Meson spectrum of $Sp(4)$ lattice gauge theory with two fundamental Dirac fermions

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Lattice 2019

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Novel strong dynamics for BSM

- Various aspects of new physics beyond the standard model (BSM) can be addressed in novel strongly coupled gauge theories.
  Composite Higgs models (pNGB) are particularly interesting.

Kaplan & Georgi (1984)

Solutions for (little) hierarchy problem

Quark mass via partial compositeness

Kaplan (1993)

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Kaplan (1993)

UV completion

5D

Contino, Nomura & Pomarol (2003)

4D

Ferretti & Karateev (2014)

UV complete Composite Higgs scenarios

Coset	HC	ψ	χ	$-q_\chi/q_\psi$	Baryon	Name	Lattice
$\frac{SU(5)}{SO(5)} \times \frac{SU(6)}{SO(6)}$	SO(7)	$5 \times \mathbf{F}$	$6 \times \mathbf{Sp}$	5/6	$\psi\chi\chi$	M1	
	SO(9)			5/12		M2	
	SO(7)	$5 \times \mathbf{Sp}$	$6 \times \mathrm{F}$	5/6	$\psi\psi\chi$	M3	
	SO(9)			5/3		M4	
$\frac{SU(5)}{SO(5)} \times \frac{SU(6)}{Sp(6)}$	Sp(4)	$5 \times \mathbf{A}_2$	$6 \times \mathbf{F}$	5/3	$\psi\chi\chi$	M5	$\surd$
$\frac{SU(5)}{SO(5)} \times \frac{SU(3)^2}{SU(3)}$	SU(4)	$5 \times \mathbf{A}_2$	$3 \times (\mathbf{F}, \overline{\mathbf{F}})$	5/3	$\psi\chi\chi$	M6	$\surd$
	SO(10)	$5 \times \mathbf{F}$	$3 \times (\mathbf{Sp}, \overline{\mathbf{Sp}})$	5/12		M7	
$\frac{SU(4)}{Sp(4)} \times \frac{SU(6)}{SO(6)}$	Sp(4)	$4 \times \mathbf{F}$	$6 \times \mathbf{A}_2$	1/3	$\psi\psi\chi$	M8	$\surd$
	SO(11)	$4 \times \mathbf{Sp}$	$6 \times \mathbf{F}$	8/3		M9	
$\frac{SU(4)^2}{SU(4)} \times \frac{SU(6)}{SO(6)}$	SO(10)	$4 \times (\mathbf{Sp}, \overline{\mathbf{Sp}})$	$6 \times \mathbf{F}$	8/3	$\psi\psi\chi$	M10	
	SU(4)	$4 \times (\mathbf{F}, \overline{\mathbf{F}})$	$6 \times \mathbf{A}_2$	2/3		M11	$\surd$
$\frac{SU(4)^2}{SU(4)} \times \frac{SU(3)^2}{SU(3)}$	SU(5)	$4 \times (\mathbf{F}, \overline{\mathbf{F}})$	$3 \times (\mathbf{A}_2, \overline{\mathbf{A}_2})$	4/9	$\psi\psi\chi$	M12	

Global symmetry: minimal, but large enough to take account for both EW & Color cosets

Gauge symmetry: Asymptotically free & non-conformal

Cacciapaglia, Ferretti, Flacke & Serodio (2019) arXiv:1902.06890

Why Sp(4)? - SU(4)/Sp(4) CH

- UV realization of SO(6)/SO(5) CH model from Sp(2N) gauge theory

Barnard, Gherghetta & Ray (2014)

Global symmetry

2 Dirac flavors in fund. rep.

$SU(4)/Sp(4) \sim SO(6)/SO(5) \times SU(6)/SO(6)$

3 Dirac flavors in anti-sym. rep.

SM EW

SM Strong

$SU(2)_L \times U(1)_Y \subset$ Sp(4)

$SU(3)_c \times U(1)_Y \subset SO(6)$

4 of 5 PNGBs: Higgs doublets

Top partner = Chimera baryon

e.g. $\hat{\Psi}^{ab\alpha} \equiv \left(q^a \chi^\alpha q^b\right)$ carry color charge

- N=2 to make it minimal & near conformal

Why Sp(4)? - SU(4)/Sp(4) CH

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SM EW

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4 of 5 PNGBs: Higgs doublets

- N=2 to make it minimal & near conformal

More on Sp(2N)

- Both light composite Higgs & top partner based on Sp(6) gauge theory with multi-reps

  Gertov, Nelson, Perko, Walker (2019)

- SIMP dark matter: 3-2 number-changing scattering process

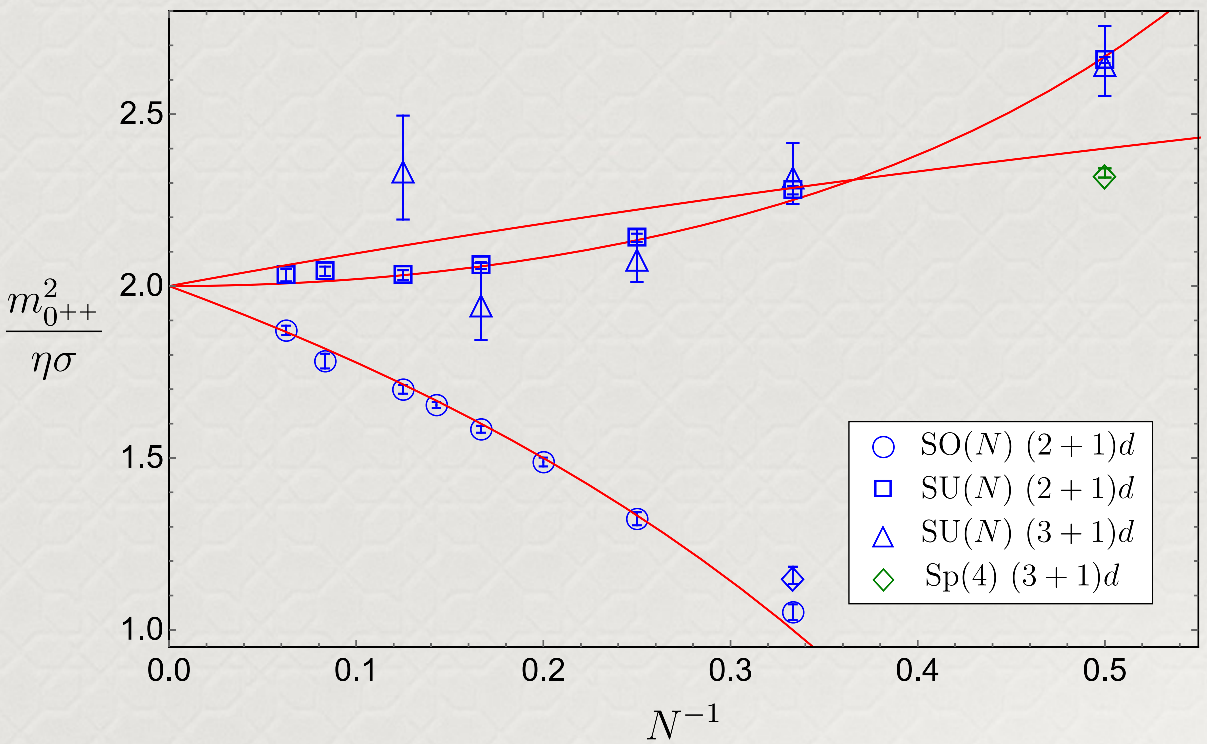
  Hochberg et al (2015)

  Hansen, Langable, Sannino (2015)

- Casimir scaling: Universality in pure SU(N), SO(N), Sp(2N) Yang-Mills

$$\frac{m_{0^{++}}^2}{\sigma} = \eta \frac{C_2(A)}{C_2(F)}$$

Hong et al (2017)



Talk by Jack @ 14:40, Friday

Lattice details

- Standard (unimproved) Wilson gauge & fermion actions

$$S = \beta \sum_x \sum_{\mu<\nu} \left(1 - \frac{1}{4}\mathrm{Re}\,\mathrm{Tr}\mathcal{P}_{\mu\nu}\right) + a^3 \sum_x \bar{\psi}(x)\,(4 + am_0)\,\psi(x)$$
$$-\frac{1}{2}a^3 \sum_{x,\mu} \bar{\psi}(x)\left((1-\gamma_\mu)U_\mu(x)\psi(x+\hat{\mu}) + (1+\gamma_\mu)U_\mu^\dagger(x-\hat{\mu})\psi(x-\hat{\mu})\right)$$

where $\beta = 4N/g^2$ and the plaquette is

$$\mathcal{P}_{\mu\nu}(x) = U_\mu(x)U_\nu(x+\hat{\mu})U_\mu^\dagger(x+\hat{\nu})U_\nu^\dagger(x)$$

The link variables $U_\mu(x)$ are elements of Sp(4) gauge group.

Bennett et al (2018)

- Gauge configurations are generated by using hybrid Monte Carlo algorithm implemented in the modified HiRep code.

