Constraining EFT's in a Theory with a Light Scalar

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[for the LSD Collaboration]

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When is a scalar light?

- A scalar meson is light when it has a mass of order $F\pi$ and small compared to $4\pi\ F\pi$, so it should be present in any low-energy EFT.

- The Standard Model Higgs is light: $M_H \sim F\pi / 2$.

- In QCD, $M\sigma \sim 4\text{-}5\ F\pi$ [see plenary by Edwards] so it's not very light so we don't usually include it in the EFT and is probably not a good candidate for composite Higgs.

- But, there are confining theories with light scalars. This is a short talk, so I'll just discuss SU(3) with N_f=8.

- Big question: What is the EFT?

Lattice Strong Dynamics Collaboration

James Osborn
Xiao-Yong Jin

Joe Kiskis

Evan Weinberg

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Anna Hasenfratz
Ethan Neil
Oliver Witzel

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Claudio Rebbi

Michael Buchoff
Chris Schroeder
Pavlos Vranas

Enrico Rinaldi

Tom Appelquist
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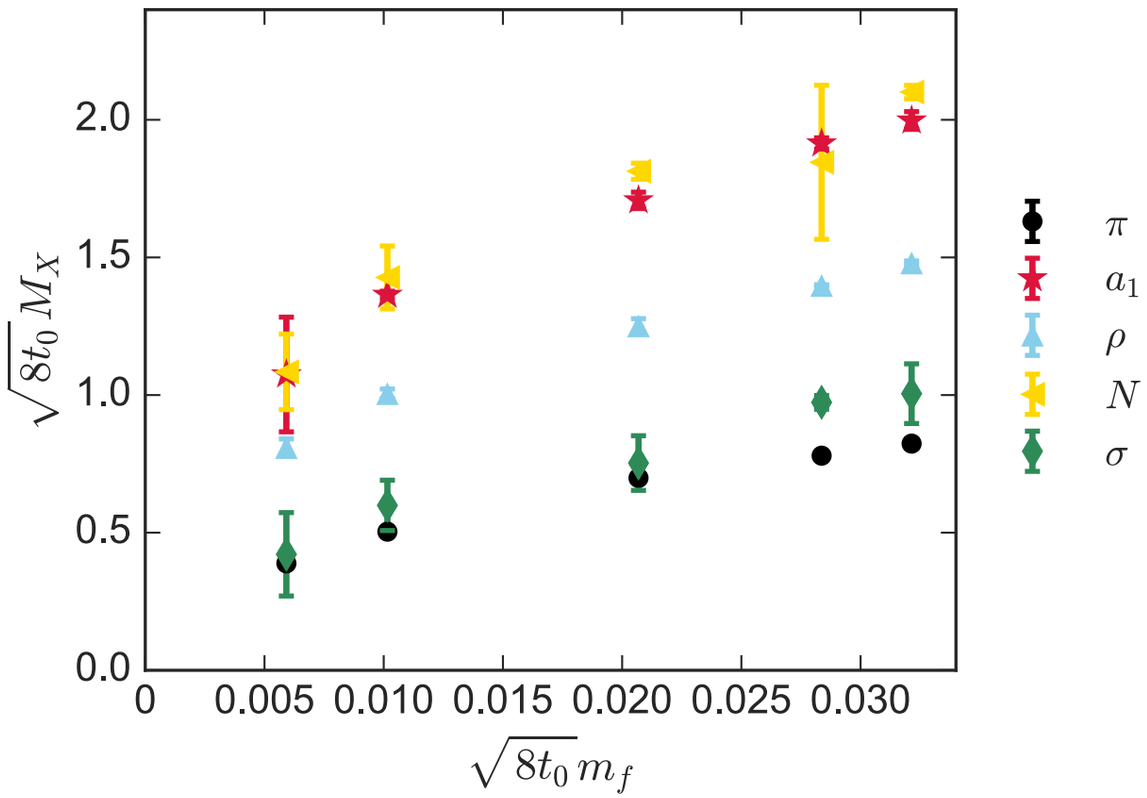
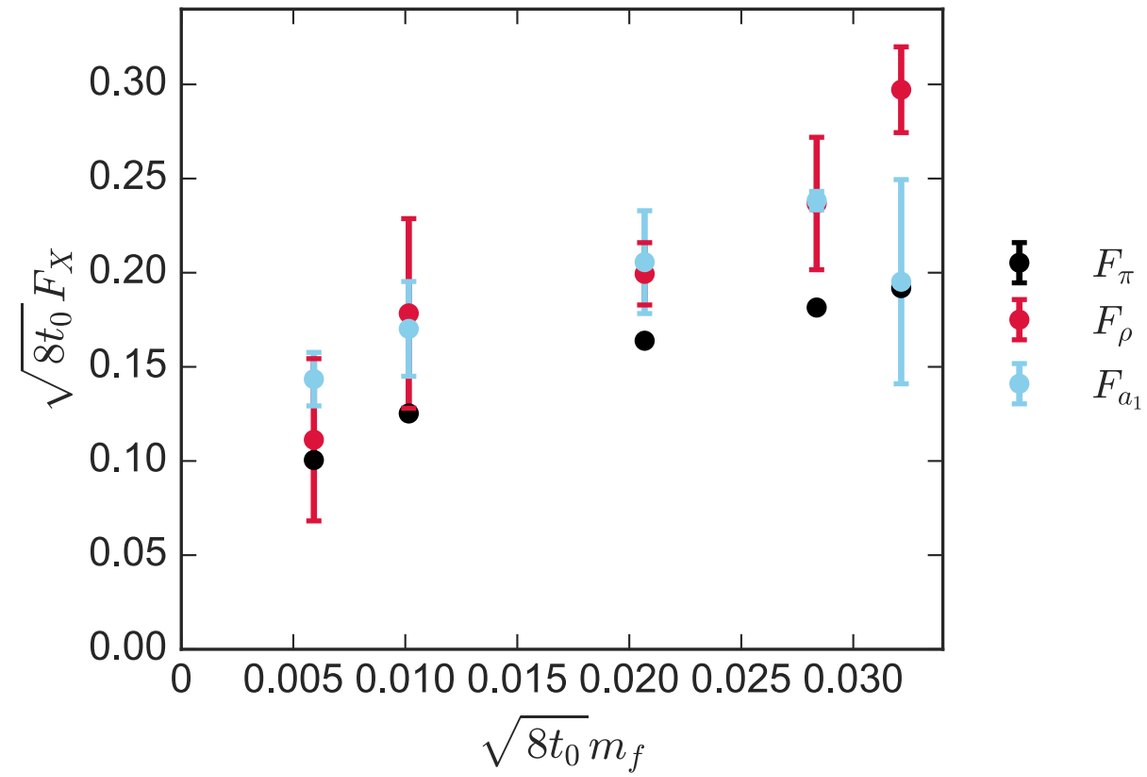
Kimmy Cushman

with James Ingoldby (Yale)

Evidence for Light Scalar in SU(3) Nf=8

LSD Collaboration, Phys.Rev. D99 (2019) 014509



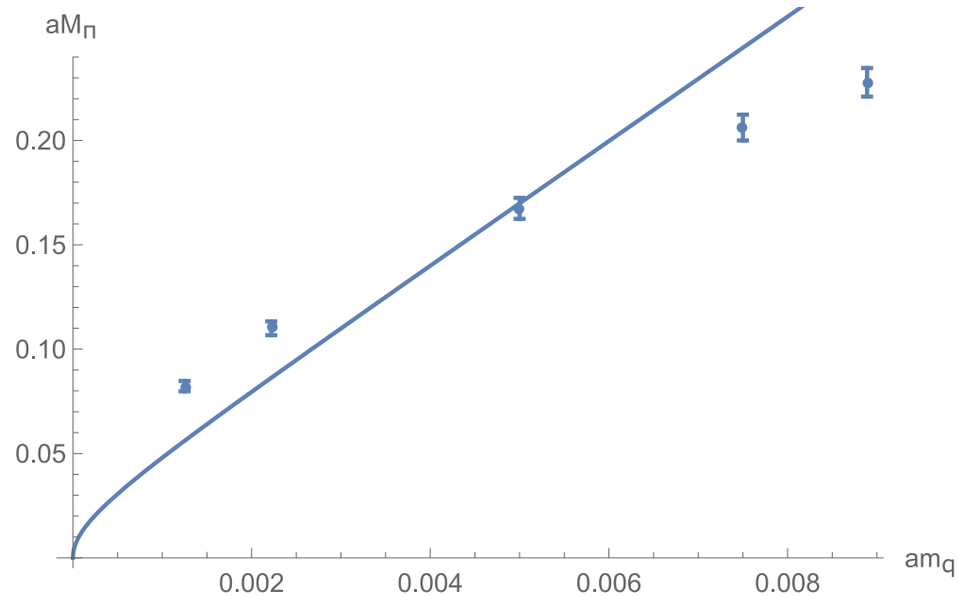


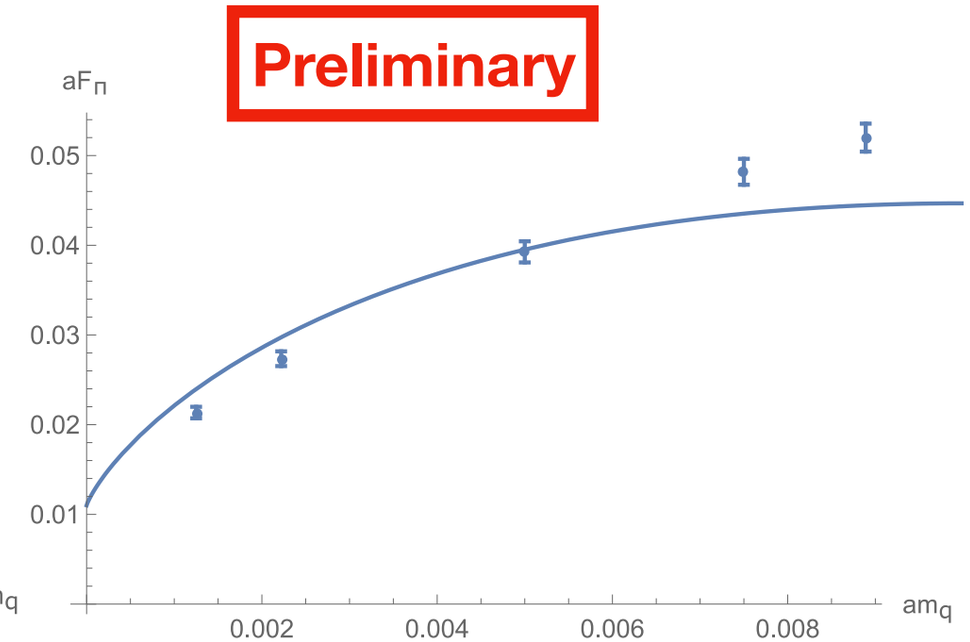
- In LO χPT, $F_\pi(m_q) \sim f_\pi$. The lattice results show NLO >> LO for $F_\pi(m_q)$, but $M_\sigma \sim M_\pi << M_\rho$.

