Chiral Condensate and Susceptibility of SU(2) $n_f = 8$ Naive Staggered System

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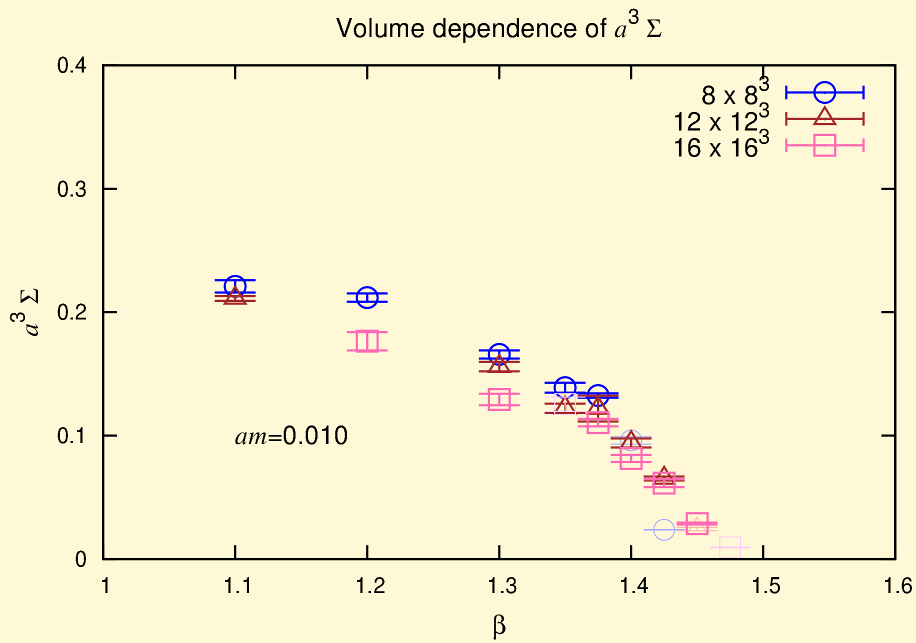
Outline

1. Motivation
2. Random Matrix Theory
3. Chiral Condensate
4. Chiral Susceptibility
5. Conclusions

Motivation

- SU(2) $N_f = 8$ system: IR fixed point Leino et al. 2017
  composit Higgs model, theoreitcal interest,...
- our previous report (lattice 2015) with naive staggered fermion:
  a bulk transition at $\beta \simeq 1.4$



lattice 2015 talk by C.-J. D. Lin

- purpose of this talk:
  $\boxed{\text{a closer look at this transition using chiral condensate}}$
  (using a new result from random matrix theory)

chiral Random Matrix Theory

- (low lying) Dirac spectra can be describe with chiral Random Matrix Theory ($N = \infty$) Shuryak-Verbaarschot 1993
- distribution of individual eigenvalues
  $\Rightarrow$ *infinite volume* chiral condensate

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  chGSE (Dyson index $\beta = 4$) with $m_f \neq 0$
  analytical formula for individual eigenvalue distribution (Damgaard-Nishigaki 2001) does not apply to integer $n_f > 0$

$\Rightarrow$brute force HMC estimation of $N = 400$ RMT (lattice. 2015)

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  $\Rightarrow$brute force HMC estimation of $N = 400$ RMT (lattice. 2015)
- $\boxed{\text{New Tool}}$ Fuji-I.K.-Nishigaki, arXiv:1903.07176

  compact analytic expression + numerical estimation

  $\Rightarrow$accurate numerical estimation for distribution of individual eigenvalues

Method: chiral condensate by using RMT

The distribution of individual small eigenvalue in the *broken* phase

$$\rho_{\mathrm{QCD}}(m;\lambda_i) = \rho_{\mathrm{RMT}}(\mu = V\Sigma_{\mathrm{param}} m, \zeta_i = V\Sigma_{\mathrm{param}}\lambda_i)$$

One parameter (Σ_{param}) fit of the distrib. $\Rightarrow$ $\boxed{\text{chiral condensate}}$

- λ_i: eigenvalue of the Dirac operator
- ζ_i: eigenvalue of the RMT
- μ: mass parameter for RMT
- Σ_{param}: chiral condensate [rescaling factor btw QCD(-like)/RMT]

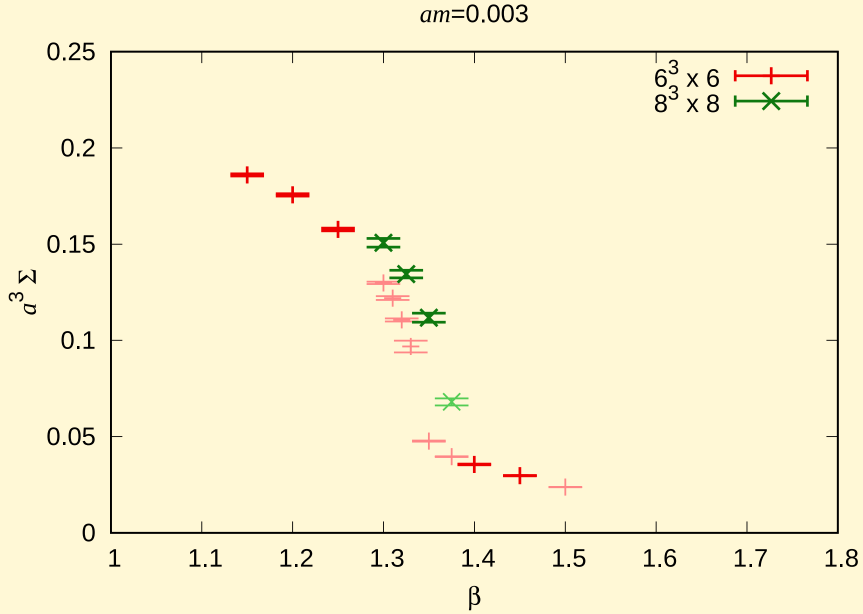
Lattice setup

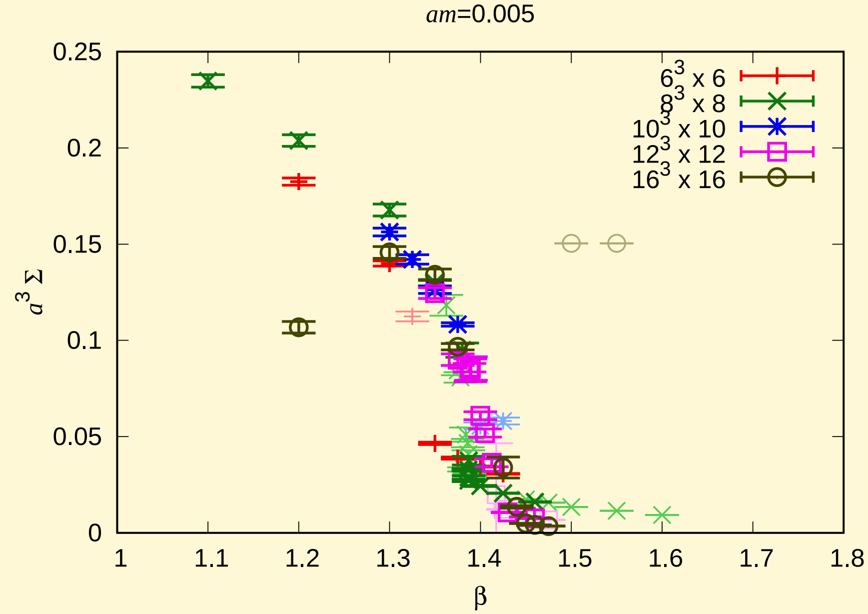
- Action: plaquette + naive staggered fermion
- periodic in all directions
- $L^3 \times T = 6^3 \times 6 – 16^3 \times 16$
- $am = 0.003, 0.005, 0.010(, 0.015)$
- fundamental $n_f = 8$
- gauge group: SU(2)