Del Debbio, Patella, Pica (2010)

- Periodic B.C. for spacial directions & anti-periodic B.C. for temporal direction

Ensembles

Ensemble	β	am_0	$N_t \times N_s^3$	N_{configs}	δ_{traj}	$m_{\text{PS}} L$	$f_{\text{PS}} L$
DB1M1	6.9	-0.85	32×16^3	100	24	13.351(17)	2.290(10)
DB1M2	6.9	-0.87	32×16^3	100	24	11.845(19)	2.079(17)
DB1M3	6.9	-0.89	32×16^3	100	20	10.042(23)	1.836(13)
DB1M4	6.9	-0.9	32×16^3	100	20	9.00(3)	1.683(18)
DB1M5	6.9	-0.91	32×16^3	100	20	7.701(16)	1.509(10)
DB1M6	6.9	-0.92	32×24^3	80	28	9.28(26)	1.977(13)
DB1M7	6.9	-0.924	32×24^3	62	12	8.13(3)	1.835(14)
DB2M1	7.05	-0.835	36×20^3	100	20	8.752(28)	1.645(17)
DB2M2	7.05	-0.85	36×24^3	100	24	7.946(26)	1.609(13)
DB2M3	7.05	-0.857	36×32^3	102	20	8.732(27)	1.958(12)
DB3M1	7.2	-0.7	36×16^3	100	20	11.043(18)	1.590(14)
DB3M2	7.2	-0.73	36×16^3	100	20	9.437(21)	1.449(13)
DB3M3	7.2	-0.76	36×16^3	100	20	7.521(21)	1.235(10)
DB3M4	7.2	-0.77	36×24^3	100	20	10.133(18)	1.743(8)
DB3M5	7.2	-0.78	36×24^3	96	12	8.884(21)	1.598(9)
DB3M6	7.2	-0.79	36×24^3	100	20	7.568(22)	1.448(9)
DB3M7	7.2	-0.794	36×28^3	195	12	8.048(19)	1.611(8)
DB3M8	7.2	-0.799	40×32^3	150	12	8.102(24)	1.714(10)
DB4M1	7.4	-0.72	48×32^3	150	12	10.208(15)	1.745(7)
DB4M2	7.4	-0.73	48×32^3	150	12	8.663(20)	1.600(9)
DB5M1	7.5	-0.69	48×24^3	100	12	7.835(23)	1.270(9)

Weak coupling regime

$$\beta \gtrsim 6.8$$

Bennett et al (2018)

Negligible FV effects

$$m_{\text{PS}} L \gtrsim 7.5$$

Lee et al (Lattice 2018)

Low-energy EFT

$$f_{\text{PS}} L > 1$$

Remarks on the lattice spacing & fermion mass

- Scale setting: Luscher's gradient flow scales *Luscher (2010)* *Luscher & Weise (2011)*

$$\mathcal{W}(t) \equiv t \frac{\mathrm{d}\mathcal{E}(t)}{\mathrm{d}t}$$ *Borsanyi et al (2012)*

$$E(t,x) = -\frac{1}{2} \mathrm{tr} G_{\mu\nu}(t,x) G_{\mu\nu}(t,x)$$

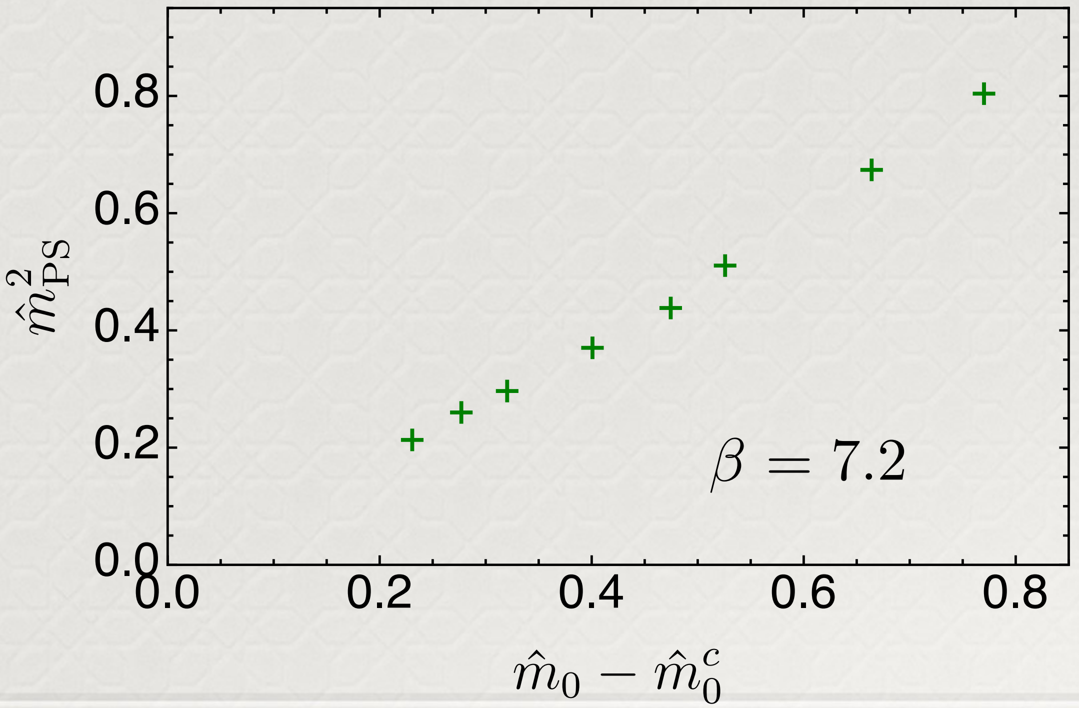
$$\mathcal{W}(t)|_{t=w_0^2} = \mathcal{W}_0 = 0.35$$

$$t^2 \langle E(t) \rangle \equiv \mathcal{E}$$

- Dimensionful quantities: $\hat{m}_M \equiv m_M w_0 = m_M^{\mathrm{lat}} w_0^{\mathrm{lat}}$ & $\hat{f}_M \equiv f_M w_0 = f_M^{\mathrm{lat}} w_0^{\mathrm{lat}}$

- Fermion mass is replaced by pseudoscalar mass squared

$$m_{\mathrm{PS}}^2 = 2Bm_f$$



Observables: flavored mesons

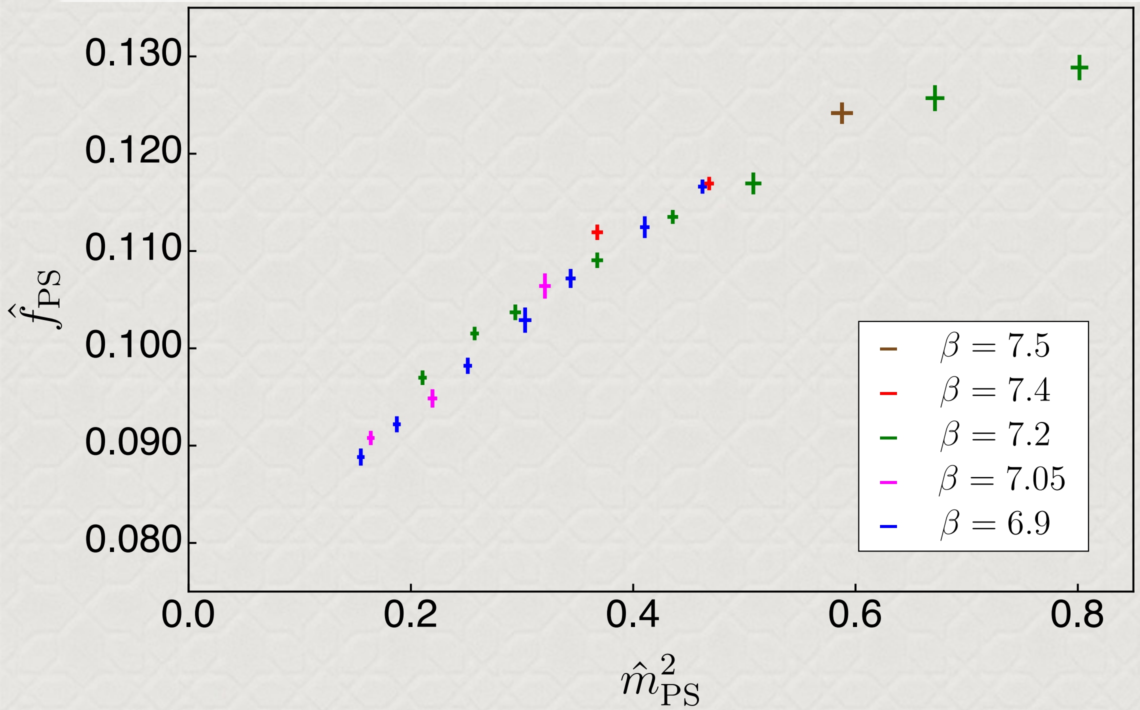
- Interpolating operators of flavored spin-0 and spin-1 mesons ($i \neq j$)