- **Notational convention**: chiral limit m_x , finite quark mass M_x

SU(3) Nf=8 NLO Fits

Lattice units
χ^2/dof=29





- Large slope on $F\pi$ leads to poor fit for χPT in current mass range.
- Adding a general scalar to chiral Lagrangian will not solve this problem although the fits might have low χ^2/dof due to many fit parameters.

Soto, Talavera, Tarrus, Nucl. Phys. B 866 (2013) 270
Hansen, Langeable, Sannino Phys. Rev. D 95 (2017) 036005
Hansen, Langeable, Sannino arXiv:1810.11993 [hep-ph]
Cata, Mueller arXiv:1906.01879 [hep-ph]

Dilaton EFT

Goldberger, Grinstein, Skiba, Phys.Rev.Lett. 100 (2008) 111802
Matsuzaki, Yamawaki, Phys.Rev.Lett. 113 (2014) 082002
Golterman, Shamir, Phys.Rev. D94 (2016) 054502
Appelquist, Ingoldby, Piai, JHEP 1707 (2017) 035

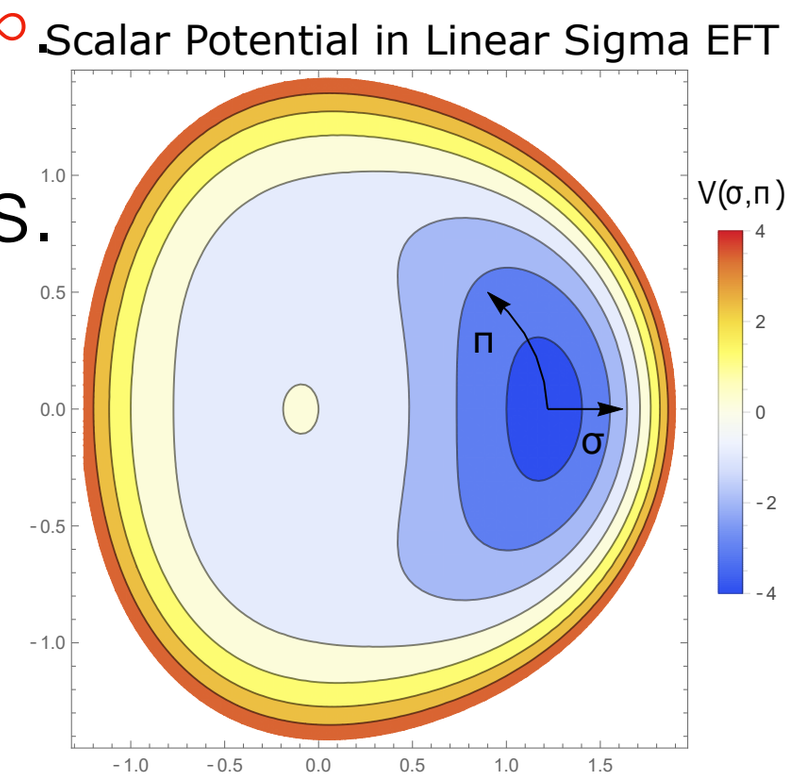
- A general feature of Dilaton EFT's is that the dilaton has a separate breaking potential with it's own condensate f_d (not unlike χPT+scalar).
- But, the EFT is predictive because the requirement that the scalar couple to NGBs as conformal compensator fixes most LECs.
- The near-degeneracy of $M\pi \sim M\sigma$ over a range of masses (*i.e. same slope*) has flavor-dependent consequence for f_d:
  $M\pi^2 \sim m_q$, $M_d^2 \sim N_f\,(f\pi^2 / f_d^2)\, m_q$.
- Implies $f_d \sim \mathrm{sqrt}(N_f)\, f\pi$. This will lead to deviations in scalar self-couplings from Higgs-like behavior. *n.b.* for sextet model N_f=2.

$$M_\pi^2 = BF_\pi^{p-2}, \quad M_d^2 = \frac{yN_f f_\pi^2}{2f_d^2}(p-y)BF_\pi^{p-2}.$$

Linear Sigma Model EFT

LSD Collaboration, Phys.Rev. D98 (2018) 114510
Floor, Gustafson, Meurice, Phys.Rev. D98 (2018) 094509

- Gell-Mann-Levy linear sigma model was early EFT for QCD. $N_f=2$ version isomorphic to O(4). Only π and σ included. Naively renormalizable as $\Lambda\to\infty$.
- $N_f>2$ requires additional dof: a_0, η'. Removing heavy η' means no longer renormalizable as $\Lambda\to\infty$.
- Can naturally incorporate light a_0 mesons.
- Very predictive as vev of σ tied to χSB.
- Naive problem with slopes:
  $M\pi^2 \sim m_q$, $(M\sigma^2 - m\sigma^2) \sim 3\, m_q$.



Explicit Symmetry Breaking

Spurion $\chi = B\, m_q$, $B \sim \langle qq \rangle / f_\pi^2$

$$V_{SB} = -\sum_{i=1}^{9} \tilde{c}_i O_i(x),$$

Relative size of χSB:

$$\frac{m_q B_\pi}{\Lambda^2} \sim \left(\frac{M_\sigma}{\Lambda}\right)^\alpha \ll 1$$

Estimate: $\Lambda \sim M_\rho$ when $M_\rho = 2\, M_\pi$

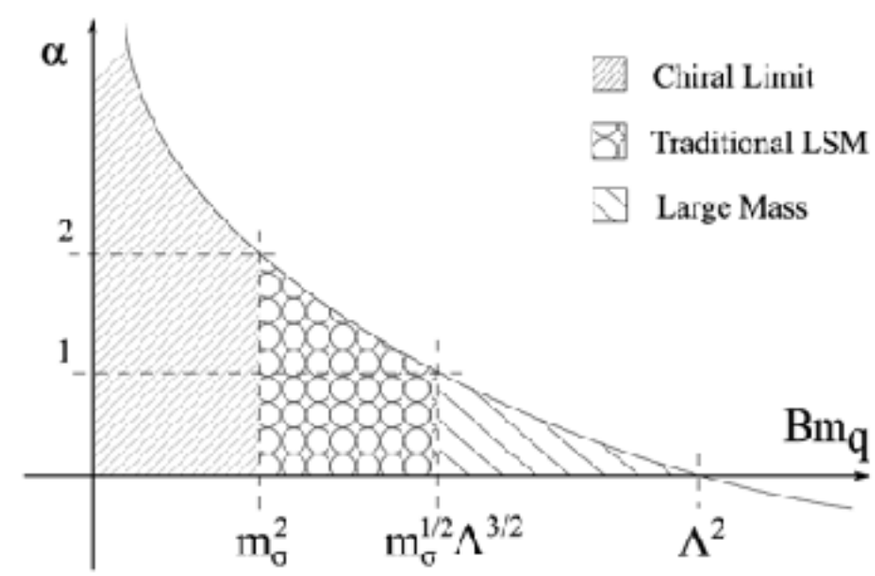
Symbol	Operator	$\alpha \lesssim 1$	$1 < \alpha \leq 2$
O_1	$\langle \chi^\dagger M + M^\dagger \chi \rangle$	✓	✓
O_2	$\langle M^\dagger M \rangle \langle \chi^\dagger M + M^\dagger \chi \rangle$	✓	X
O_3	$\langle (M^\dagger M)(\chi^\dagger M + M^\dagger \chi) \rangle$	✓	X
O_4	$\langle \chi^\dagger M + M^\dagger \chi \rangle^2$	✓	X
O_5	$\langle \chi^\dagger \chi M^\dagger M \rangle$	✓	X
O_6	$\langle \chi^\dagger \chi \rangle \langle M^\dagger M \rangle$	✓	X
O_7	$\langle \chi^\dagger M \chi^\dagger M + M^\dagger \chi M^\dagger \chi \rangle$	✓	X
O_8	$\langle \chi^\dagger \chi \rangle \langle \chi^\dagger M + M^\dagger \chi \rangle$	✓	X
O_9	$\langle (\chi^\dagger \chi)(\chi^\dagger M + M^\dagger \chi) \rangle$	✓	X

$$\frac{F^2}{f^2} = 1 + \frac{2}{m_\sigma^2}\left[2Bm_q\frac{f}{F} + 6Bm_q(c_2+c_3)F + 2B^2m_q^2(4c_4+c_5+c_6+2c_7) + 2B^3m_q^3\frac{c_8+c_9}{F}\right],$$

$$M_\pi^2 = 2Bm_q\frac{f}{F} + 2Bm_q(c_2+c_3)F + 8B^2m_q^2(c_4+c_7) + 2B^3m_q^3\frac{c_8+c_9}{F},$$

$$M_\sigma^2 = m_\sigma^2 + 6Bm_q\frac{f}{F} + 6Bm_q(c_2+c_3)F + 4B^2m_q^2(4c_4+c_5+c_6+2c_7) + 6B^3m_q^3\frac{c_8+c_9}{F},$$

$$M_a^2 = m_a^2\frac{F^2}{f^2} + 4Bm_q\frac{f}{F} + 8Bm_q c_2 F + 2B^2m_q^2(8c_4+c_5+c_6+2c_7) + 4B^3m_q^3\frac{c_8+c_9}{F}.$$



$M_\sigma^2 \geq 3\, M_\pi^2$?

