Continuum limit of SU(3) $\mathcal{N} = 1$ supersymmetric Yang-Mills theory and supersymmetric gauge theories on the lattice

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Why study supersymmetric Yang-Mills theory on the lattice?

- Non-perturbative SUSY:
  1. BSM physics: Supersymmetric particle physics requires breaking terms based on an unknown non-perturbative mechanism.
     $\Rightarrow$ need to understand non-perturbative SUSY
  2. Supersymmetry is a general beautiful theoretical concept: (Extended) SUSY simplifies theoretical analysis and leads to new non-perturbative approaches.

     $\Rightarrow$ need to bridge the gap between "beauty" and **"reality"**

$\Rightarrow$ Before studying SQCD and extended SUSY theories: simplest 4D SUSY gauge theory: $\mathcal{N} = 1$ SYM

$\mathcal{N} = 1$ super Yang-Mills theory

Supersymmetric Yang-Mills theory:

$$\mathcal{L} = \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} \bar{\lambda} (\not{D} + m_g) \lambda$$

- supersymmetric counterpart of Yang-Mills theory; but in several respects similar to QCD
- λ Majorana fermion in the adjoint representation
- SUSY transformations: $\delta A_\mu = -2i\bar{\lambda}\gamma_\mu\varepsilon$, $\delta\lambda = -\sigma_{\mu\nu}F_{\mu\nu}\varepsilon$
- current approach: one-loop $O(a)$ improved Wilson fermions, tuning of m_g to restore chiral symmetry and SUSY

Finalization of our investigations of SYM bound states

- preparatory studies SU(2) SYM: development of techniques for SUSY Ward-identities and bound state spectrum
- recent improvement of bound states: enlarged operator basis and mixing analysis $\Rightarrow$ see talk by P. Scior
- final objective: concluded analysis of SU(3) SYM bound state spectrum
- ongoing follow up studies: phase diagram of SYM and SQCD

SUSY restoration and the bound state spectrum of SYM

Signals for SUSY restoration:

- SUSY Ward identities
- degenerate bound states (scalar,pseudoscalar,fermion)

Conjectured multiplets:

	multiplet[1]	**multiplet**[2]
scalar	meson a$-f_0$	glueball 0^{++}
pseudoscalar	meson a$-\eta'$	glueball 0^{-+}
fermion	gluino-glue	gluino-glue

Does not exhaust all possibilities: e. g. exotic baryon states

[1] [Veneziano, Yankielowicz, Phys.Lett.B113 (1982)]

[2] [Farrar, Gabadadze, Schwetz, Phys.Rev. D58 (1998)]