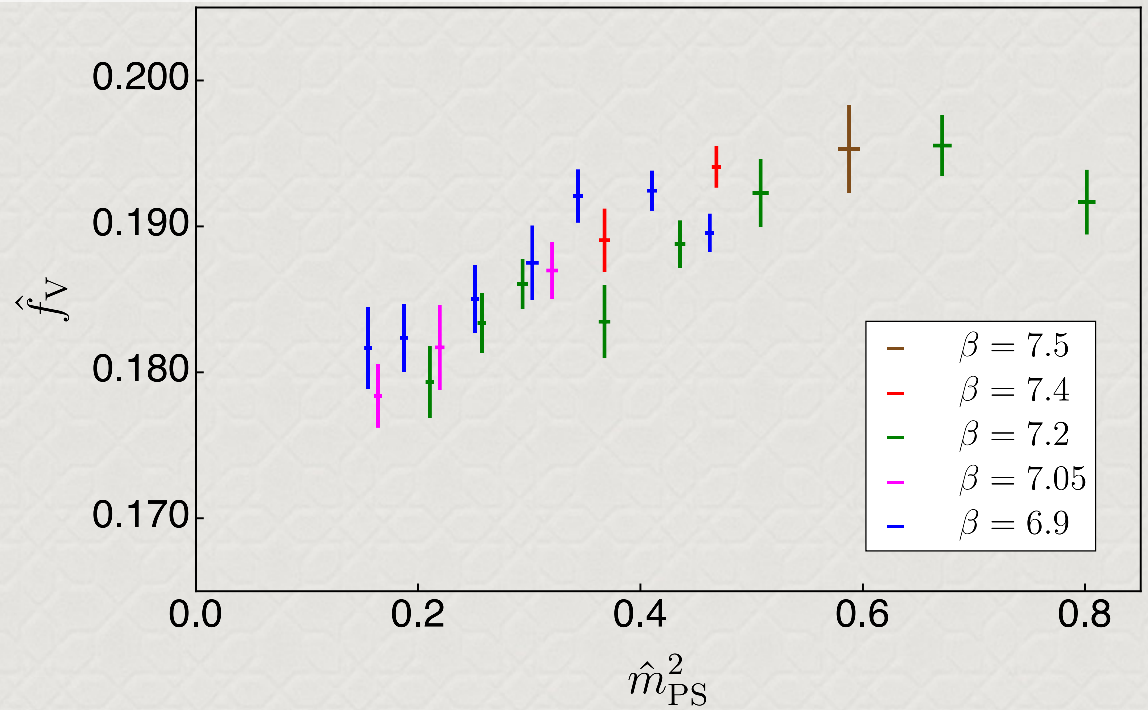
Label	Interpolating operator ($\mathcal{O}_M$)	Meson	J^P
PS	$\overline{Q^i}\gamma_5 Q^j$	π	0^-
S	$\overline{Q^i} Q^j$	a_0	0^+
V	$\overline{Q^i}\gamma_\mu Q^j$	ρ	1^-
T	$\overline{Q^i}\gamma_0\gamma_\mu Q^j$	ρ	1^-
AV	$\overline{Q^i}\gamma_5\gamma_\mu Q^j$	a_1	1^+
AT	$\overline{Q^i}\gamma_5\gamma_0\gamma_\mu Q^j$	b_1	1^+

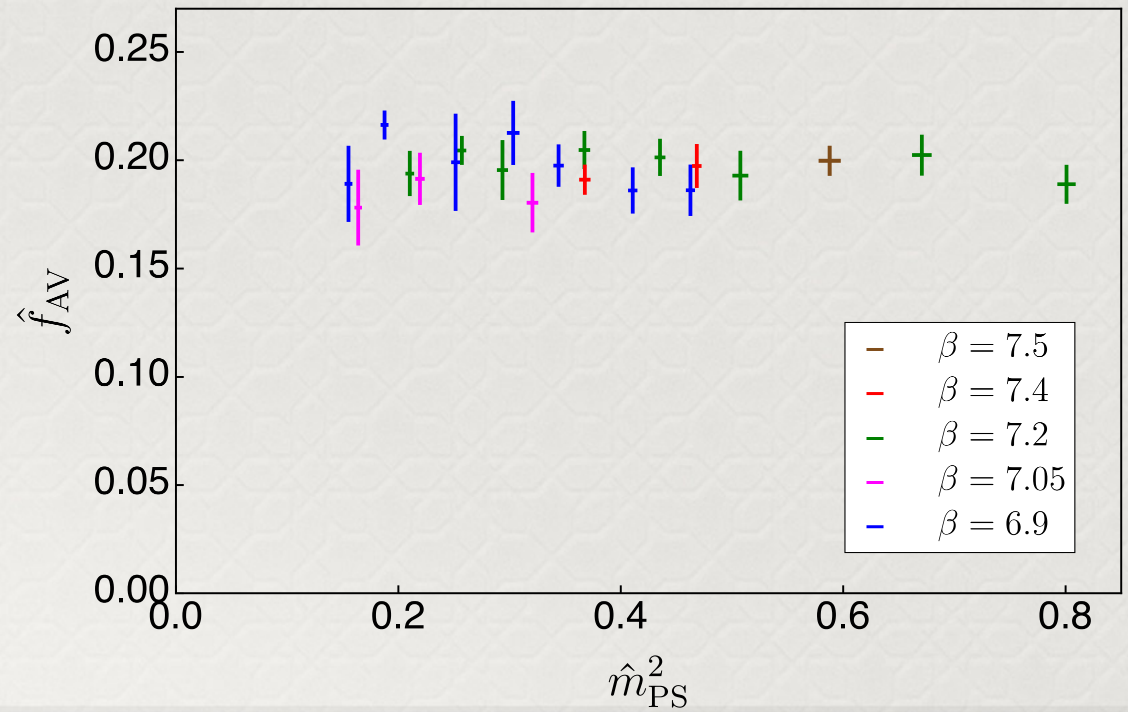
- Masses and decay constants (only for PS, V and AV) are extracted from two-point correlation functions as usual.

- Decay constants are renormalized by using the one-loop perturbative matching with tad-pole improvement. QCD analog of f_π in our convention is $f_\pi \simeq 93\,\mathrm{MeV}$.