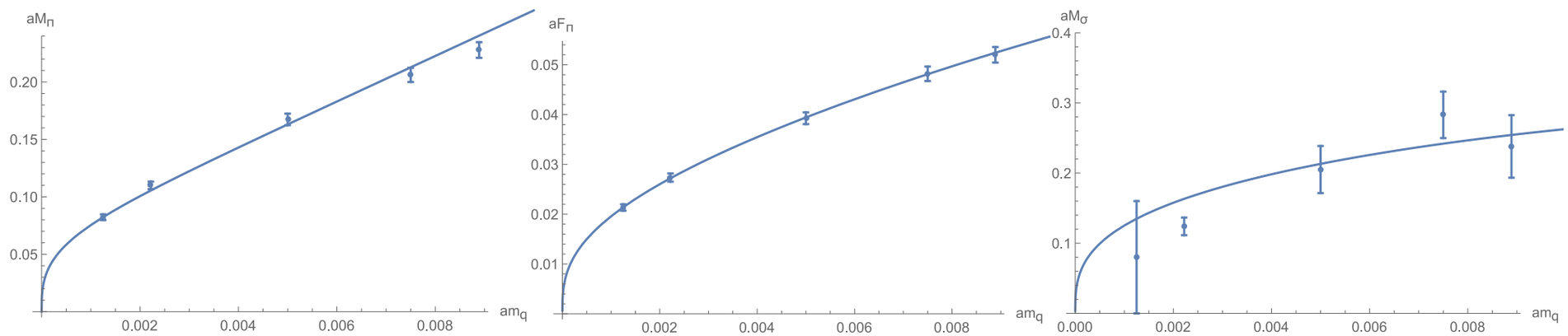
$$3M_\pi^2 - M_\sigma^2 + m_\sigma^2 = 4B^2m_q^2(2c_4 - c_5 - c_6 + 4c_7),$$

- If m_σ is light ($\sim f_\pi$) new kinematic regimes are opened in the linear sigma model. These new regimes match lattice results.

SU(3) Nf=8 LSM9 LO Fits

Preliminary

Lattice units
χ^2/dof=1.30



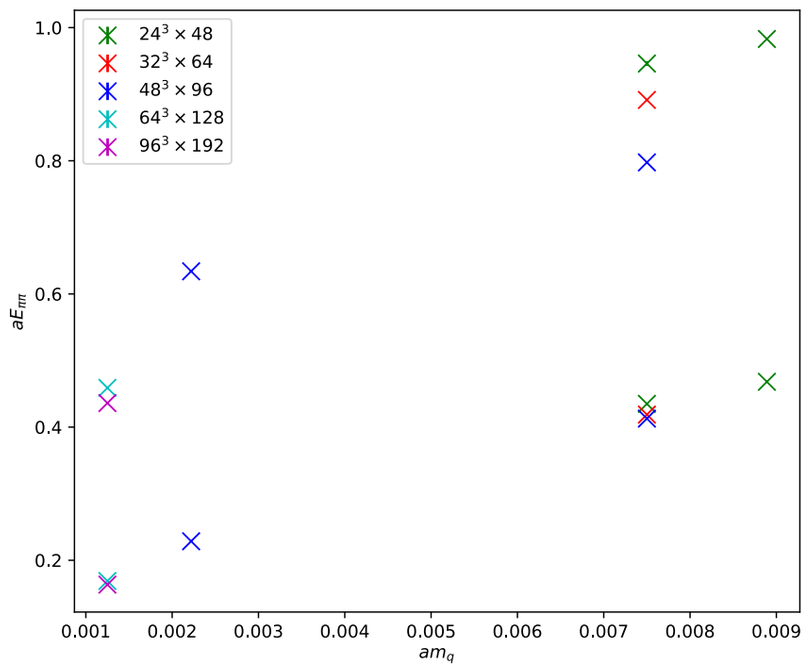
- LSM with 9 LO breaking terms, required when $M\sigma \sim M\pi$, so far is good description of lattice results.

- James Ingoldby has computed dilaton and LSM EFT expressions for $l=0,1,2$ $\pi\pi$ scattering lengths a and effective ranges b, scalar decay constant $F\sigma \sim \langle\sigma|\psi\psi|0\rangle$. Lattice calculations underway.

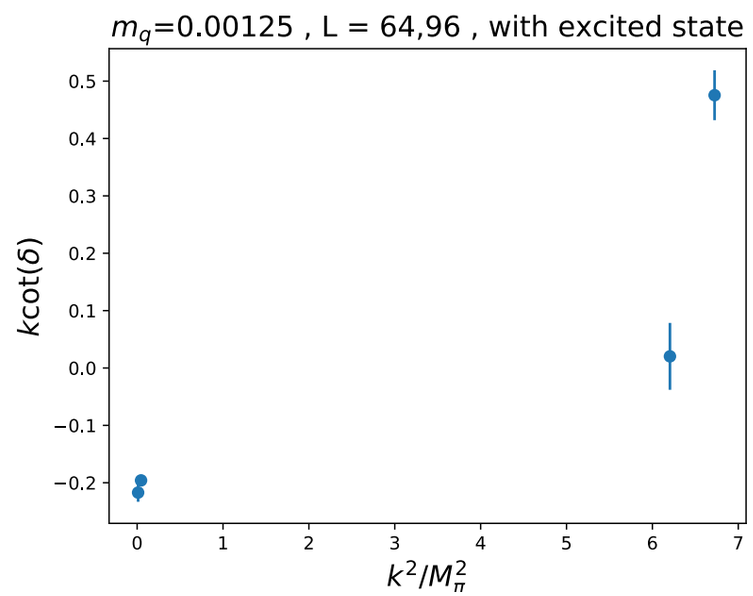
$$a_{PP}M_\pi = -\frac{M_\pi^2}{16\pi F_\pi^2} + \frac{c_4 B^2 m_q^2}{\pi N_f F_\pi^2} + \frac{c_7 B^2 m_q^2}{2\pi F_\pi^2} + \frac{\left[-M_\pi^2 + 4(c_2+c_3)F_\pi\sqrt{\frac{N_f}{2}}Bm_q + 8(c_4+c_7)B^2 m_q^2\right]^2}{8\pi N_f F_\pi^2 M_\sigma^2}.$$

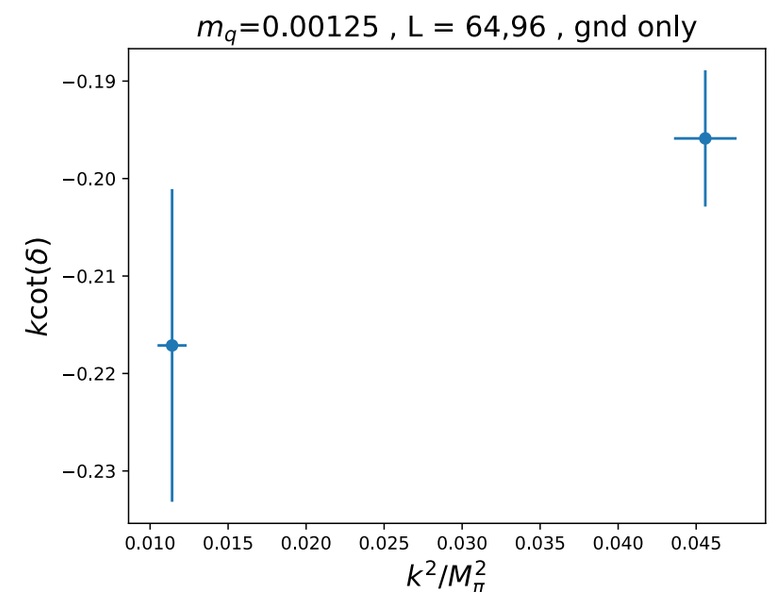
Preliminary

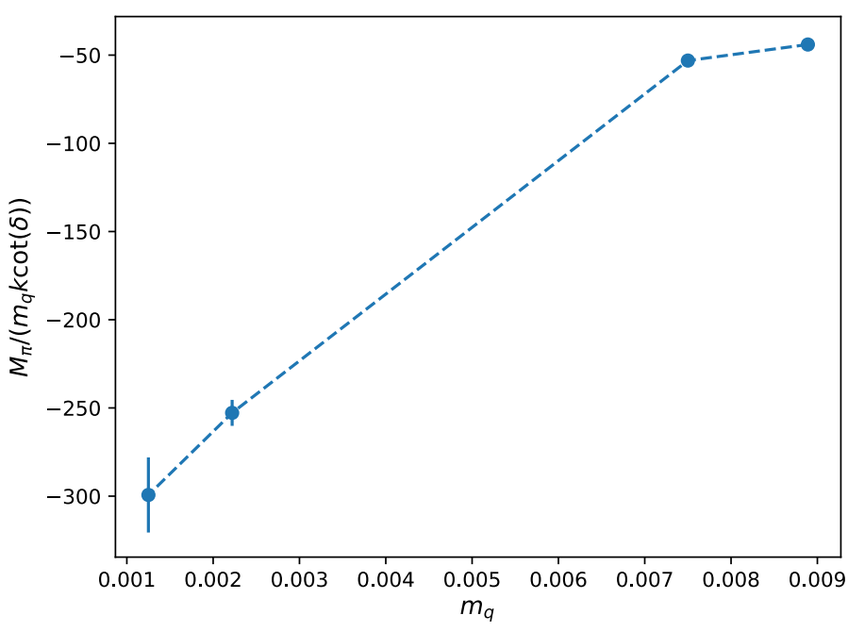
I=2 $\pi\pi$ scattering



Two-pion energy levels in various box sizes.
Big gap in scattering k^2 using excited states.
Working on using moving frames.
Still some sensitivity to effective range.







