Gauge-invariant path-integral measure for the Overlap Weyl fermions in 16 of SO(10) and the Standard Model

Y. Kikukawa

Institute of Physics, University of Tokyo

based on :
Y.K., arXiv:1710.11618[hep-lat], arXiv:1710.11101[hep-lat]

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the Standard Model / SO(10) chiral gauge theory

SU(3)xSU(2)xU(1) xU(1)$_{B-L}$

(3 ,2) 1/6		(1,2) -1/2	
(3*,1) -2/3	(3*,1) 1/3	(1,1) 1	(1,1) 0

SO(10)

16

- **Complex, but free from gauge anomalies, both local and global ones**

$$\mathrm{Tr}\{\mathrm{P}_+\Sigma_{a_1b_1}[\Sigma_{a_2b_2}\Sigma_{a_3b_3}+\Sigma_{a_3b_3}\Sigma_{a_2b_2}]\}=0$$

$$\Sigma_{ab}=-\tfrac{i}{4}[\Gamma^a,\Gamma^b] \quad \{\Gamma^a \,|\, a=1,2,\cdots,10\}$$

$$\mathrm{P}_+=\frac{1+\Gamma^{11}}{2}, \qquad \Gamma^{11}=-i\Gamma^1\Gamma^2\cdots\Gamma^{10}$$

$\Omega^{spin}_5(BSpin(10)) = 0$
$\Omega_5(\mathrm{Spin}(5)\times Spin(10)/Z_2) = Z_2$

[*Garcia-Etxebarria-Montero, Wang-Wen-Witten (2018)*]

- **U(1) fermion symmetry broken by chiral anomaly**
  => zero modes (4 x m / SU(2) instanton)
  => <0| 't Hooft vertex |0>
- **'t Hooft vertex for 16 : 16 x 16 x 16 x 16 => 1**

(16 x 16 => 10)

$$T_-(x)=\frac{1}{2}V^a_-(x)V^a_-(x) \qquad V^a_-(x)=\psi_-(x)^{\mathrm{T}}i\gamma_5 C_D \mathrm{T}^a\psi_-(x)$$

$$\bar{T}_-(x)=\frac{1}{2}\bar{V}^a_-(x)\bar{V}^a_-(x) \qquad \bar{V}^a_-(x)=\bar{\psi}_-(x)i\gamma_5 C_D \mathrm{T}^{a\dagger}\bar{\psi}_-(x)^{\mathrm{T}}$$

$$\mathrm{T}^a=\mathrm{C}\Gamma^a \quad \mathrm{T}^{aT}=\mathrm{T}^a$$

Overlap Weyl fermions [Narayanan-Neuberger] [Luscher]

$$S_{\rm w} = a^4 \sum_x \bar{\psi}_-(x) D \psi_-(x)$$

$$\psi_-(x) = \hat{P}_- \psi(x) \quad \bar{\psi}_-(x) = \bar{\psi}(x) P_+$$

$$D = \frac{1}{2a}\left(1 + X \frac{1}{\sqrt{X^\dagger X}}\right), \quad X = a D_{\rm w} - m_0$$

$$\gamma_5 D + D\gamma_5 = 2aD\gamma_5 D$$

$$\hat{\gamma}_5 = \gamma_5(1 - 2aD) \quad \hat{\gamma}_5{}^2 = \mathbb{I}$$

$$\hat{P}_\pm = \left(\frac{1 \pm \hat{\gamma}_5}{2}\right), \quad P_\pm = \left(\frac{1 \pm \gamma_5}{2}\right)$$

Path Integral measure

$$e^{\Gamma_W[U]} \equiv \int \mathcal{D}[\psi_-]\mathcal{D}[\bar{\psi}_-]\, e^{-S_W[\psi_-,\bar{\psi}_-]}$$

$$e^{\Gamma_W[U]} = \det(\bar{v} D v)$$

$$\mathcal{D}_\star[\psi_-]\mathcal{D}_\star[\psi_-] \equiv \prod_j dc_j \prod_k d\bar{c}_k$$

$$(\bar{v} D v)_{ki} \quad (k = 1, \cdots, n/2; i = 1, \cdots, n/2 + 8Q)$$

(matrix shape is variable, can be rectangular)

$$\psi_-(x) = \sum_j v_j(x) c_j, \quad \bar{\psi}_-(x) = \sum_k \bar{c}_k \bar{v}_k(x)$$

