SU(3) gauge system with twelve fundamental flavors

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GRADIENT FLOW STEP SCALING FUNCTION [1,2]

Gradient Flow step scaling function

We study the finite volume step scaling function [4,5]

$$\beta_{c,s}(g_c^2; L) = \frac{g_c^2(sL) - g^2(L)}{\log(s^2)},$$

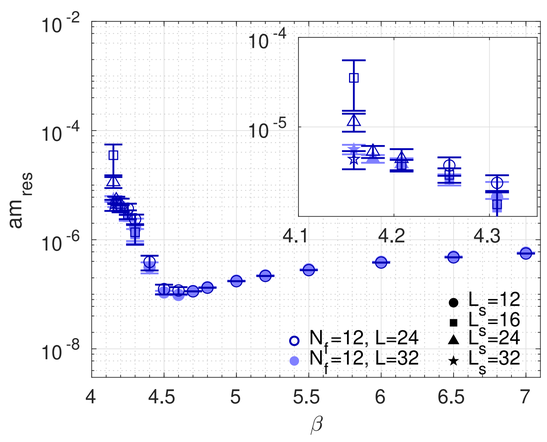
with $s = 2$, using the renormalized GF coupling

$$g_c^2(L) = \frac{128\pi^2}{3(N_c^2-1)} \frac{1}{C} t^2 \langle E(t) \rangle$$

- The flow time is set by the volume, $\sqrt{8t} = cL$
- C: tree-level normalization $C(c, L)$ [6] or zero mode correction $(1 + \delta(t/L^2))$ [5]

Simulation details

- `Grid`[7] code fully optimized for KNL
- Symanzik gauge action
- 3 × stout smeared Möbius domain wall fermions
- L^4 volumes, $L = 8, 10, 12, 14, 16, 20, 24, 28, 32$
- Antiperiodic BC in all four directions
- Massless: $am_f = 0$
- L_s grows from 12 to 32 keeping $am_{\rm res} < 10^{-5}$



Advantages of Domain Wall Fermions

- Preserves full SU(N)×SU(N) flavor symmetry even at finite gauge coupling
- Effective gauge term generated by fermions and smearing is very small due to Pauli-Villars term
  - reduced cut-off effects
  - increased region of perturbative improvement

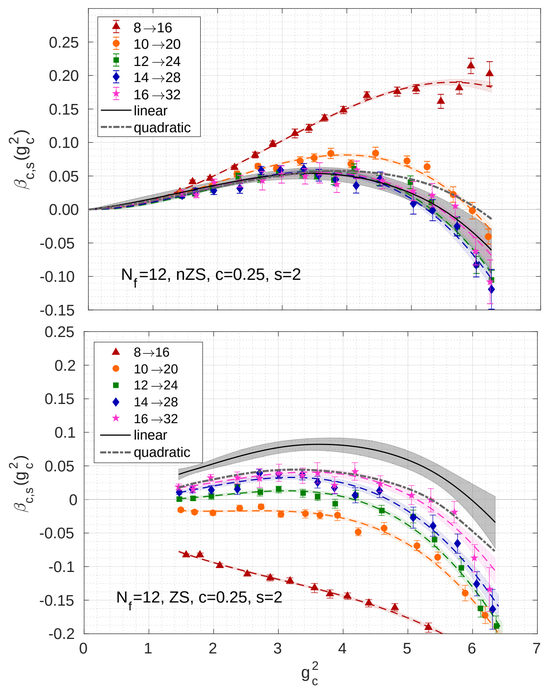
Gradient flow coupling

- Fully $\mathcal{O}(a^2)$ Symanzik improved set-up
  - Symanzik (S) gauge action
  - Zeuthen (Z) flow [8]
  - Symanzik (S) operator
- Consistent with different gradient flows Wilson (W), Symanzik (S), Zeuthen (Z) [8] and/or operators Wilson-plaquette (W), Symanzik (S), clover (C)
- Include tree-level normalization [6]