Continuum and chiral extrapolations

SUSY restored in continuum limit at chiral point

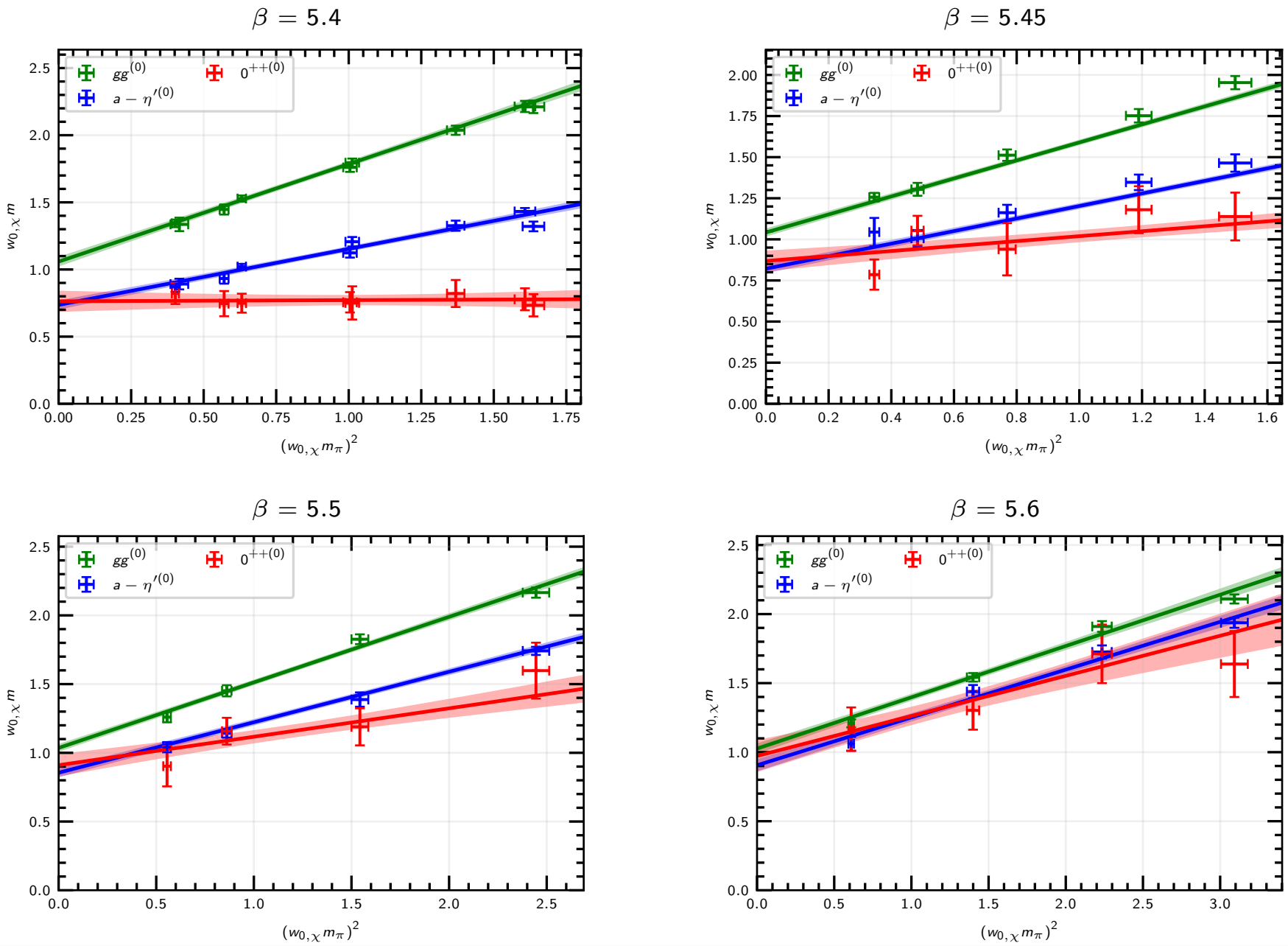
Two-step approach:

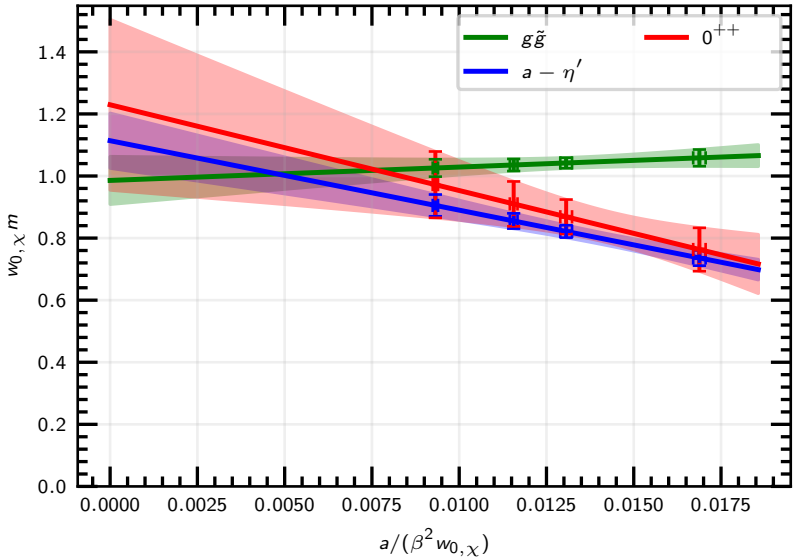
1. chiral extrapolation at each lattice spacing (tuning of m_g)
2. continuum extrapolations of "chiral results"

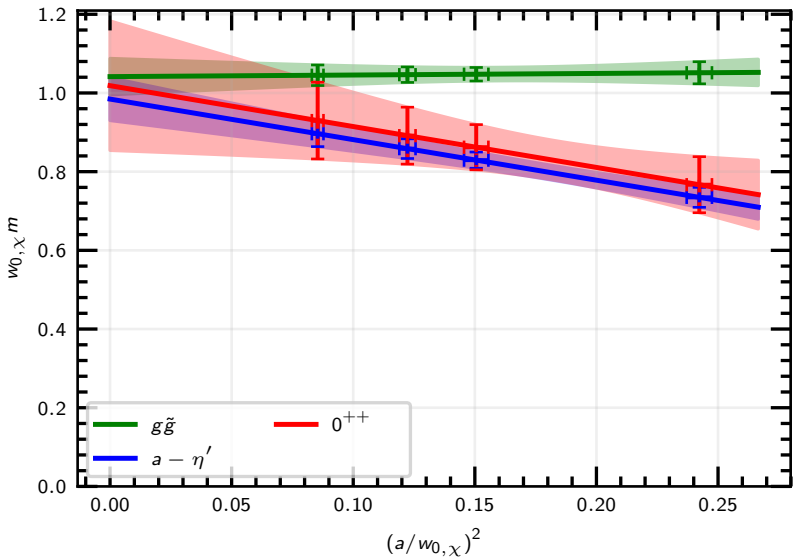
More reliable approach: one step global fit

