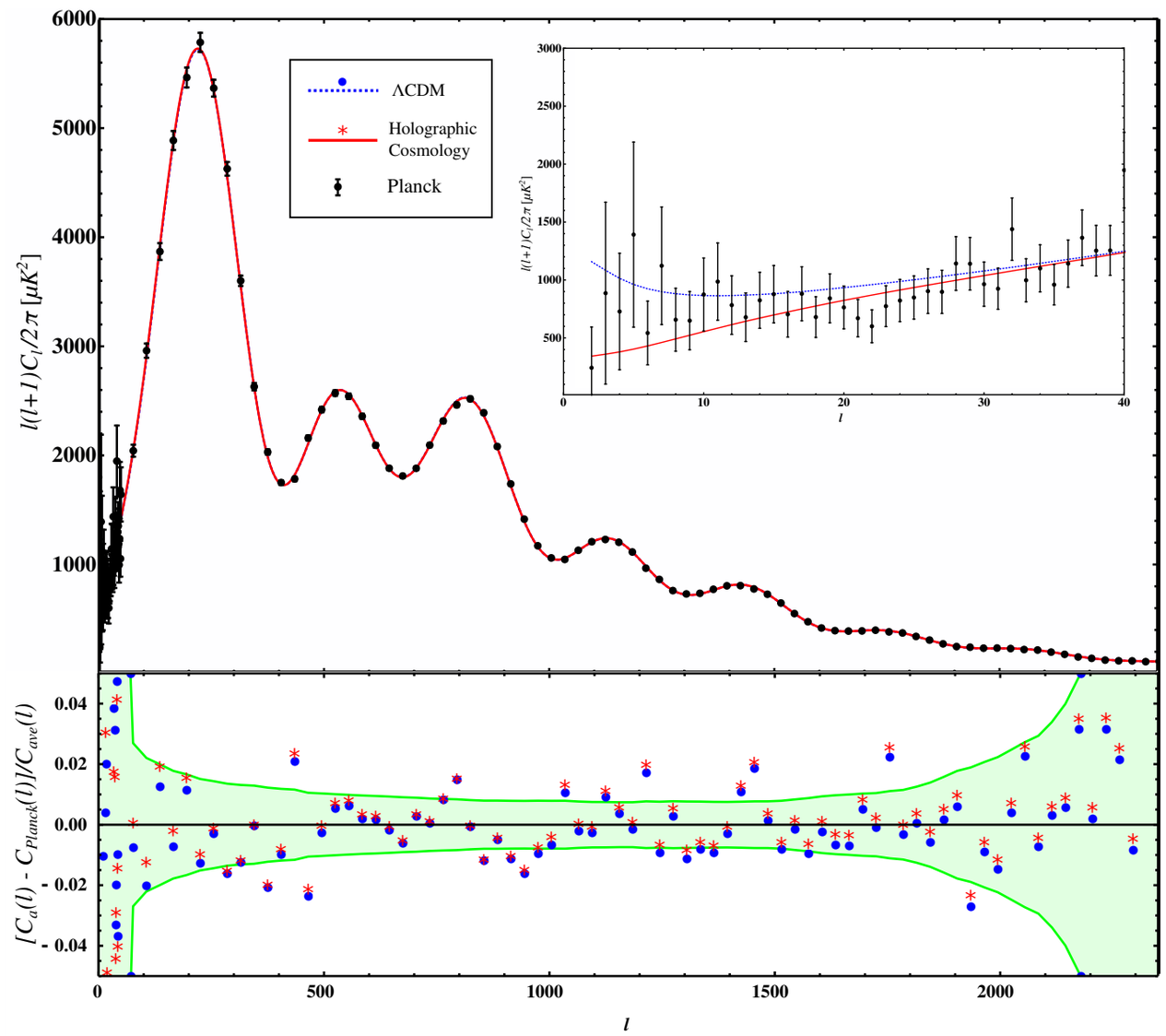
- vs ΛCDM

$$\Delta^2_{\mathcal{R}-\Lambda\mathrm{CDM}}(q) = \Delta_0^2\left(\frac{q}{q_*}\right)^{n_s-1}$$

Confronting Planck data

- Competitive with ΛCDM (within regime of PT validity)
- No fermions
- Large N





[Afshordi et al., PRL 118(4) & PRD 95(1), 2017]

Conclusion of Planck analysis

- Dual theory massless and super-renormalisable

  → Dimensionful coupling $[g] = 1$

  → Correlation function expansion in $g_{\rm eff} = \dfrac{g}{|q|}$

  → Each order of PT: degree of divergence

  → IR divergences in PT when $g_{\rm eff}(q) \gtrsim 1$

UV ↓

IR ↑

Conclusion of Planck analysis

- Planck fit suggests dual theory becomes non-perturbative around $l \sim 35$

[Afshordi et al., PRL 118(4) & PRD 95(1), 2017]