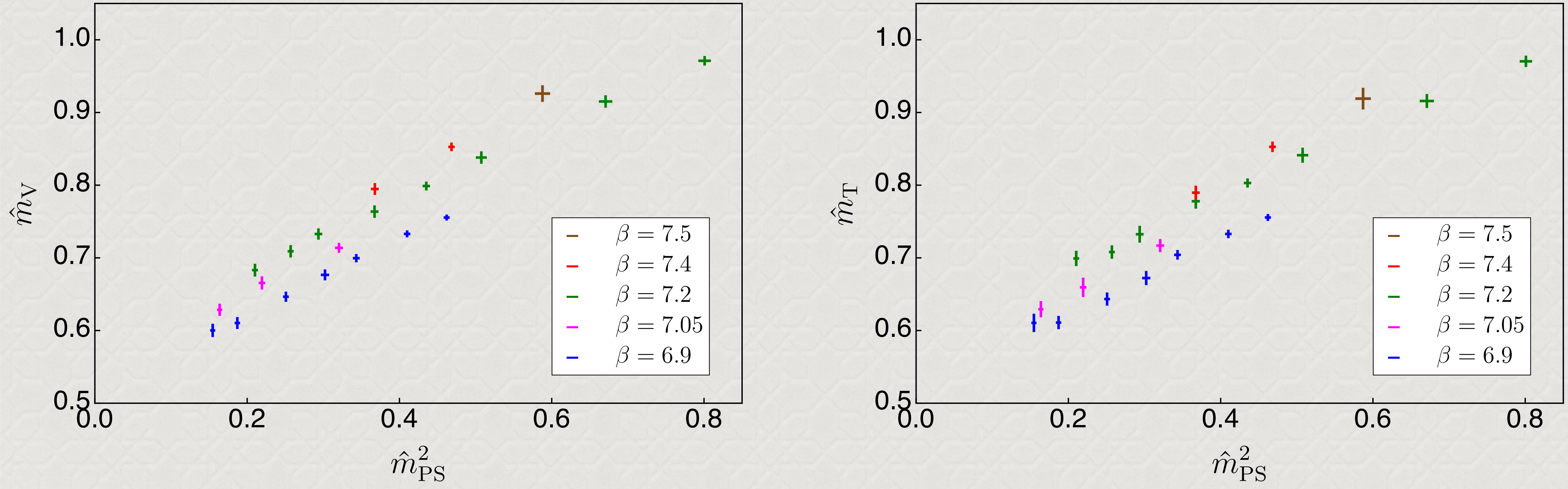
Decay constants



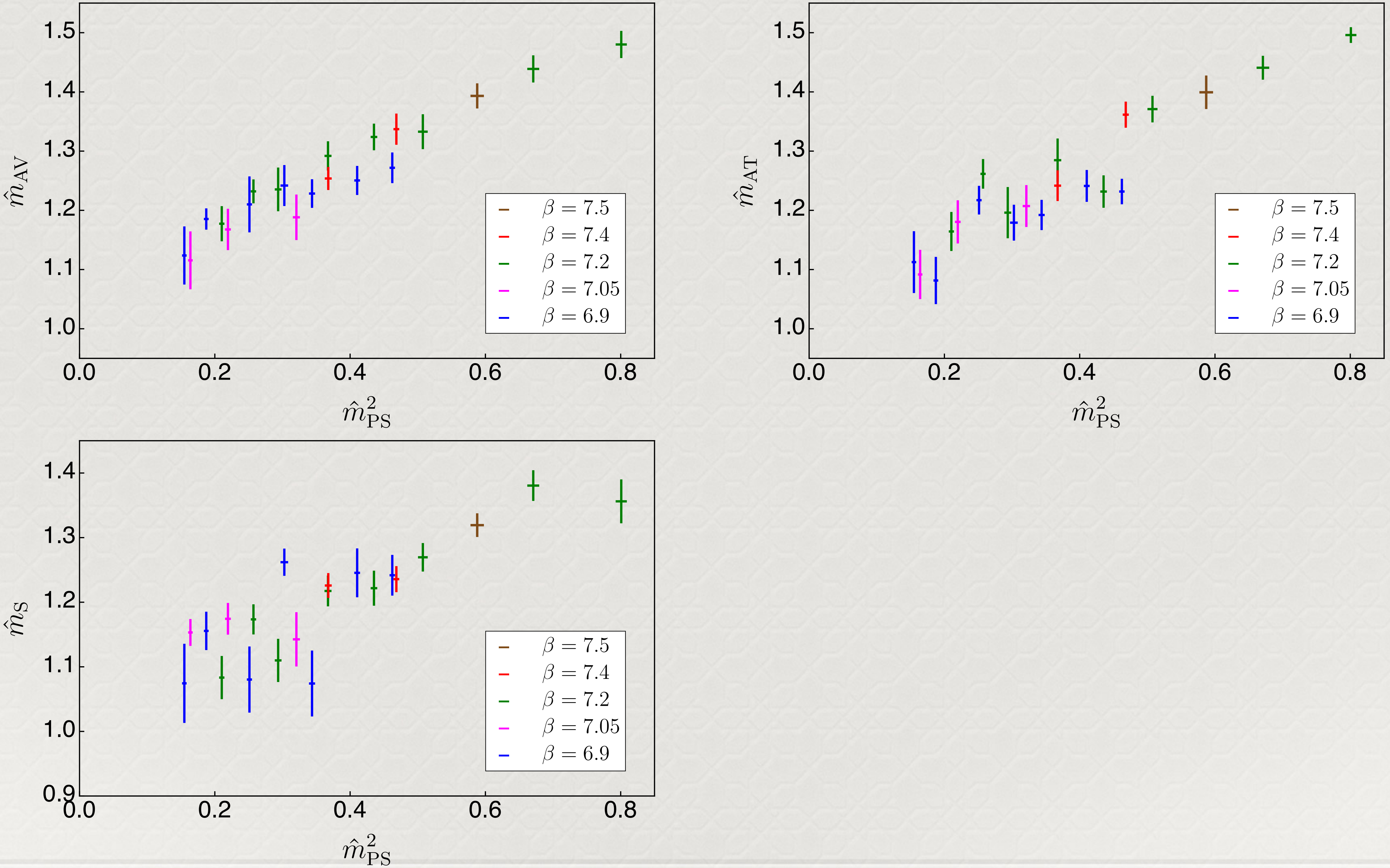




Masses: vector and tensor



Masses: scalar, axial-vetor and axial-tensor



Continuum and massless extrapolation

- (tree-level) Wilson-like chiral perturbation theory at NLO

$$\hat{f}_{\rm PS}^{\rm NLO} = \hat{f}^{\chi}\left(1 + \hat{b}_f^{\chi}\hat{m}_{\rm PS}^2\right) + \hat{W}_f^{\chi}\hat{a} \qquad \hat{a} \equiv a/w_0 = 1/w_0^{\rm lat}$$

- Power counting

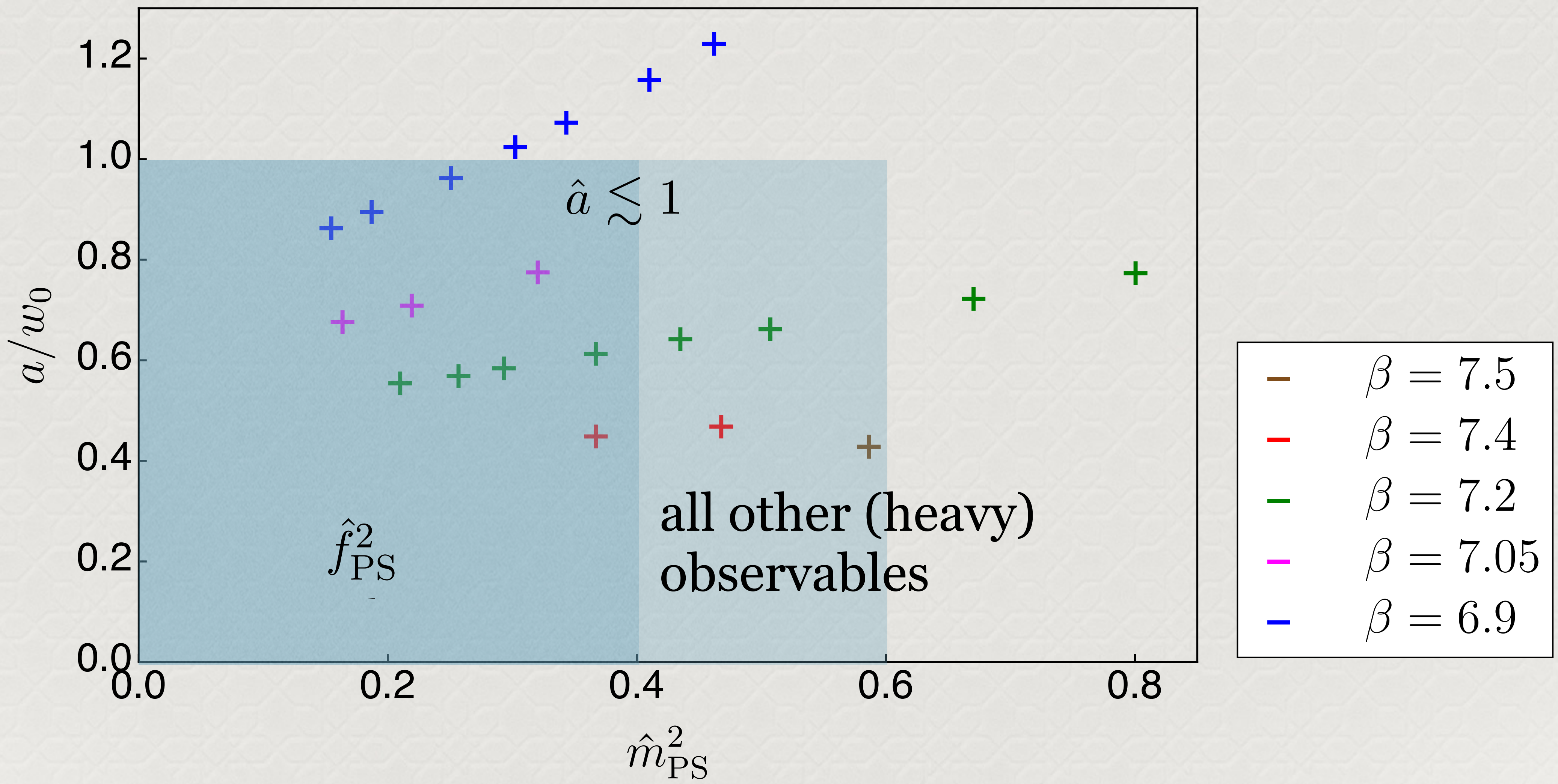
$$\frac{p^2}{\Lambda_\chi^2} < \frac{m_{\rm PS}^2}{\Lambda_\chi^2} \sim a\Lambda_\chi < 1$$

