Lattice spectroscopy of SU(2) Adjoint Higgs model

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Lattice 2019, 19.6.2019

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Outline and Motivation

Goal:

- **Verify the presence of a massless vector state in SU(2)+adjoint scalar theory.**
- **Show how an effective QED can arise in a non Abelian gauge theory.**

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Outline:

- Fröhlich-Morchio-Strocchi mechanism.
- SU(2) with adjoint Higgs spectrum.
- Lattice preliminary results.

Gauge-invariant perturbation theory

FMS Mechanism

- Elementary fields are treated as observable in PT, even if not gauge invariant.
- The real physical objects must be described by gauge invariant composite operators with definite quantum numbers. [Fröhlich,Morchio,Strocchi-Nucl.Phys.B190(1981)553-582]

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- Correlators of gauge-invariant J^P $\rightarrow$ comparison with ordinary perturbation theory J^P states.
- FMS example: SU(N) + fundamental Higgs

$$O_{0^+}(x) = (\phi^\dagger \phi)(x)$$

- Fix the gauge to non-vanishing vev: $\phi(x) = vn + h(x)$
- Expand the correlator

$$\langle O^\dagger_{0^+}(x) O_{0^+}(y) \rangle = 4v^2 \langle h(x)^\dagger h(y) \rangle + O(h^4)\,.$$

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- Poles of bound states are at same position as elementary fields. [Maas,Mufti-1412.6440(hep-lat)]

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- States are mapped from multiplets of local $SU(2)$ to multiplets of global custodial $SU(2)$.
- Poles of bound states are at same position as elementary fields. [Maas,Mufti-1412.6440(hep-lat)]
- FMS mechanism can be applied to fermions (no lattice results so far).

Review: [Maas-1712.04721(hep-ph)]

SU(2) adjoint Higgs model

SU(2) Gauge theory coupled with an adjoint Scalar

- It is a toy model of a GUT with a low energy QED:

$$\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} + \mathrm{tr}\Big[(D_\mu \Phi)^\dagger (D^\mu \Phi) \Big] - V(\Phi)\,.$$

- Potential:

$$V = -\mu^2 \,\mathrm{tr}\, \Phi^2 + \frac{\lambda}{2} (\mathrm{tr}\, \Phi^2)^2\,.$$

- $\Phi(x)$ is the scalar field in the adjoint representation.
- Symmetry of the theory under: $\Phi(x) \to U(x)\Phi(x)U(x)^\dagger$.

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- We look for potentials that allow the BEH effect.

Brout-Englert-Higgs Effect - Perturbation theory approach

- The only relevant breaking pattern which lead to a potential with a minimum is $SU(2) \to U(1)$.
- Split scalar field in vev and fluctuations:

$$\Phi(x) = \langle\Phi\rangle + \phi(x) \equiv w\Phi_0 + \phi(x)\,.$$

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Vector channel:

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\begin{aligned}
O^{\mu}_{1^-} =& \frac{\partial_\nu}{\partial^2}\,\mathrm{tr}[\Phi F^{\mu\nu}] \\
=& -w\,\mathrm{tr}\big[\Phi_0 A^{\mu}_{\perp}\big](x) + \mathcal{O}(A,\phi) \\
=& -w\,\mathrm{tr}\Big[\Phi_0(\delta^{\mu}_{\nu} - \partial^{\mu}\partial_{\nu}/\partial^2)A^{\nu}\Big](x) + \mathcal{O}(A,\phi)\,.
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[Maas,Sondenheimer,Toerek-1709.07477(hep-ph)]

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We predict a massless composite vector bound state.