Neuberger & Narayanan showed the chiral determinant as vacuum overlap reproduces zero-modes and VEV of 't Hooft vertex.

$$v_j(x) \to \tilde{v}_j(x) = v_l(x) Q^{-1}_{lj}[U]$$

$$\prod_j dc_j \to \prod_j d\tilde{c}_j = \prod_j dc_j \times \det Q[U]$$

Luscher formulated a method to reconstruct the chiral basis preserving locality, lattice symmetries and exact gauge-invariance. successful for the U(1), $SU(2)_L$ x $U(1)_Y$ cases, but not yet for non-Abelian cases.

Overlap Weyl fermions in 16 of SO(10)

cf. [*Neuberger-Narayanan*] [*Luscher*]

$$S_{\rm w} = a^4 \sum_x \bar{\psi}_-(x) D \psi_-(x)$$

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$$\psi_+(x) = \hat{P}_+ \psi(x) \quad \bar{\psi}_+(x) = \bar{\psi}(x) P_-$$

$$D = \frac{1}{2a}\left(1 + X \frac{1}{\sqrt{X^\dagger X}}\right), \quad X = a D_{\rm w} - m_0$$

$$\gamma_5 D + D\gamma_5 = 2aD\gamma_5 D$$

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$$\hat{P}_\pm = \left(\frac{1 \pm \hat{\gamma}_5}{2}\right), \quad P_\pm = \left(\frac{1 \pm \gamma_5}{2}\right)$$

Path Integral measure for the 16

$$e^{\Gamma_W[U]} \equiv \int \mathcal{D}[\psi_-]\mathcal{D}[\bar{\psi}_-]\, e^{-S_W[\psi_-,\bar{\psi}_-]}$$

$$\mathcal{D}[\psi_-]\mathcal{D}[\bar{\psi}_-] \equiv \mathcal{D}[\psi]\mathcal{D}[\bar{\psi}] \prod_x F[T_+(x)] \prod_x F[\bar{T}_+(x)]$$

$$\mathcal{D}[\psi]\mathcal{D}[\bar{\psi}] \equiv \prod_{x\in\Lambda}\prod_{\alpha=1}^{4}\prod_{s=1}^{16} d\psi_{\alpha s}(x) \prod_{x\in\Lambda}\prod_{\alpha=1}^{4}\prod_{s=1}^{16} d\bar{\psi}_{\alpha s}(x)$$

$$T_+(x) = \frac{1}{2} V_+^a(x) V_+^a(x), \quad V_+^a(x) = \psi_+(x)^{\rm T} i\gamma_5 C_D {\rm T}^a \psi_+(x)$$

$$\bar{T}_+(x) = \frac{1}{2} \bar{V}_+^a(x) \bar{V}_+^a(x), \quad \bar{V}_+^a(x) = \bar{\psi}_+(x) i\gamma_5 C_D {\rm T}^a \bar{\psi}_+(x)^{\rm T}$$

$$F(w) \equiv 4!\,(z/2)^{-4} I_4(z)\Big|_{(z/2)^2=w}$$

$$F(w)\Big|_{w=(1/2)u^a u^a} = (\pi^5/12)^{-1} \int \prod_{a=1}^{10} de^a \delta(\sqrt{e^b e^b} - 1)\, e^{e^c u^c}$$

Right-handed part of the Dirac measure is "saturated" completely by "Right-handed" 't Hooft verteces .

The measure correctly reproduces zero-modes and VEV of 't Hooft vertex

Exact gauge inv. is manifest and CP inv. can be proved.