Over the small mass region

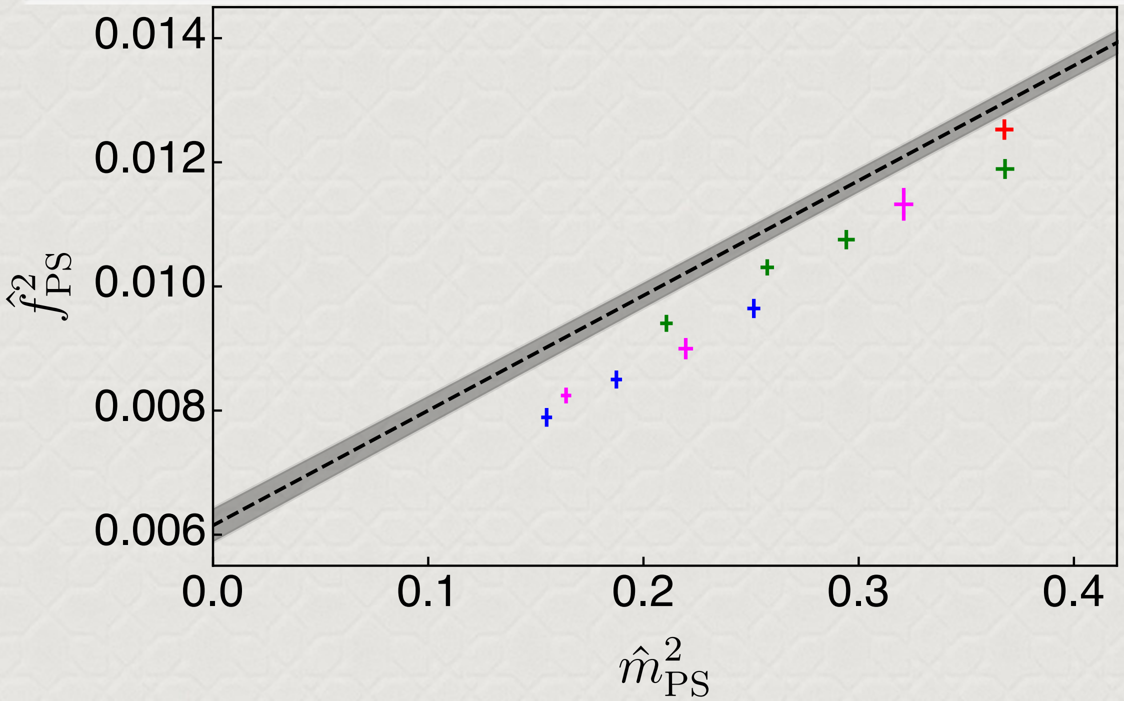
$$\frac{p^2}{\Lambda_\chi^2} : 0.06 \sim 0.12, \;\; \frac{m_{\rm PS}^2}{\Lambda_\chi^2} : 0.13 \sim 0.2, \;\; \text{and } a\Lambda_\chi : 0.6 \sim 1.4$$

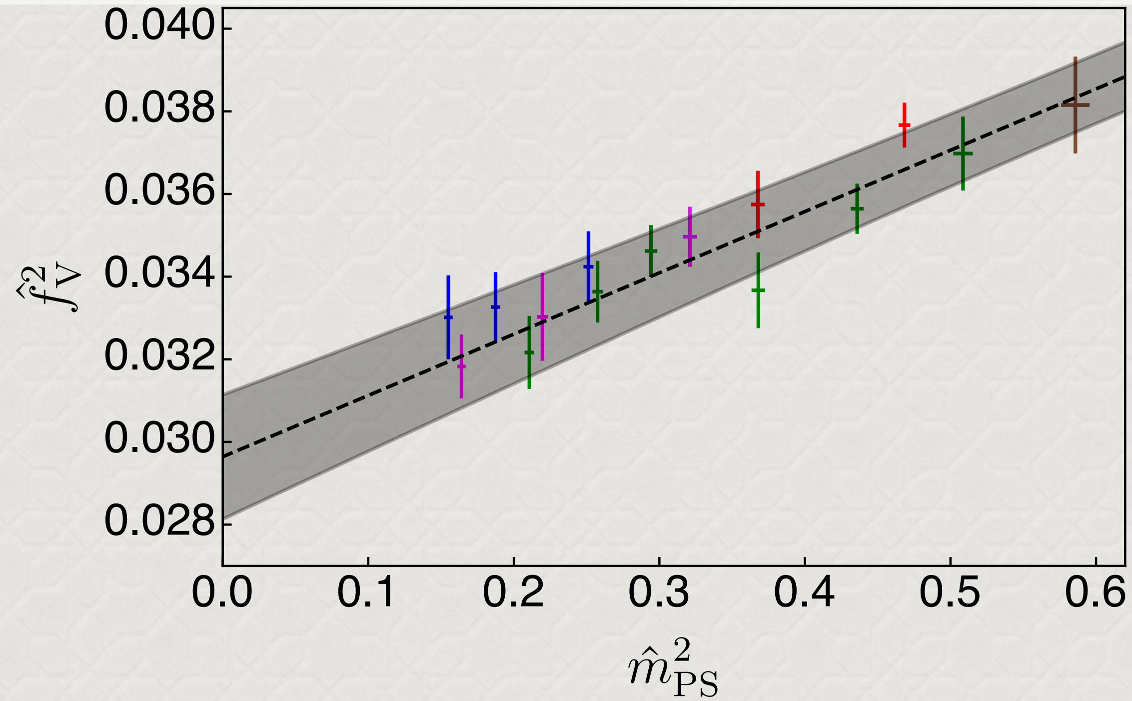
- Exclude coarse lattices from the fits → $a\Lambda_\chi \lesssim 1.1$

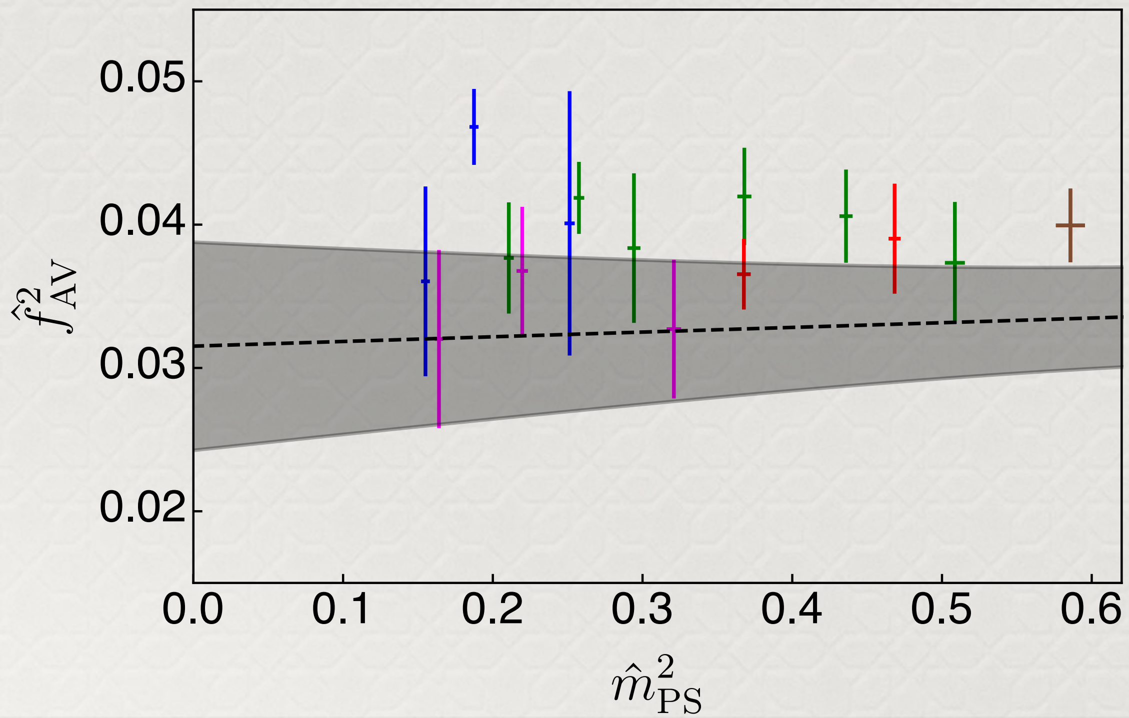
Ensembles used for continuum & massless extrapolations



