Walking, the dilaton, and complex CFT (II)

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Lattice Higgs Collaboration (LatHC)

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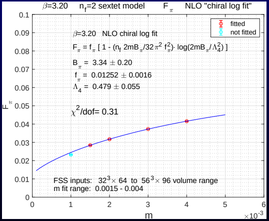
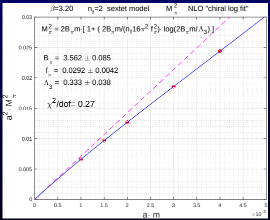
Outline

- Dilaton Effective Field Theory
- Implicit Maximum Liklihood Estimate
- Conclusion

Dilaton Effective Field Theory

- Sextet model: SU(3) $N_f = 2$ fermions in two-index symmetric representation
- Near conformal window $\Rightarrow$ candidate of walking theory
- Emergent 0^{++} scalar gets lighter as conformal window is approached
- In the pion mass range of our simulation, the scalar mass is comparable with the pions' $\Rightarrow$ χPT cannot work properly
- Previous attempt of χPT analysis:
  Inconsistent values of f_π are obtained from M_π and F_π





Dilaton Effective Field Theory

- σ model: masses of σ and π comparable $\Rightarrow$ Non-linear σ model may work, but not linear
- Dilaton hypothesis: The scalar acts as a dilaton from scale symmetry breaking
- Lagrangian of Dilaton Effective Field Theory:

$$L = \frac{1}{2}\,\partial_\mu \chi\,\partial_\mu \chi - V(\chi) + \frac{f_\pi^2}{4}\left(\frac{\chi}{f_d}\right)^2 \mathrm{Tr}\left[\partial_\mu\,\Sigma^\dagger\,\partial_\mu \Sigma\right] + \frac{f_\pi^2\,m_\pi^2}{4}\left(\frac{\chi}{f_d}\right)^y \mathrm{Tr}\left(\,\Sigma + \Sigma^\dagger\,\right)$$

- $y = 3 - \gamma,\ \ \gamma:$ mass anomalous dimension
- $\chi(x) = f_d\; e^{\;\sigma(x)\;/f_d}$, $\sigma(x)$: Dilaton field
- $\Sigma = e^{\;i\;\pi^a\;\;\tau^a\;\;/f_\pi}\;:\;\tau^a$ = Pauli matrices, $m_\pi^2 = 2B_\pi m$

Dilaton Effective Field Theory

- In this talk, we fit our data against two representative forms of V and compare the corresponding results
  - Deformation of CFT parametrically

    [e.g. M. Golterman and Y. Shamir,Phys. Rev. D94 (2016) 054502]

$$V_d = \frac{m_d^2}{16 f_d^2}\,\chi^4\left(4\ln\frac{\chi}{f_d} - 1\right)$$

  - Linear-σ model inspired potential

    [T. Appelquist et al, JHEP 07 (2017) 035; T. Appelquist et al, JHEP 07 (2018) 039;W. D. Goldberger et al, Phys. Rev. Lett. 100 (2008) 111802]

$$V_\sigma = \frac{m_d^2}{8 f_d^2}\left(\chi^2 - f_d^2\right)^2$$

- Previous similar attempt of analysis with our data by another group [T. Appelquist et al, JHEP 03 (2018) 039] only fitted against the assumption of $V \sim \chi^4$ asymptotically and was not comprehensive
- We are fitting our own data against the two explicit forms of V
- We investigate with implicit maximum likelihood analysis (different from [Z. Fodor et al, PoS LATTICE2018 (2019) 196])

Tree Level predictions:

V-independent scaling relation :

$$M_\pi^2\, F_\pi^{\gamma-1} - 2B_\pi\, f_\pi^{\gamma-1}\, m = 0$$

Expanding:

$$\frac{f_\pi^2\, m_\pi^2}{4}\left(\frac{\chi}{f_d}\right)^y \mathrm{Tr}\left(\Sigma+\Sigma^\dagger\right) = \frac{n_f\, m_\pi^2\, f_\pi^2}{2}\left(\frac{\chi}{f_d}\right)^y + \ldots$$

One can define:

$$W(\chi) \equiv V(\chi) - (n_f\, m_\pi^2\, f_\pi^2\, /\, 2)(\chi\, /\, f_d)^y,\; W'(F_d) = 0,\; W''(F_d) = M_d^2$$