- IR divergences expected to be an artefact of PT

[Jackiw, Templeton, PRD 23(10), 1981]
[Appelquist, Pisarski, PRD 23(10), 1981]

- IR region describe low-multipole region of CMB spectrum

→ Motivation for non-perturbative lattice calculation

The model

- 3D $\mathfrak{su}(N)$-valued massless scalar matrix $\phi_i^j(x) = \phi^a(x)\,(T^a)_i^j$

- Action:

$$S = \frac{N}{g} \int d^3x \, \mathrm{Tr} \left(\left(\delta_\mu \phi(x)\right)^2 + m_c^2 \phi^2(x) + \phi^4(x) \right)$$

  - Set critical mass m_c^2 for massless theory (earlier talk by Andreas Jüttner)

- First simulation: SU(2) theory

  - 4 lattice spacings

  - 2 masses close to critical mass

Non-perturbative window

- Non-perturbative (small momentum) window:

$$g_{\text{eff}} > 1$$

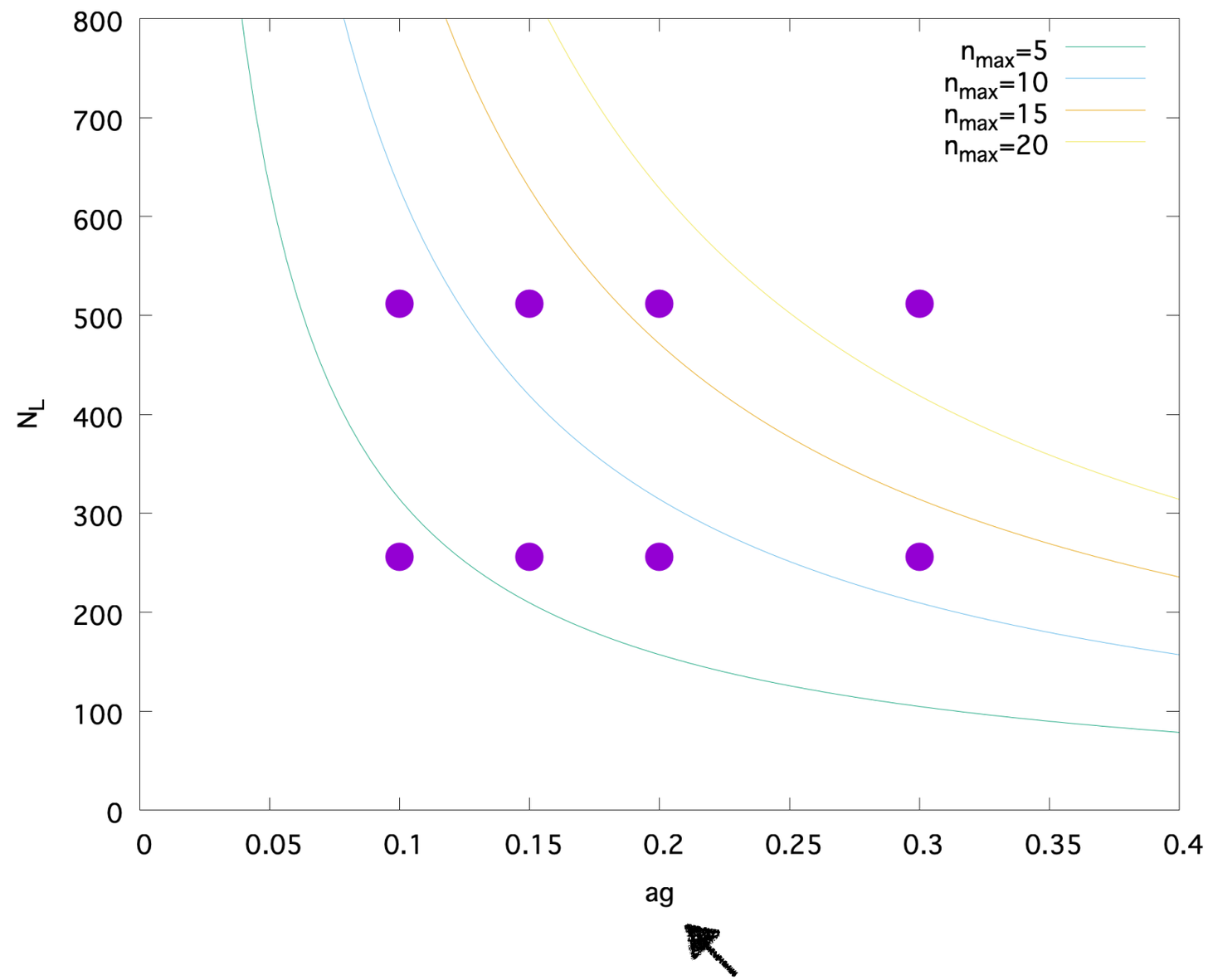
- Momentum quantisation → discrete values