[*YK 2017*]

cf.

$$\hat{P}_+^T i\gamma_5 C_D P_+ {\rm T}^a E^a(x) \hat{P}_+ = (1-D)^T i\gamma_5 C_D P_+ {\rm T}^a E^a(x)(1-D)$$

The saturation of the "right-handed" measures due to 't Hooft vertices

cf. [Eichten-Preskill(1986)]

$$\mathrm{e}^{\Gamma_W[U]} \equiv \int \mathcal{D}[\psi]\mathcal{D}[\bar{\psi}] \prod_x F[T_+(x)] \prod_x F[\bar{T}_+(x)]\, \mathrm{e}^{-S_W[\psi_-,\bar{\psi}_-]}$$

$$\equiv \int \mathcal{D}_\star[\psi_-]\mathcal{D}_\star[\bar{\psi}_-]\, \mathrm{e}^{-S_W[\psi_-,\bar{\psi}_-]} \times \int \mathcal{D}_\star[\psi_+] \prod_x F[T_+(x)] \times \int \mathcal{D}_\star[\bar{\psi}_+] \prod_x F[\bar{T}_+(x)]$$

$$\psi_+(x) = \sum_j u_j(x) b_j, \quad \bar{\psi}_+(x) = \sum_k \bar{b}_k \bar{u}_k(x)$$

$$\mathcal{D}_\star[\psi_+] \equiv \prod_j db_j$$

$$\mathcal{D}_\star[\bar{\psi}_+] \equiv \prod_k d\bar{b}_k = \prod_x \prod_{\alpha=3}^{4} \prod_{s=1}^{16} d\bar{\psi}_{\alpha s}(x)$$

$$\int \mathcal{D}_\star[\bar{\psi}_+] \prod_x F[\bar{T}_+(x)] = 1$$

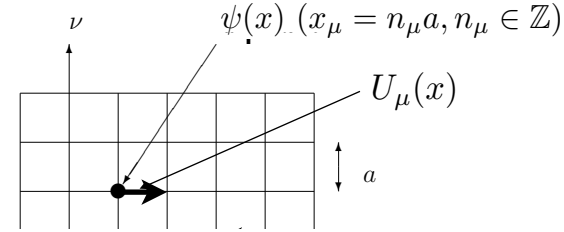
$$\int \mathcal{D}_\star[\psi_+] \prod_x F[T_+(x)] = \int \mathcal{D}[E^a]\, \mathrm{pf}(u^T i\gamma_5 C_D T^a E^a u) \quad (\neq 0)$$

$$(E^a(x)E^a(x) = 1)$$

16 *has 32-components at a site !*

$$\int \prod_{x\in\Lambda} \prod_{\alpha=3}^{4} \prod_{s=1}^{16} d\bar{\psi}_{\alpha s}(x) \prod_{x\in\Lambda} \frac{4!}{8!12!} \left\{ \frac{1}{2}\, \bar{\psi}(x) P_- i\gamma_5 C_D \mathrm{T}^a \bar{\psi}(x)^{\mathrm{T}}\, \bar{\psi}(x) P_- i\gamma_5 C_D \mathrm{T}^a \bar{\psi}(x)^{\mathrm{T}} \right\}^8 = 1$$

[Eichten-Preskill(1986)]



$$\left(u^{\mathrm{T}} i\gamma_5 C_D \mathrm{T}^a E^a u\right)_{ij} \qquad (i,j = 1,\cdots,n/2 - 8Q)$$

(matrix shape is variable, but even square)

$$(u^{\mathrm{T}}\, i\gamma_5 C_D \mathrm{T}^a E^a u) = \mathcal{C} \times (u^\dagger \Gamma^{10} \Gamma^a E^a u)$$

$$= (u^\dagger \Gamma^{10} \Gamma^a E^a u)^T \times \mathcal{C}$$

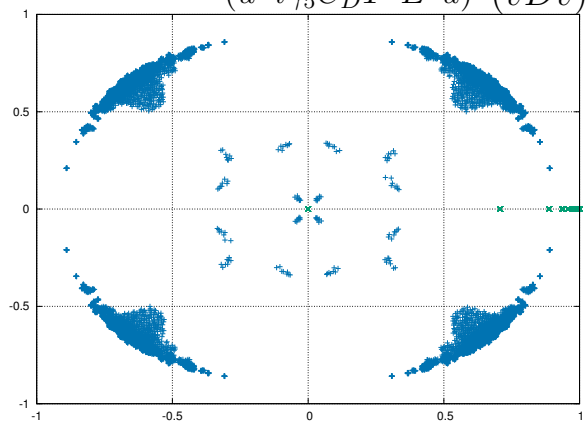
$\{(\tilde{\lambda}_i, -\tilde{\lambda}_i)\,|\, i = 1,\cdots,n/4-4Q\}$ $\quad$ $\{(\lambda_i, \lambda_i)\,|\, i = 1,\cdots,n/4-4Q\}$

$$\{\lambda, \lambda, \lambda^*, \lambda^*\} \qquad \mathrm{pf}(u^T i\gamma_5 C_D T^a E^a u) \geq 0$$