$W'_d(\ \chi = F_d\ ) = 0$:

$$F_\pi^{\gamma+1}\, \ln(F_\pi/f_\pi) - (3-\gamma)\, n_f\, f_\pi^{\gamma-1}\, B_\pi\, m\, (m_d/f_\pi)^{-2}\, (f_d/f_\pi)^{-2} = 0$$

$W''_d(\ \chi = F_d\ ) = M_d^2$:

$$2\,(F_\pi^2/M_\pi^2)\,(3\,\ln(F_\pi/f_\pi)+1)\,(m_d/f_\pi)^2 - 2\,(M_d^2/M_\pi^2) - (3-\gamma)\,(2-\gamma)\,n_f\,(f_d/f_\pi)^{-2} = 0$$

$W'_\sigma(\ \chi = F_d\ ) = 0$:

$$F_\pi^{\gamma+1}\,(1 - f_\pi^2/F_\pi^2) - 2\,(3-\gamma)\,n_f\,f_\pi^{\gamma-1}\,B_\pi\,m\,(m_d/f_\pi)^{-2}\,(f_d/f_\pi)^{-2} = 0$$

$W''_\sigma(\ \chi = F_d\ ) = M_d^2$:

$$\left(3F_\pi^2/M_\pi^2 - f_\pi^2/M_\pi^2\right)\,(m_d/f_\pi)^2 - 2\,M_d^2/M_\pi^2 - (3-\gamma)\,(2-\gamma)\,n_f\,(f_d/f_\pi)^{-2} = 0$$

Implicit Maximum Likelihood Estimate

- From numerical data, we estimate M_π^{data}, F_π^{data}, M_d^{data} in the infinite volume limit at each m. Then minimize the following χ^2

$$\chi^2 = \sum_m \frac{\left(M_\pi^{\text{data}}(m) - M_\pi(m)\right)^2}{\Delta_{M_\pi}(m)} + \frac{\left(F_\pi^{\text{data}}(m) - F_\pi(m)\right)^2}{\Delta_{F_\pi}(m)} + \frac{\left(M_d^{\text{data}}(m) - M_d(m)\right)^2}{\Delta_{M_d}(m)}$$

$\Delta_{M_\pi}, \Delta_{F_\pi}, \Delta_{M_d}$: variances

Subject to the constraints:

$$V = V_d \begin{cases} M_\pi^2\, F_\pi^{\gamma-1} - 2B_\pi\, f_\pi^{\gamma-1}\, m = 0 \\ F_\pi^{\gamma+1}\, \ln(F_\pi/f_\pi) - (3-\gamma)\, n_f\, f_\pi^{\gamma-1}\, B_\pi\, m\, (m_d/f_\pi)^{-2}\, (f_d/f_\pi)^{-2} = 0 \\ 2\,(F_\pi^2/M_\pi^2)\,(3\,\ln(F_\pi/f_\pi)+1)\,(m_d/f_\pi)^2 - 2\,(M_d^2/M_\pi^2) - (3-\gamma)\,(2-\gamma)\,n_f\,(f_d/f_\pi)^{-2} = 0 \end{cases}$$

or

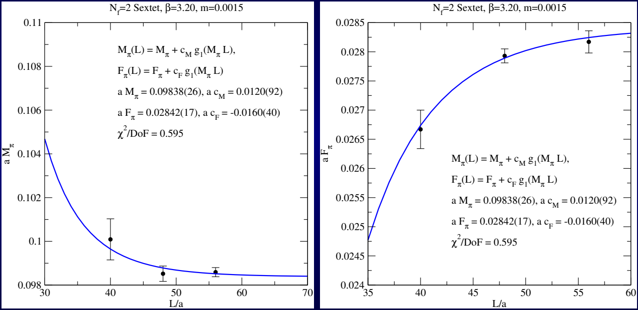
$$V = V_\sigma \begin{cases} M_\pi^2\, F_\pi^{\gamma-1} - 2B_\pi\, f_\pi^{\gamma-1}\, m = 0 \\ F_\pi^{\gamma+1}\, (1 - f_\pi^2/F_\pi^2) - 2\,(3-\gamma)\, n_f\, f_\pi^{\gamma-1}\, B_\pi\, m\, (m_d/f_\pi)^{-2}\, (f_d/f_\pi)^{-2} = 0 \\ \left(3F_\pi^2/M_\pi^2 - f_\pi^2/M_\pi^2\right)\,(m_d/f_\pi)^2 - 2\,M_d^2/M_\pi^2 - (3-\gamma)\,(2-\gamma)\,n_f\,(f_d/f_\pi)^{-2} = 0 \end{cases}$$

