Resonance study of SU(2) model with 2 flavours of fermions in the fundamental representation

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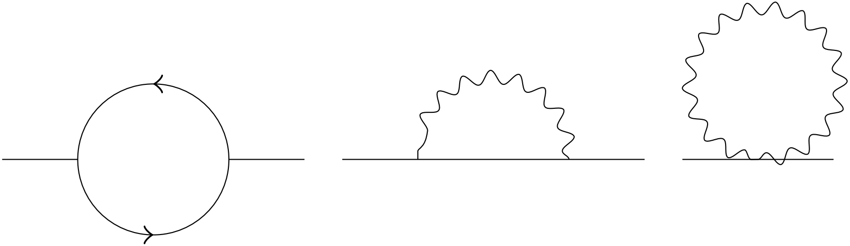
Problems with SM Higgs

SM Higgs mass quadratically sensitive to the cutoff (i.e. new physics scale) of the theory.



Let Λ be the scale up to which SM is valid.

$$m_H^2 = m_R^2(\Lambda) - \Sigma(\Lambda)$$

with

$$\delta m_H^2 \propto \Lambda^2.$$

But $m_H^2 = 126\mathrm{GeV} \rightarrow$ large cancellation between UV (m_R^2) and IR (Σ) contributions?

Possible solution: **make Higgs composite** (goldstone boson)

SU(2) with 2 fundamental flavours

SU(2) model, 2 Dirac fermions in fundamental representantion [1402.0233].
$SU(2) = Sp(2) \sim SO(3)$ smallest non-abelian Lie group.

$$\mathcal{L} = -\frac{1}{4} F^{a\mu\nu} F^a_{\mu\nu} + i\bar{U}\gamma^\mu D_\mu U + i\bar{D}\gamma^\mu D_\mu D$$

Fundamental representation of SU(2) is pseudo-real $\rightarrow$ we can construct a flavour multiplet

$$Q = \begin{pmatrix} u_L \\ d_L \\ -i\sigma^2 C \bar{u}_R^T \\ -i\sigma^2 C \bar{d}_R^T \end{pmatrix}$$

$\mathcal{L}$ is symmetric under SU(4) flavour group (NOT $SU(2) \times SU(2)$ one would get in QCD)

SU(2) with 2 fundamental flavours

SU(4) symmetry is broken spontaneously by a fermion condensate $\Sigma^{ab} = \langle Q^a(i\sigma_c^2)CQ^b\rangle$ [1109.3513] to the subgroup which leaves it invariant:

$$U^T\Sigma U = \Sigma \quad U \in Sp(4) \sim SO(5).$$

- The flavour symmetry breaking pattern is $SU(4) \to Sp(4)$, equivalent to $SO(6) \to SO(5)$.
- This produces 5 Goldstone bosons ("pions") - 3 of them 'eaten' by W and Z.
- On the lattice we add an explicit mass term:

$$-m(\bar{u}u + \bar{d}d) = \frac{m}{2}Q^T(-i\sigma^2)CEQ + h.c.$$

$$E = \begin{pmatrix} 0 & 1_2 \\ -1_2 & 0 \end{pmatrix}$$

Need to take the 'chiral' $m \to 0$ limit later.

Scattering in composite Higgs models

Why should we care about scattering in composite Higgs models?

- We can predict the resonance spectrum in vector boson scattering
- This follows from Goldstone boson equivalence theorem: at large energies, external vector boson states equivalent to Goldstone boson states
- e.g. $\rho \to \pi^+\pi^-$ corresponds to vector resonance in W^+W^- scattering
- need to calculate the resonance mass and width.