$$O(m^2_{\mathsf{a}-\pi}, w_{0,\chi}) = O_{\chi,\mathrm{cont.}} + c^{(1)}x + c^{(2)}y + c^{(3)}xy,$$

Chiral limit: $x = (w_{0,\chi} m_{\mathsf{a}-\pi})^2$
Continuum limit: $y = \frac{a}{w_{0,\chi}\beta^2}$ or $y = \frac{a^2}{w^2_{0,\chi}}$ (one-loop $O(a)$ improved)
Parameter range: $0.2 < am_{\mathsf{a}-\pi} < 0.7$; $a \sim [0.053, 0.082]$fm
Lattice sizes: $12^3 \times 24$ to $24^3 \times 48$


$\beta = 5.4$
$\beta = 5.45$
$\beta = 5.5$
$\beta = 5.6$
$gg^{(0)}$
$0^{++(0)}$
$a - \eta'^{(0)}$
$w_{0,\chi} m$
$(w_{0,\chi} m_\pi)^2$





Fit	$w_0 m_{g\tilde{g}}$	$w_0 m_{0^{++}}$	$w_0 m_{a-\eta'}$
linear fit	0.917(91)	1.15(30)	1.05(10)
quadratic fit	0.991(55)	0.97(18)	0.950(63)
SU(2) SYM	0.93(6)	1.3(2)	0.98(6)

More details about Ward identities to appear soon.

([Eur.Phys.J. C78 (2018) no.5, 404])

Our final conclusions for the bound state spectrum of SYM

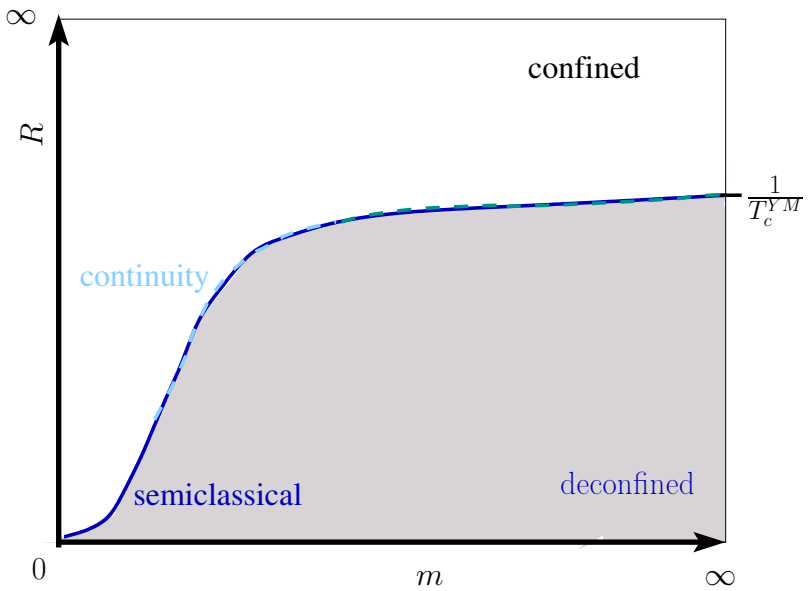
- SUSY breaking under control in well adjusted parameter region
- improved action essential, otherwise required lattice sizes min. $\sim 24^3 \times 48$
- challenging to obtain reliable signal for bound states (large statistics required 5k to 10k configurations)
- more details: Pfaffian sign, finite size effects, topological freezing see [JHEP 1803 (2018) 113]