- Fitted parameters: f_π, B_π, γ, m_d/f_π, f_d/f_π

Implicit Maximum Likelihood Estimate

- M_π^{data} and F_π^{data} values are defined in infinite volume limit, while simulations are done in finite volumes $\Rightarrow$ Extrapolation into infinite volume limit is required
- Ansatz:

$$M\pi(L) = M\pi(L\to\infty) + c_M\, g_1(M\pi L, \eta = N_t/N_L), \quad F\pi(L) = F\pi(L\to\infty) + c_F\, g_1(M\pi L, \eta = N_t/N_L)$$



- In the presence of light dilaton $M_d \sim M_\pi$, the particle being exchanged can also be dilaton. However, the combined correction term would still be g_1
- M_d is noisier $\Rightarrow$ FSS of M_d is more difficult, M_d at largest volume is used instead

Implicit Maximum Likelihood Estimate

- Markov Chain Monte Carlo (MCMC)
  - Using the means and covariances of $\{M_\pi(m), F_\pi(m)\}$, and the means and variances of $M_d(m)$, the posterior distributions of $\{M_\pi, F_\pi, M_d\}(m)$ are generated with MCMC

$$P(\{M_\pi, F_\pi, M_d\})|_m \sim \exp(-1/2((((M_\pi - \bar{M_\pi})^2 \\ +(F_\pi - \bar{F_\pi})^2)/\Sigma(M,F) + (M_d - \bar{M_d})^2/\Delta_{M_d}))|_m$$

  - Markov Chain Monte Carlo algorithms sample a desired distribution by constructing a Markov Chain with such distribution as the equilibrium distribution
  - A simple implementation is the familiar Metropolis-Hastings algorithm using a Gaussian proposal density
  - Using the posterior distributions as input, one can obtain the Implicit Maximum Likelihood estimate of the parameters

Implicit Maximum Likelihood Estimate

- A sample $\{M_\pi^{\text{draw}}, F_\pi^{\text{draw}}, M_d^{\text{draw}}\}$ from the posterior distribution at each m is drawn, then the following is minimized:

$$\chi^2 = \sum_m \frac{\left(M_\pi^{\text{draw}}(m) - M_\pi(m)\right)^2}{\Delta_{M_\pi}(m)} + \frac{\left(F_\pi^{\text{draw}}(m) - F_\pi(m)\right)^2}{\Delta_{F_\pi}(m)} + \frac{\left(M_d^{\text{draw}}(m) - M_d(m)\right)^2}{\Delta_{M_d}(m)}$$

$\Delta_{M_\pi}, \Delta_{F_\pi}, \Delta_{M_d}$: variances of the posterior distribution

Subject to the constraints:

$$V = V_d \begin{cases} M_\pi^2\, F_\pi^{\gamma-1} - 2B_\pi\, f_\pi^{\gamma-1}\, m = 0 \\ F_\pi^{\gamma+1}\, \ln(F_\pi/f_\pi) - (3-\gamma)\, n_f\, f_\pi^{\gamma-1}\, B_\pi\, m\, (m_d/f_\pi)^{-2}\, (f_d/f_\pi)^{-2} = 0 \\ 2\,(F_\pi^2/M_\pi^2)\,(3\,\ln(F_\pi/f_\pi)+1)\,(m_d/f_\pi)^2 - 2\,(M_d^2/M_\pi^2) - (3-\gamma)\,(2-\gamma)\,n_f\,(f_d/f_\pi)^{-2} = 0 \end{cases}$$

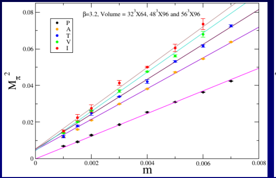
or

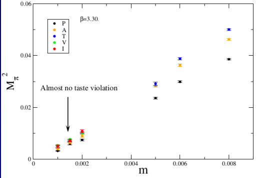
$$V = V_\sigma \begin{cases} M_\pi^2\, F_\pi^{\gamma-1} - 2B_\pi\, f_\pi^{\gamma-1}\, m = 0 \\ F_\pi^{\gamma+1}\,(1 - f_\pi^2/F_\pi^2) - 2\,(3-\gamma)\, n_f\, f_\pi^{\gamma-1}\, B_\pi\, m\, (m_d/f_\pi)^{-2}\, (f_d/f_\pi)^{-2} = 0 \\ \left(3F_\pi^2/M_\pi^2 - f_\pi^2/M_\pi^2\right)\,(m_d/f_\pi)^2 - 2\,M_d^2/M_\pi^2 - (3-\gamma)\,(2-\gamma)\,n_f\,(f_d/f_\pi)^{-2} = 0 \end{cases}$$