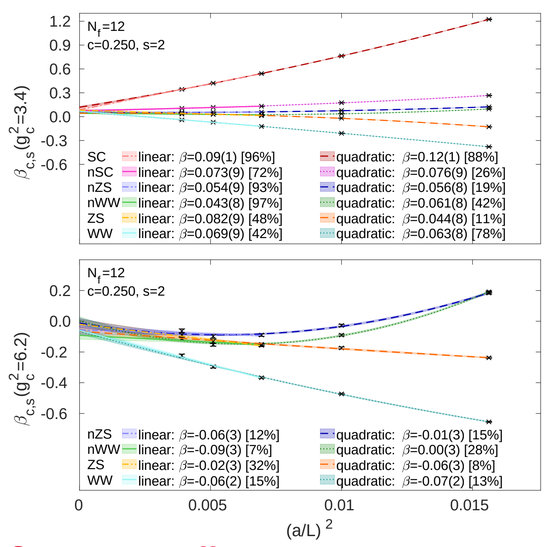
Analysis steps

- Calculate $\beta_{c,s}(g_c^2)$ for all volume pairs at all bare couplings
- Interpolate volume pairs using a 3rd order polynomial in g_c^2
- Perform $L \to \infty$ continuum extrapolations on interpolated $\beta_{c,s}(g_c^2)$
- Account for systematic effects using the envelope of nZS and ZS

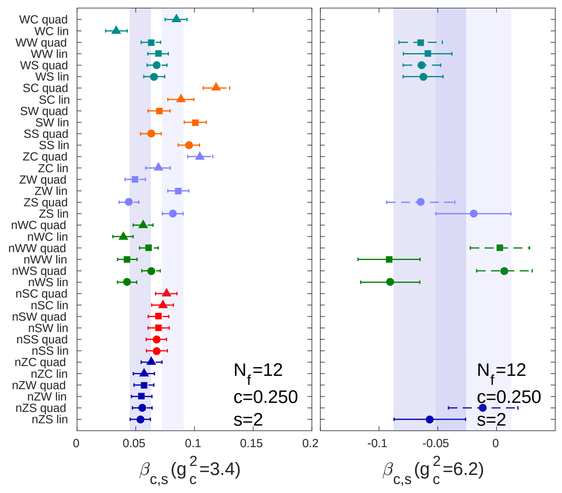
Preferred nZS and ZS analysis



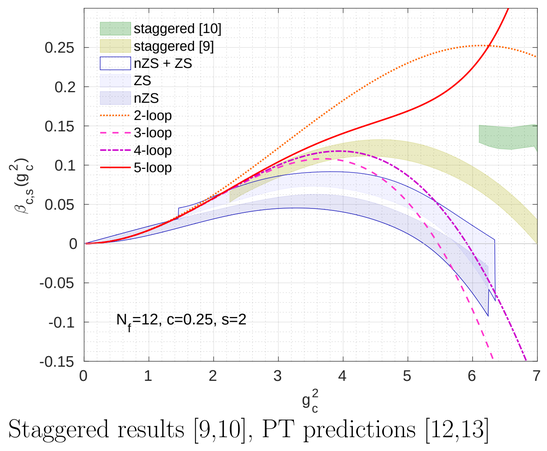
Continuum $L \to \infty$ extrapolations



Systematic effects



Final result for $c = 0.250$



Staggered results [9,10], PT predictions [12,13]

Summary

- $N_f = 12$ step scaling function is very small, numerical simulations are challenging
- Necessary to consider systematical effects and reduce cut-off effects
  - DWF reduce cut-off effects from gauge fields
  - ZS is fully $O(a^2)$ improved
  - Different flows/operators indicate systematical errors
- $c = 0.250$ scheme: IRFP at $5.2 \le g_c^2 \le 6.4$
- Similar conclusions for $c = 0.275$, 0.3
- Tension with staggered results [9,10] persists

References

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CONTINUOUS GRADIENT FLOW β FUNCTION [3]

Drawbacks of step scaling function

- GF flow time grows $t \propto L^2$
- $\beta_{c,s}(g_c^2; L)$ defined as difference of two volumes
- Cut-off effects are dominated by smallest volumes