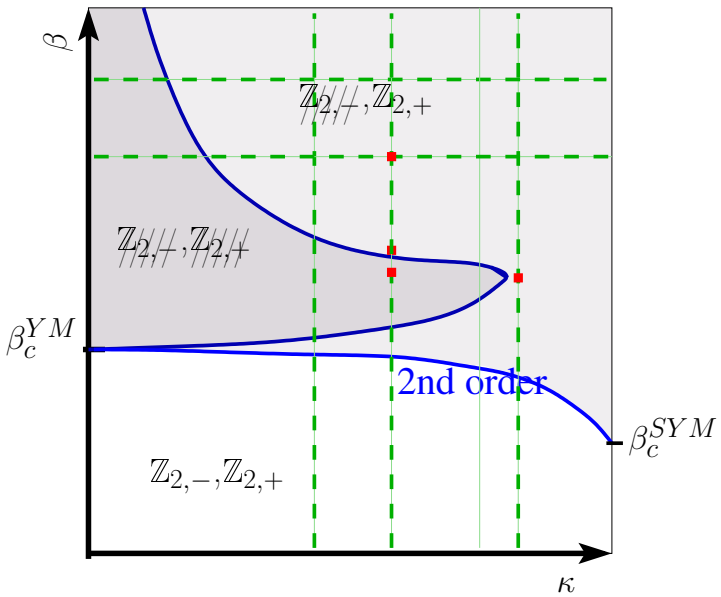
We can see SUSY, what else can we learn?

- extend tuning approach to other SUSY gauge theories (e. g. SQCD)
- SYM phase diagram and controlled confinement
- better handle on chiral properties: Overlap fermions

Absence of confinement and semiclassical predictions

- SYM on $R^3 \times S^1$: absence of confinement down to semiclassical regime (small Radius R of S^1)
- perturbative cancellation of gluon and gluino contributions
- nice picture of instanton gas at work [Ünsal, Yaffe, Shifman,Poppitz ...]

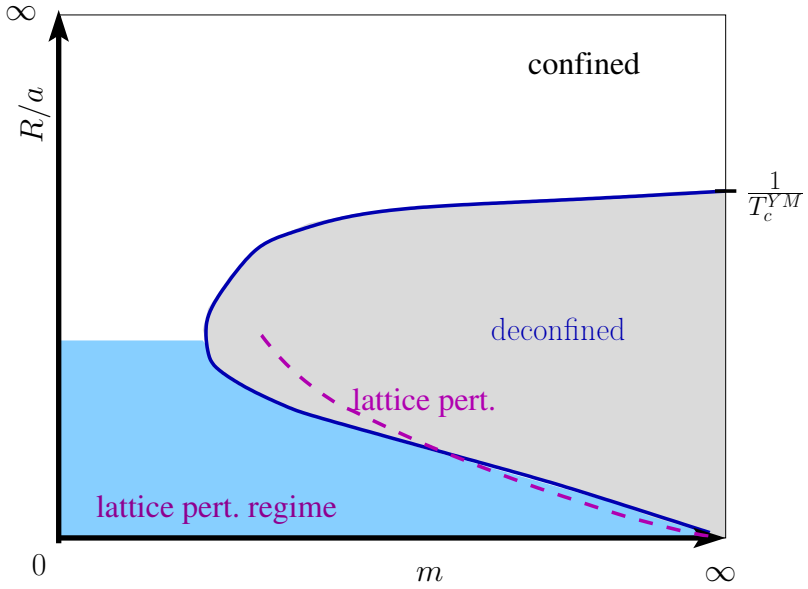


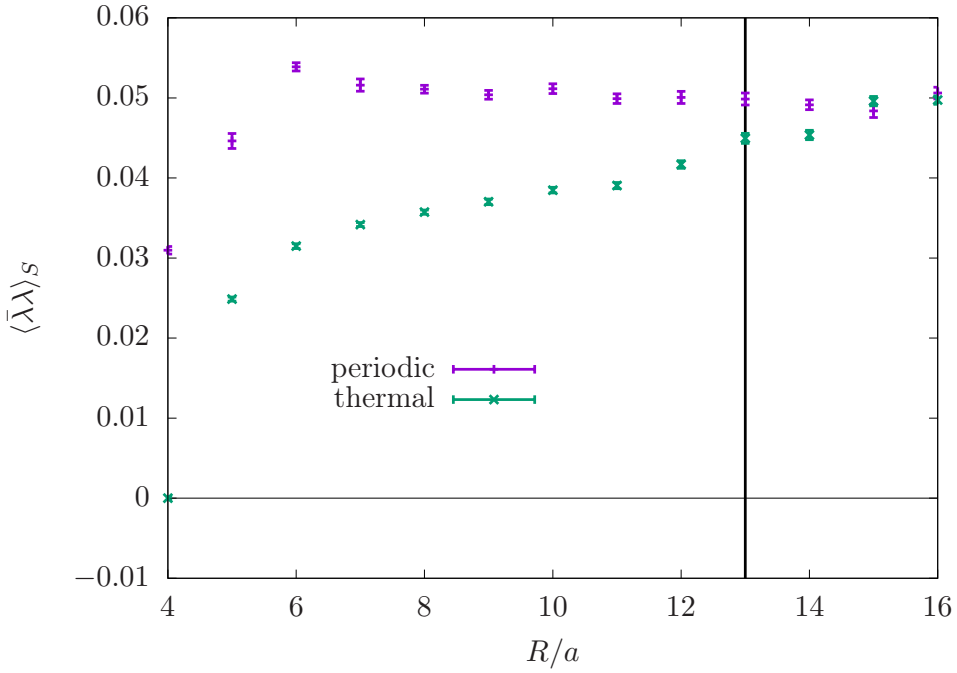


[JHEP 1412 (2014) 133]