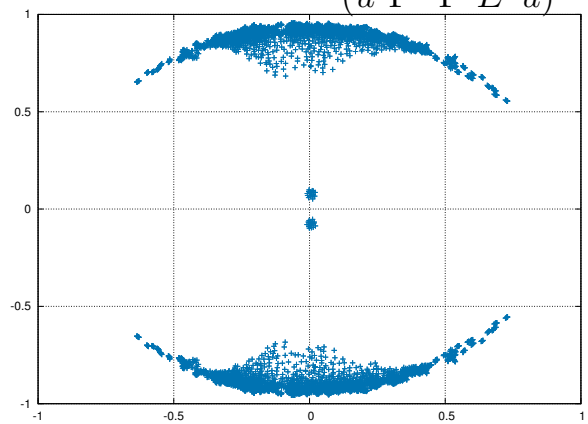
($E^a(x)$ randomly generated)

$U(x,\mu) = 1$

$(u^T i\gamma_5 C_D T^a E^a u)\ (\bar{v} D v)$



$(u^\dagger \Gamma^{10} \Gamma^a E^a u)$



The saturation of the "right-handed" measures due to 't Hooft vertices

cf. [Eichten-Preskill(1986)]

$$e^{\Gamma_W[U]} \equiv \int \mathcal{D}[\psi]\mathcal{D}[\bar{\psi}] \prod_x F[T_+(x)] \prod_x F[\bar{T}_+(x)]\, e^{-S_W[\psi_-,\bar{\psi}_-]}$$

$$\equiv \int \mathcal{D}_\star[\psi_-]\mathcal{D}_\star[\bar{\psi}_-]\, e^{-S_W[\psi_-,\bar{\psi}_-]} \times \int \mathcal{D}_\star[\psi_+] \prod_x F[T_+(x)] \times \int \mathcal{D}_\star[\bar{\psi}_+] \prod_x F[\bar{T}_+(x)]$$

$$\psi_+(x) = \sum_j u_j(x) b_j, \quad \bar{\psi}_+(x) = \sum_k \bar{b}_k \bar{u}_k(x)$$

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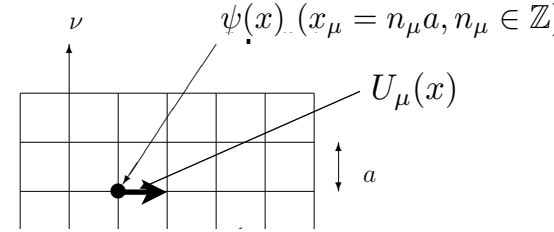

$$\int \prod_{x\in\Lambda} \prod_{\alpha=3}^{4} \prod_{s=1}^{16} d\bar{\psi}_{\alpha s}(x) \prod_{x\in\Lambda} \frac{4!}{8!12!} \left\{ \frac{1}{2} \bar{\psi}(x) P_- i\gamma_5 C_D \mathrm{T}^a \bar{\psi}(x)^{\mathrm{T}}\, \bar{\psi}(x) P_- i\gamma_5 C_D \mathrm{T}^a \bar{\psi}(x)^{\mathrm{T}} \right\}^8 = 1$$

