SU(3) gauge system with twelve fundamental flavors

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**GRADIENT FLOW STEP SCALING FUNCTION [1,2]**

Gradient flow step scaling function

\[ \beta_{g_2}(L) = \frac{g_2(L) - g_2(0)}{g_2(0)} \]

with \( s = 2 \), using the renormalized GF coupling

\[ g_2(L) = \frac{32L^2}{N(N-1)(N+2)} (E(0)) \]

- The flow time is set by the volume: \( cL = sL \)
- C: two-loop normalization \( C(L) \) [6]
- or zero mode correction \( 1 + \delta/t(L)^2 \) [5]

**Simulation details**

- \( \sigma_{\text{K}3}/\text{g} \) code fully optimized for KNL
- Symmetric gauge action
- \( 3 \times 4 \) stout smeared Monday domain wall fermions
- \( 4 \times 3 \) volumes, \( L = 8, 10, 12, 14, 16, 20, 24, 28, 32 \)
- Antiperiodic BC in all four directions
- Massless: \( \text{am} = 0 \)
- \( L_0 \) grows from 12 to 32 keeping \( \text{am}_{\text{max}} < 10^{-5} \)

Advantages of Domain Wall Fermions

- Preserves full SU(3) \& SU(3) flavor symmetry even at finite gauge coupling
- Effective gauge term generated by fermions and smearing is very small due to Punk-Villain term
  - reduced cut-off effects
  - increased region of perturbative improvement

Gradient flow coupling

- Fully \( O(3) \) Symmam improved set-up
- Symmetric (S) gauge action
  - Zeuthen (Z) flow [6]
- Symmetric (S) operator
- Consistent with different gradient flows
  - Wilson (W), Symmam (S), Zeehnth (Z) [6]
- and/or operators
  - Wilson-plaquette (W), Symmam (S), clover (C)
- Include tree-level normalization [6]

Analysis steps

- Calculate \( \beta_{g_2}(L) \) for all volume pairs at all bare couplings
- Interpolate volume pairs using a 3rd order polynomial in \( g_2^2 \)
- Perform \( L \to \infty \) continuum extrapolation on interpolated \( \beta_{g_2}(L) \)
- Account for systematic effects using the envelope of all

Final result for \( c = 0.250 \)

- \( N_f = 12 \) step scaling function is very small
- Numerical simulations are challenging
- Necessary to consider systematic effects and reduce cut-off effects
  - DWF reduce cut-off effects from gauge fields
  - DWF is fully \( O(3) \) improved
  - Different flows/operators indicate systematic errors
- \( c = 0.250 \) scheme: \( \beta_{g_2} \) at 5.2 \( \leq g_2^2 \leq 6.4 \)
- Similar conclusions for \( c = 0.275, 0.3 \)

**CONTINUOUS GRADIENT FLOW \( \beta \) FUNCTION [3]**

Continuous \( \beta \) function

\[ \beta(g_2) = \frac{dg_2}{d\beta} = -\frac{d^2g_2}{d\beta^2} \]

- \( L/a \to \infty \)
- \( g_2^t \) at fixed \( t/a^2 \)

Analysis steps

- Interpolate \( \beta(g_2, g_2^t) \) vs. \( g_2^2(t/a^2, L/a) \)
  - For fixed \( t \), \( L \), \( g_2 \) extrapolate \( L/a \to \infty \)
  - Table the \( g_2^t \) at 0 continuum limit
  - Improve by combining different operators
  - Investigate systematic effects in fit ranges

Details of analysis

- Finite volume effects are \( O(1/L^2) \)
- Physical volume is fixed at \( L/a \to \infty \) at fixed \( g_2^2 \)
  - Only volumes \( \text{am} = 0 \), chirally symmetric regime
  - Continuum limit extrapolation
    \[ \beta(g_2) = \beta(g_2 t/a^2) + \xi (t/a^2)^{1+\nu} + \text{h.o.t.} \]
  - \( (t/a^2)^{1+\nu} \) describes leading irrelevant operator
  - Control cut-off effects and infinite volume extrapolation errors by limiting the \( t/a^2 \) range
  - \( \beta(g_2) \): continuum limit continuous \( \beta \) function
  - Independent of operator used to define \( \beta_{g_2} \)

Systematic effects

- Study different operators
- Very fit range in flow time
- Explore different infinite volume extrapolations

**Summary**

- \( N_f = 12 \) step scaling function is very small
- Numerical simulations are challenging
- Necessary to consider systematic effects and reduce cut-off effects
- DWF reduces cut-off effects from gauge fields
- DWF is fully \( O(3) \) improved
- Different operators indicate systematic errors
- \( c = 0.250 \) scheme: \( \beta_{g_2} \) at 5.2 \( \leq g_2^2 \leq 6.4 \)
- Similar conclusions for \( c = 0.275, 0.3 \)

- Tension with staggered results [9,10], PT predictions [12,13]

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**Interpolation in \( g_2^2 \) at fixed \( t/a^2 \)**

**Final result**

- \( L/a \to \infty \) at fixed \( t/a^2, g_2^2 \)

- \( a^2/t \to 0 \) continuum extrapolation

**Outlook**

- New method, easy-to-generalize, e.g. determination of running anomalous dimensions [15]
- Talk by Ali on Monday on \( N_f = 2 \) flavor QCD
- Continuous \( \beta \) function has significantly reduced uncertainties
- \( a^2/t \) extrapolation is via correlated continuous variable leading to reduced errors
- Aims for quick resumé of existing GF step scaling data

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