- Fitted parameters: $f_\pi,\ B_\pi,\ \gamma,\ m_d/f_\pi,\ f_d/f_\pi$

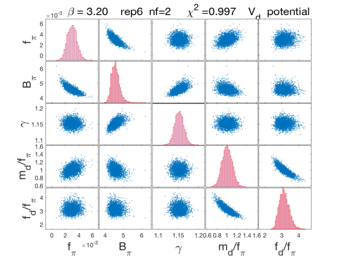
Results and Implications

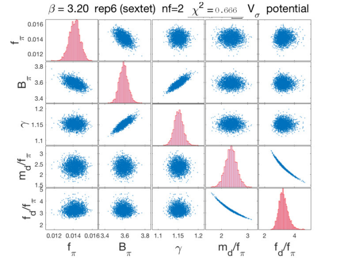
- Tree-level Symanzik-Improved gauge action with 2-step $\rho = 0.15$ stout-smeared Staggered $N_f = 2$ SU(3) Sextet fermion
- In order to study the taste breaking effects, we have dataset of $\beta = 3.20, 3.25, 3.30$





- As a pilot study, we present here only $\beta = 3.20$, using $m = 0.0015, 0.002, 0.003, 0.004$





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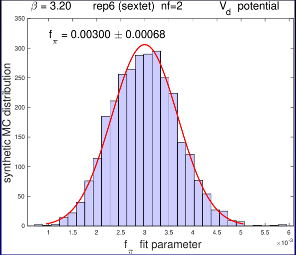
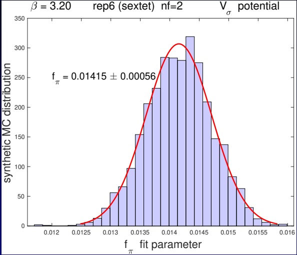
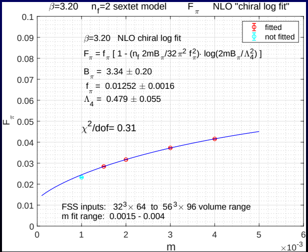
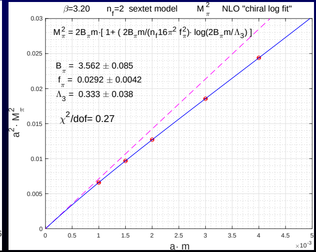




- V_d : f_π is very low $\Rightarrow$ validity of tree-level calculation is questionable
- V_σ : f_π Similar to previous χ PT fits





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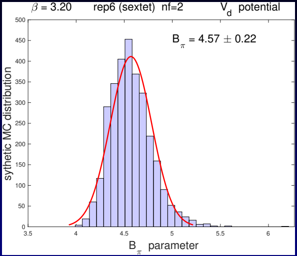
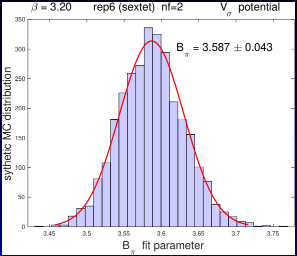
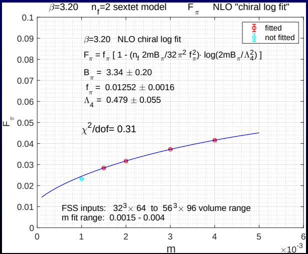
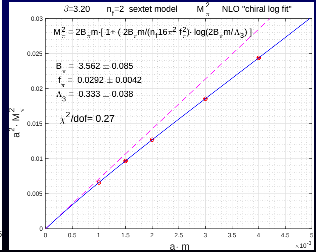




- V_d : B_π Higher than previous χPT fits
- V_σ : B_π Consistent with previous χPT fits