The ratio on the left should be constant at LO in chiPT.
Fits to other EFT's in progress:

$a\pi\pi\, M\pi = -\, M\pi^2/(16\pi\, F\pi^2) + \ldots$

LSD Collaboration, Phys.Rev.D 85 (2012) 074505
Gasbarro, Fleming, PoS LATTICE2016 (2017) 242

Future of SU(3) N_f=8

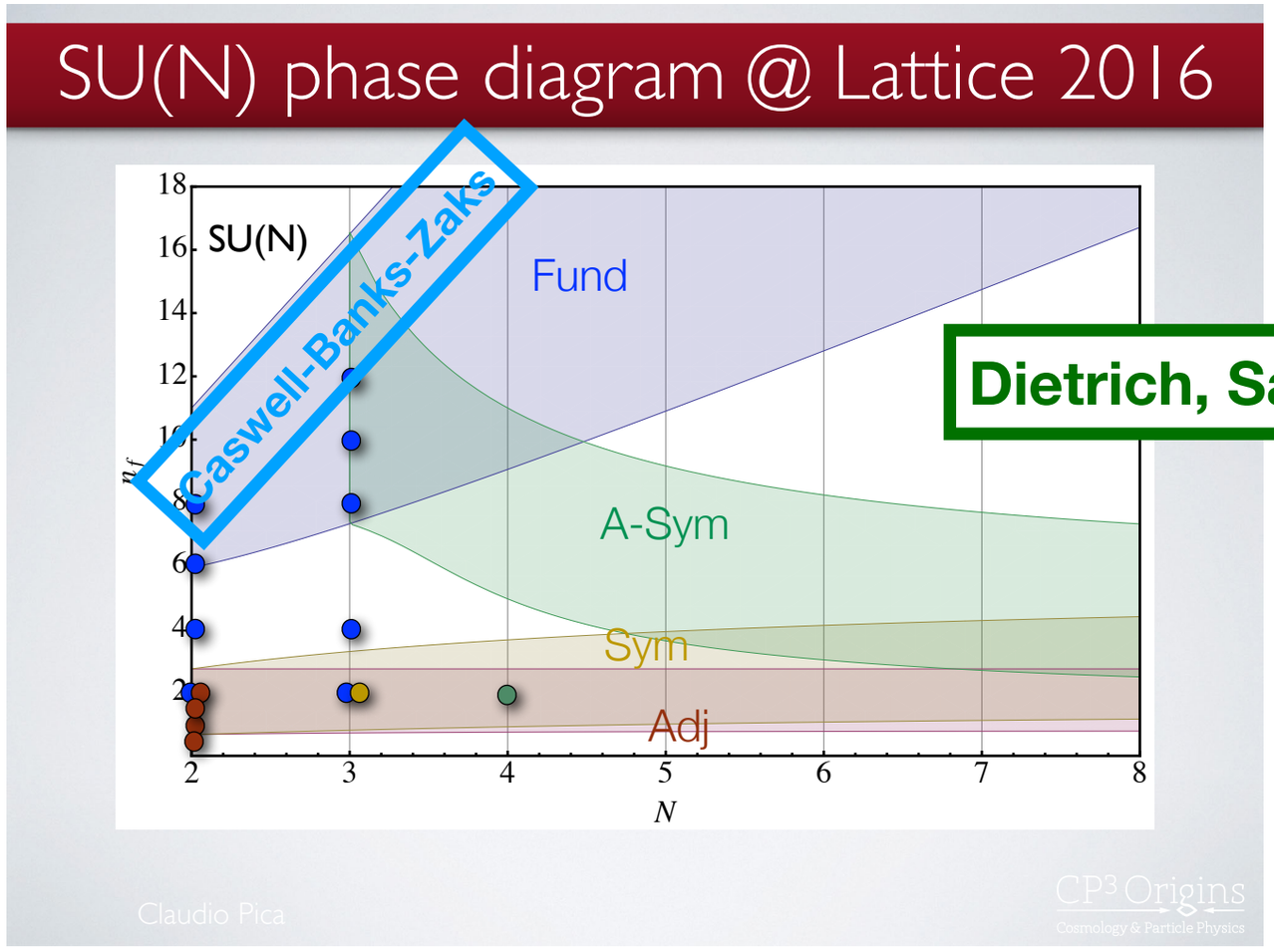
- Finishing I=2 scattering studies on $96^3 \times 192$ volumes, $M\pi \times L \sim 7.9$.

- Planning $96^3 \times 192$ lattice generation this year, $M\pi \times L \sim 5.3$. Should give $M\pi / M\rho \sim 0.41$.

- Given excellent performance of QUDA on Lassen/Summit, it's possible we can generate $128^3 \times 256$ lattices with $M\pi / M\rho \sim 0.35$ year after next.

Backup Slides

Conformal Windows



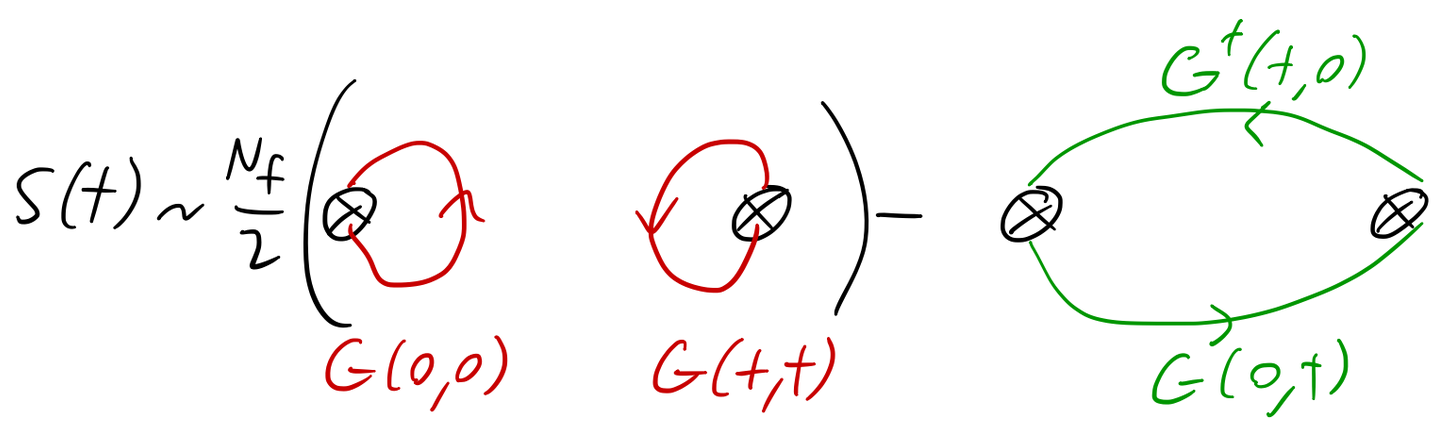
Dietrich, Sannino, Tuominen '05

- Shown estimate of lower edge (strongly coupled IRFP) is unreliable.
- Just below window is near-conformal region. A light scalar (pseudo-dilaton) might exist in this region [Yamawaki, Bando, Matutumo '86]
- Why are light scalars interesting? M_{Higgs} ~ (1/2) vev. So, a light composite scalar could be a composite Higgs candidate.

Hadron Mass Inequalities: Can the scalar be lighter than the pion?

- Rigorous hadron mass inequalities exist for flavored mesons in confining vector-like theories [Weingarten ('83), Witten ('83), Nussinov ('83), Detmold ('14)]

- In particular, $M\pi \leq Ma_0$, so for σ/f_0 to be as light as π the valence-disconnected diagram in correlator must dominate. Can this feature be related to dilaton?



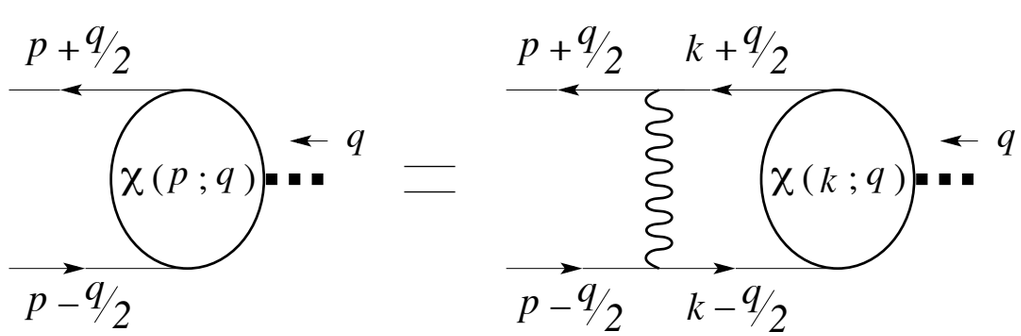
$\equiv 2D(t) - C(t)$

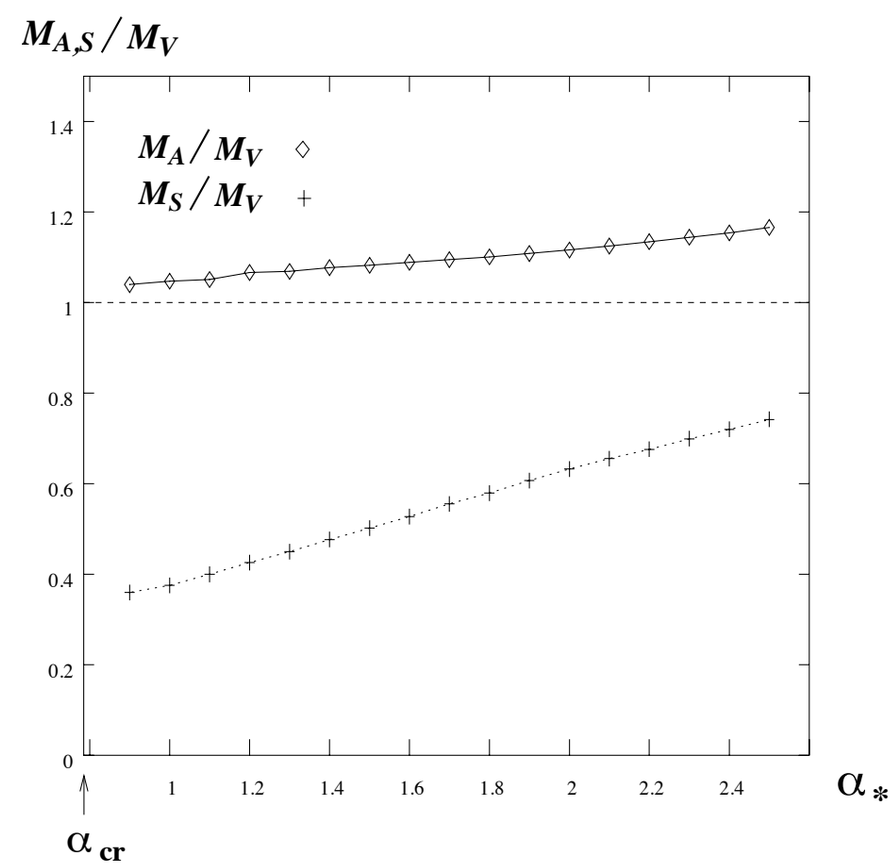
E. Neil

Other conjectured features of near-conformal theories

- Solving Schwinger-Dyson and Bethe-Salpeter equations suggests parity-doubled spectrum and light *flavored* scalar a_0 [Shrock, Kurachi ('06)]