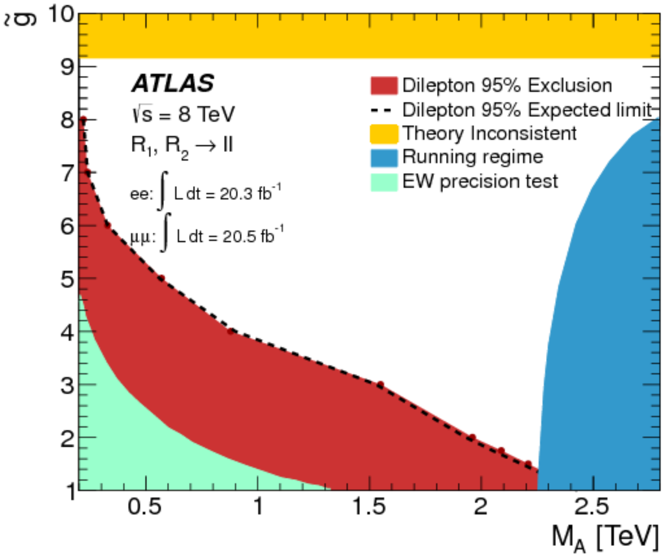
Effective Lagrangian:

$$\mathcal{L}_{eff} = g_{\rho\pi\pi}\rho^{\mu}_{[ij]}\partial_{\mu}\pi_i\pi_j$$

In QCD $\rho^{\mu}_{[ab]} = f_{abc}\rho^{\mu}_{c}$.

Constraint example

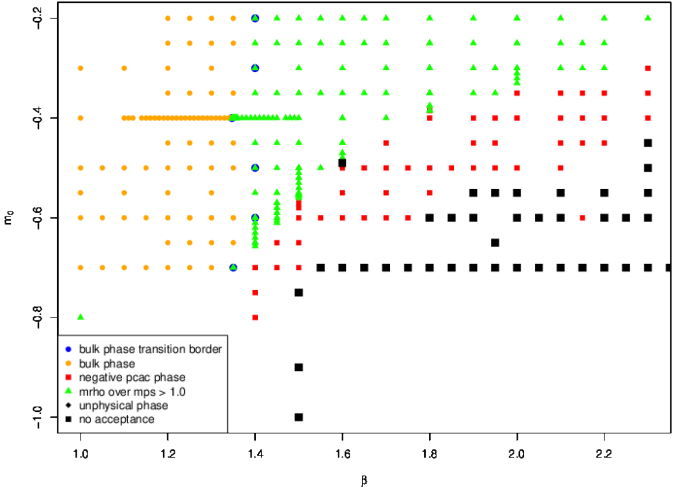
Example: Minimal walking technicolor.



Computational strategy

1. Generate SU(2) ensembles with unstable ρ.
2. Calculate the energy spectrum of states with ρ quantum numbers.
3. Use Lüscher formula to convert evergies into phase shifts.
4. Fit the phase shifts to obtain $g_{\rho\pi\pi}$ and resonance mass.

Wilson clover ($c_{sw} = 1$) + Symanzik improved gauge



Our ensemble

Wilson clover fermions + Symanzik improved gauge action

β	1.45	
m_0	-0.6050	
c_{sw}	1.0	
volumes	$16^3 \times 32$	$24^3 \times 48$
am_π	0.20213(5)	
am_ρ^{naive}	0.444(9)	
am_{pcac}^0	0.01110(7)	
af_π	0.0564(3)	
$m_\pi L$	3.5	5
# trajectories	1354	2551

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GEVP

We follow the PACS-CS procedure in 1106.5365.
The correlation functions are given by

$$C_{ij}(t) \equiv \langle 0 \mid O_i^\dagger(t) O_j(0) \mid 0 \rangle = \sum_{n,m} \langle 0 \mid O_i^\dagger \mid n \rangle (e^{-E_n t} \delta_{mn}) \langle m \mid O_j \mid 0 \rangle$$

U and V are square matrices assuming higher-energy states don't contribute.
Then

$$C_{ij}^{-1}(t_0) C_{jk}(t) = V_{in}^{-1} diag \left(e^{-E_n (t - t_0)} \right)_{nm} V_{mj}$$

The spectrum can be extracted from the eigenvalues of $C^{-1}(t_0)C(t)$.