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- A mass for the scalar ground state m_H^2.
- One massless vector boson (from first order expansion).
- One next level scattering state with mass $4g^2 w^2$ (leptoquark).
  [Maas,Sondenheimer,Toerek-1709.07477(hep-ph)]

Lattice spectroscopy

Lattice action

A multihit Montecarlo has been implemented, with action

$$S[\Phi, U] = S_W[U] + \sum_x 2\,\mathrm{tr}(\Phi(x)\Phi(x)) + \lambda(2\,\mathrm{tr}(\Phi(x)\Phi(x)) - 1)^2 \\ - 2\kappa \sum_\mu \mathrm{tr}\Big(\Phi(x) U_\mu(x) \Phi(x+\hat{\mu}) U_\mu^\dagger(x)\Big)$$

Center symmetry Z_2.

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Explicitating the generators of the algebra

$$S[\phi, U] = S_W[U] + \sum_{x,a} \Phi^a(x)\Phi^a(x) + \lambda(\Phi^a(x)\Phi^a(x) - 1)^2$$
$$- 2\kappa \sum_{\mu,a,b} \Phi^a(x)V_\mu^{ab}(x)\Phi^b(x+\hat{\mu})$$

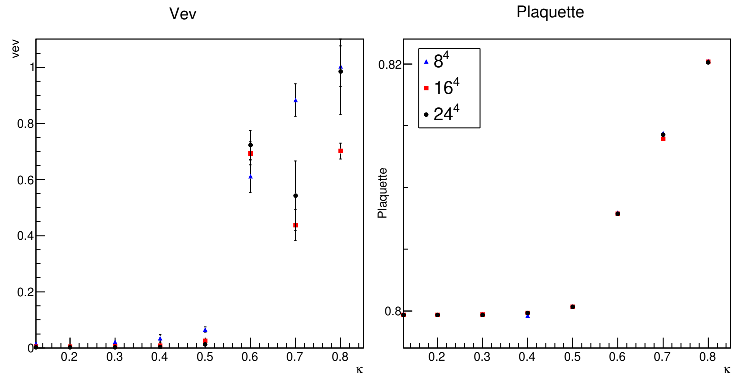
where

$$V_\mu^{ab}(x) = \mathrm{tr}\Big(T^a U_\mu(x) T^b U_\mu^\dagger(x)\Big),.$$

Scan of the phase diagram

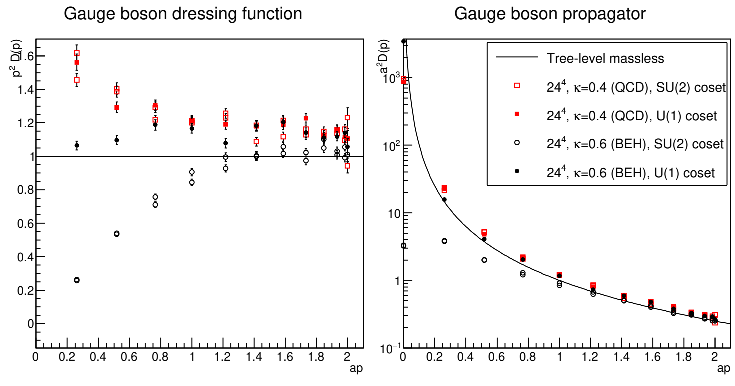
- Lattice results point to phase transition: QCD-like vs. BEH. [Baier-Lang-Reusch Nuclear Physics B 305.3 (1988): 396-416.]
- We can use vev and the plaquette as order parameters.
- We fixed $\beta = 4, \lambda = 1$ and we varied κ over $[0.1, 0.8]$.
- Lattice size: $8^4, 16^4, 24^4, 32^4$.

Scan of the phase diagram



- The vev and plaquette variable show a second order phase transition around $\kappa \sim 0.5$ (with β, λ fixed).
- We expect to be in the BEH phase for $\kappa > 0.5$.

Gauge boson propagator



- Phase diagram and FMS RHS result, using gauge propagator in a fixed gauge, at $\kappa = 0.4$ and $\kappa = 0.6$.
- The split of the two cosets at $\kappa = 0.6$ is a strong hint of the BEH phase.

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- Massless vector bound state is predicted by FMS.