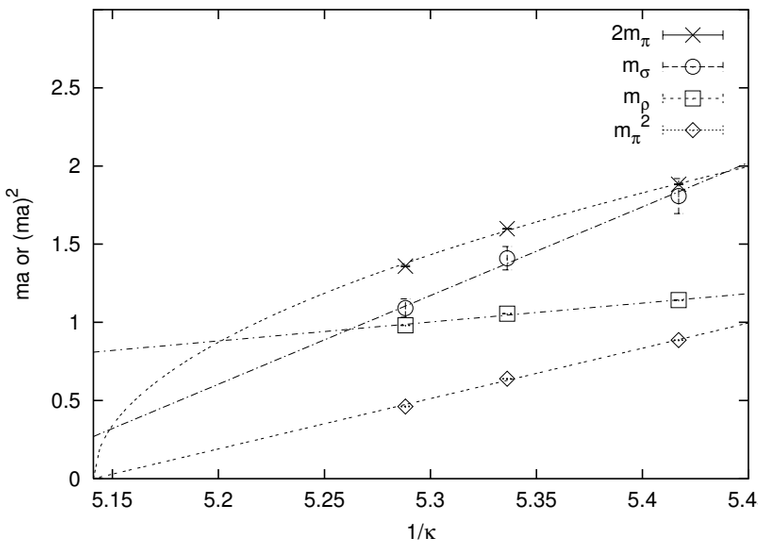
- Parity doubling is good for small S parameter but if the flavored scalar is really this light, shouldn't it be included in any EFT?



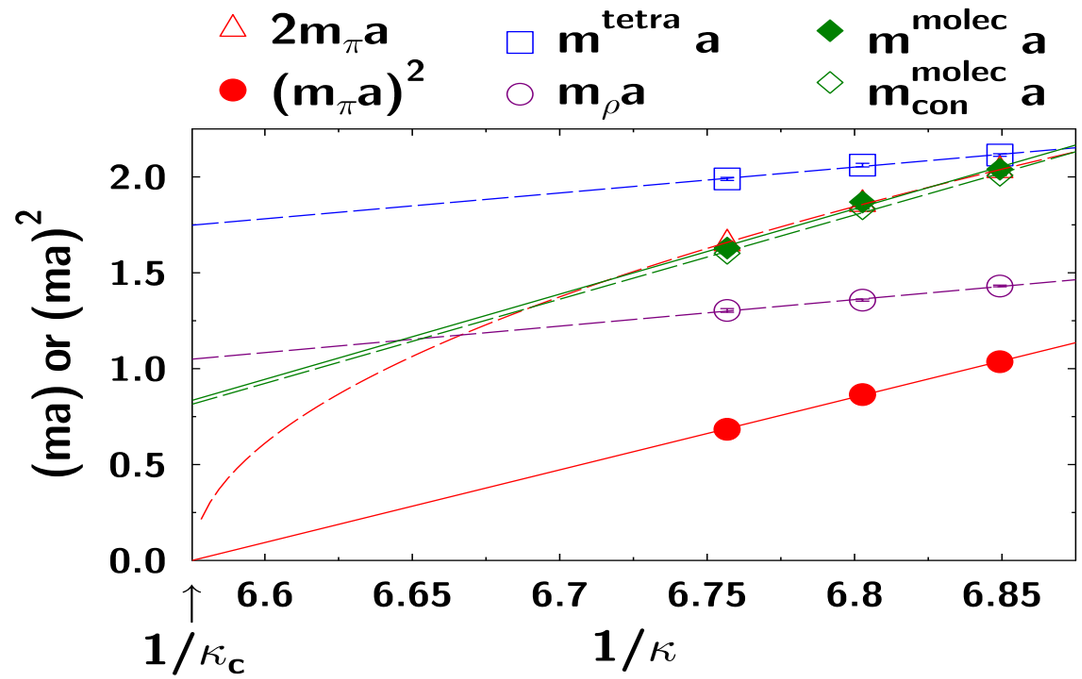


Scalar Sector of QCD

- Some heavy quark results from lattice SCALAR collaboration:

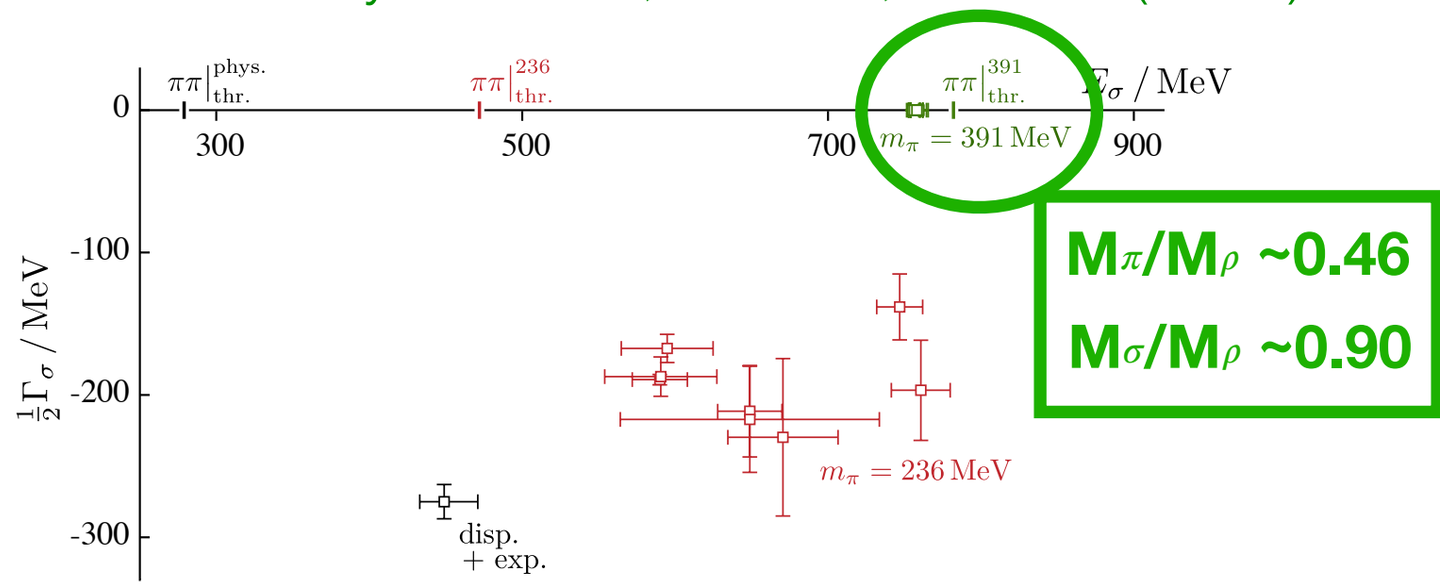


T. Kunihiro et al, PRD **70**, 034504 (2004)



M. Wakayama et al, PRD **91**, 094508 (2015)

- New result from Hadron Spectrum Collaboration



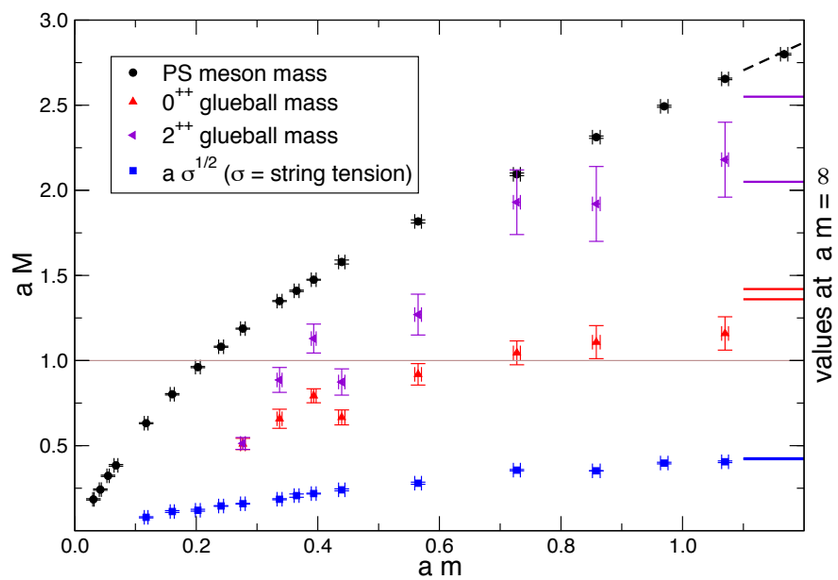
R. Briceno et al, PRL **118**, 022002 (2017)

- Bottom line: $M_\sigma \sim M_\rho$.

