Constraining EFT’s in a Theory with a Light Scalar

George T. Fleming (Yale)
[for the LSD Collaboration]

Lattice 2019
Wuhan, China
June 20, 2019
When is a scalar light?

- A scalar meson is light when it has a mass of order $F\pi$ and small compared to $4\pi F\pi$, so it should be present in any low-energy EFT.

- The Standard Model Higgs is light: $M_H \sim F\pi / 2$.

- In QCD, $M_\sigma \sim 4-5 F\pi$ [see plenary by Edwards] so it’s not very light so we don’t usually include it in the EFT and is probably not a good candidate for composite Higgs.

- But, there are confining theories with light scalars. This is a short talk, so I’ll just discuss SU(3) with $N_f=8$.

- Big question: What is the EFT?
In LO $\chi$PT, $F_\pi(m_q) \sim f_\pi$. The lattice results show NLO $\gg$ LO for $F_\pi(m_q)$, but $M\sigma \sim M\pi \ll M\rho$.

- **Notational convention**: chiral limit $m_x$, finite quark mass $M_x$
SU(3) Nf=8 NLO Fits

• Large slope on $F_\pi$ leads to poor fit for $\chi$PT in current mass range.

• Adding a general scalar to chiral Lagrangian will not solve this problem although the fits might have low $\chi^2$/dof due to many fit parameters.

A general feature of Dilaton EFT’s is that the dilaton has a separate breaking potential with it’s own condensate $f_d$ (not unlike $\chi$PT+scalar).

But, the EFT is predictive because the requirement that the scalar couple to NGBs as conformal compensator fixes most LECs.

The near-degeneracy of $M_\pi \sim M_\sigma$ over a range of masses (i.e. same slope) has flavor-dependent consequence for $f_d$:

$$M_\pi^2 \sim m_q, \quad M_d^2 \sim N_f \left( \frac{f_\pi^2}{f_d^2} \right) m_q.$$

Implies $f_d \sim \sqrt{N_f} \, f_\pi$. This will lead to deviations in scalar self-couplings from Higgs-like behavior. n.b. for sextet model $N_f=2$.

$$M_\pi^2 = BF_\pi^{p-2}, \quad M_d^2 = \frac{y N_f f_\pi^2}{2 f_d^2} (p-y) BF_\pi^{p-2}.$$
Linear Sigma Model EFT

Floor, Gustafson, Meurice, Phys.Rev. D98 (2018) 094509

- Gell-Mann-Levy linear sigma model was early EFT for QCD. \( N_f=2 \) version isomorphic to \( O(4) \). Only \( \pi \) and \( \sigma \) included. Naively renormalizable as \( \Lambda \to \infty \).

- \( N_f>2 \) requires additional dof: \( a_0, \eta' \). Removing heavy \( \eta' \) means no longer renormalizable as \( \Lambda \to \infty \).

- Can naturally incorporate light \( a_0 \) mesons.

- Very predictive as vev of \( \sigma \) tied to \( \chi_{SB} \).

- Naive problem with slopes: 
  \[ M\pi^2 \sim m_q, (M\sigma^2 - m\sigma^2) \sim 3 \, m_q. \]
Explicit Symmetry Breaking

Spurion $\chi = B m_q, B \sim \langle qq \rangle / f_{\pi}^2$

Relative size of $\chi_{SB}$:

$$\frac{m_q B_{\pi}}{\Lambda^2} \sim \left( \frac{M_\sigma}{\Lambda} \right)^\alpha \ll 1$$

Estimate: $\Lambda \sim M_\rho$ when $M_\rho = 2 M_\pi$

\[
\frac{F^2}{f^2} = 1 + \frac{2}{m_\sigma^2} \left[ 2 B m_q \frac{f}{F} + 6 B m_q (c_2 + c_3) F + 2 B^2 m_q^2 (4 c_4 + c_5 + c_6 + 2 c_7) + 2 B^3 m_q^3 \frac{c_8 + c_9}{F} \right],
\]

\[
M_\pi^2 = 2 B m_q \frac{f}{F} + 2 B m_q (c_2 + c_3) F + 8 B^2 m_q^2 (c_4 + c_7) + 2 B^3 m_q^3 \frac{c_8 + c_9}{F},
\]

\[
M_\sigma^2 = m_\sigma^2 + 6 B m_q \frac{f}{F} + 6 B m_q (c_2 + c_3) F + 4 B^2 m_q^2 (4 c_4 + c_5 + c_6 + 2 c_7) + 6 B^3 m_q^3 \frac{c_8 + c_9}{F},
\]

\[
M_a^2 = m_a^2 \frac{F^2}{f^2} + 4 B m_q \frac{f}{F} + 8 B m_q c_2 F + 2 B^2 m_q^2 (8 c_4 + c_5 + c_6 + 2 c_7) + 4 B^3 m_q^3 \frac{c_8 + c_9}{F}.
\]