Regime for lattice and semiclassics

- Wilson fermions: at smaller R/a lattice artefacts effectively lead to larger N_f
- fermion effects dominate, no cancellation at very small R ([JHEP 1811 (2018) 092])
- but approximate cancellation in intermediate regime





Overlap operator and SYM

Gluino condensation and chiral symmetry realization

- indications of same T_c for chiral and deconfinement transition
- to understand gluino condensation: need better control of chiral symmetry
- option 1: gradient flow (talk be C. Lopez)
- option 2: overlap fermions

Advantages of overlap fermions for SUSY gauge theories

- well defined chiral symmetry
- reduced tuning, interesting option for theories beyond SYM

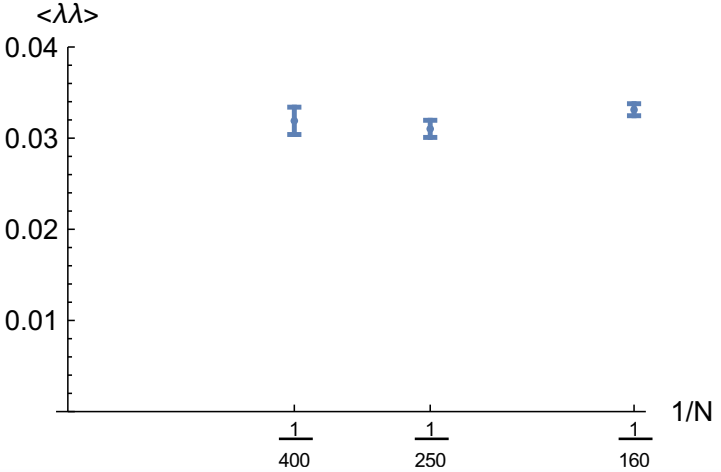
Overlap fermions and SYM

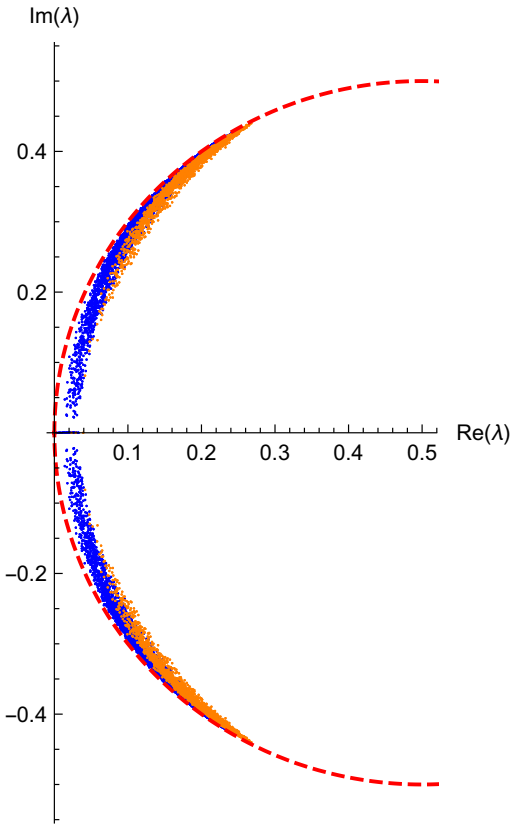
Disadvantages of overlap fermions

- zero modes: problems with tunnelling between topological sectors
- Pfaffian of SYM requires implementation in RHMC

Polynomial approximations

- approximation reduces zero mode problem
- straight forward implementation in RHMC





see also [S. W. Kim et al. arXiv: 1111.2180]

Conclusions

- final results for SU(3) SYM with one-loop clover improved Wilson fermions show symmetry restoration in bound state spectrum and Ward identities
- parameter range estimation for reliable simulations of SYM
- simulation of SQCD and theories with extended SUSY possible based on these findings
- investigations of special properties of SYM: phase transitions, gluino condensations . . .
- new techniques: gradient flow and overlap fermions for better control of chiral symmetry breaking