

Fits of $SU(3)$ $N_f = 8$ data to dilaton-pion effective field theory

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(Thanks to Ethan Neil for discussions)

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Aims and assumptions

- Test systematic EFT of pions and a dilaton (MG & Shamir, '16) on data
Use LSD data for $SU(3)$, $N_f = 8$ theory (Appelquist *et al.* '18)
Salient feature: 0^{++} state with mass \sim pion mass
- Assume tree-level describes these data well
(naïve expansion parameter $M_\pi^2 / (4\pi F_\pi)^2 \leq 0.1$)
- Some results obtained by Fodor *et al.* '19 – agree where overlap
- New: predictions for taste-breaking effects – **unlike** QCD
(LSD used staggered fermions)
- Use published data (stat. correlations are small (private comm. Ethan Neil))

EFT for pions and a dilaton

(MG & Shamir, '16, '18)

Assumptions:

- Scale invariance gets restored as we take the theory closer to the conformal window. For N_f fundamental flavors in an $SU(N_c)$ theory this happens when N_f crosses into the conformal region. Technically, $n_f \equiv N_f/N_c \uparrow n_f^*$ in the limit $N_f, N_c \rightarrow \infty$.
- The theory contains pions associated with chiral symmetry breaking, and a dilaton associated with breaking of scale symmetry, which becomes massless for $n_f \rightarrow n_f^*$ (and $m \rightarrow 0$).
- The dilaton potential has a zero, as a function of the dilaton field τ .

The EFT for dilatons and pions (dChPT)

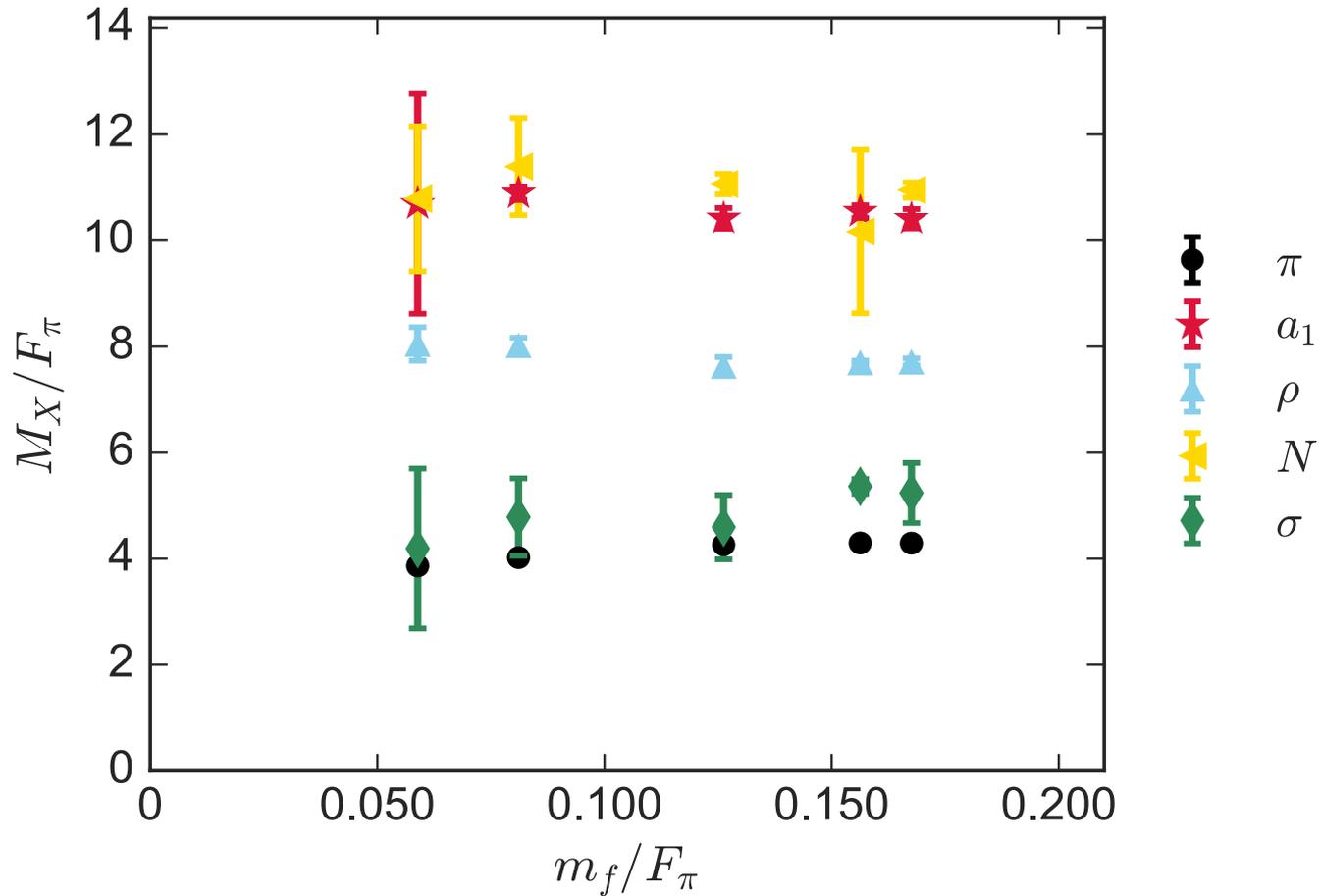
Leading-order Lagrangian:

$$\mathcal{L} = \frac{\hat{f}_\pi^2}{4} e^{2\tau} \text{tr}(\partial_\mu \Sigma^\dagger \partial_\mu \Sigma) + \frac{\hat{f}_\tau^2}{2} e^{2\tau} (\partial_\mu \tau)^2$$
$$- \frac{\hat{f}_\pi^2 \hat{B}_\pi m}{2} e^{(3-\gamma_*)\tau} \text{tr}(\Sigma + \Sigma^\dagger)$$
$$+ \hat{f}_\tau^2 \hat{B}_\tau e^{4\tau} c_1 (\tau - 1/4)$$

- m is the fermion mass, γ_* is its anom. dim. at $n_f = n_f^*$ (i.e., at the IRFP)
- The parameters $c_1 \propto n_f - n_f^*$ and m are assumed small (in the “large-mass” regime, which has approximate hyper-scaling, $c_1 \log m \ll 1$ (MG & Shamir, '18))
- Field redefinition such that $v \equiv \langle \tau \rangle$ vanishes (at tree level) for $m = 0$

“Large mass” regime

(MG & Shamir, '18)



From LSD (courtesy E. Neil). Evidence for hyper-scaling, m dominates breaking of scale invariance.

Tree level predictions

- Minimize potential: $\frac{m}{c_1 \hat{\mathcal{M}}} = v e^{(1+\gamma_*)v}$, $\hat{\mathcal{M}} = \frac{4\hat{f}_\tau^2 \hat{B}_\tau}{\hat{f}_\pi^2 \hat{B}_\pi N_f (3 - \gamma_*)}$
- Pion decay constant: $F_\pi = \hat{f}_\pi e^{v(m)}$
- Pion mass: $M_\pi^2 = 2\hat{B}_\pi m e^{(1-\gamma_*)v(m)}$
- (Dilaton mass: $M_\tau^2 = 4c_1 \hat{B}_\tau e^{2v(m)} (1 + (1 + \gamma_*)v(m))$)

These results lead to $(\sqrt{t_0} M_\pi)^2 (\sqrt{t_0} F_\pi)^{\gamma_* - 1} = C (t_0/a^2)^{\gamma_*/2} (\sqrt{t_0} m)$

and $\frac{m}{F_\pi} = D_2 \frac{M_\pi^2}{F_\pi^2} \exp\left(D_1 \frac{M_\pi^2}{F_\pi^2}\right)$

which can be fit to the data if we assume that a is independent of m (check!)