- Number of points in non-perturbative window:

$$n_{\max} = \frac{N_L a g}{2\pi}$$

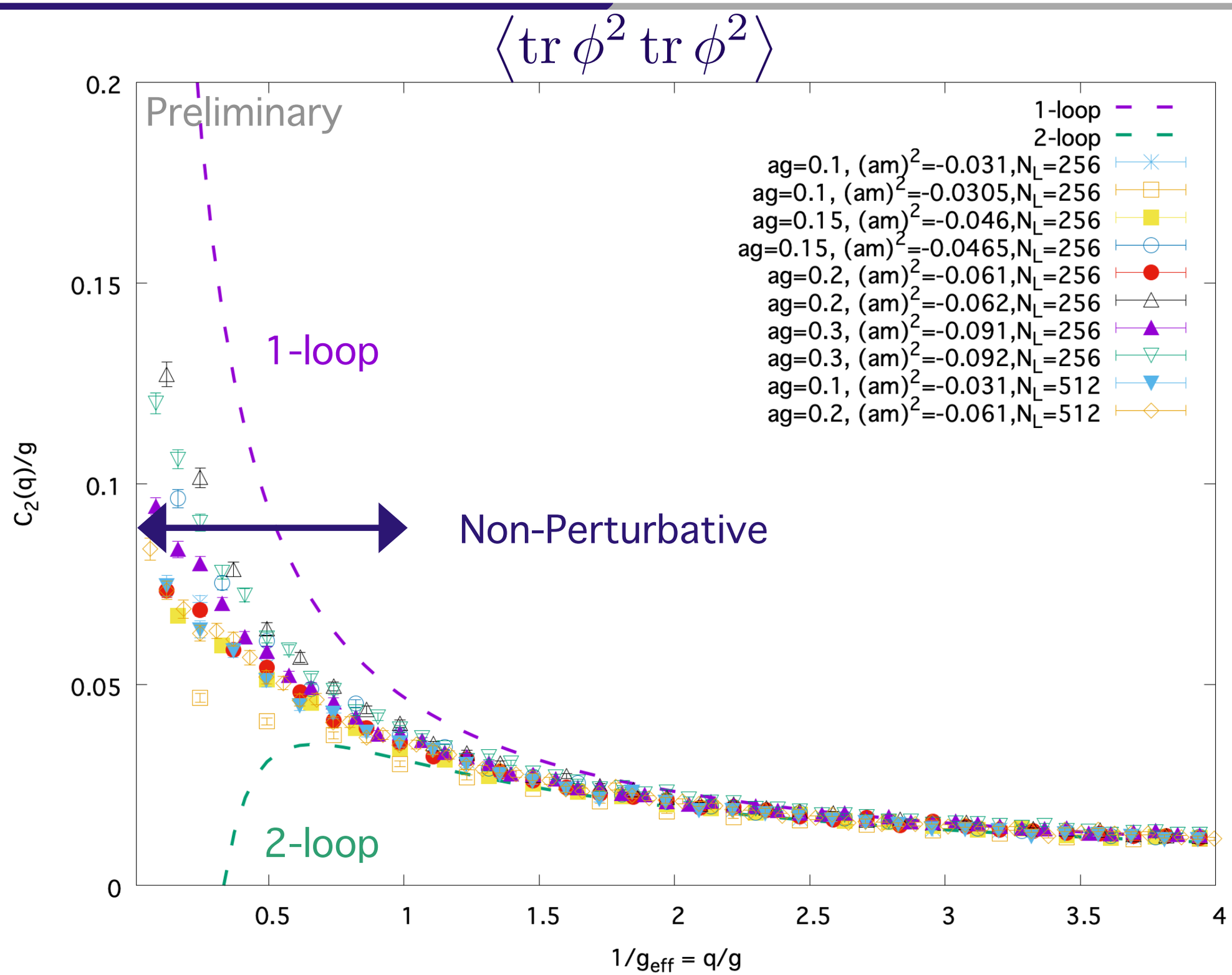
- Large lattice volumes:

$$256^3 \quad 512^3$$



Dimensionless lattice spacing

Non-perturbative window



Renormalisation Strategy

- Goal: to renormalise $\langle T_{\mu\nu} T_{\rho\sigma} \rangle$

- 2 sources of divergence

  I) $T_{\mu\nu}$ operator mixing

  - Impose Ward identity on $\langle T_{\mu\nu} \, \mathrm{tr}\, \phi^2 \rangle$

  II) Bi-linear correlator divergence

  - Subtract $\frac{1}{a^3}$ constant divergence with 0 momentum value

  - Subtract $\frac{1}{a} \, q^2$ divergence by fit function

Lattice Energy-Momentum Tensor

- First candidate: massless SU(N) scalar matrix theory

- Naive discretisation of EMT:

$$\hat{T}_{\mu\nu} = \frac{g}{N} \operatorname{tr} \left\{ 2 \left(\overline{\delta}_\mu \phi \right) \left(\overline{\delta}_\nu \phi \right) - \delta_{\mu\nu} \left[\sum_\rho \left(\overline{\delta}_\rho \phi \right)^2 + m_c^2 \phi^2 + \phi^4 \right] \right\}$$