Continuum & massless extrapolations: Decay constants





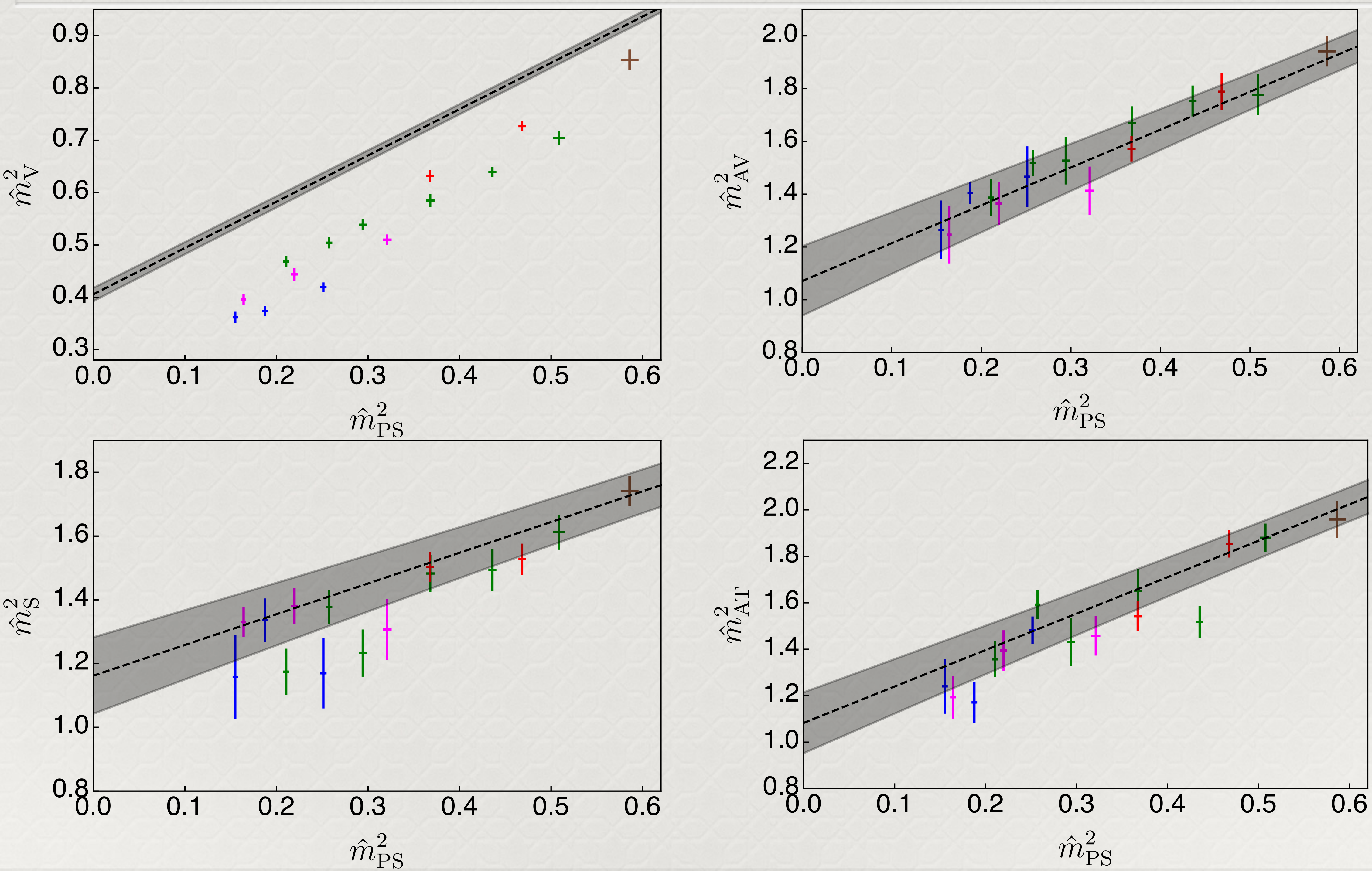


Fit function:

$$\hat{f}_M^{2,\mathrm{NLO}} = \hat{f}_M^{2,\chi}\left(1 + L^0_{f,M}\hat{m}^2_{\mathrm{PS}}\right) + W^0_{f,M}\hat{a}$$

$$\hat{m}_M^{2,\mathrm{NLO}} = \hat{m}_M^{2,\chi}\left(1 + L^0_{m,M}\hat{m}^2_{\mathrm{PS}}\right) + W^0_{m,M}\hat{a}$$

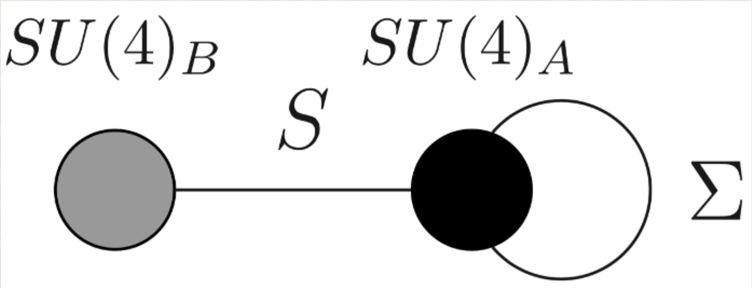
Continuum & massless extrapolations: Masses



Hidden local symmetry and EFT

- Weakly gauging the SU(4) global symmetry & providing mass to V and AV by $\langle S \rangle \neq 0$

Bando, Kugo & Yamawaki (1985)



- NLO EFT Lagrangian from Hidden local symmetry (HLS)

$$\begin{aligned}
\mathcal{L} = & -\frac{1}{2}\mathrm{Tr}\ A_{\mu\nu}A^{\mu\nu} - \frac{\kappa}{2}\mathrm{Tr}\ \{A_{\mu\nu}\Sigma(A^{\mu\nu})^T\Sigma^*\} \\
& + \frac{f^2}{4}\mathrm{Tr}\left\{ D_\mu\Sigma\,(D^\mu\Sigma)^\dagger\right\} + \frac{F^2}{4}\mathrm{Tr}\left\{ D_\mu S\,(D^\mu S)^\dagger\right\} \\
& + b\frac{f^2}{4}\mathrm{Tr}\left\{D_\mu(S\Sigma)\,(D^\mu(S\Sigma))^\dagger\right\} + c\frac{f^2}{4}\mathrm{Tr}\left\{D_\mu(S\Sigma S^T)\,\left(D^\mu(S\Sigma S^T)\right)^\dagger\right\} \\
& - \frac{v^3}{8}\mathrm{Tr}\left\{ M\,S\,\Sigma\,S^T\right\} + \mathrm{h.c.} \\
& - \frac{v_1}{4}\mathrm{Tr}\left\{ M\,(D_\mu S)\,\Sigma\,(D^\mu S)^T\right\} - \frac{v_2}{4}\mathrm{Tr}\left\{ M\,S\,(D_\mu\Sigma)\,(D^\mu S)^T\right\} + \mathrm{h.c.} \\
& - \frac{y_3}{8}\mathrm{Tr}\ \{A_{\mu\nu}\Sigma\left[(A^{\mu\nu})^T S^T M S - S^T M S A^{\mu\nu}\right]\} + \mathrm{h.c.} \\
& - \frac{y_4}{8}\mathrm{Tr}\ \{A_{\mu\nu}\Sigma\left[(A^{\mu\nu})^T S^T M S + S^T M S A^{\mu\nu}\right]\} + \mathrm{h.c.} \\
& - \frac{v_5^2}{128}\left(\mathrm{Tr}\, M S\Sigma S^T + \mathrm{h.c.}\right)^2 .
\end{aligned}$$

LO (first four lines)

NLO (last four lines)

JWL & Lucini, (Lattice 2017)

Bennett et al (2018)