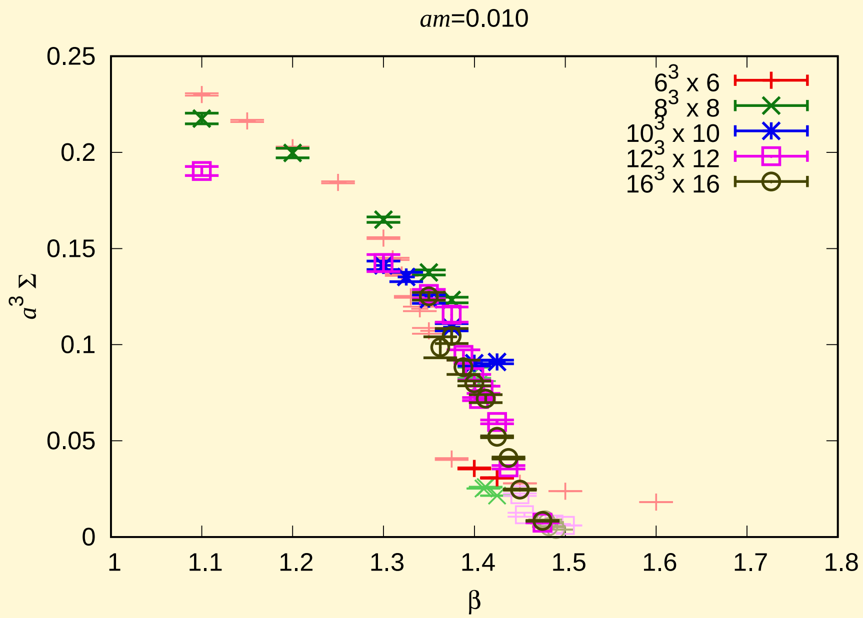
to apply RMT...

- $$n_f = 8 \quad \Rightarrow \quad 8 \quad \underbrace{\times\frac{1}{4}}_{\text{taste breaking}} \quad \underbrace{\times 2}_{\text{SU(2): extra degeneracy}} \quad \overset{n_f \text{ for RMT}}{=} \boxed{4}$$
- RMT works only in the broken phase. We expect poor fitting in the symmetric phase

Chiral Condensate (update)

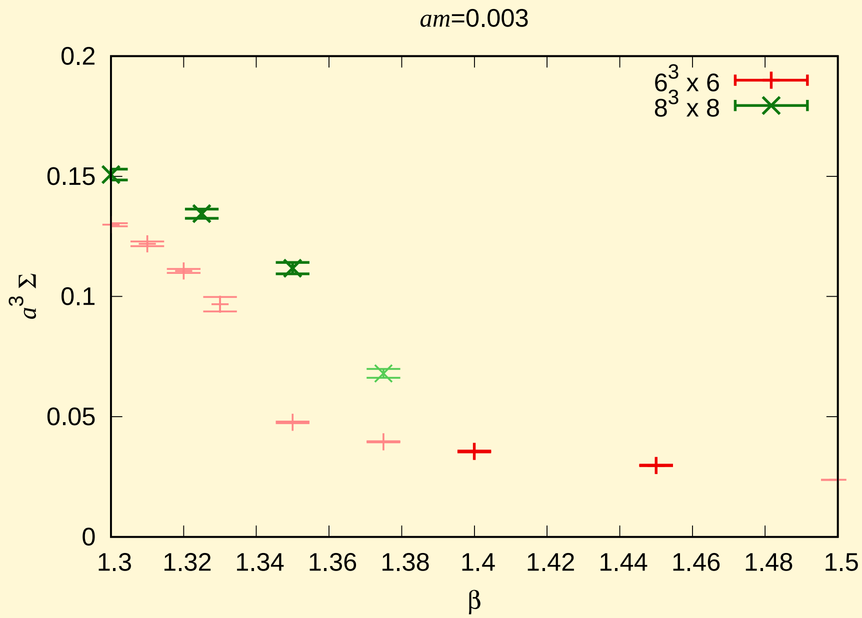


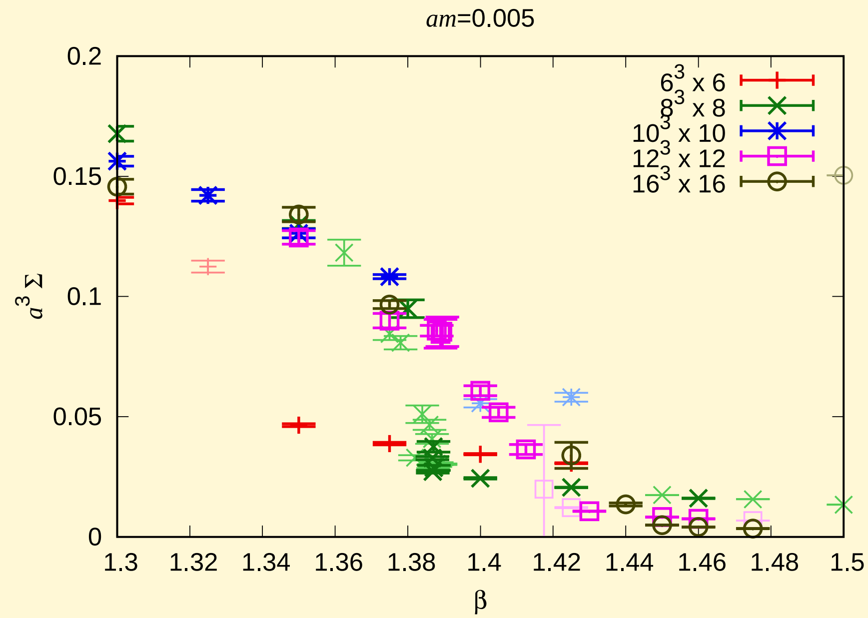


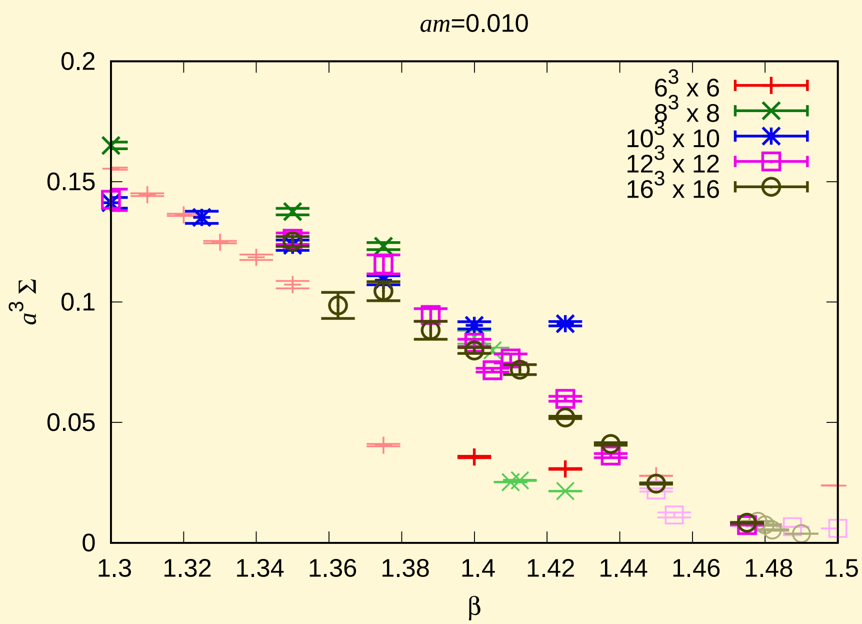


- pale colors: χ^2/d.o.f > 1.5
- as β goes larger (strong coupling to weak coupling), the condensate almost disappears and ends up with poor fitting
- almost no volume dependence in this scale

Chiral Condensate (update; zoom up)







- pale colors: $\chi^2/\text{d.o.f} > 1.5$
- $L = 6$ and $L = 8$ give smaller condensate

Can we say something more?

we can say...

- a bulk transition btw broken/symmetric phase of chiral symm.
- Simulation for conformal window:

  must be performed in the symmetric phase

  (i.e., weak coupling side)

...something more?

