Continuum limit of SU(3) $\mathcal{N} = 1$ supersymmetric Yang-Mills theory and supersymmetric gauge theories on the lattice

Georg Bergner
FSU Jena, WWU Münster

DESY-Münster-Regensburg-Jena Collaboration
Wuhan: June 19, 2019

PRL 122, no. 22, 221601 (2019) [arXiv:1902.11127]
Why study supersymmetric Yang-Mills theory on the lattice?

- Non-perturbative SUSY:
  1. BSM physics: Supersymmetric particle physics requires breaking terms based on an unknown non-perturbative mechanism.
     ⇒ need to understand non-perturbative SUSY
  2. Supersymmetry is a general beautiful theoretical concept: (Extended) SUSY simplifies theoretical analysis and leads to new non-perturbative approaches.
     ⇒ need to bridge the gap between "beauty" and "reality"
  ⇒ Before studying SQCD and extended SUSY theories: simplest 4D SUSY gauge theory: $\mathcal{N} = 1$ SYM
\( \mathcal{N} = 1 \) super Yang-Mills theory

Supersymmetric Yang-Mills theory:

\[ \mathcal{L} = \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} \bar{\lambda} (\mathcal{D} + m_g) \lambda \]

- supersymmetric counterpart of Yang-Mills theory; but in several respects similar to QCD
- \( \lambda \) Majorana fermion in the adjoint representation
- SUSY transformations: \( \delta A_\mu = -2i \bar{\lambda} \gamma_\mu \varepsilon \), \( \delta \lambda = -\sigma_{\mu\nu} F_{\mu\nu} \varepsilon \)
- current approach: one-loop \( O(a) \) improved Wilson fermions, tuning of \( m_g \) to restore chiral symmetry and SUSY
Finalization of our investigations of SYM bound states

- preparatory studies SU(2) SYM: development of techniques for SUSY Ward-identities and bound state spectrum
- recent improvement of bound states: enlarged operator basis and mixing analysis ⇒ see talk by P. Scior
- final objective: concluded analysis of SU(3) SYM bound state spectrum
- ongoing follow up studies: phase diagram of SYM and SQCD
SUSY restoration and the bound state spectrum of SYM

Signals for SUSY restoration:
- SUSY Ward identities
- degenerate bound states (scalar, pseudoscalar, fermion)

Conjectured multiplets:

<table>
<thead>
<tr>
<th>multiplet 1</th>
<th>multiplet 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>scalar</td>
<td>meson $a-f_0$</td>
</tr>
<tr>
<td>pseudoscalar</td>
<td>meson $a-\eta'$</td>
</tr>
<tr>
<td>fermion</td>
<td>gluino-glue</td>
</tr>
</tbody>
</table>

Does not exhaust all possibilities: e.g. exotic baryon states

---

1 [Veneziano, Yankielowicz, Phys.Lett.B113 (1982)]
Continuum and chiral extrapolations

SUSY restored in continuum limit at chiral point

Two-step approach:
1. chiral extrapolation at each lattice spacing (tuning of $m_g$)
2. continuum extrapolations of “chiral results”

More reliable approach: one step global fit

$$O(m_{a-\pi}^2, w_{0,\chi}) = O_{\chi,\text{cont.}} + c^{(1)} x + c^{(2)} y + c^{(3)} xy,$$

Chiral limit: $x = (w_{0,\chi} m_{a-\pi})^2$
Continuum limit: $y = \frac{a}{w_{0,\chi} \beta^2}$ or $y = \frac{a^2}{w_{0,\chi}^2}$ (one-loop $O(a)$ improved)

Parameter range: $0.2 < am_{a-\pi} < 0.7; a \sim [0.053, 0.082] \text{fm}$
Lattice sizes: $12^3 \times 24$ to $24^3 \times 48$
More details about Ward identities to appear soon.

Our final conclusions for the bound state spectrum of SYM

- SUSY breaking under control in well adjusted parameter region
- improved action essential, otherwise required lattice sizes min. $\sim 24^3 \times 48$
- challenging to obtain reliable signal for bound states (large statistics required 5k to 10k configurations)
- more details: Pfaffian sign, finite size effects, topological freezing see [JHEP 1803 (2018) 113]

We can see SUSY, what else can we learn?

- extend tuning approach to other SUSY gauge theories (e. g. SQCD)
- SYM phase diagram and controlled confinement
- better handle on chiral properties: Overlap fermions
Absence of confinement and semiclassical predictions

- SYM on $R^3 \times S^1$: absence of confinement down to semiclassical regime (small Radius $R$ of $S^1$)
- perturbative cancellation of gluon and gluino contributions
- nice picture of instanton gas at work ["Unsal, Yaffe, Shifman, Poppitz ..."]

[JHEP 1412 (2014) 133]
Regime for lattice and semiclassics

- Wilson fermions: at smaller $R/a$ lattice artefacts effectively lead to larger $N_f$
- Fermion effects dominate, no cancellation at very small $R$ ([JHEP 1811 (2018) 092])
- But approximate cancellation in intermediate regime
Overlap operator and SYM

Gluino condensation and chiral symmetry realization
- indications of same $T_c$ for chiral and deconfinement transition
- to understand gluino condensation: need better control of chiral symmetry
- option 1: gradient flow (talk be C. Lopez)
- option 2: overlap fermions

Advantages of overlap fermions for SUSY gauge theories
- well defined chiral symmetry
- reduced tuning, interesting option for theories beyond SYM
Overlap fermions and SYM

Disadvantages of overlap fermions

- zero modes: problems with tunnelling between topological sectors
- Pfaffian of SYM requires implementation in RHMC

Polynomial approximations

- approximation reduces zero mode problem
- straight forward implementation in RHMC

see also [S. W. Kim et al. arXiv: 1111.2180]
Conclusions

- final results for SU(3) SYM with one-loop clover improved Wilson fermions show symmetry restoration in bound state spectrum and Ward identities
- parameter range estimation for reliable simulations of SYM
- simulation of SQCD and theories with extended SUSY possible based on these findings
- investigations of special properties of SYM: phase transitions, gluino condensations . . .
- new techniques: gradient flow and overlap fermions for better control of chiral symmetry breaking