- Ward identities in the continuum:

$$\partial_\mu \left\langle T_{\mu\nu} P \right\rangle = - \left\langle \delta_{x,\nu} P \right\rangle \quad \text{where} \quad \delta_{x,\nu} P = \frac{\delta P}{\delta \phi(x)} \partial_\nu \phi(x)$$

$\rightarrow$ $\langle T_{\mu\nu} P \rangle$ UV finite

Lattice Energy-Momentum Tensor

- Ward identity on the lattice:

$$\left\langle \hat{P}\left(\overline{\delta}_\mu \hat{T}_{\mu\nu} + \hat{R}_\nu\right)\right\rangle = -\left\langle \overline{\delta}_{x,\rho}\hat{P}\right\rangle$$

- Mixing with irrelevant operators $\hat{R} \rightarrow \hat{T}$ renormalises

- Divergence proportional to $\delta_{\mu\nu}\,\mathrm{tr}\,\phi^2$

[Caracciolo et al., NPB 309(4), 1988]

$$\hat{T}^{(R)}_{\mu\nu} = \frac{g}{N}\,\mathrm{tr}\left\{2\left(\overline{\delta}_\mu\phi\right)\left(\overline{\delta}_\nu\phi\right) - \delta_{\mu\nu}\left[\sum_\rho\left(\overline{\delta}_\rho\phi\right)^2 + \left(m_c^2 + c_3\right)\phi^2 + \phi^4\right]\right\}$$

Operator Mixing

$$\hat{T}^{(R)}_{\mu\nu} = \frac{g}{N}\,\mathrm{tr}\left\{2\left(\overline{\delta}_\mu\phi\right)\left(\overline{\delta}_\nu\phi\right) - \delta_{\mu\nu}\left[\sum_\rho\left(\overline{\delta}_\rho\phi\right)^2 + \left(m_c^2 + c_3\right)\phi^2 + \phi^4\right]\right\}$$

- Determine operator mixing c_3 by imposing Ward identities on:

$$C_{\mu\nu}(q) = \langle \hat{T}^{(R)}_{\mu\nu}(q)\,\mathrm{tr}\,\phi^2(-q)\rangle$$

$$q_\mu C_{\mu\nu} = 0$$

$$\rightarrow C^{(R)}_{\mu\nu}(q) := \left(F(q) - F(0)\right)\left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) = \left(F(q) - F(0)\right)\pi_{\mu\nu}$$

Remove constant divergence

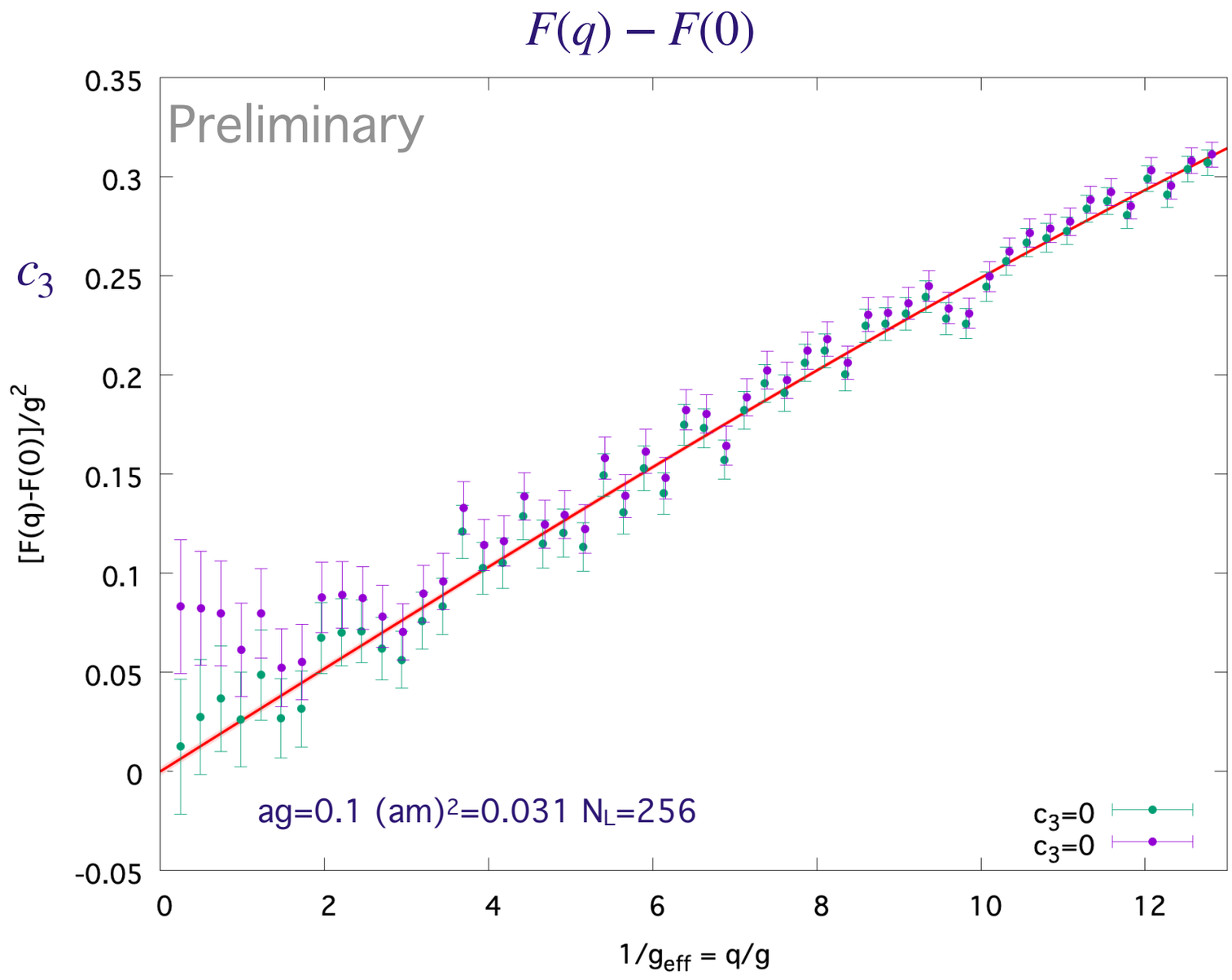
$\langle T_{\mu\nu} \mathrm{tr}\phi^2 \rangle$