- how about the other of the transition?
- can we use RMT more?

susceptibility with RMT

in the ϵ-regime:
$Z_{\mathrm{QCD}}(m;\lambda_i) = Z_{\mathrm{RMT}}(\mu = V\Sigma_{\mathrm{param}} m; \zeta_i = V\Sigma_{\mathrm{param}}\lambda_i)$

$$\begin{aligned}\chi &= -\frac{1}{n_f}\frac{1}{V}\frac{\partial^2}{\partial^2 m}\ln Z_{\mathrm{QCD}} = -\frac{1}{n_f}V\Sigma_{\mathrm{param}}\frac{\partial^2}{\partial\mu^2}\ln Z_{\mathrm{RMT}} \\ &= V\Sigma_{\mathrm{param}}^2\left\{\langle A(\mu)\rangle_{\mathrm{RMT}} + n_f\left(\langle B(\mu)B(\mu)\rangle_{\mathrm{RMT}} - \left(\langle B(\mu)\rangle_{\mathrm{RMT}}\right)^2\right)\right\} \\ &= \frac{\Sigma_{\mathrm{param}}}{m}\mu\left\{\langle A(\mu)\rangle_{RMT} + n_f\left(\langle B(\mu)B(\mu)\rangle_{\mathrm{RMT}} - \left(\langle B(\mu)\rangle_{\mathrm{RMT}}\right)^2\right)\right\}\end{aligned}$$

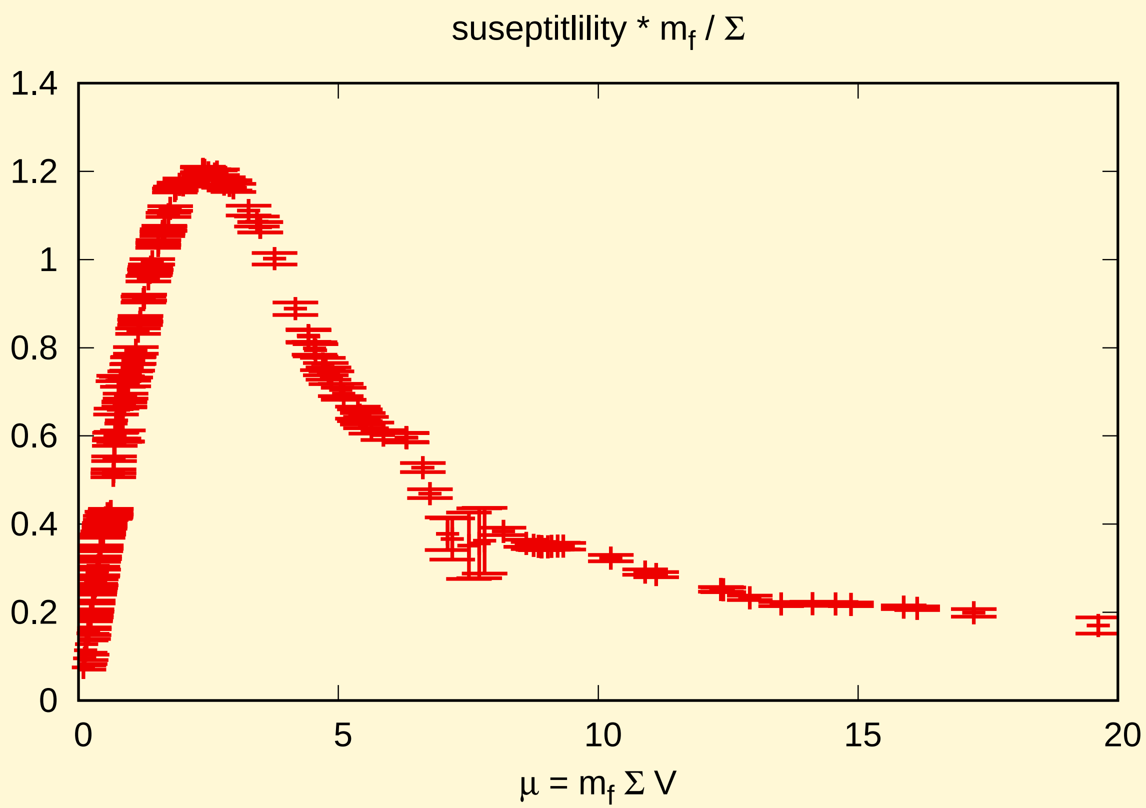
with $A(\mu) = \sum_i\left[\frac{2}{\zeta_i^2+\mu^2} - \frac{4\mu^2}{\left(\zeta_i^2+\mu^2\right)^2}\right]$, $\qquad B(\mu) = \sum_i \frac{2\mu}{\zeta_i^2+\mu^2}$

cf. M.E. Berbenni-Bitsch et al, 1999

- Once Σ_{param} (and thus μ) is obtained by fit, we can calculate χ
- To estimate $\langle\bullet\rangle_{\mathrm{RMT}}$, we use HMC with $N = 2000$

speculation

- RMT describes the system only for small μ cf. M.E. Berbenni-Bitsch et al., 1999
  (smaller than the Thouless energy)



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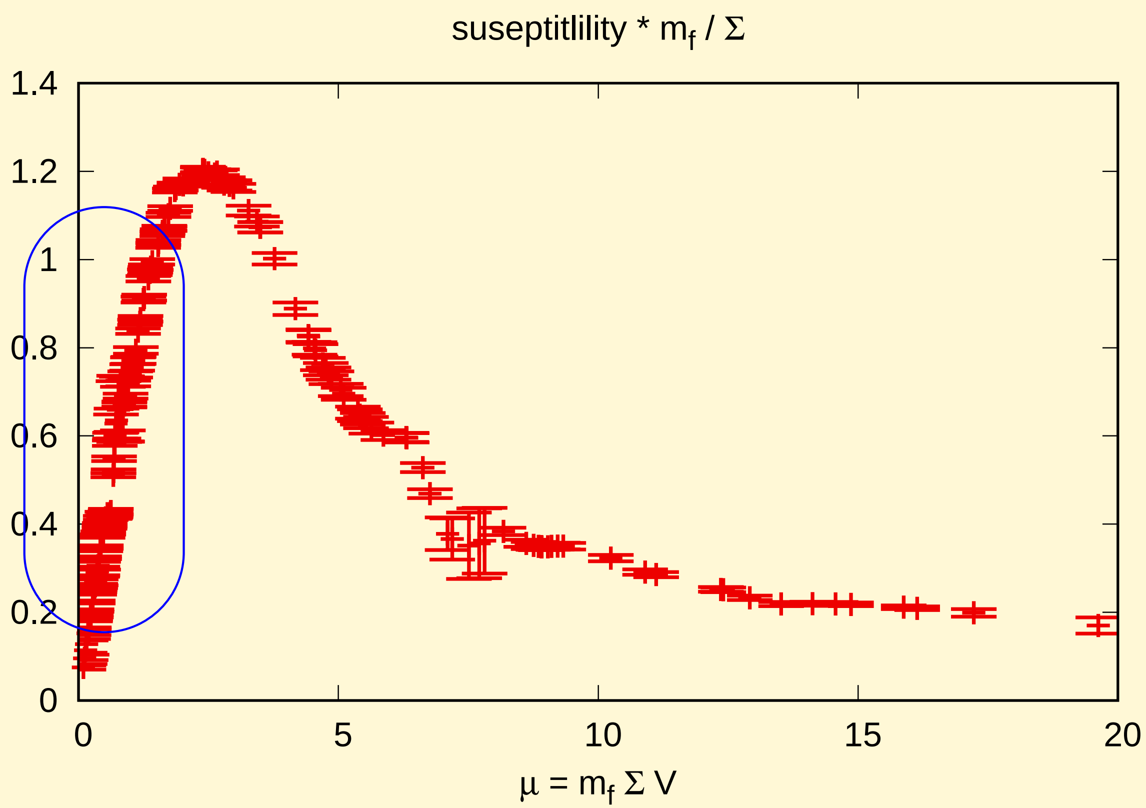
- Fixed m_f and Σ ($\simeq$ fixed β):
  the susceptibility is linear in μ ($\lesssim 2$)$\Rightarrow$ $\boxed{\text{linear in the volume}}$
  Is this also true for the *peak*?