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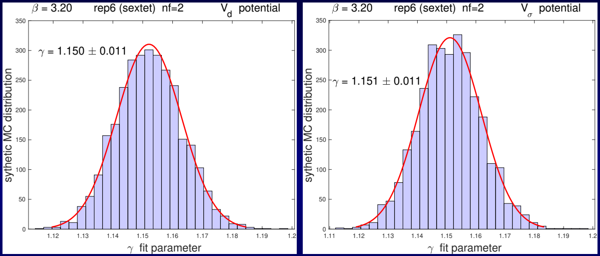
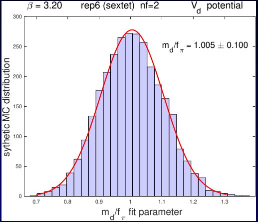
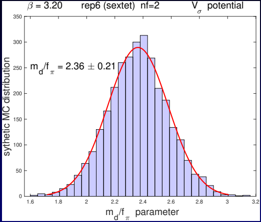


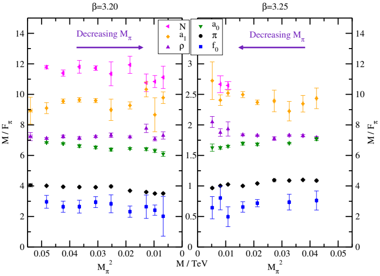
- Both V_d and V_σ give the same value of γ
- Further study needed: Which scale does this γ correspond to?

Results and Implications





- m_d/f_π is more sensitive to the $M_d(m)$ input which is of less control
- V_d : m_d/f_π dramatically lower than previous extrapolated results
- V_σ : m_d/f_π Closer to previous estimations



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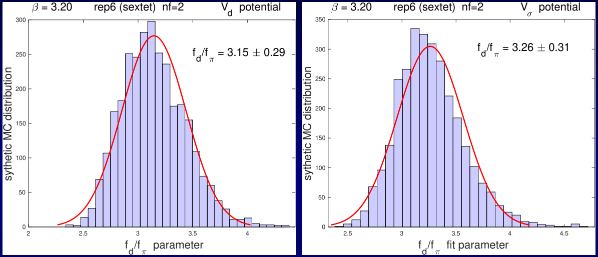
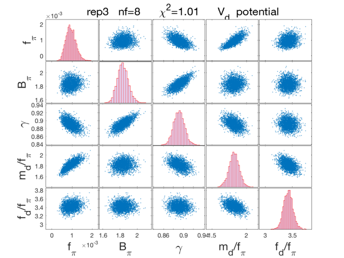


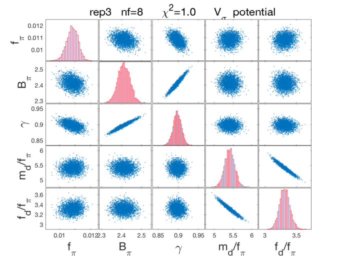
- Both V_d and V_σ give similar $f_d/f_\pi \sim 3 >> 1$
  $\Rightarrow$ phenomenological difficulties for potential BSM applications

[e.g. J. Ellis and T. You JHEP06(2012)140]

Results and Implications

- Similar situation for Fund $N_f = 8$ model, at $\beta = 4.8$ (Data are taken from LSD collaboration [T. Appelquist et al, Phys. Rev. D 99, 014509])





	V_d	V_σ
f_π	0.000947(24)	0.01082(36)
B_π	1.833(64)	2.413(30)
γ	0.891(11)	0.897(10)
m_d/f_π	1.821(73)	5.41(13)
f_d/f_π	3.424(86)	3.315(81)

Conclusion

	Sextet $N_f = 2$, $\beta = 3.20$		Fund $N_f = 8$, $\beta = 4.8$	
	V_d	V_σ	V_d	V_σ
f_π	0.00300(68)	0.01415(56)	0.000947(24)	0.01082(36)
B_π	4.57(22)	3.587(43)	1.833(64)	2.413(30)
γ	1.150(11)	1.151(11)	0.891(11)	0.897(10)
m_d/f_π	1.005(100)	2.36(21)	1.821(73)	5.41(13)
f_d/f_π	3.15(29)	3.26(31)	3.424(86)	3.315(81)

- The sextet model and fund $N_f = 8$ model are analyzed under the dilaton hypothesis with two typical dilaton potentials
- The linear-σ model inspired potential form V_σ seems to fit the data well, while V_d does not seem to work
- f_d/f_π and m_d/f_π are particularly important but more sensitive to the noisy M_d inputs
- A full analysis in which M_d inputs are better controlled and taste breaking effects are taken into account with different β's, would shed new light on crucial properties of the models.