Continuous β function

$$\beta(g^2) = \mu \frac{dg^2}{d\mu} = -2t \frac{dg^2}{dt}$$

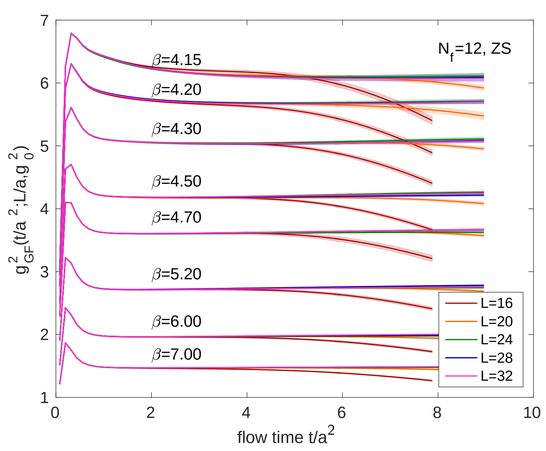
$L/a \to \infty$, $t/L^2 \to 0$, with g^2 the GF coupling

$$g_{GF}^2(t/a^2; L/a, g_0^2) = \frac{128\pi^2}{3(N^2-1)} \frac{1}{1+\delta(t/L^2)} \langle t^2 E(t) \rangle$$

- $\beta(g^2; L)$ is defined for each ensemble
- Correlated t/a^2 values reduce statistical uncertainty
- Flow time t/a^2 is fixed, not growing with volume
- Only large volumes, reducing cut-off effects
- Definition of continuous $\beta(g^2)$ differs from step scaling function by a factor -2

For alternative ideas see [13,14]

Renormalized coupling vs. flow time



Analysis steps

- Interpolate $\beta(g^2, g_0^2)$ vs. $g^2(t/a^2, L/a)$ at fixed t/a^2, L/a
- For fixed $t \ll L^2$ and g^2 extrapolate $L/a \to \infty$
- Take the $a^2/t \to 0$ continuum limit
- Improve by combining different operators
- Investigate systematic effects in fit ranges

Details of analysis

- Finite volume effects are $\mathcal{O}(t/L^2)$
- Physical volume is fixed as $L/a \to \infty$ at fixed g^2 ($am = 0$, chirally symmetric regime)
- Continuum limit extrapolation

$$\beta(g^2) = \beta(g^2; t/a^2) + \xi(a^2/t)^{1+p} + \text{h.o.t}$$

$(a^2/t)^{1+p}$ describes leading irrelevant operator

- Control cut-off effects and infinite volume extrapolation errors by limiting the a^2/t range
- $\beta(g^2)$: continuum limit continuous β function
- Independent of operator used to define g_{GF}^2

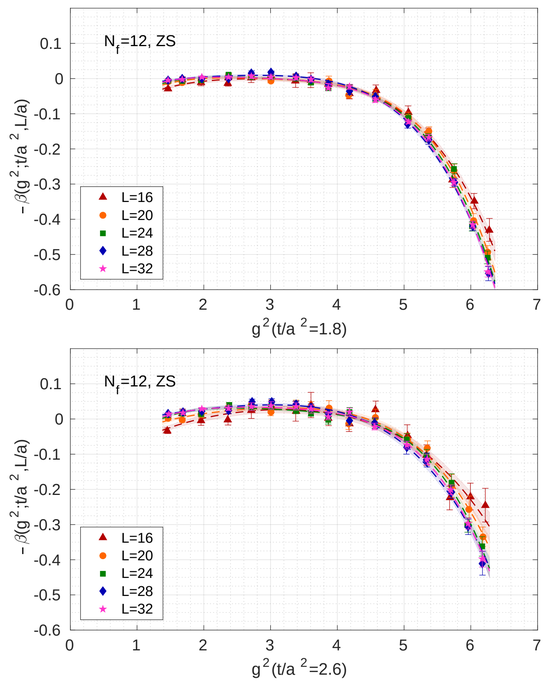
Systematic effects

- Study different operators
- Vary fit range in flow time
- Explore different infinite volume extrapolations