$M_\sigma^2 \geq 3 M_\pi^2$ ?

\[
3 M_\pi^2 - M_\sigma^2 + m_\sigma^2 = 4 B^2 m_q^2 (2 c_4 - c_5 - c_6 + 4 c_7),
\]

- If $m_\sigma$ is light ($\sim f_\pi$) new kinematic regimes are opened in the linear sigma model. These new regimes match lattice results.
SU(3) Nf=8 LSM9 LO Fits

- LSM with 9 LO breaking terms, required when $M_\sigma \sim M_\pi$, so far is good description of lattice results.

- James Ingoldby has computed dilaton and LSM EFT expressions for $I=0,1,2 \pi\pi$ scattering lengths $a$ and effective ranges $b$, scalar decay constant $F_\sigma \sim \langle \sigma | \psi \psi | 0 \rangle$. Lattice calculations underway.

\[
 a_{PP} M_\pi = -\frac{M_\pi^2}{16\pi F_\pi^2} + \frac{c_4 B^2 m_q^2}{\pi N_f F_\pi^2} + \frac{c_7 B^2 m_q^2}{2\pi F_\pi^2} + \left[ -\frac{M_\pi^2}{8\pi N_f F_\pi^2 M_\sigma^2} + 4(c_2 + c_3) F_\pi \sqrt{\frac{N_f}{2}} B m_q + 8(c_4 + c_7) B^2 m_q^2 \right]^2.
\]
\( l=2 \pi\pi \) scattering

Two-pion energy levels in various box sizes. Big gap in scattering \( k^2 \) using excited states. Working on using moving frames. Still some sensitivity to effective range.

The ratio on the left should be constant at LO in chiPT. Fits to other EFT’s in progress:

\[ a_{\pi\pi} M_\pi = -M_\pi^2/(16\pi F_\pi^2) + \ldots \]

Future of SU(3) $N_f=8$

- Finishing $l=2$ scattering studies on $96^3 \times 192$ volumes, $M\pi \times L \sim 7.9$.

- Planning $96^3 \times 192$ lattice generation this year, $M\pi \times L \sim 5.3$. Should give $M\pi / M\rho \sim 0.41$.

- Given excellent performance of QUDA on Lassen/Summit, it’s possible we can generate $128^3 \times 256$ lattices with $M\pi / M\rho \sim 0.35$ year after next.
Backup Slides
Conformal Windows

- Shown estimate of lower edge (strongly coupled IRFP) is unreliable.
- Just below window is near-conformal region. A light scalar (pseudo-dilaton) might exist in this region [Yamawaki, Bando, Matutumo ’86]
- Why are light scalars interesting? $M_{\text{Higgs}} \sim (1/2) \text{ vev}$. So, a light composite scalar could be a composite Higgs candidate.
Hadron Mass Inequalities: Can the scalar be lighter than the pion?

- Rigorous hadron mass inequalities exist for flavored mesons in confining vector-like theories [Weingarten ('83), Witten ('83), Nussinov ('83), Detmold ('14)]

- In particular, \( M_\pi \leq M_{a_0} \), so for \( \sigma/f_0 \) to be as light as \( \pi \) the valence-disconnected diagram in correlator must dominate. Can this feature be related to dilaton?

\[
S(t) \sim \frac{N_f}{2} \left( G(0,0) - G(t,t) \right) - G^+(t,0) \quad G(0,0) \quad G(t,t) \quad G^+(0,t) \\
\equiv 2D(t) - C(t)
\]

E. Neil
Other conjectured features of near-conformal theories

- Solving Schwinger-Dyson and Bethe-Salpeter equations suggests parity-doubled spectrum and light *flavored* scalar $a_0$ [Shrock, Kurachi (’06)]