- Ward identity → $C_{22}\left(q = (0, 0, q_2)\right) = 0$

- Determine operator mixing c_3

- Fit: $C_{\mu\nu}(q) - C_{\mu\nu}(0) = g\hat{q}\alpha_{\mu\nu}$

$$\hat{q} = \frac{2}{a}\sin\left(\frac{aq}{2}\right)$$

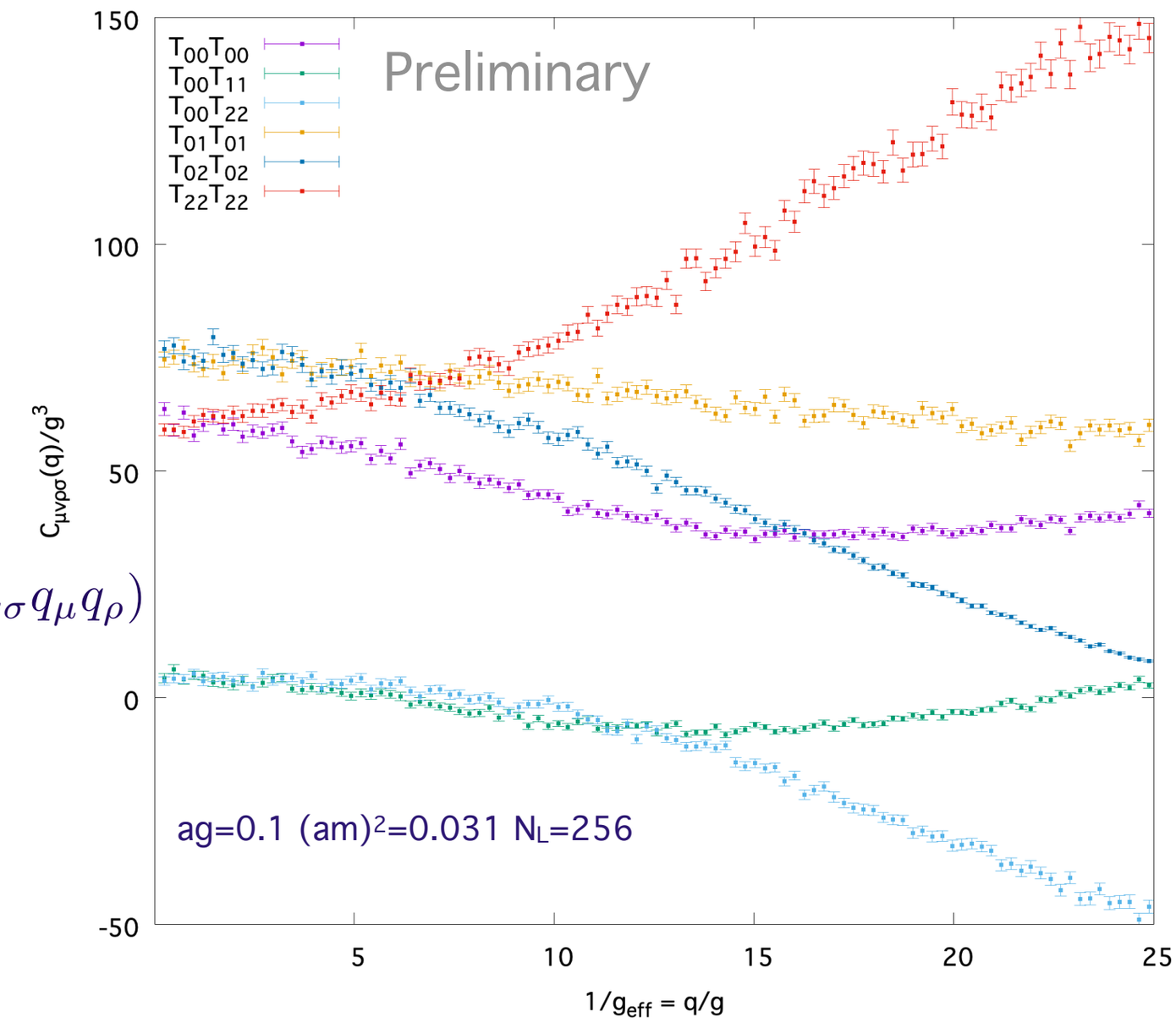


$\langle T_{\mu\nu} T_{\rho\sigma} \rangle$

- Energy-momentum 2-point function

$$C_{\mu\nu\rho\sigma}(q) = \left\langle \hat{T}^{(R)}_{\mu\nu}(q) \hat{T}^{(R)}_{\rho\sigma}(-q) \right\rangle$$

$$= G_1(q)\delta_{\mu\nu}\delta_{\rho\sigma}$$
$$+G_2(q)(\delta_{\mu\rho}\delta_{\nu\sigma} + \delta_{\mu\sigma}\delta_{\nu\rho})$$