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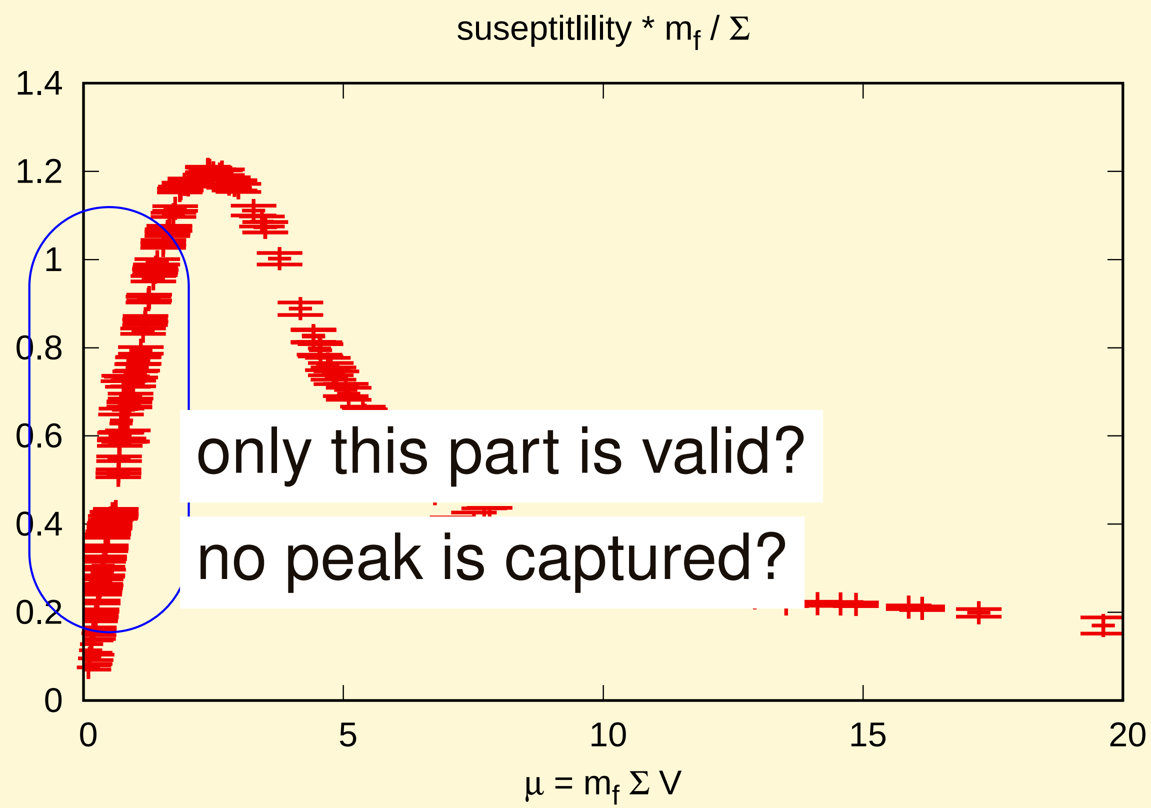

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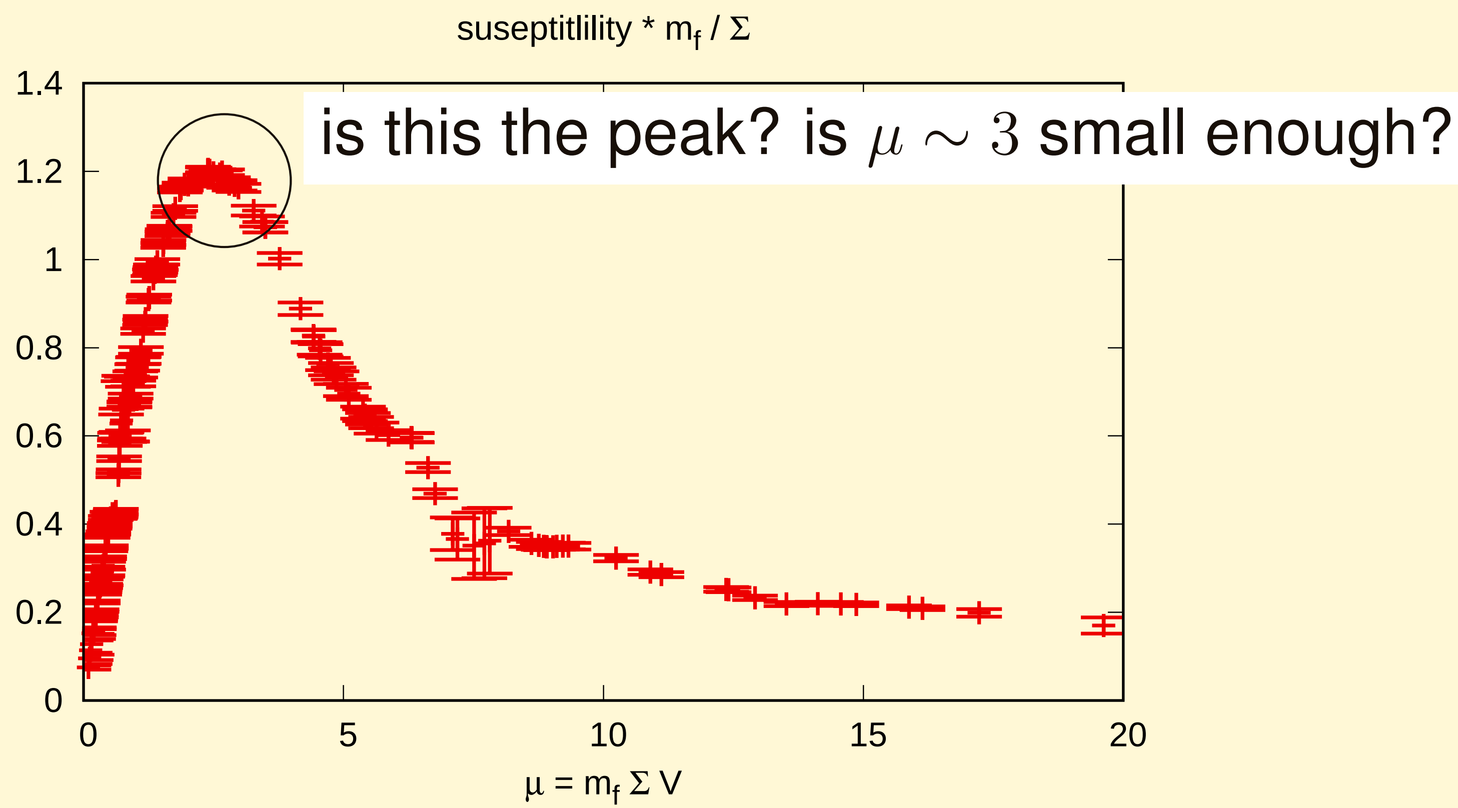
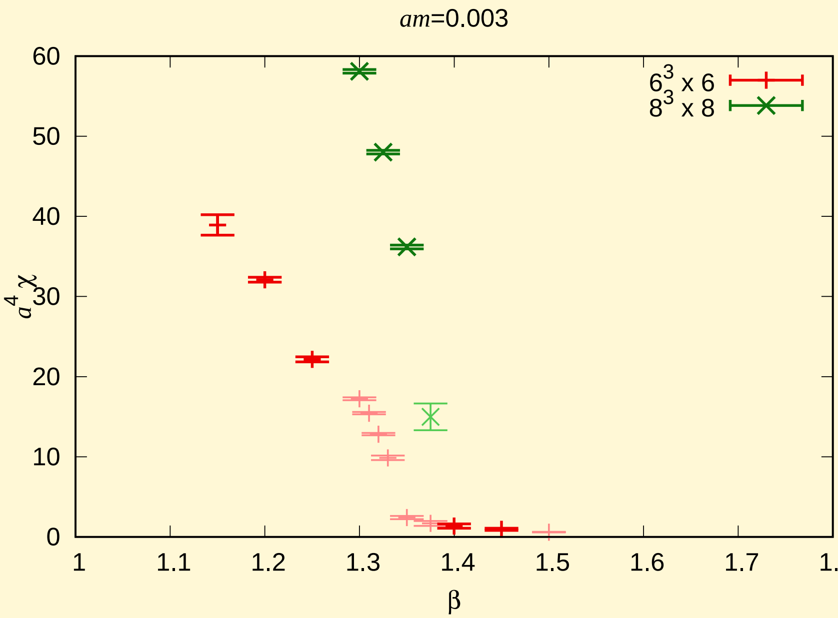
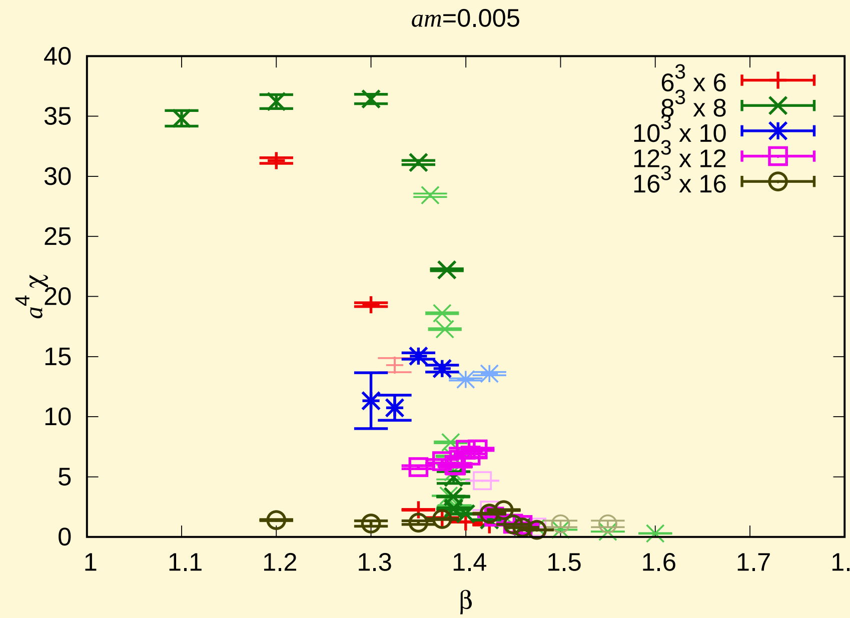
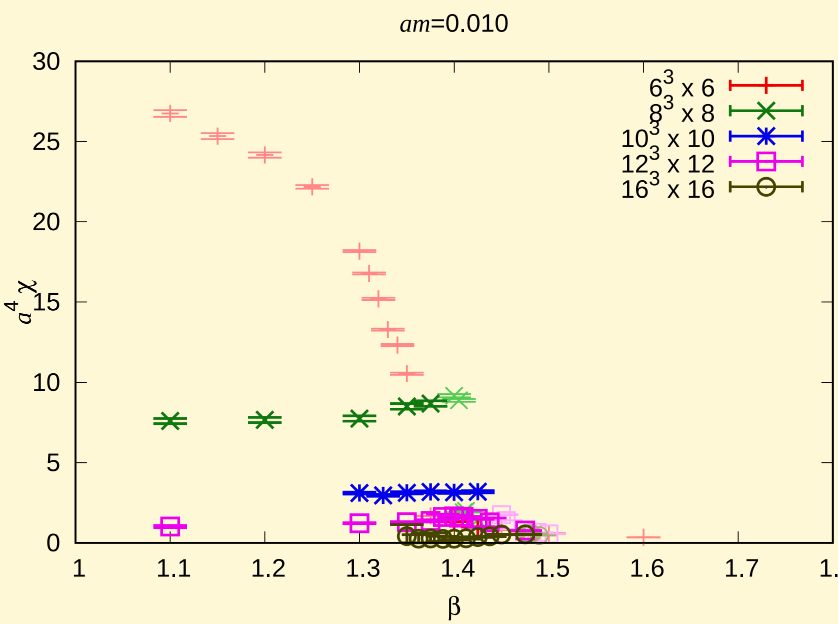
- Fixed m_f and V, varying μ is varying $\beta = 4/g^2$ through Σ_{param}