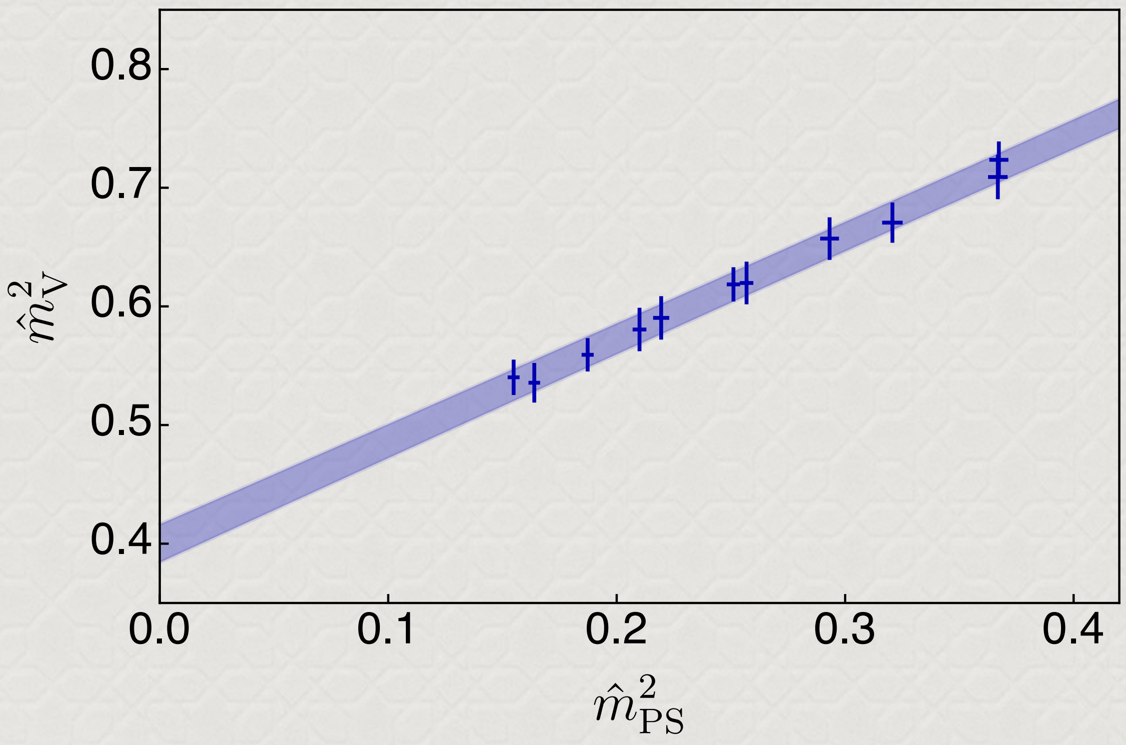
Mesons in NLO EFT

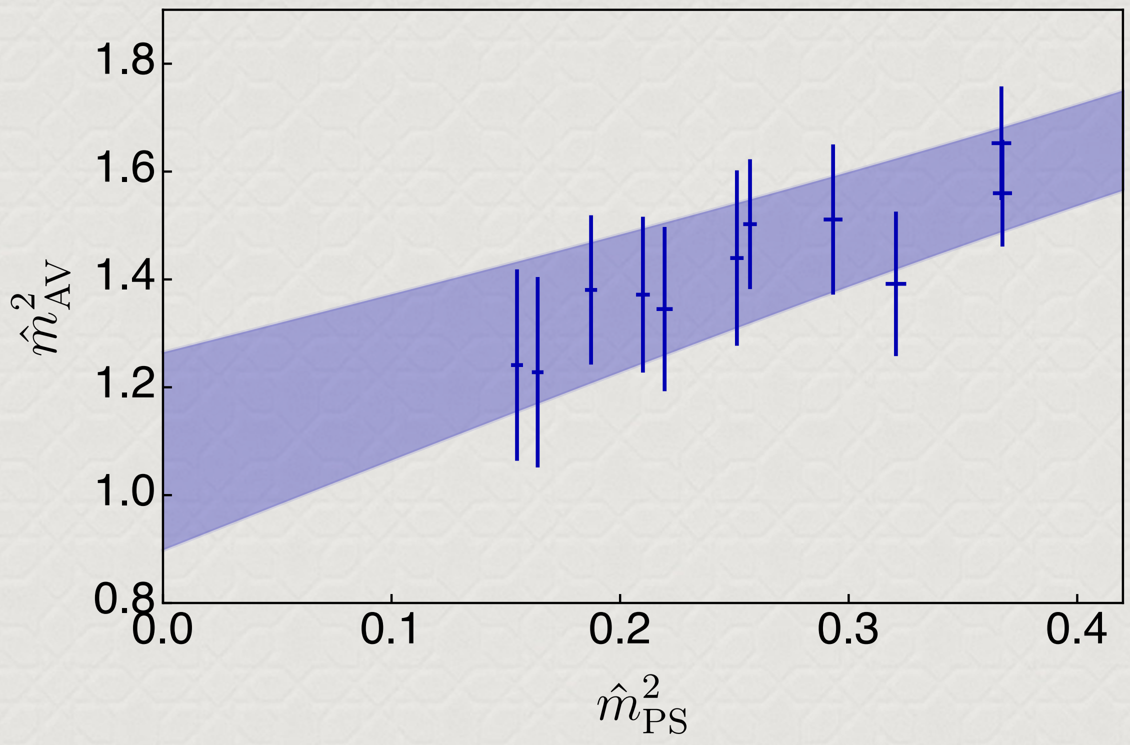
- NLO HLS EFT relates meson masses and decay constants with LECs.
- Using the LO mass relations & linearization $m_{\rm PS}^2 = 2Bm_f$

$$
\begin{aligned}
\hat{m}_{\rm V}^2 &= \frac{g_{\rm V}^2(b\hat{f}^2+\hat{F}^2)}{4(1+\kappa)} + \frac{2\hat{v}_1(\kappa+1)-\hat{y}_3(b\hat{f}^2+\hat{F}^2)}{4(\kappa+1)^2} g_{\rm V}^2 \hat{m}_{\rm PS}^2, \\
\hat{m}_{\rm AV}^2 &= \frac{(b+4)\hat{f}^2+\hat{F}^2}{4(1-\kappa)} g_{\rm V}^2 + \frac{\left((b+4)\hat{f}^2+\hat{F}^2\right)\hat{y}_4 - 2(1-\kappa)(\hat{v}_1-2\hat{v}_2)}{4(1-\kappa)^2} g_{\rm V}^2 \hat{m}_{\rm PS}^2, \\
\hat{f}_{\rm V}^2 &= \frac{1}{2}(b\hat{f}^2+\hat{F}^2) + \hat{v}_1 \hat{m}_{\rm PS}^2, \\
\hat{f}_{\rm AV}^2 &= \frac{(\hat{F}^2-b\hat{f}^2)^2}{2((b+4)\hat{f}^2+\hat{F}^2)} - \frac{((3b+8)\hat{v}_1-4(b+2)\hat{v}_2)\hat{f}^2+\hat{F}^2\hat{v}_1}{((b+4)\hat{f}^2+\hat{F}^2)^2}(\hat{F}^2-b\hat{f}^2)\hat{m}_{\rm PS}^2, \\
\hat{f}_{\rm PS}^2 &= \hat{F}^2 + (b+2c)\hat{f}^2 - \hat{f}_{\rm V}^2 - \hat{f}_{AV}^2,
\end{aligned}
$$

- 5 measurements to determine 10 low-energy constants (LECs).