Interpolating operators

We use the following two interpolating operators

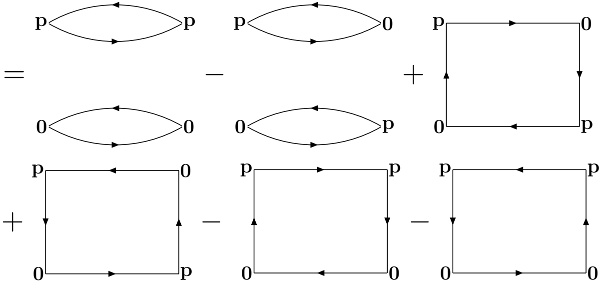
$$O_1(t) = \left(\sum_x \bar{\psi}(x)\gamma^5\psi(x)e^{i\mathbf{p}\cdot\mathbf{x}}\right)\left(\sum_y \bar{\psi}(y)\gamma^5\psi(y)e^{i\mathbf{y}\cdot\mathbf{0}}\right)$$

$$O_2(t) = \sum_x \bar{\psi}(x)(\gamma\cdot\hat{P})\psi(x)e^{i\mathbf{P}\cdot\mathbf{x}}$$

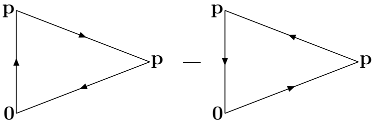
- $\mathbf{P} = \mathbf{p} + \mathbf{0}$
- Two moving frames, $p = (0,0,1)$ in MF1 and $p = (1,1,0)$ in MF2
- P+A boundary conditions in the time direction - effectively doubles the time extent of the lattice

Contractions

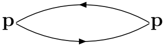
$C_{11}(t) =$



$C_{12}(t) = -C^*_{21}(t) =$

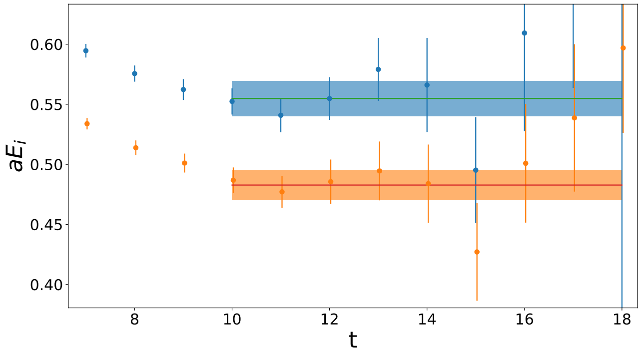


$C_{22}(t) =$



Effective energies

$E_{eff\,i}(t) = log(\lambda_i(t)/\lambda_i(t+1))$
Example: 24^3, $P = (0,0,1)$



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Moving frames

Phase shift formula depends on the frame and the representation:

P	group	repr	$\tan\delta_1$
$(0,0,1)$	D_{4h}	A_2^-	$\frac{\pi^{3/2}q}{Z_{00}(1;q^2)+\frac{2}{\sqrt{5}q^2}Z_{20}}$
$(1,1,0)$	D_{2h}	B_1^-	$\frac{\pi^{3/2}q}{Z_{00}(1;q^2)-\frac{1}{\sqrt{5}q^2}Z_{20}-i\frac{\sqrt{3}}{\sqrt{10}q^2}\left(Z_{22}(1;q^2)-Z_{2(-2)}(1;q^2)\right)}$

$$Z_{lm}(s,q^2)=\sum_{n\in P_d}\frac{Y_{lm}(n)}{(q^2-n^2)^s}$$

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Relating phase shifts to $g_{\rho\pi\pi}$ and resonance mass

$$\tan\delta_1 = \frac{g_{\rho\pi\pi}^2}{6\pi}\frac{p_*^3}{E_{CM}(M_\rho^2 - E_{CM}^2)}, \; p_* = \sqrt{E_{CM}^2/4 - m_\pi^2}$$