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  [Lee,Shigemitsu-Nucl.Phys.B263(1986)280-294]

$$B^i(x) = \frac{1}{\sqrt{2\,\mathrm{Tr}(\Phi^2)}}\,\mathrm{Im}\,\mathrm{Tr}\Big(\Phi(\vec{x},t)U^{jk}(\vec{x},t)\Big)\,.$$

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- We give the operator a non-zero momentum via

$$B^j(\vec{p},t) = \frac{1}{\sqrt{V_{\vec{x}}}}\,\mathrm{Re}\sum_{\vec{x}} B^j(\vec{x},t)e^{i\vec{p}\cdot\vec{x}}\,.$$

- We choose as momentum the smallest one in the z direction

$$\vec{p}_z = \left(0, 0, \frac{2\pi}{N_z}\right)\,.$$

Transverse and Longitudinal Correlator

We split the correlator in the transverse and the longitudinal part

$$C_{\perp}(t) = \frac{1}{N_t} \sum_{t'=0}^{N_t-1} \sum_{j=1}^{2} \langle B^j(\vec{p}_z, t') B^j(\vec{p}_z, t+t') \rangle \ ,$$

$$C_{\parallel}(t) = \frac{1}{N_t} \sum_{t'=0}^{N_t-1} \langle B^3(\vec{p}_z, t') B^3(\vec{p}_z, t+t') \rangle \ .$$

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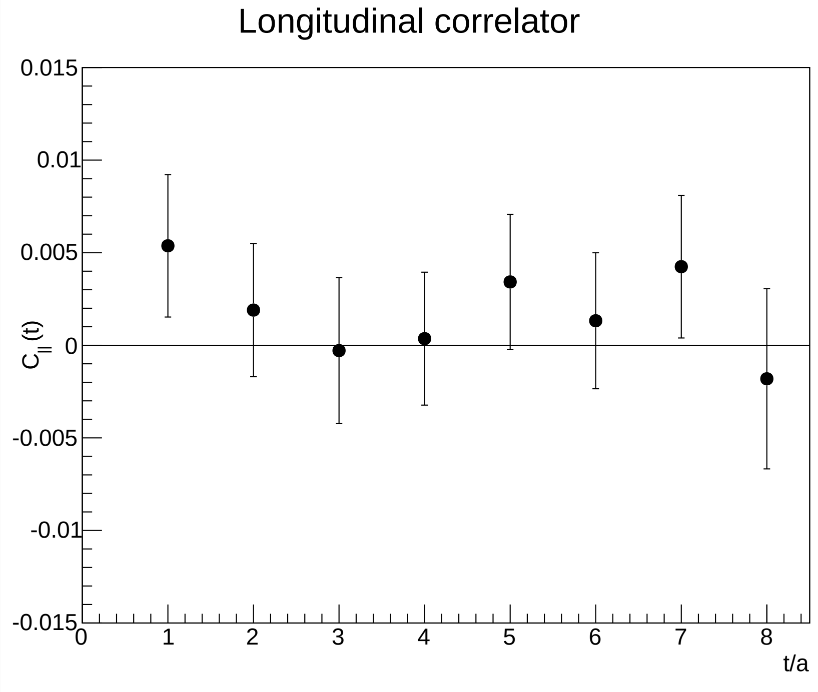
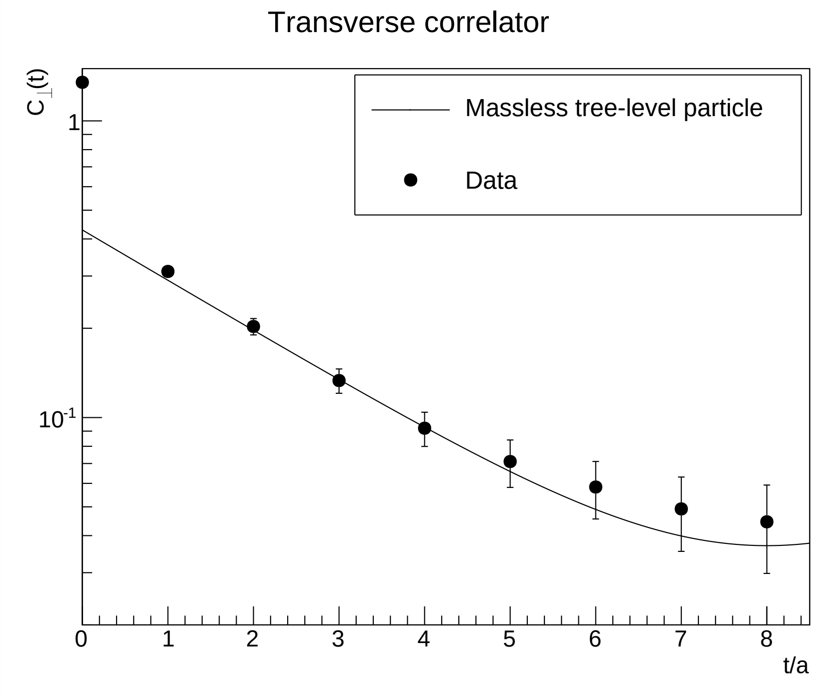
We expect the transverse correlators to behave as