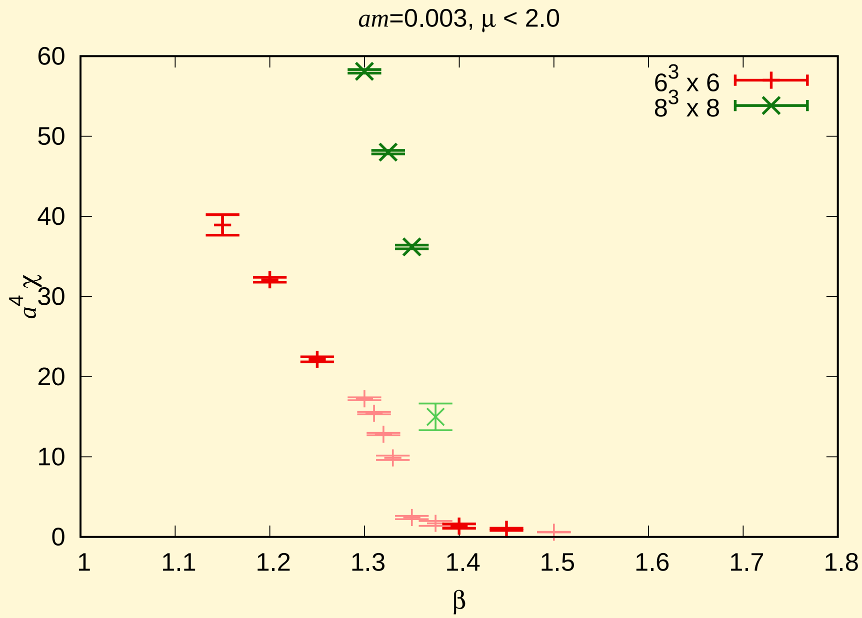
plot of the susceptibility

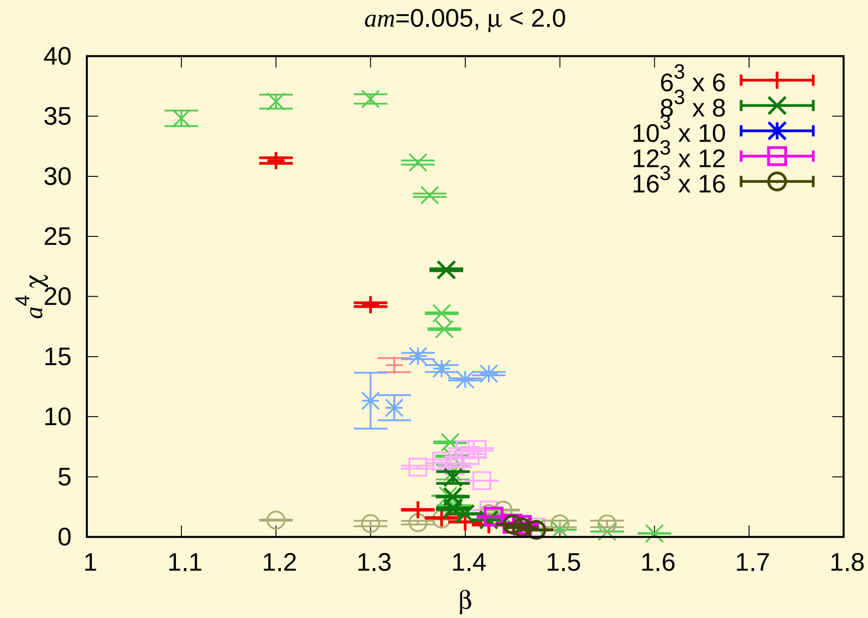


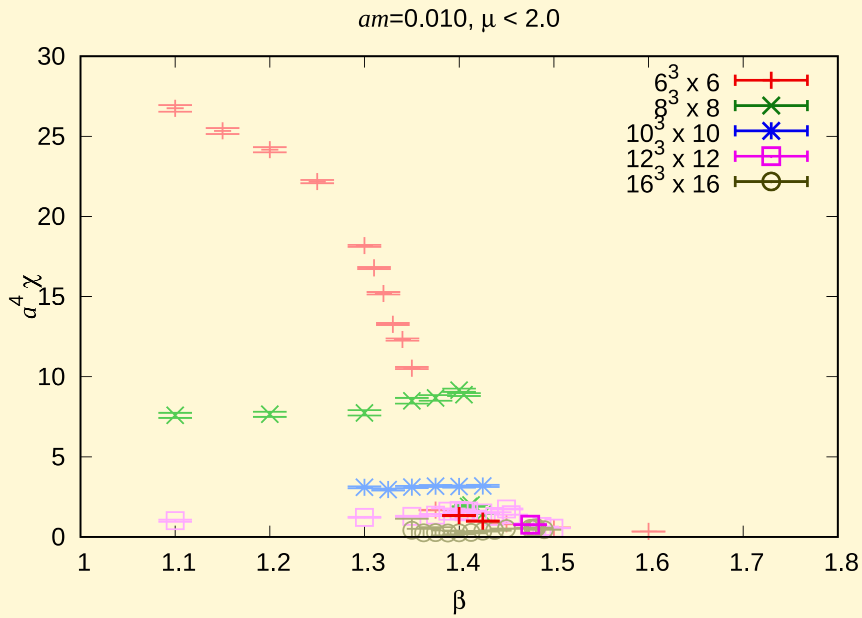




- no clear peak
- for $am = 0.005, 0.010$, larger volume gives smaller susceptibility







- data with $\mu < 2$ is quite limited
- no peak
  $\Rightarrow$ *not* first order