[Eichten-Preskill(1986)]

$$\left(u^{\mathrm{T}} i\gamma_5 C_D \mathrm{T}^a E^a u\right)_{ij} \qquad (i,j = 1,\cdots,n/2 - 8Q)$$

$$U(x,\mu) = e^{i\theta_{12}(x,\mu)\Sigma^{12}} \quad (Q=2)$$

(matrix shape is variable, but even square)

$$(u^{\mathrm{T}}\, i\gamma_5 C_D \mathrm{T}^a E^a u) = \mathcal{C} \times (u^\dagger \Gamma^{10} \Gamma^a E^a u)$$
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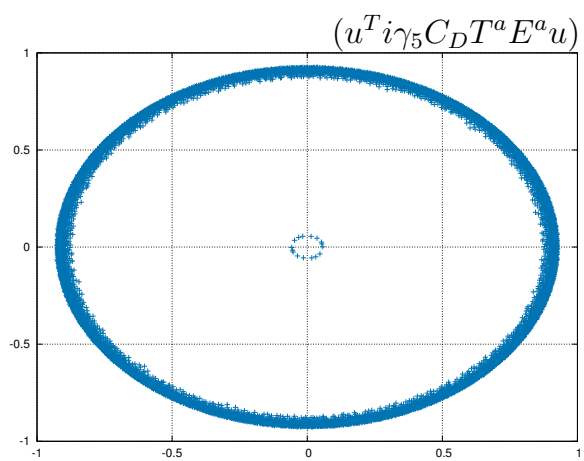

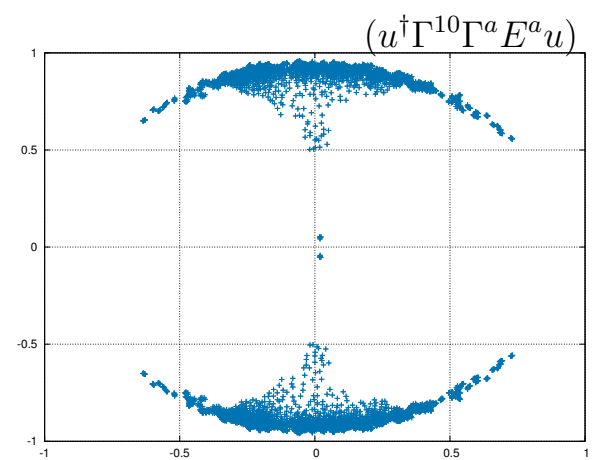

Overlap Weyl fermions in 16 of SO(10)

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Path Integral measure for the 16

$$e^{\Gamma_W[U]} \equiv \int \mathcal{D}[\psi_-]\mathcal{D}[\bar{\psi}_-]\, e^{-S_W[\psi_-,\bar{\psi}_-]}$$

$$e^{\Gamma_W[U]} = \det(\bar{v} D v) \times \int \mathcal{D}[\bar{E}]\, {\rm pf}(u^T i\gamma_5 C_D T^a E^a u)$$

$$\mathcal{D}[\psi_-]\mathcal{D}[\bar{\psi}_-] \equiv \mathcal{D}[\psi]\mathcal{D}[\bar{\psi}] \prod_x F[T_+(x)] \prod_x F[\bar{T}_+(x)]$$

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The measure correctly reproduces zero-modes and VEV of 't Hooft vertex

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cf.

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cf. Overlap Weyl fermions in 2 of SU(2) / Global SU(2) anomaly [*Bar-Campos(2000)*]

$$\mathrm{Tr}\{\tau^{a'}(\tau^{b'}\tau^{c'}+\tau^{c'}\tau^{b'})\}=0$$

$$\left[\psi_+^{\mathrm{T}}(x)i\gamma_5 C_D(i\tau_2\tau^{a'})\psi_+(x)\right]^2 \qquad \left[\bar{\psi}_+(x)i\gamma_5 C_D(i\tau_2\tau^{a'})^\dagger\bar{\psi}_+(x)^T\right]^2$$

2 x 2 => 3 3 x 3 => 1

$$i\tau^2\tau^{a'}$$

$$\mathrm{e}^{\Gamma W[U]} = \det(\bar{v}Dv)\times\int\mathcal{D}[E']\,\mathrm{pf}(u^T i\gamma_5 C_D(i\tau^2\tau^{a'})E^{a'}u)$$

the pfaffian is real

$$u_j(x)^T i\gamma_5 C_D(i\tau_2)=\mathcal{C}_{jk}u_k(x)^\dagger$$

$$\det\mathcal{C}=1$$

$$\int\mathcal{D}[E']\,\mathrm{pf}(u^T i\gamma_5 C_D(i\tau^2\tau^{a'})E^{a'}u)=0 \quad \text{for} \quad {}^\exists U\in SU(2)$$