Fits with LSD data

(Appelquist *et al.* '18)

- Fits to data at 5 different masses, $10^3 am = (1.25, 2.22, 5.00, 7.50, 8.89)$

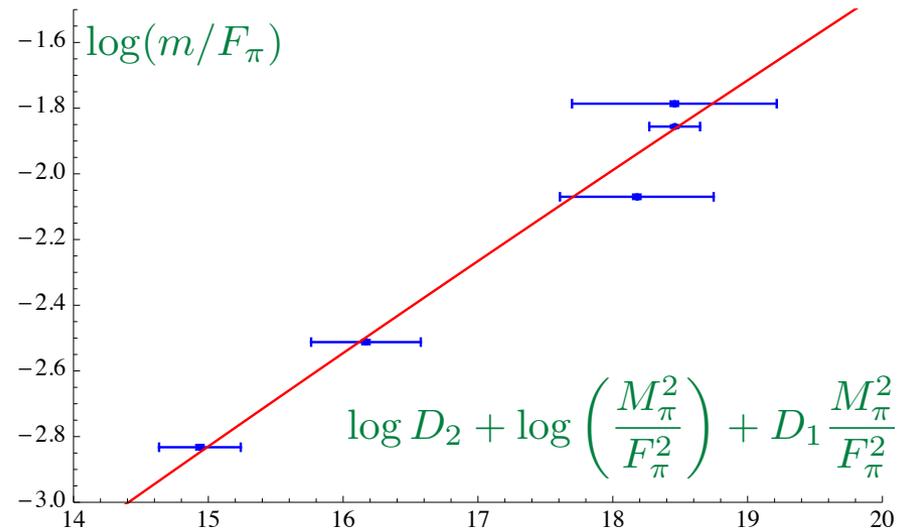
- $\gamma_* = 0.936(19)$, $C = 1.931(61)$ ($\chi^2/\text{dof} = 0.784/3$)

- $D_1 = 0.220(27)$, $\log D_2 = -8.83(47)$ ($\chi^2/\text{dof} = 0.886/3$)

- Determine $a\hat{B}_\pi$ from aM_π , aF_π , and am for each data point, using γ_* :

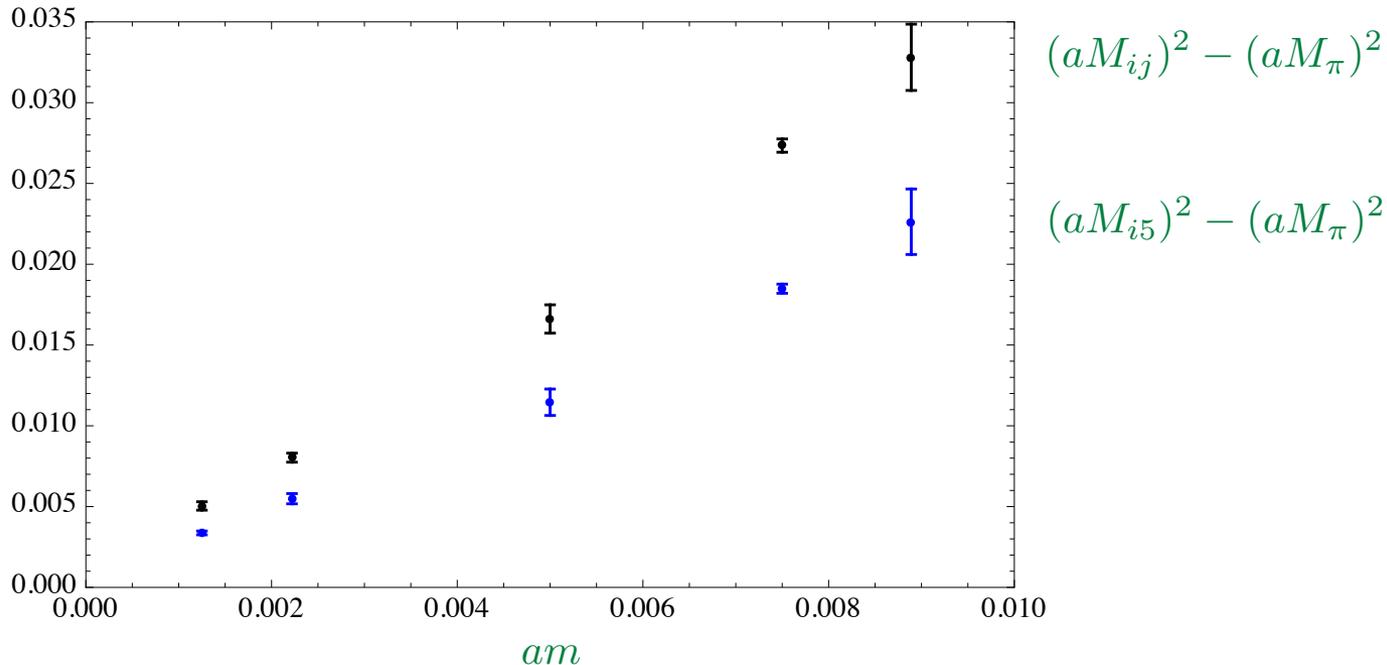
$$a\hat{B}_\pi = (2.15, 2.14, 2.17, 2.15, 2.21)$$