Conclusions

SU(2) fundamental $n_f = 8$ system with naive staggered fermion

- bulk transition at $\beta = 4/g^2 \sim 1.4$:

  chiral sym. is broken at strong coupling
- used new estimations of RMT for chGSE ($\beta = 4$)
- RMT analysis of chiral susceptibility:

  no peak appears, not first order

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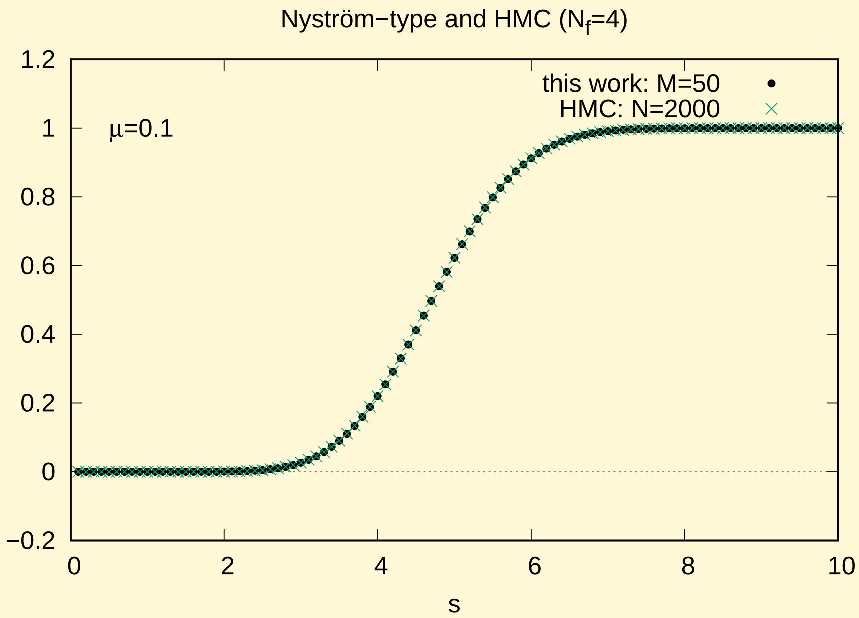
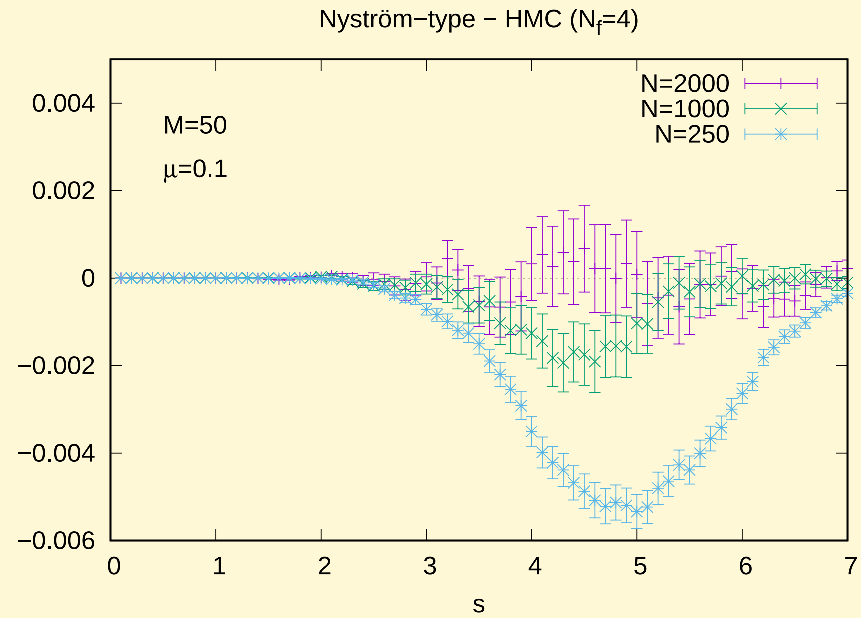
Thank you.

Backup Slides

RMT: integrated dist. of the smallest eigenvalue

estimation with HMC vs. Nyström type discretization





Fuji-I.K.-Nishigaki, arXiv:1903.07176

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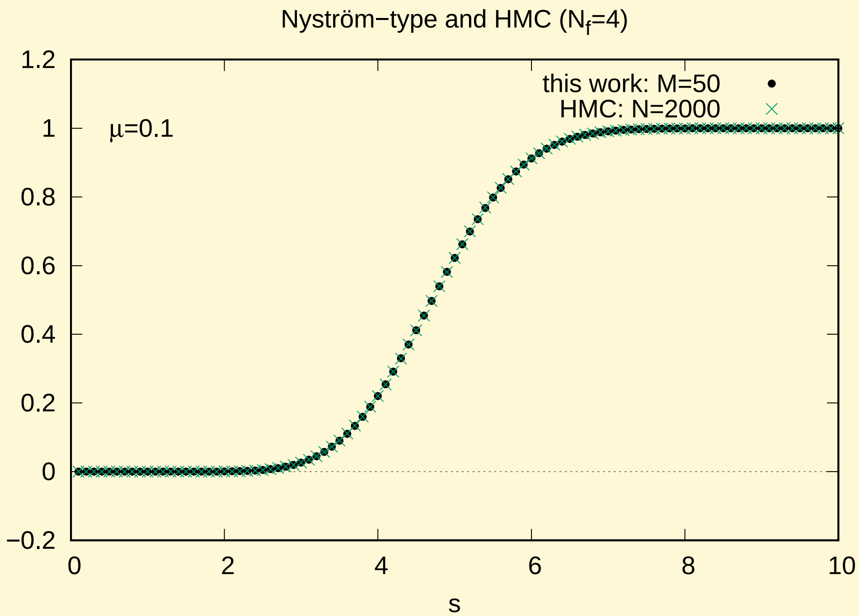
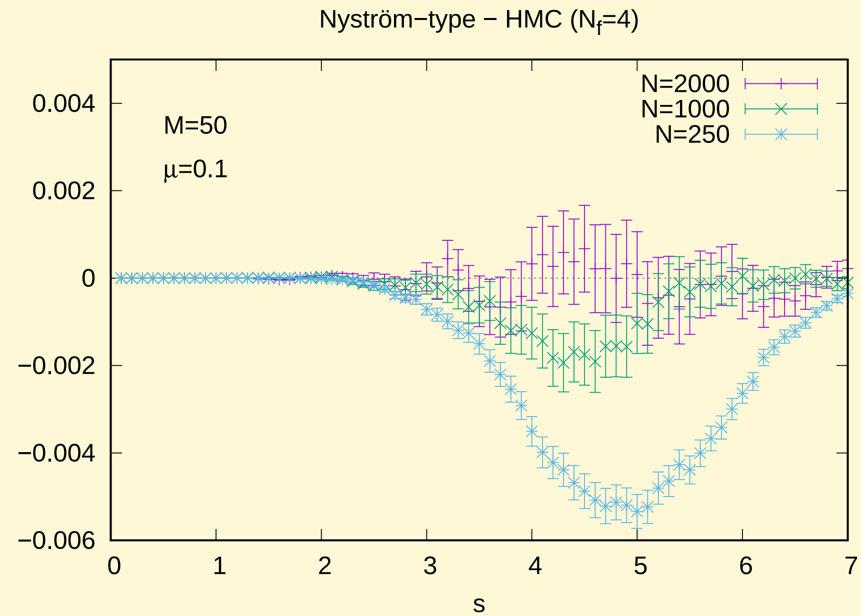
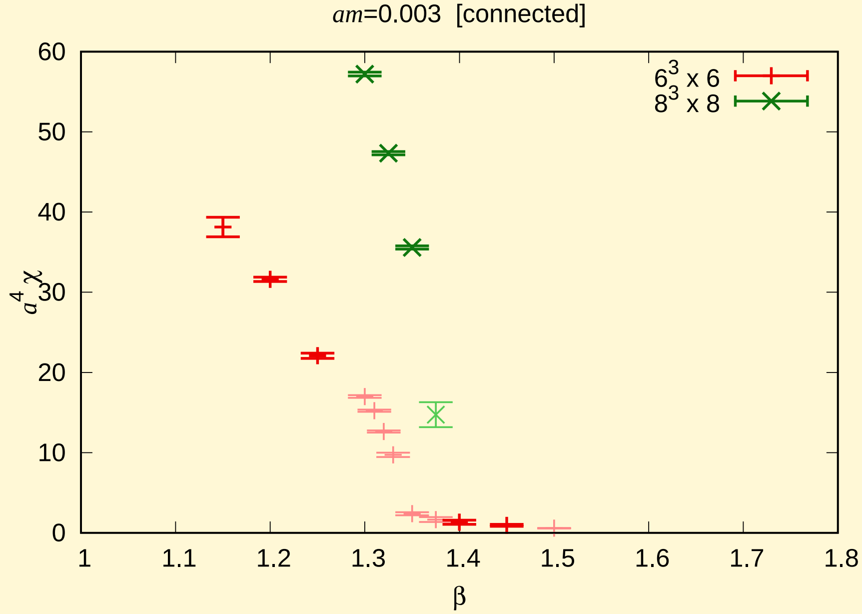