$Q=$ odd integer

$(u^T i\gamma_5 C_D(i\tau^2\tau^{a'})E^{a'}u)_{ij} \qquad (i,j=n/2,\cdots,Q)$

(matrix shape is variable and square, but can be odd)

$Q=0$ [*Bar-Campos(2000)*]

$U_t(x,\mu)$ $(t\in[0,1])$ loop in the space of SU(2) link fields with a homotopically non-trivial continuum gauge transformation $g(x)$

$$c_t=\int\mathcal{D}[E']\mathrm{pf}(u_t^T i\gamma_5 C_D(i\tau_2\tau^{a'})E^{a'}u_t) \qquad t\in[0,1] \qquad (u_t)_j(x)=Q_t(u_0)_j(x) \qquad \hat{P}_tQ_t=Q_t\hat{P}_0$$

$$c_1=(-1)\,c_0 \qquad \mathcal{T}=\det(1-\hat{P}_++\hat{P}_+Q_1)=-1$$

Given the absence of global anomaly in SO(10) CGT, the saturation should be a robust property of overlap Weyl fermions in 16 of SO(10)

Overlap Weyl fermions in 16 and the Standard Model

[YK 2017]

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16 x 3 (three families)
SO(10) —> SU(3)xSU(2)xU(1)
Higgs scalar **(1,2)**$_{1/2}$ & Yukawa int.

$$S_Y = \sum_x \Big[y_u\, \bar{q}_-^i(x)\tilde{\phi}(x) u_+^i(x) + y_u^*\, \bar{u}_+^i(x)\tilde{\phi}(x)^\dagger q_-^i(x)$$
$$+ y_d\, \bar{q}_-^i(x)\phi(x) d_+^i(x) + y_d^*\, \bar{d}_+^i(x)\phi(x)^\dagger q_-^i(x)$$
$$+ y_l\, \bar{l}_-(x)\phi(x) e_+(x) + y_l^*\, \bar{e}_+(x)\phi(x)^\dagger l_-(x)\Big]$$

Exact gauge inv. is manifest and CP violations comes from KM, PMNS matrixes and theta terms.

cf. [*Ishibashi-Fujikawa-Suzuki(2002)*]

Summary and Discussions

- We defined path integral measure for overlap Weyl fermions in **16** of SO(10) / the SM
- manifestly gauge-invariant by using full Dirac-field measure, but **saturating the right-handed part with 't Hooft vertices completely !**
- covers **all possible topological sectors**
- reproduces **zero modes, 't Hooft vertex VEV, fermion number non-conservation**
- **CP invariant** $\Gamma_W[U^{\rm CP}] = \Gamma_W[U]$
- **Locality** : $-i\mathfrak{T}_\eta$ should be a local functional of link field, Testable, Need a rigorous proof

$$-i\mathfrak{T}_\eta \equiv -2\,\mathrm{Tr}\{\delta_\eta \hat{P}_+ \langle \psi_+ [\psi_+^{\rm T} i\gamma_5 C_D \mathrm{T}^a E^a] \rangle_F\} / \langle 1 \rangle_F$$
$$= \langle \mathrm{Tr}\{(u^T i\gamma_5 C_D T^a E^a \delta_\eta \hat{P}_+ u)(u^T i\gamma_5 C_D T^a E^a u)^{-1}\} \rangle_E / \langle 1 \rangle_E$$

$$\| \langle \psi_+(x) [\psi_+^{\rm T} i\gamma_5 C_D \mathrm{T}^a E^a(y)] \rangle_F / \langle 1 \rangle_F \| \;\leq\; C\,|x-y|^\sigma\, \mathrm{e}^{-|x-y|/\xi}$$

the similar bounds for the variations w.r.t. the link field

In weak gauge-coupling limit, MC studies feasible without sign problem, **Checked in 2D**

- Making the 't Hooft vertex terms **well-defined in large coupling limit**,

Established the relations with GW Mirror-fermion model [*Poppitz et al (2006)*]

DW fermion with boundary EP terms [*Creutz et al (1997)*]