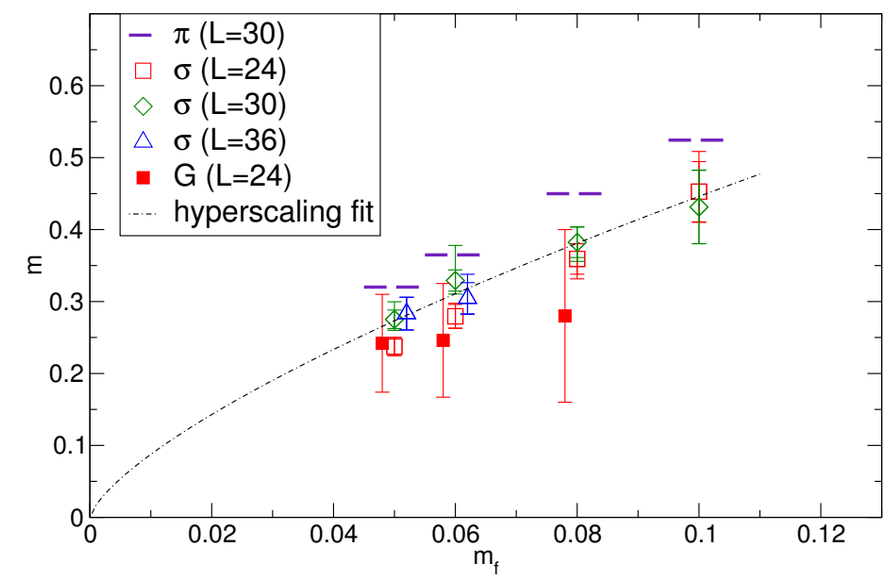
Theories with Light Scalars

- Mass-deformed IRFP theories with very light scalars.

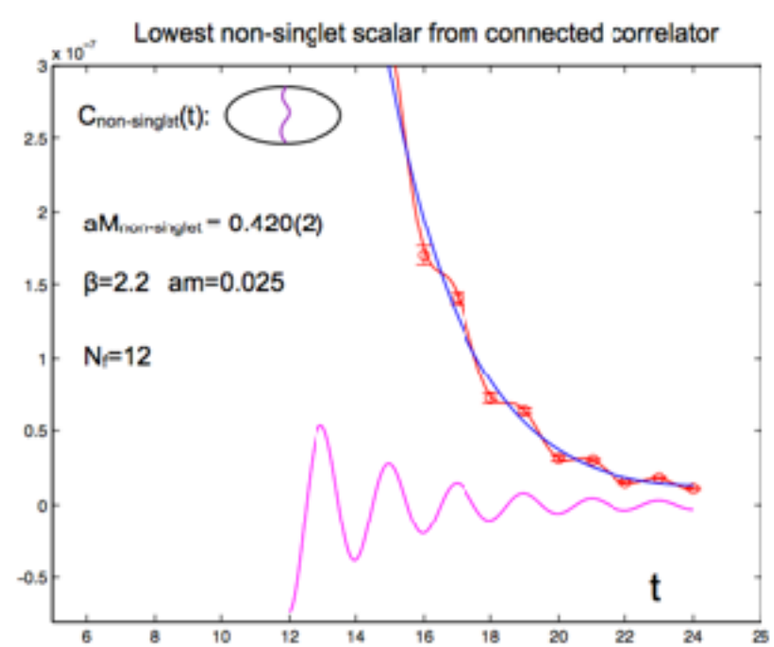
SU(2) N_f=2 adj (Edinburgh)
Phys. Rev. D 82, 014510 (2010)

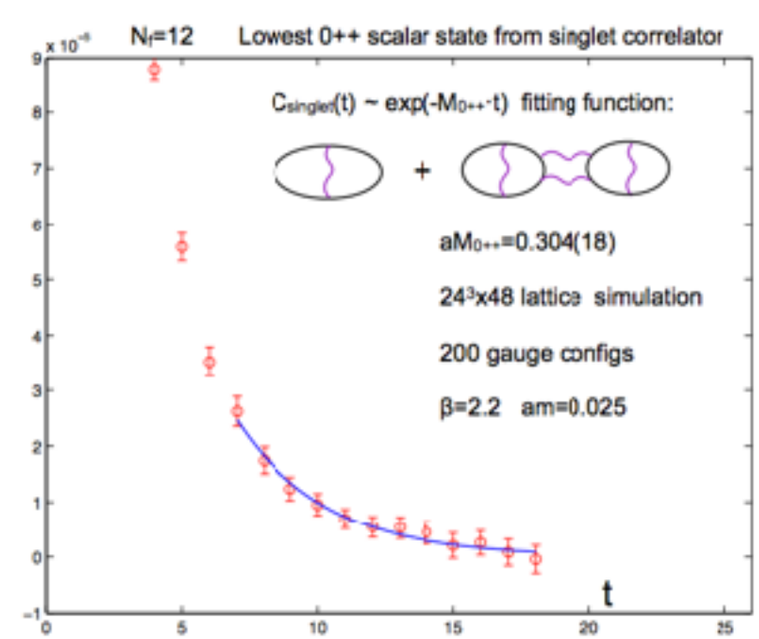


SU(3) N_f=12 fund (LatKMI)
Phys. Rev. Lett. 111, 162001 (2013)