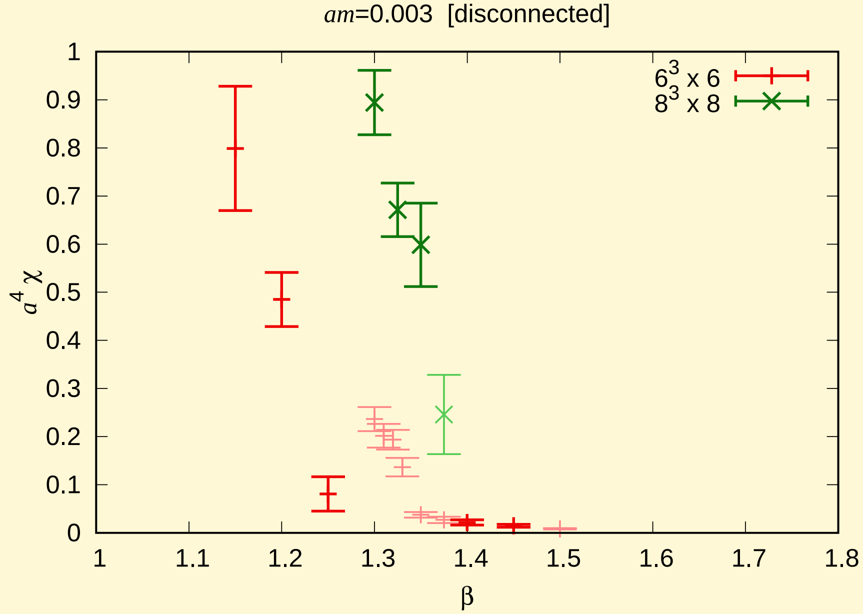


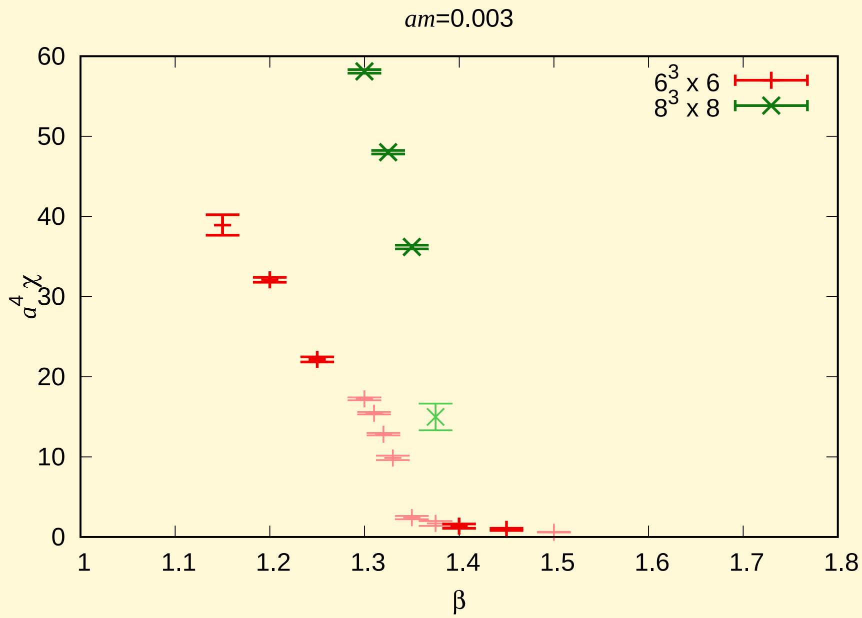
Fuji-I.K.-Nishigaki, arXiv:1903.07176

...seems HMC with $N = 400$ (lat.2015) was not very accurate, though the difference is within the error from the lattice simulation