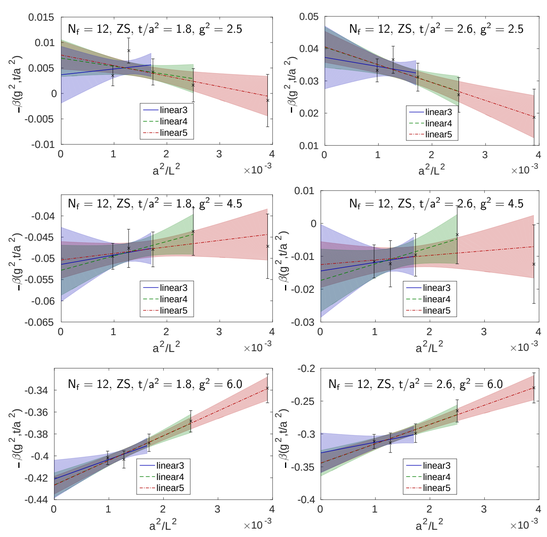
Outlook

- New method, easy to generalize e.g. determination of running anomalous dimensions [15] (Talk by AH on Monday on $N_f = 2$ flavor QCD)
- Continuous β function has significantly reduced uncertainties
- a^2/t extrapolation is via correlated continuous variable leading to reduced errors
- Allows for quick reanalysis of existing GF step-scaling data

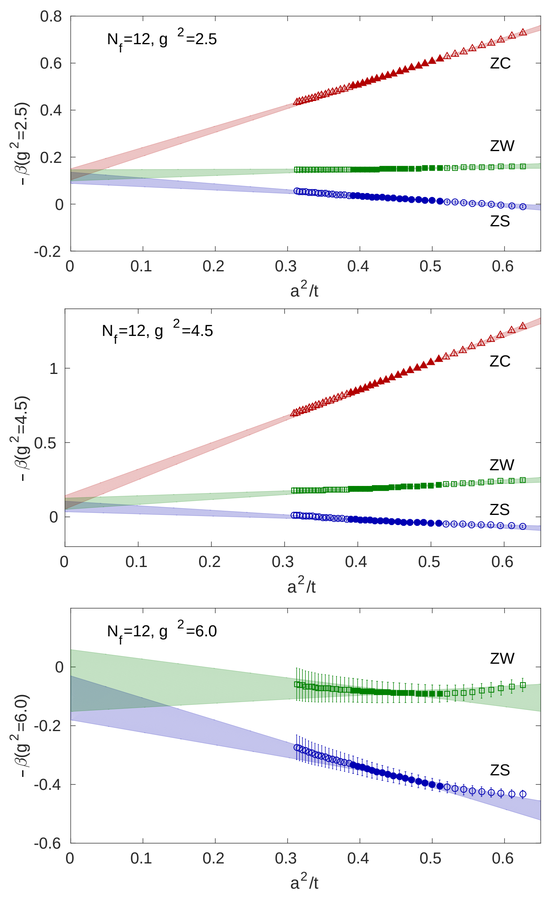
Interpolatation in g^2 at fixed t/a^2



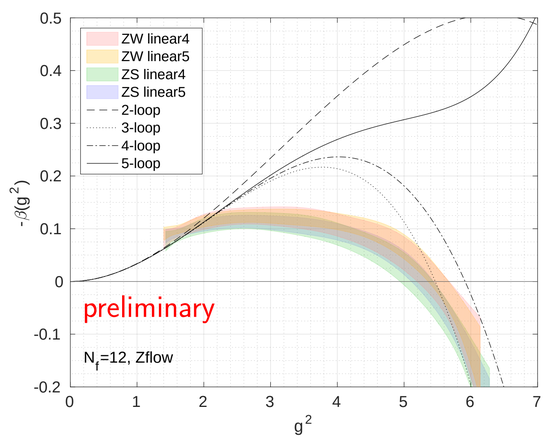
$L/a \to \infty$ at fixed t/a^2, g^2



$a^2/t \to 0$ continuum extrapolation



Final result



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