$$+G_3(q)(\delta_{\mu\nu}q_\rho q_\sigma + \delta_{\rho\sigma}q_\mu q_\nu)$$
$$+G_4(q)(\delta_{\mu\rho}q_\nu q_\sigma + \delta_{\mu\sigma}q_\nu q_\rho + \delta_{\nu\rho}q_\mu q_\sigma + \delta_{\nu\sigma}q_\mu q_\rho)$$
$$+G_5(q)\ q_\mu q_\nu q_\rho q_\sigma$$
$$+H_1(q)\delta_{\mu\nu\rho\sigma} + \cdots$$

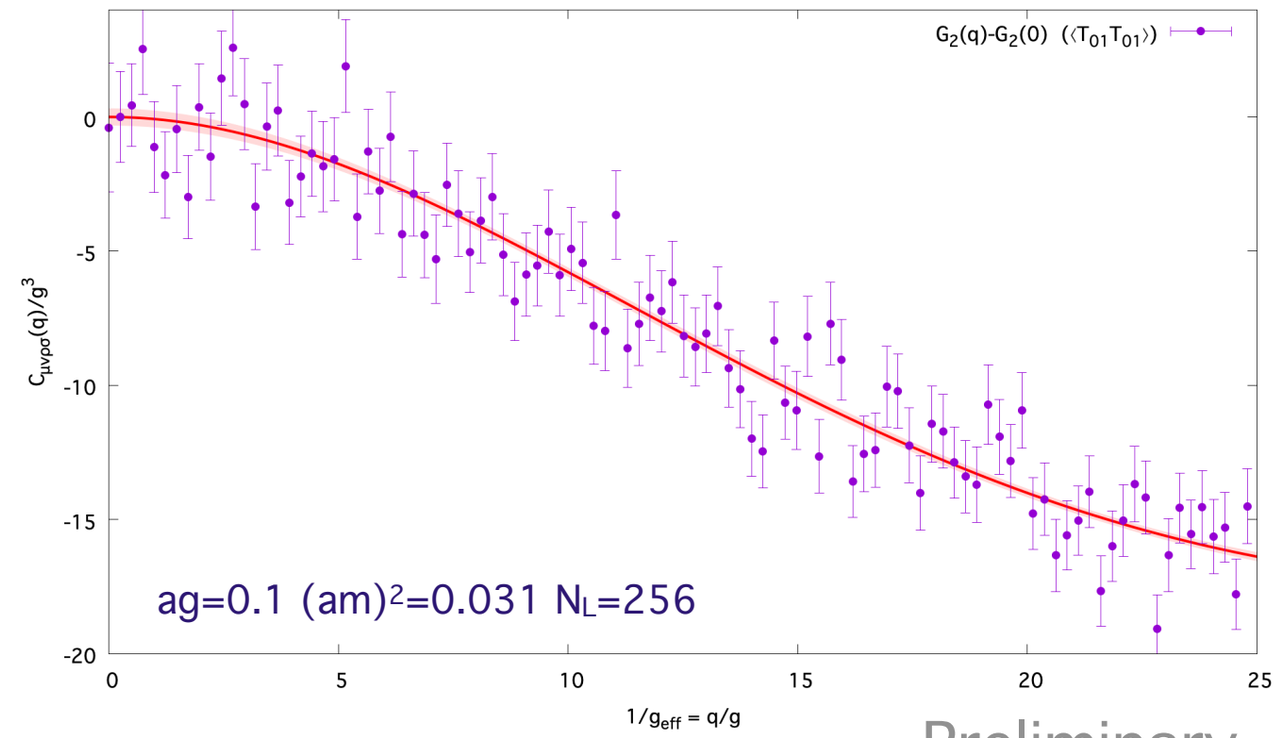


$\langle T_{\mu\nu} T_{\rho\sigma} \rangle$

- Fit: $G_i(q) - G_i(0) = \beta_i \frac{1}{a} \hat{q}^2 + \gamma_i \hat{q}^3$

- Remove $\sim \frac{1}{a} q^2$ divergence:

$$\rightarrow G_i^{(R)}(q) = \gamma_i q^3 f(g_{\text{eff}})$$



Preliminary

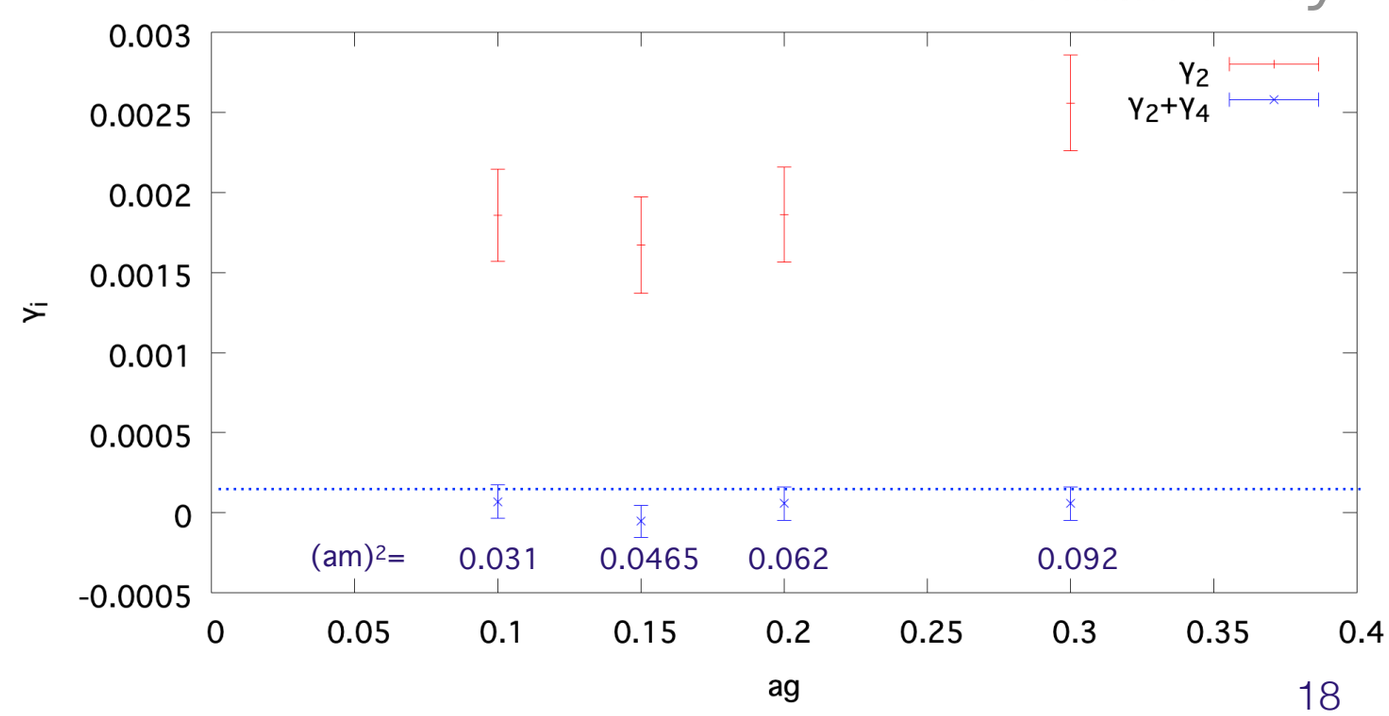
- Verify Ward identity:

$$q_\mu C_{\mu\nu\rho\sigma}(q) = q_\mu \langle \hat{T}^{(R)}_{\mu\nu}(q) \hat{T}^{(R)}_{\rho\sigma}(-q) \rangle = 0$$

$$\rightarrow G_2^{(R)}(q) + G_4^{(R)}(q) = 0$$

$$G_1^{(R)}(q) + G_3^{(R)}(q) = 0$$

$$G_5^{(R)}(q) - G_1^{(R)}(q) - 2G_2^{(R)}(q) = 0$$



Renormalised CMB spectrum

- Renormalised 2-point function:

$$\rightarrow C_{\mu\nu\rho\sigma}(q) = (A(q) - A(0))\Pi_{\mu\nu\rho\sigma} + (B(q) - B(0))\pi_{\mu\nu}\pi_{\rho\sigma}$$

$$A(q) \equiv 2G_2^{(R)}(q) \qquad B(q) \equiv G_1^{(R)}(q) + G_2^{(R)}(q)$$

Where $\Pi_{\mu\nu\rho\sigma} = \frac{1}{2}\left(\pi_{\mu\rho}\pi_{\nu\sigma} + \pi_{\mu\sigma}\pi_{\nu\rho} - \pi_{\mu\nu}\pi_{\rho\sigma}\right)$