connected vs. disconnected







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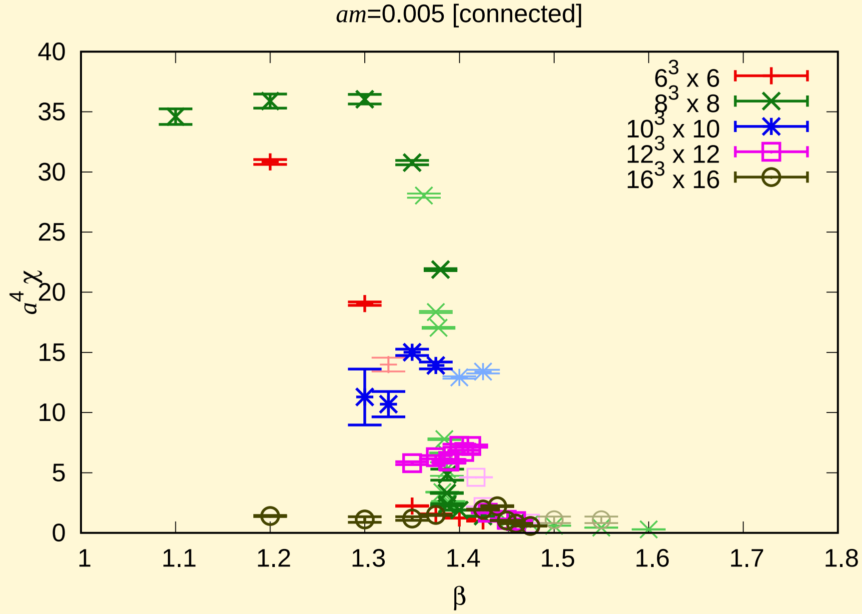
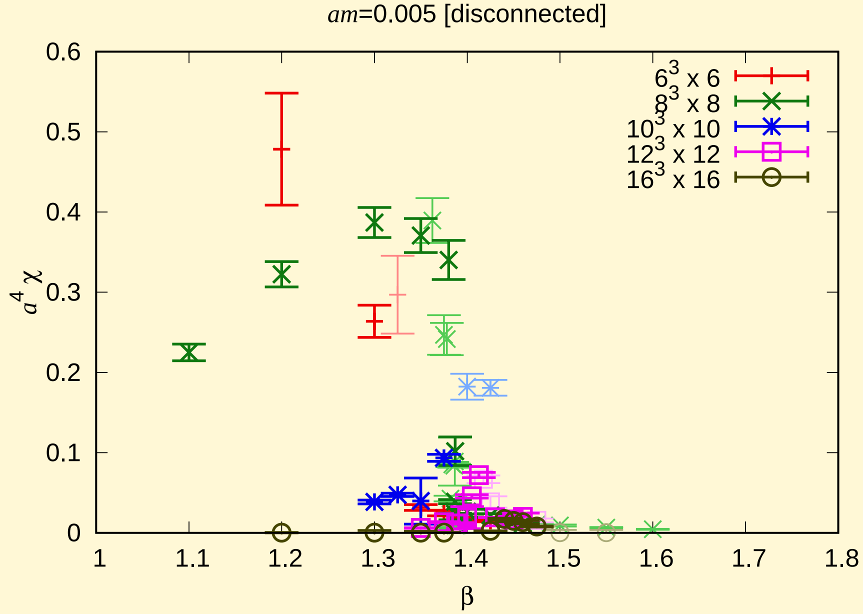
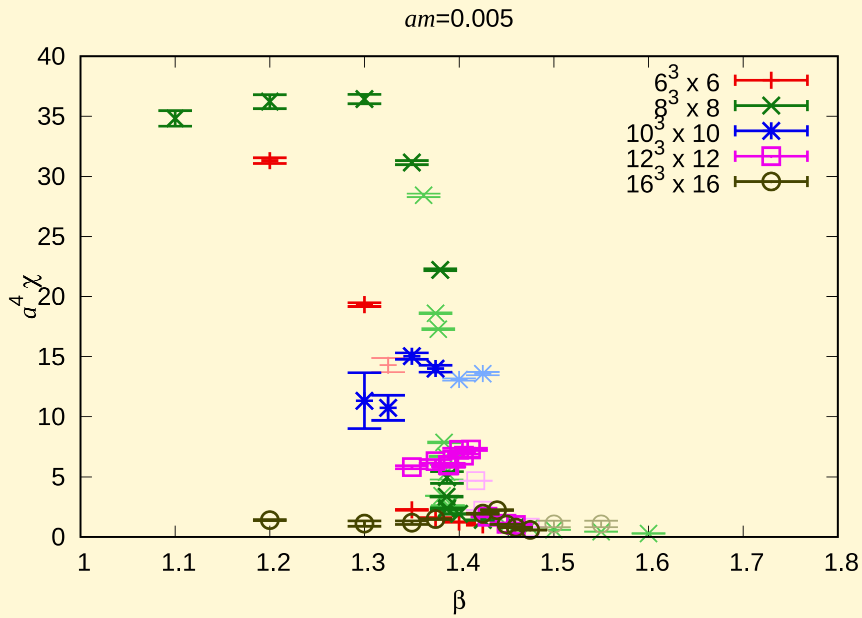
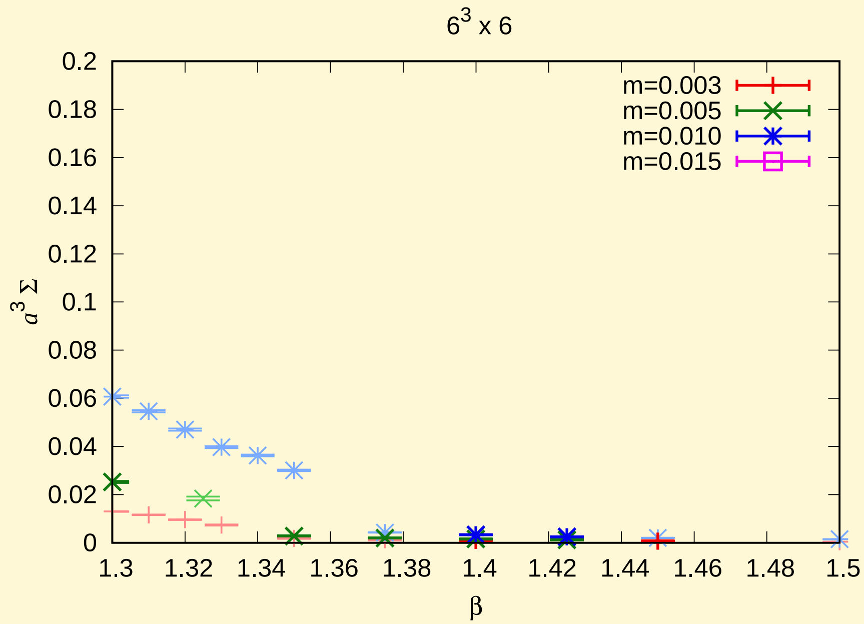
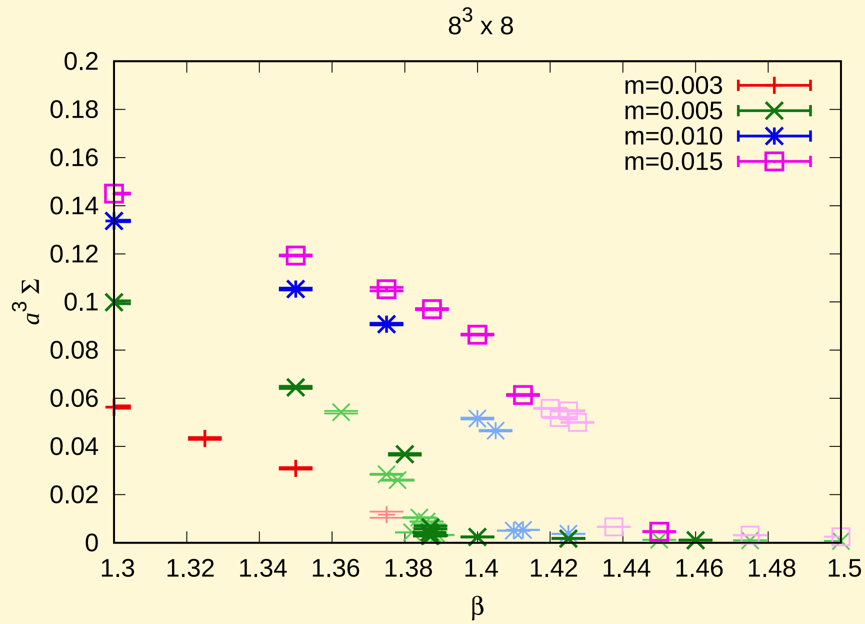


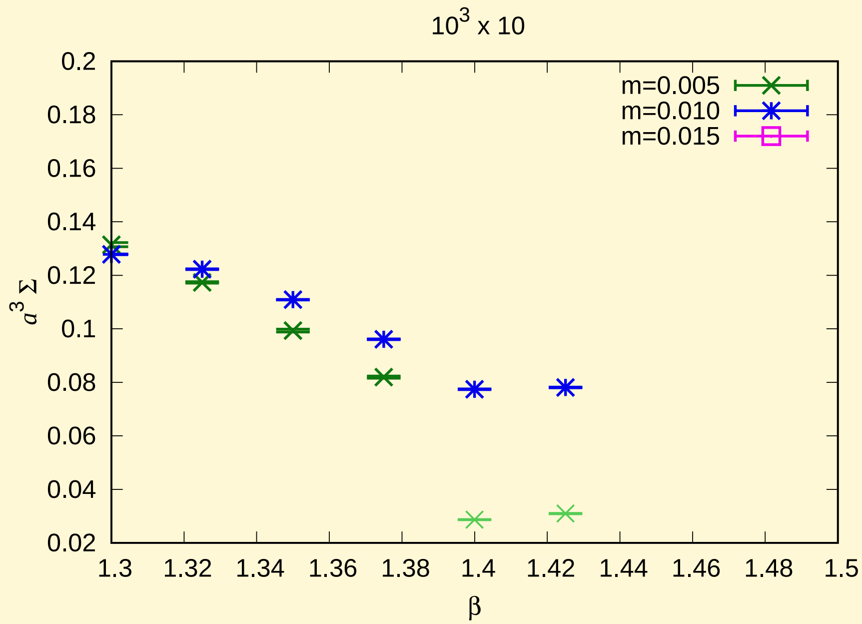


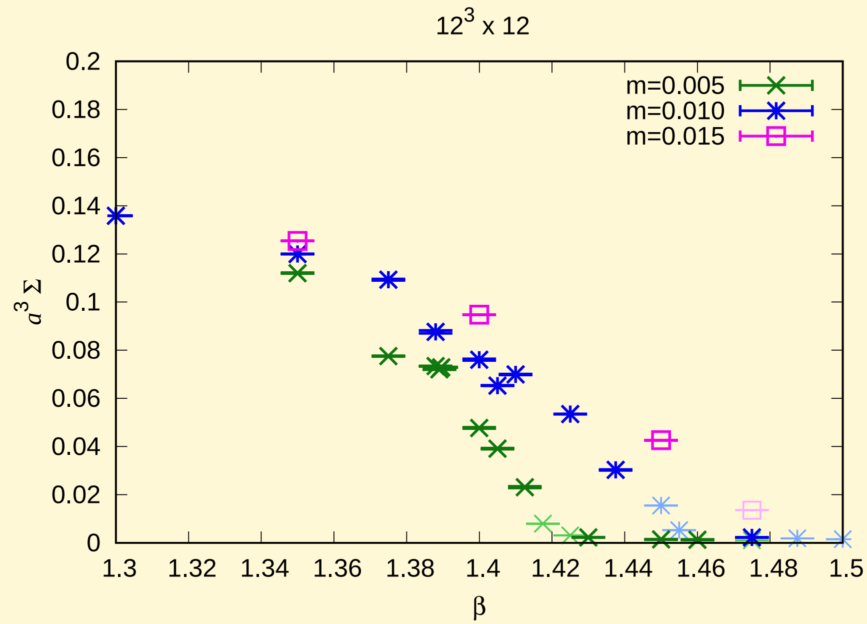


chiral condensate at fixed volume









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