SU(3) N_f=12 fund (LatHC)
USQCD White Paper 2013

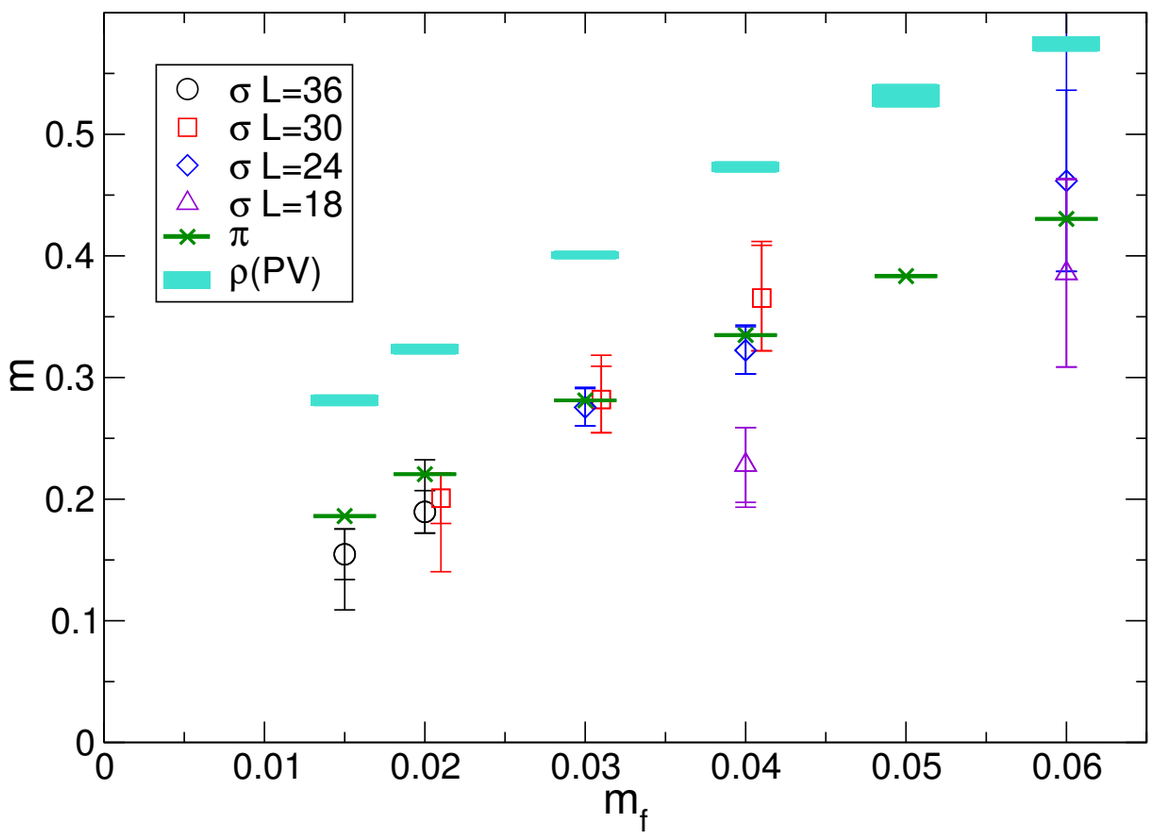




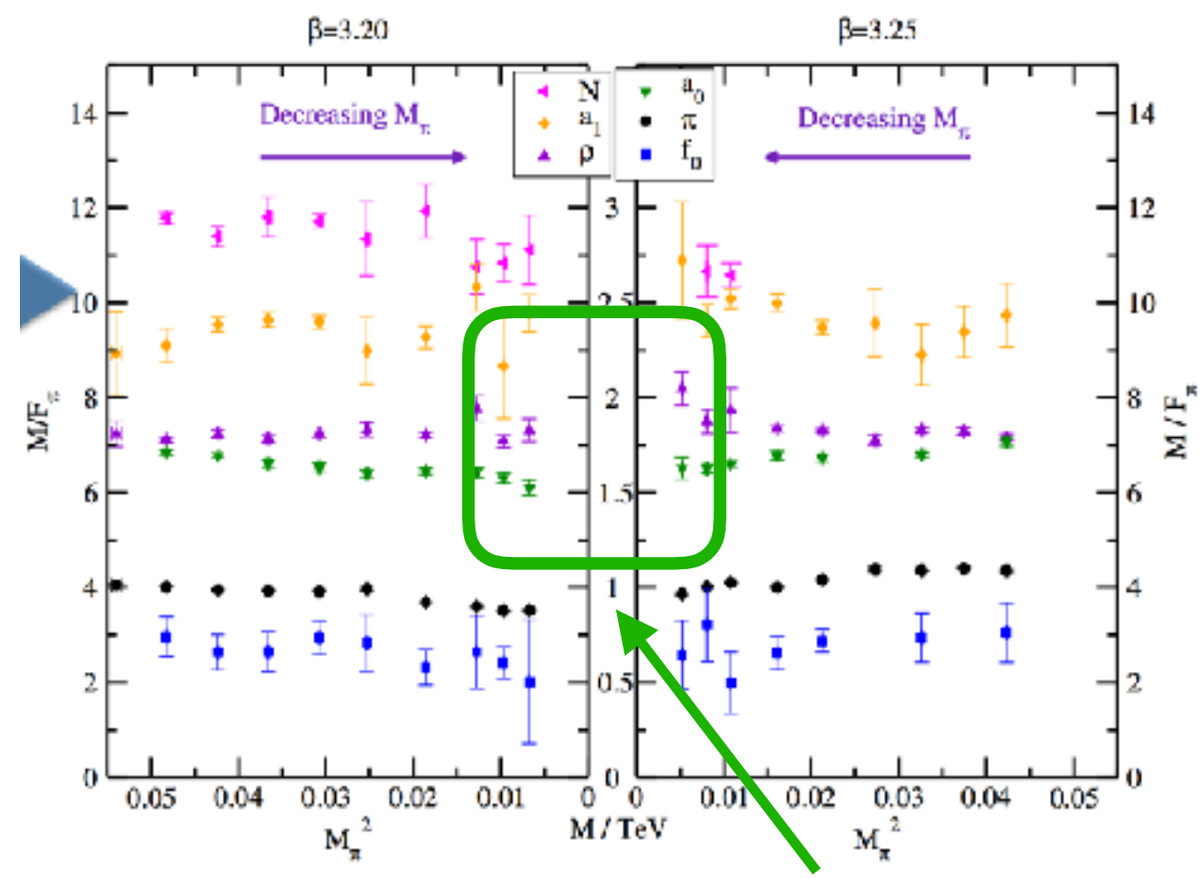
More Light Scalars

- Theories likely just below conformal window also have light scalars.

SU(3) N_f=8 fund
LatKMI (Nagoya)
Phys. Rev. D 89, 111502 (2014)



SU(3) N_f=2 sym
L_{at}HC Collaboration
LATTICE 2015

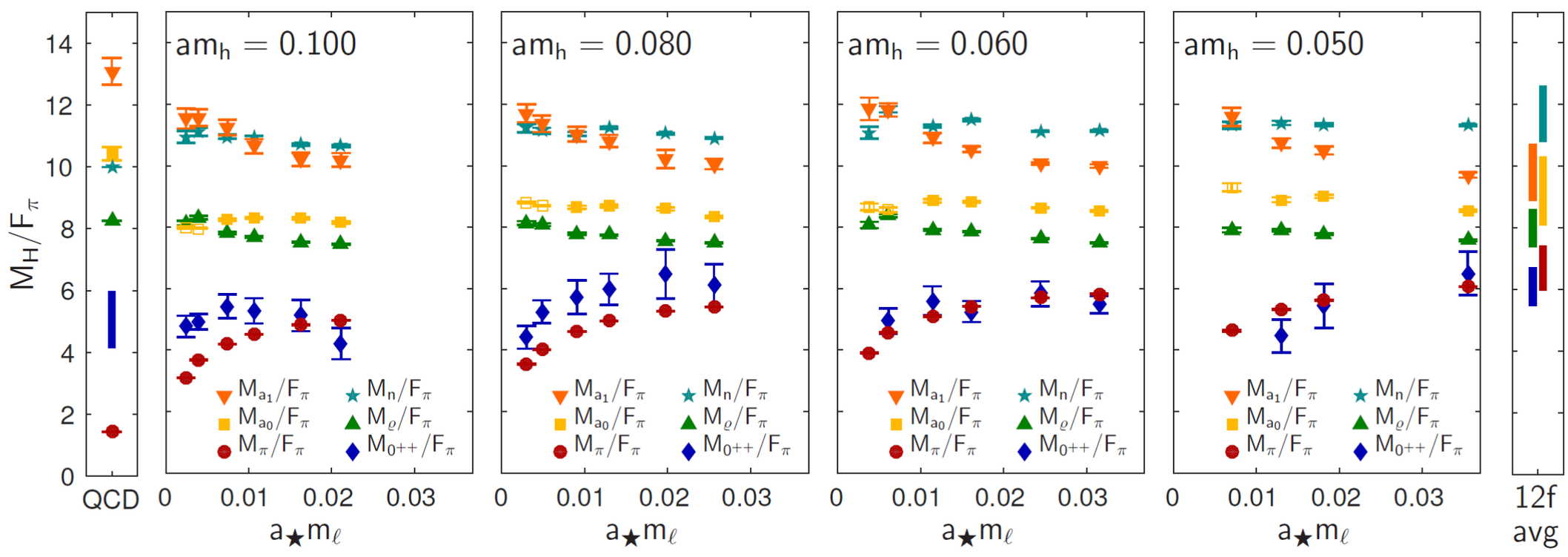


Note $M_{a_0} < M_\rho$

Mass-Split System: SU(3) N_f=4+8

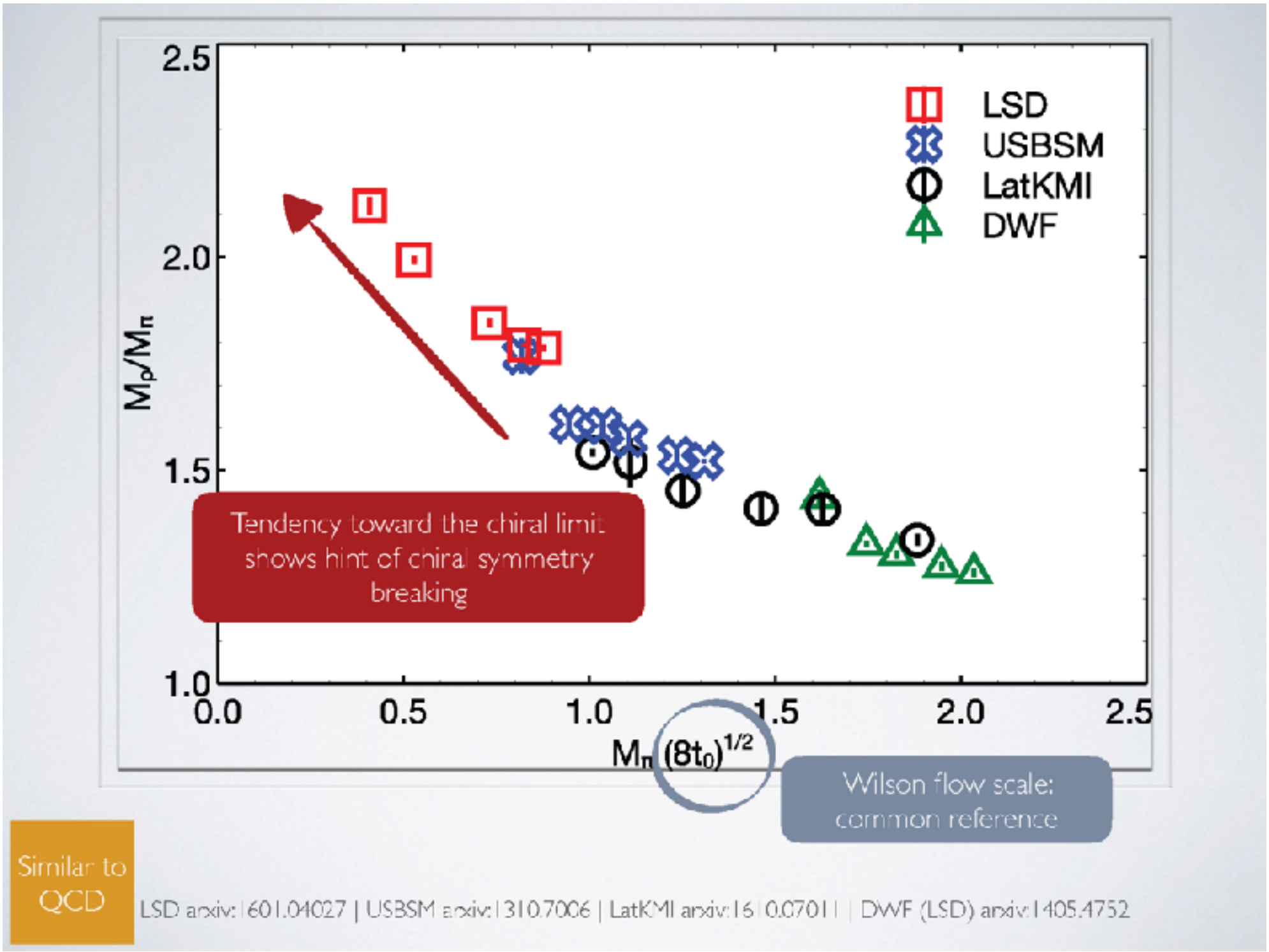
- A mass-split system would be inside the conformal window if $m_h/m_l = 1$ as $m_l \to 0$.

- But, if $m_h/m_l \sim 5$ as $m_l \to 0$, the low energy theory may be outside the conformal window with light scalar.