constant to within 3 percent



Staggered taste splittings

- LSD data were obtained with staggered fermions, and pion masses for tastes $i5$ and ij were measured.
- Tastes splittings are order a^2 effect – can dChPT describe this effect?



Taste breaking – theory

In the Symanzik effective action, taste-breaking operators show up as 4-fermion operators (Lee & Sharpe, '98):

$$a^2 (\bar{\psi} \Gamma \psi) (\bar{\psi} \Gamma \psi)$$

where Γ is a gamma-matrix acting on the taste index of ψ .

Under a scale transformation, $(\bar{\psi} \Gamma \psi) (\bar{\psi} \Gamma \psi) \rightarrow \lambda^{6-\gamma_\Gamma} (\bar{\psi} \Gamma \psi) (\bar{\psi} \Gamma \psi)$
 \Rightarrow treat a^2 as a spurion transforming as $a^2 \rightarrow \lambda^{-2+\gamma_\Gamma} a^2$.

In dChPT this operator is then represented by

$$c_\Gamma a^2 e^{(6-\gamma_\Gamma)\tau} \hat{f}_\pi^2 \Lambda^4 \text{tr}(\Sigma \Gamma \Sigma^\dagger \Gamma)$$

Tree level taste splittings

- For the taste splittings, we find

$$(aM_\Gamma)^2 - (aM_\pi)^2 = (a\Lambda)^4 \sum_{\Gamma'} c'_{\Gamma\Gamma'} e^{(4-\gamma_{\Gamma'})v(m)}$$