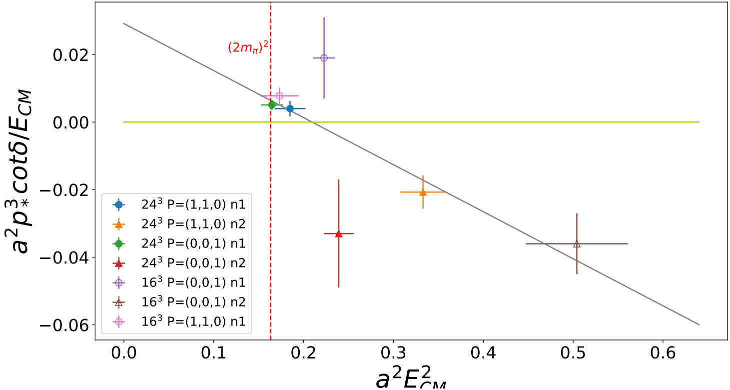
Can be rewritten as

$$\frac{p_*^3\cot\delta}{E_{CM}} = \frac{6\pi}{g_{\rho\pi\pi}^2}\left(M_\rho^2 - E_{CM}^2\right) \tag{1}$$

Plot $p_*^3\cot\delta/E_{CM}$ as a function of E_{CM}^2

- Linear plot
- $g_{\rho\pi\pi}$ can be read off from the slope
- x-intercept is M_ρ^2

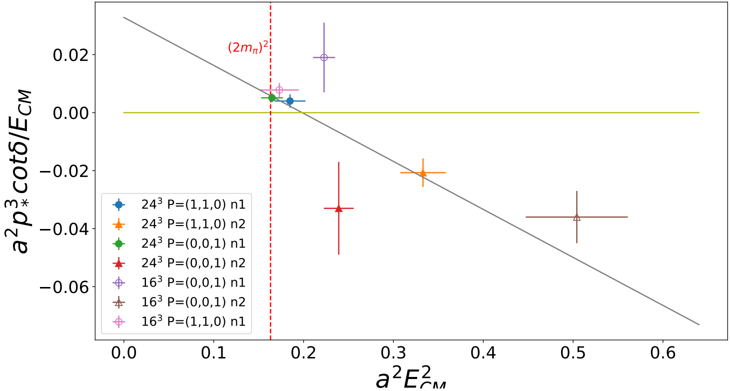
Extracting $g_{\rho\pi\pi}$ and M_ρ



$$g_{\rho\pi\pi} = 11.6(18)$$

$$aM_\rho = 0.458(67) \qquad am_\rho^{\mathrm{naive}} = 0.444(9)$$

Extracting $g_{\rho\pi\pi}$ and M_ρ - without 16^3



$$g_{\rho\pi\pi} = 10.7(23)$$

$$aM_\rho = 0.445(95) \qquad a\mathrm{m}_\rho^{\mathrm{naive}} = 0.444(9)$$

Conclusions and outlook

Conclusions:

- Composite Higgs models, which address the naturalness problem can be studied using lattice gauge theory techniques
- $\pi\pi$ scattering = W and Z scattering at high energies
- Techniques from lattice QCD can be applied directly, only flavour structure different
- First result for the phase shift in the SU(2) model with 2 fundamental flavours - $g_{\rho\pi\pi} \sim 11(2)$ somewhat larger than SU(3) value

Next steps:

- Chiral extrapolation - in progress, new ensemble in production
- Continuum limit

Backup

Phenomenology

Identify generators of SU(4) with electroweak generators. Σ_0 leaves them unbroken, Σ_{TC} breaks all.

$$\Sigma = \cos\theta \Sigma_{TC} + \sin\theta \Sigma_0$$

	$\theta = 0$	$\theta = \pi/2$
EW symmetry	unbroken	broken
model	composite Higgs	Technicolor
pions	$W^{\pm}$, Z, H + others	$W^{\pm}$, Z + others
Higgs	pion	scalar resonance

In general, Higgs is a superposition of a Goldstone boson and a σ-like resonance.

Connection to electroweak physics:

$$f_{PS} \sin\theta = v = 246\mathrm{GeV}$$

$$\Lambda_{TC} \sim f_{PS} = \frac{246\mathrm{GeV}}{\sin\theta}$$

SM interaction tend to push $\theta \to \pi/2$.