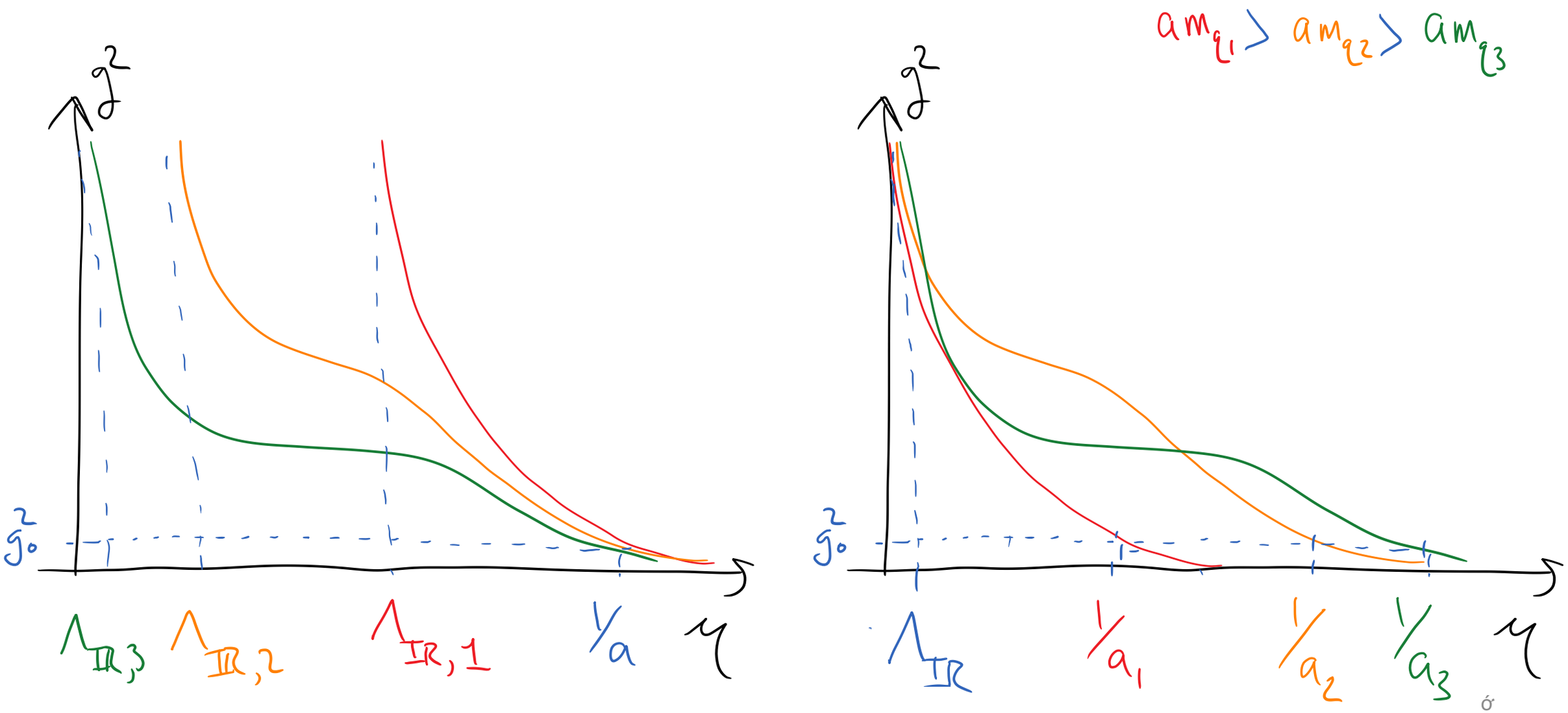
Brower, Hasenfratz, Rebbi, Weinberg, Witzel (2016)

- SU(3) Nf=8 doesn't look like mass deformed IRFP here unless there are large corrections.



E. Rinaldi

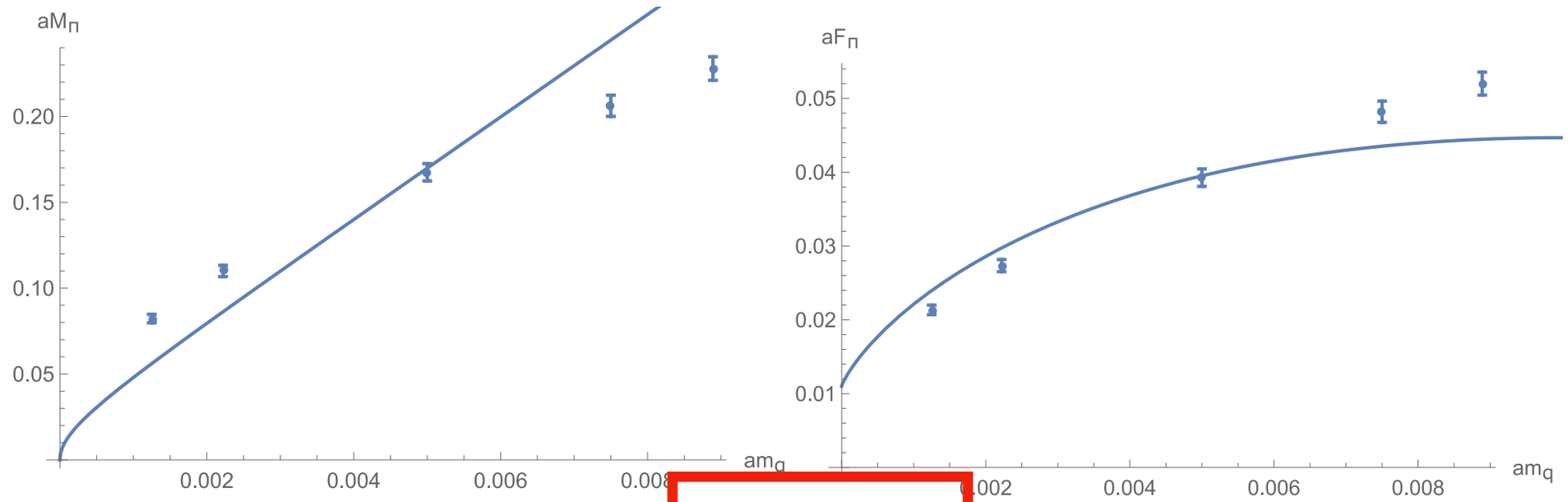
Near Conformal Effects



- Large slopes expected for IR quantities when plotted in bare lattice units $a\, m_q$.

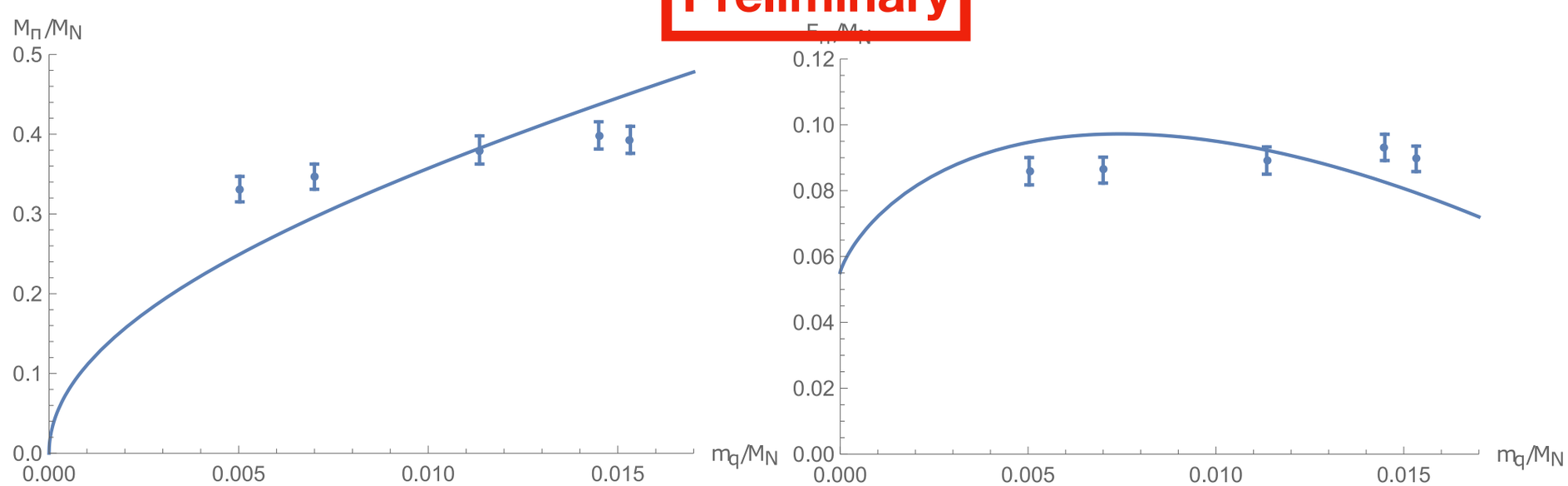
SU(3) Nf=8 NLO Fits

Lattice units
χ^2/dof=29



Preliminary

Nucleon units
χ^2/dof=7



- Fitting in nucleon vs. lattice units relieves some tension, but NLO χPT still poor description of results.

LSM Fields and Lagrangian

- Bifundamental (Nf,Nf) transforms linearly under U(Nf)×U(Nf).

$$M(x) = \mathrm{s(x)} + \mathrm{ip(x)} \qquad \mathrm{s}(x) = \frac{\tilde{\sigma}(x)}{\sqrt{N_f}} + \tilde{a}_i(x)T_i \qquad \mathrm{p}(x) = \frac{\tilde{\eta}'(x)}{\sqrt{N_f}} + \tilde{\pi}_i(x)T_i$$

- Polar (non-linear) basis enables trivial decoupling of η'.

$$M(x) = \Sigma(x)S(x) \qquad \Sigma(\mathrm{x}) = \exp\left[\frac{i\sqrt{2}}{F_\pi}\left(\frac{\eta'(x)}{\sqrt{N_f}} + \pi_i(x)T_i\right)\right] \qquad S(x) = \frac{\sigma(x)}{\sqrt{N_f}} + a_i(x)T_i$$

- General Lagrangian (no explicit symmetry breaking)

$$\mathcal{L} = \frac{1}{2}\langle \partial_\mu M^\dagger \partial^\mu M\rangle - V_0(M) \qquad \boxed{V_0 = \frac{\mu^2}{2}\langle M^\dagger M\rangle + \frac{\lambda_1}{4}\langle M^\dagger M\rangle^2 + \frac{N_f\lambda_2}{4}\left\langle (M^\dagger M)^2\right\rangle}$$

- Rewrite potential after SSB ($\mu^2 < 0$), easy to decouple a0.

$$\boxed{V_0 = \frac{-m_\sigma^2}{4}\langle M^\dagger M\rangle + \frac{m_\sigma^2 - m_a^2}{8f^2}\langle M^\dagger M\rangle^2 + \frac{N_f m_a^2}{8f^2}\left\langle (M^\dagger M)^2\right\rangle}$$