- Assume that one term in the sum dominates simplifies this to

$$(aM_\Gamma)^2 - (aM_\pi)^2 = A_\Gamma e^{(4-\gamma_\Gamma)v(m)}$$

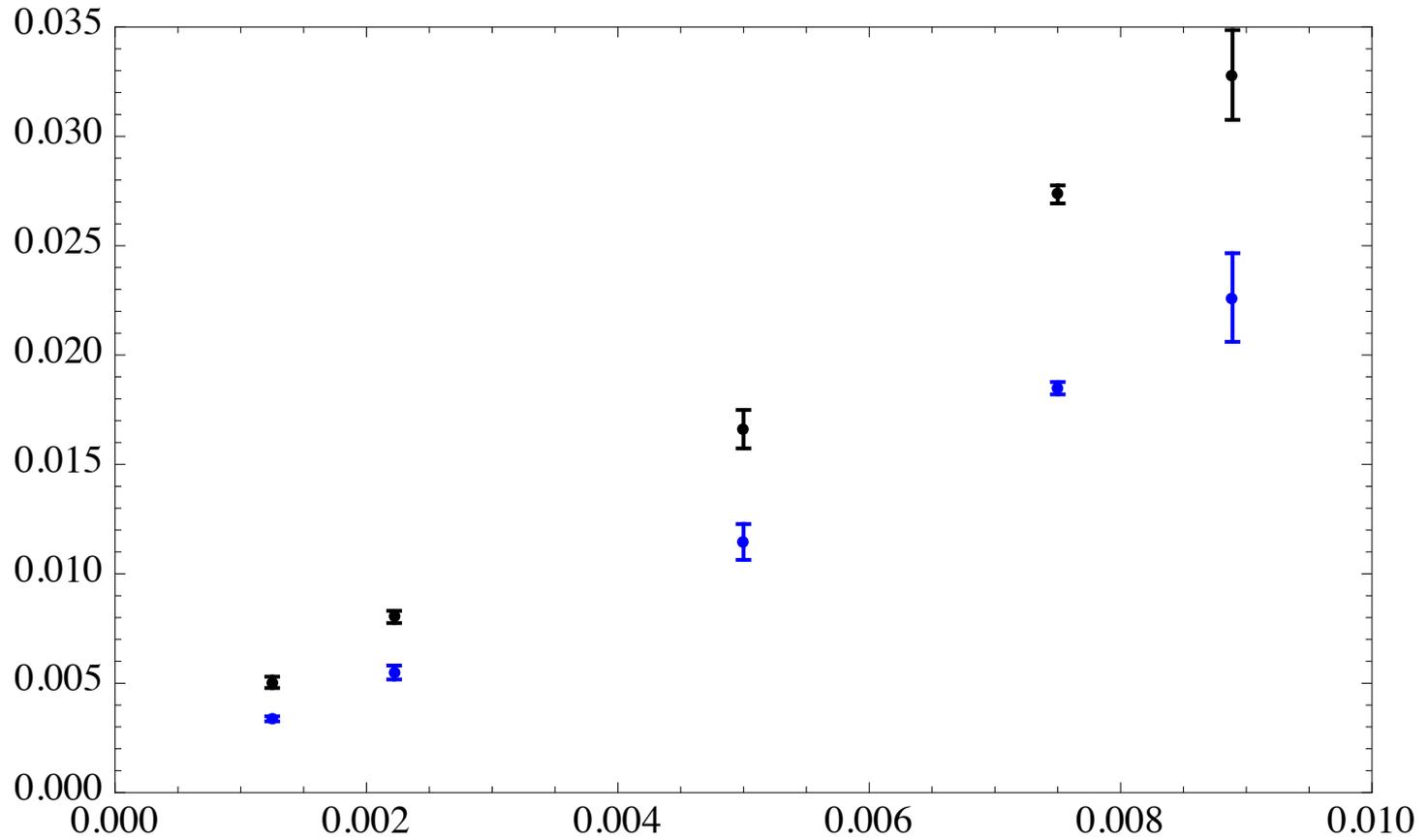
- Fits give (using the known solution for $v(m)$)

$$A_{i5} = 2.0 \times 10^{-6}, \quad \gamma_{i5} = 1.89 \quad (\chi^2/\text{dof} = 1.82/3)$$