- To obtain CMB spectrum $\rightarrow \Delta^2_{\mathcal{R}}(q) = -\frac{q^3}{16\pi^2}\frac{1}{B(q)}$

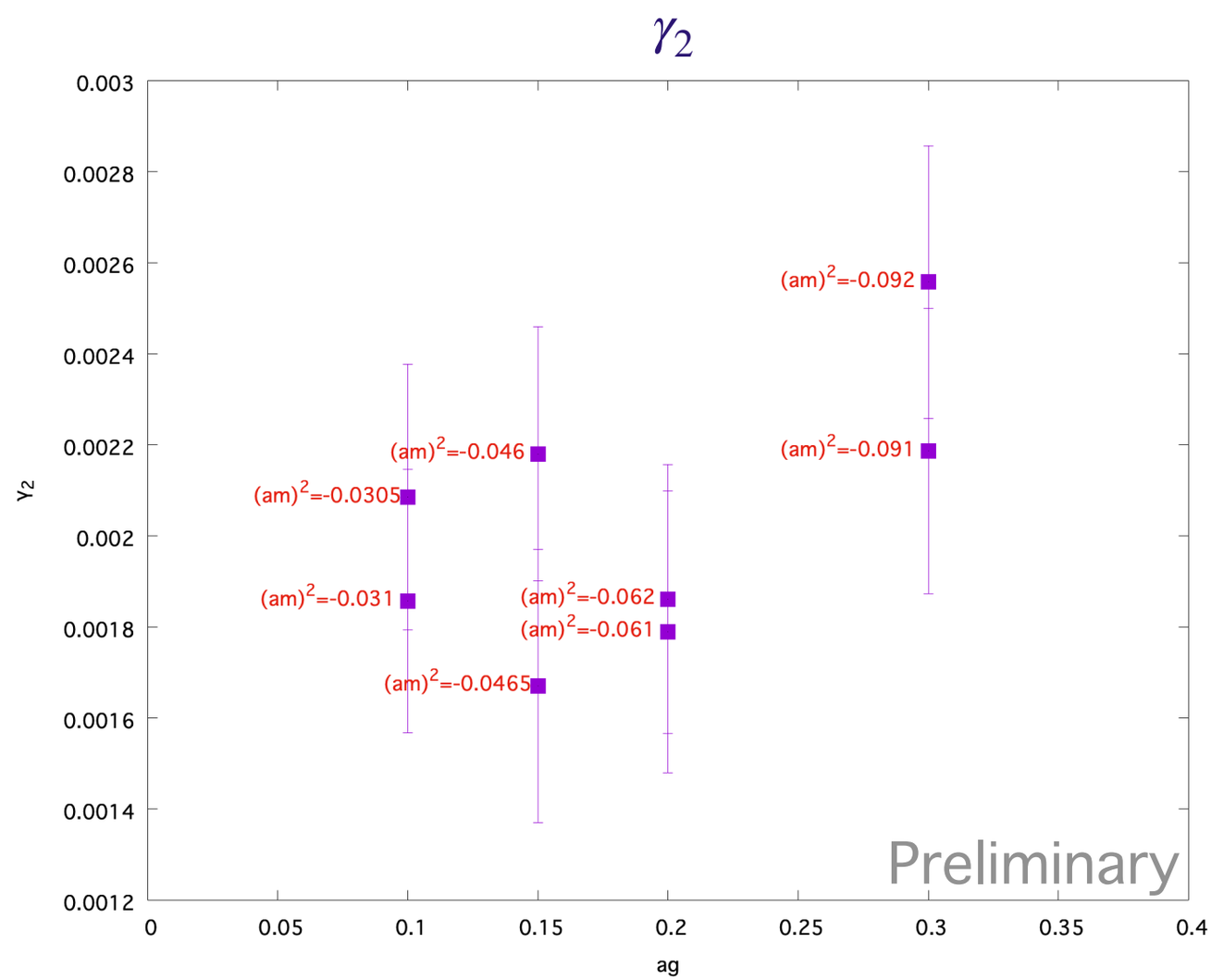
(tensor power spectrum: $\Delta^2_T(q) = -\frac{2q^3}{\pi^2}\frac{1}{A(q)}$)

Conclusion

- Reviewed holographic description of cosmology for CMB
- Explore non-perturbative region with lattice simulation
- Renormalise EMT 2-point function using Ward identities
- Fit data to obtain form factors

Outlook

- Extrapolate to massless, infinite volume, continuum limit
- Compare against Planck CMB
- Repeat for gauge + scalar theory



Form factor coefficient at different lattice spacings & masses for extrapolation

Thank you

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