- Parity doubling is good for small $S$ parameter but if the flavored scalar is really this light, shouldn’t it be included in any EFT?

\[ p + \frac{q}{2} \quad p - \frac{q}{2} \quad \chi(p; q) \quad q \]

\[ p + \frac{q}{2} \quad k + \frac{q}{2} \quad \chi(k; q) \quad q \]
Scalar Sector of QCD

• Some heavy quark results from lattice SCALAR collaboration:

\[
\begin{align*}
\frac{m_\pi}{m_\pi^0} & \sim 1.0 \\
\frac{(m_\pi a)^2}{(m_\pi a)^2} & \sim 1.0
\end{align*}
\]


• New result from Hadron Spectrum Collaboration

\[
\begin{align*}
\frac{M_\pi}{M_\rho} & \sim 0.46 \\
\frac{M_\sigma}{M_\rho} & \sim 0.90
\end{align*}
\]

R. Briceno et al, PRL 118, 022002 (2017)

• Bottom line: \(M_\sigma \sim M_\rho\).
Theories with Light Scalars

- Mass-deformed IRFP theories with very light scalars.

SU(2) \( N_f=2 \) adj (Edinburgh)

SU(3) \( N_f=12 \) fund (LatKMI)

SU(3) \( N_f=12 \) fund (LatHC)
USQCD White Paper 2013
More Light Scalars

- Theories likely just below conformal window also have light scalars.

SU(3) $N_f=8$ fund
LatKMI (Nagoya)

SU(3) $N_f=2$ sym
LHHC Collaboration
LATTICE 2015

Note $M_{a_0} < M_\rho$
Mass-Split System:
SU(3) $N_f=4+8$

- A mass-split system would be inside the conformal window if $m_h/m_l = 1$ as $m_l \to 0$.

- But, if $m_h/m_l \sim 5$ as $m_l \to 0$, the low energy theory may be outside the conformal window with light scalar.

• SU(3) Nf=8 doesn’t look like mass deformed IRFP here unless there are large corrections.
Near Conformal Effects

- Large slopes expected for IR quantities when plotted in bare lattice units $a m_q$. 

\[ a m_{q_1} > a m_{q_2} > a m_{q_3} \]
SU(3) Nf=8 NLO Fits

Lattice units
\( \chi^2/\text{dof} = 29 \)

Nucleon units
\( \chi^2/\text{dof} = 7 \)

- Fitting in nucleon vs. lattice units relieves some tension, but NLO \( \chi \)PT still poor description of results.
LSM Fields and Lagrangian

- Bifundamental $(N_f,N_f)$ transforms linearly under $U(N_f) \times U(N_f)$.

$$M(x) = s(x) + ip(x) \quad \text{where} \quad s(x) = \frac{\sigma(x)}{\sqrt{N_f}} + \bar{a}_i(x)T_i \quad \text{and} \quad p(x) = \frac{\eta'(x)}{\sqrt{N_f}} + \bar{\pi}_i(x)T_i$$

- Polar (non-linear) basis enables trivial decoupling of $\eta'$.

$$M(x) = \Sigma(x)S(x) \quad \Sigma(x) = \exp \left[ \frac{i\sqrt{2}}{F_\pi} \left( \frac{\eta'(x)}{\sqrt{N_f}} + \pi_i(x)T_i \right) \right] \quad S(x) = \frac{\sigma(x)}{\sqrt{N_f}} + a_i(x)T_i$$

- General Lagrangian (no explicit symmetry breaking)

$$\mathcal{L} = \frac{1}{2} \langle \partial_\mu M^\dagger \partial^\mu M \rangle - V_0(M) \quad \text{where} \quad V_0 = \frac{\mu^2}{2} \langle M^\dagger M \rangle + \frac{\lambda_1}{4} \langle M^\dagger M \rangle^2 + \frac{N_f \lambda_2}{4} \left( \langle M^\dagger M \rangle^2 \right)$$

- Rewrite potential after SSB ($\mu^2 < 0$), easy to decouple $a_0$.

$$V_0 = \frac{-m_\sigma^2}{4} \langle M^\dagger M \rangle + \frac{m_\sigma^2 - m_\sigma^2}{8f^2} \langle M^\dagger M \rangle^2 + \frac{N_f m_\sigma^2}{8f^2} \left( \langle M^\dagger M \rangle^2 \right)$$