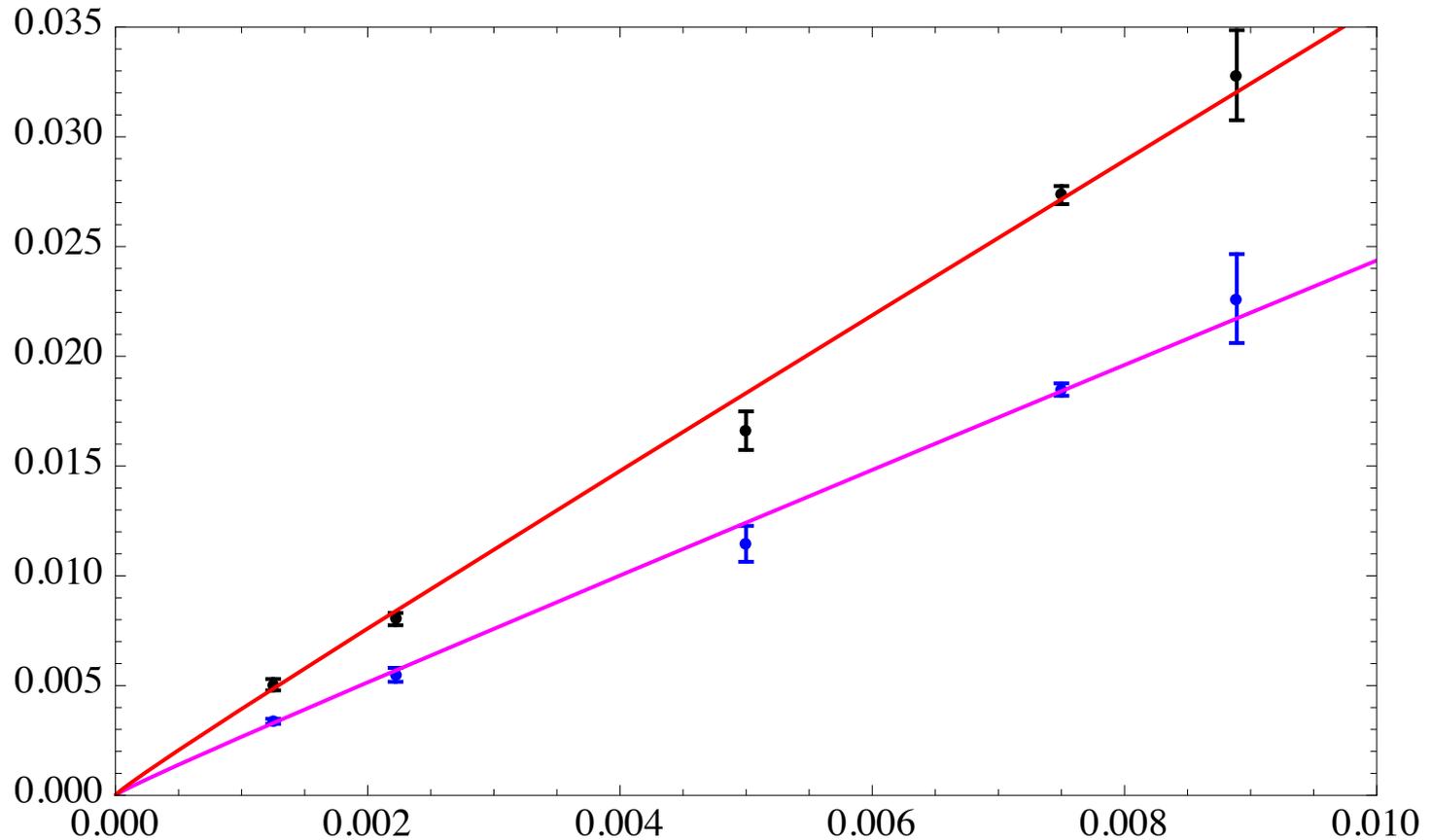
$$A_{ij} = 2.9 \times 10^{-6}, \quad \gamma_{ij} = 1.89 \quad (\chi^2/\text{dof} = 2.05/3)$$

- Note that the fit finds that $\gamma_{i5} = \gamma_{ij} = 2\gamma_*$

Tree level taste splittings



Tree level taste splittings



Note: in QCD no mass dependence at tree level in taste splittings!

Concluding remarks

- LO dChPT describes the LSD data quite well – no sign of discrepancies (more consistency checks done; dilaton mass has large errors)
- Helps understand taste breaking – very different from QCD!
- Next steps: - complete error analysis (correlations)
- NLO analysis (Hansen *et al.* '18, Catà & Mueller '19)

- LSD data are deep in the “large-mass” regime:

$$\frac{m}{c_1 \hat{\mathcal{M}}} = v e^{(1+\gamma_*)v} \sim 6 \times 10^3$$

- ϵ regime? However, find that $a \hat{f}_\pi = 0.00062(24)$, need $F_\pi L > 1$!