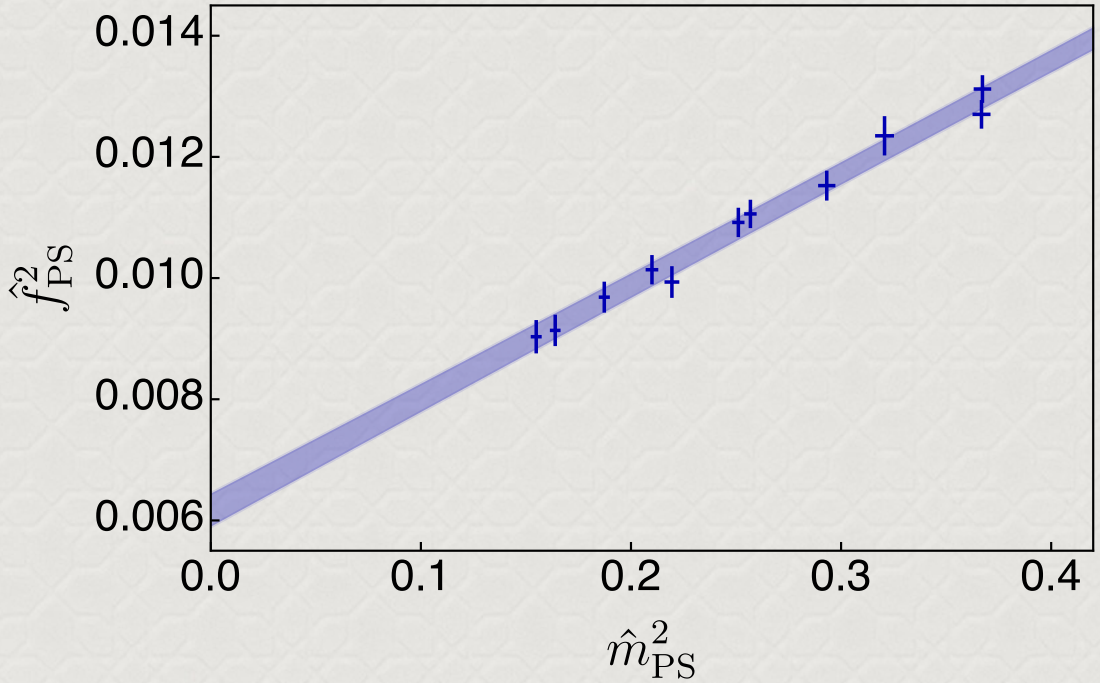
Global fit: masses

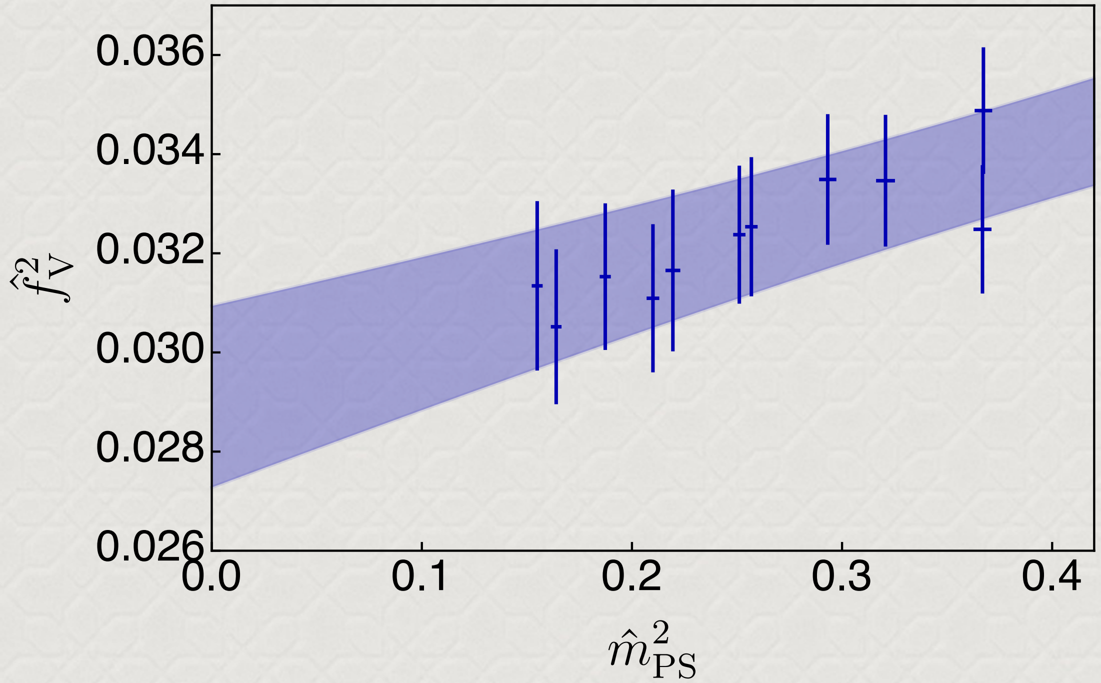


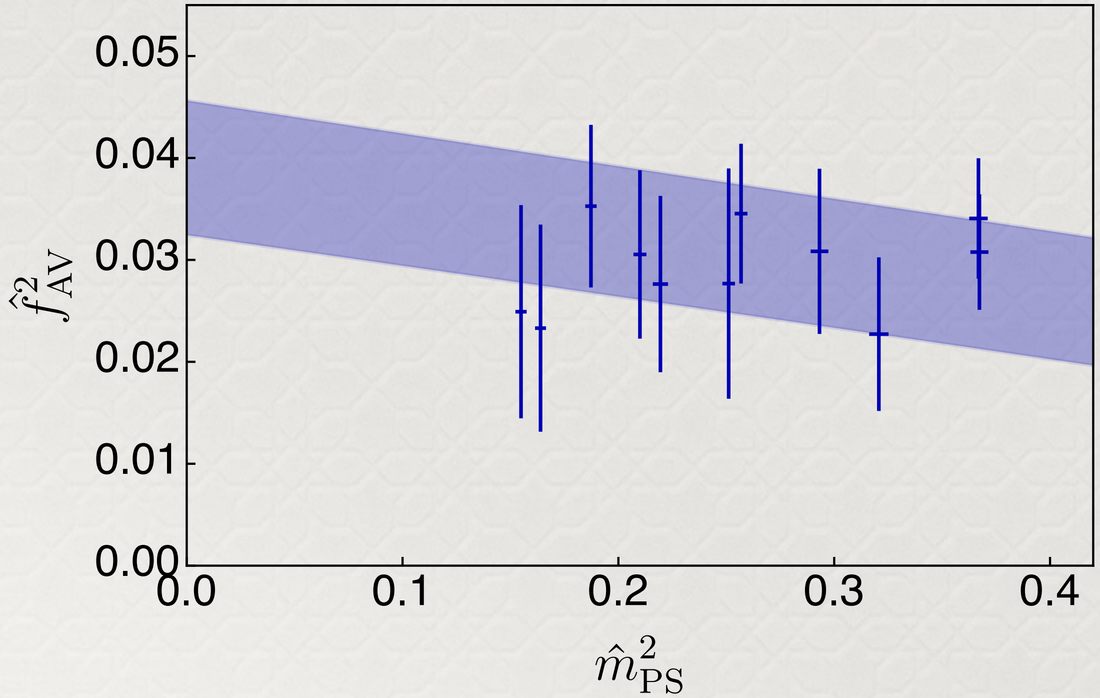


Consistent with the results of linear extrapolations

Global fit: decay constants







Consistent with the results of linear extrapolations

Linear mass dependence of $\hat{f}^2_{\rm AV}$ is better constrained by the HLS EFT.

V-PS-PS coupling constant from EFT

- (tree-level) HLS EFT predicts

$$g^{\chi}_{\mathrm{VPP}} = \frac{g_{\mathrm{V}}(b+2)(2\hat{f}^2+\hat{F}^2)(b\hat{f}^2+\hat{F}^2)}{((b+4)\hat{f}^2+\hat{F}^2)((b+(b+4)c)\hat{f}^2+(b+c+1)\hat{F}^2)\sqrt{1+\kappa}}$$

- V-PS-PS coupling constant in the massless limit: $g^{\chi}_{\mathrm{VPP}} = 6.0(4)(2)$

Limitations of NLO EFT

- Vector meson is stable.

- V-PS-PS coupling is as large as that in QCD.

- Linearization of the EFT mass relations could be questionable over the mass range considered without further assumption of cancellation from NNLO corrections. In particular,

$$m_{\rm V}^2 = \frac{1}{4(1+\kappa+my_3)} g_{\rm V}^2 (bf^2 + F^2 + 2mv_1)$$

$$\Rightarrow \quad \hat{m}_{\rm V}^2 = \frac{g_{\rm V}^2 (b\hat{f}^2 + \hat{F}^2)}{4(1+\kappa)} + \frac{2\hat{v}_1(\kappa+1) - \hat{y}_3(b\hat{f}^2+\hat{F}^2)}{4(\kappa+1)^2} g_{\rm V}^2 \hat{m}_{\rm PS}^2 + \mathcal{O}(\hat{m}_\pi^4)$$

requires $|y_3 m_{\rm PS}^2| \ll |1+\kappa|$ numerically we found $\hat{m}_{\rm PS}^2 \ll 0.67$

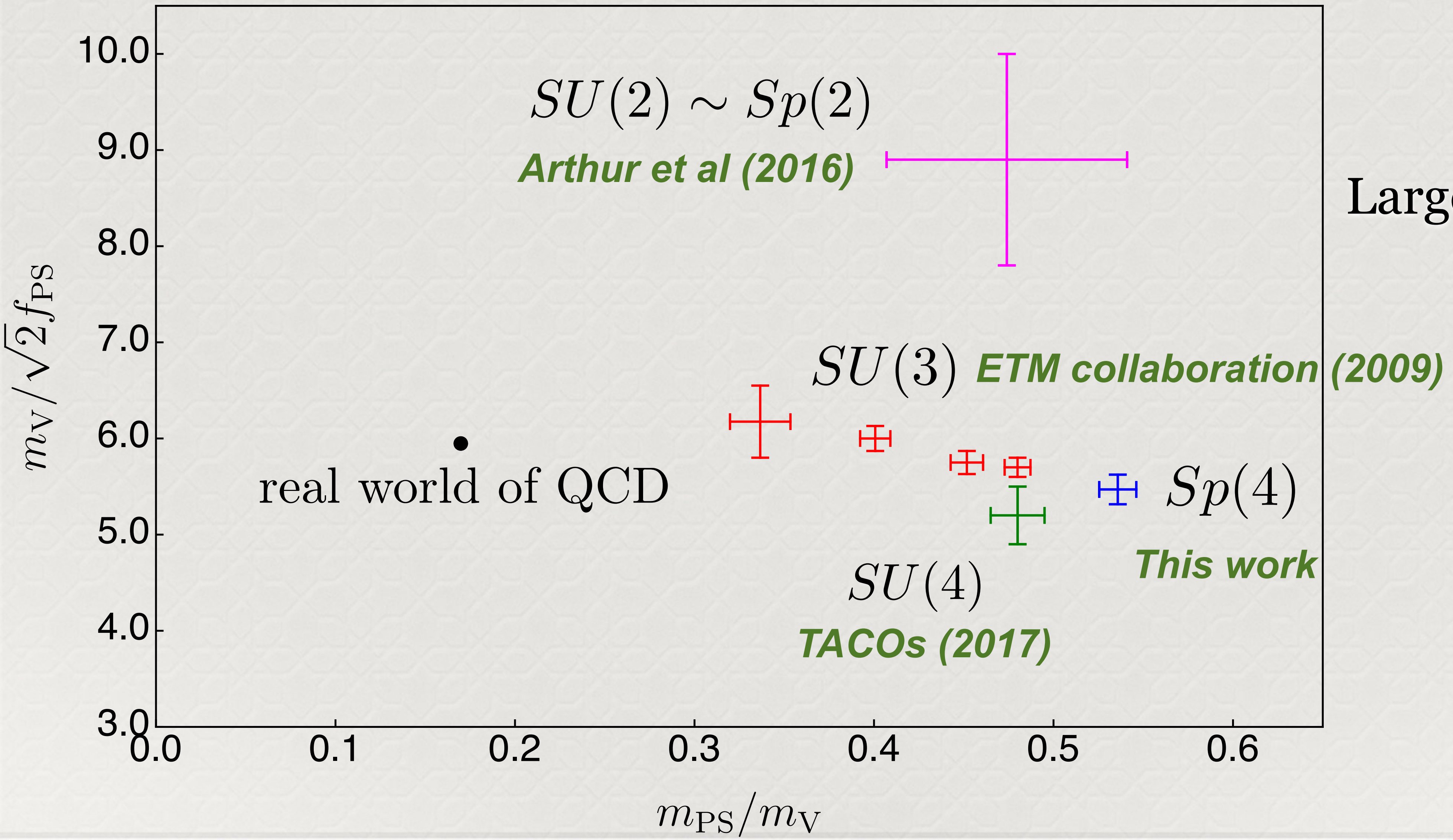
NNLO corrections become compatible with stat. error only for the lightest ensemble.

Comparison with other gauge groups

- The hypothesis vector meson dominance leads to the KSRF relation

Kowarabayashi & Suzuki (1966)
Riazuddin & Fayyazuddin (1966)

$$g_{VPP} = \frac{m_{\rm V}}{\sqrt{2} f_{\rm PS}}$$



Large Nc argument:

$$f_{\rm PS} \sim \sqrt{N_c}$$

Summary & outlook

- Dynamical calculations of `Sp(4)` with 2 fund. Dirac flavors: continuum & massless extrapolations of meson masses & decay constants for the first time.
- Performed a global fit by using (tree-level) NLO EFT based on HLO with some limitations.
- (Roughly) consistent with the large Nc argument.
- Larger volume calculations with smaller masses & finer lattices are underway.
- Explore the meson spectra of `Sp(4)` with 3 anti-sym. flavors of dynamical Dirac fermions toward partial compositeness.

Thank you for your attention!