$$C_\perp(t) \propto \exp(-E(P_z)t)\,,$$
$$C_\parallel(t) \propto \delta(t=0)\,.$$


Longitudinal correlator
0.015
0.01
0.005
0
-0.005
-0.01
-0.015
0
1
2
3
4
5
6
7
8
t/a
C∥(t)


Transverse correlator
$C_\perp(t)$
1
10^{-1}
Massless tree-level particle
Data
0
1
2
3
4
5
6
7
8
t/a

Massless state investigation

For a massless state,in a homogenous lattice $V = 16^4$, we expect

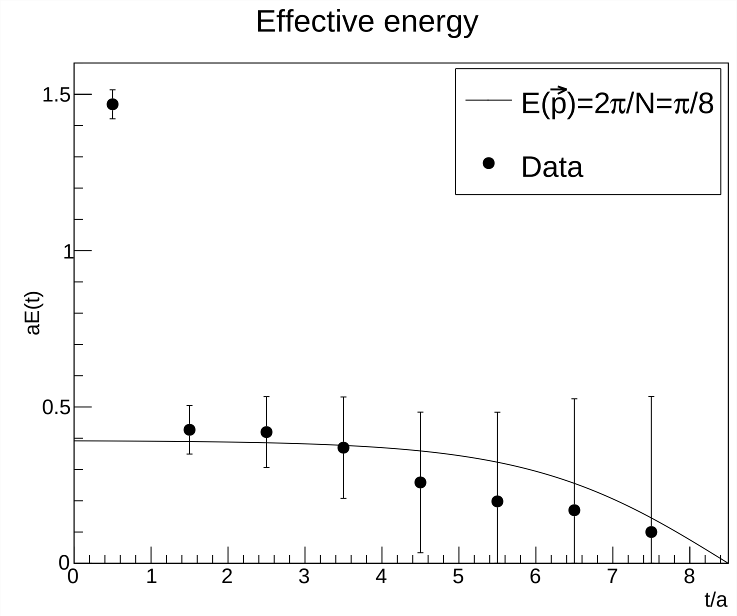
$$E(\vec{P_z}) = |\vec{P_z}| = \frac{2\pi}{16} = \frac{\pi}{8}$$

We can extract the energy using

$$E_{eff}(t + 0.5) = \log\left(\frac{C_{\perp}(t)}{C_{\perp}(t+1)}\right)$$

We can confront it with the expected cosh behaviour for a massless state.

Preliminary spectroscopy results



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Outlook:

- Extend analysis to other BSM models using FMS mechanism.
- Make predictions and confront them with the phenomenology. [Egger,Maas,Sondenheimer-1701.02881(hep-ph)]

Thanks!