4D TI/TSC with Gapped boundary phase

[*Wen(2013), You-BenTov-Xu(2014),You-Xu (2015)*]

[*Creutz, Horvath(1994)*] [*Qi, Hughes, Zhang (2008)*]

cf. $\hat{P}_+^T i\gamma_5 C_D P_+ \mathrm{T}^a E^a(x) \hat{P}_+ = (1-D)^T i\gamma_5 C_D P_+ \mathrm{T}^a E^a(x)(1-D)$

Discussions

Relation to the reconstruction approach by Luscher

$$\mathrm{e}^{\Gamma_W[U]} = \det(\bar{v}Dv) \times \int \mathcal{D}[E]\,\mathrm{pf}(u^T i\gamma_5 C_D T^a E^a u) \qquad \langle\mathcal{O}\rangle_E \equiv \int \mathcal{D}[E]\,\mathrm{pf}(u^T i\gamma_5 C_D T^a E^a u)\,\mathcal{O}$$

Link-field dependence of Effective action

$$\delta_\eta \Gamma_W[U] = \mathrm{Tr}\{P_+ \delta_\eta D D^{-1}\} - i\mathfrak{T}_\eta \qquad (\,\delta_\eta U_\mu(x) = i\eta_\mu(x) U_\mu(x)\,) \qquad \sum_j (u_j, \delta_\eta u_j) + \sum_j (v_j, \delta_\eta v_j) = 0$$

$$-i\mathfrak{T}_\eta = \left\langle \mathrm{Tr}\{(u^T i\gamma_5 C_D T^a E^a \delta_\eta \hat{P}_+ u)(u^T i\gamma_5 C_D T^a E^a u)^{-1}\}\right\rangle_E \Big/ \langle 1\rangle_E$$

cf. measure term (local counter terms)

$$-i\mathfrak{L}_\eta = \sum_j (v_j, \delta_\eta v_j)$$

[*Luscher (1999, 2000)*]

1) gauge-covariant current, smooth and local w.r.t. link field
2) anomalous conservation law
3) integrability conditions (local and global ones)

- $\langle 1\rangle_E$ is non-vanishing over the space of admissible link fields
- $-i\mathfrak{T}_\eta$ is smooth and local with respect to gauge link field

$$\mathfrak{L}_\eta = \mathrm{Re}\{\mathfrak{T}_\eta\}$$

If these conditions are fulfilled, one can reconstruct the chiral basis preserving locality, lattice symmetries and exact gauge-invariance, following the method by Luscher

Relation to the other approaches

Overlap Weyl fermions in 16 of SO(10)

[YK 2017]

$$e^{\Gamma_W[U]} \equiv \int \mathcal{D}[\psi]\mathcal{D}[\bar{\psi}] \prod_x F[T_+(x)] \prod_x F[\bar{T}_+(x)]\, e^{-S_W[\psi_-,\bar{\psi}_-]}$$

- Eichten-Preskill model (species doublers)
  —> realized by the Overalp Fermions/GW rel.

[*Eichten, Preskill(1986)*]

- Mirror Fermion model (mirror modes)
  using the Overalp Fermions/GW rel.
  —> non-singular and well-defined large limit of
  Majorana-Yukawa coupling in PMS phase

[*Montvay(1987)*]
[*Poppitz et al (2006)*]

[*Kaplan(1992), Shamir(1993)*]

- 4+1 dim. DWF w/ boundary Eichten-Preskill term
- 4dim. TI/TSC w/ gapped boundary phase
  (Hamiltonian formalism of 4+1 dim. DWF)
  [*Creutz, Horvath(1994)*] [*Qi, Hughes, Zhang (2008)*]

[*Creutz et al (1997)*]
[*Wen(2013),
You, BenTov, Xu(2014),
You, Xu (2015)*]

—> Low energy effective *4d lattice model*
through non-singular boundary terms

- Recent studies on PMS/"Mass without Symmetry Breaking"
  —> O(10) spin model defined with 't Hooft vertex pfaffian

[Ayyar,
Chandrasekharan (2015,16)]
[Catterall,
Schaich,But*(2016,17)*]

$$\mathrm{e}^{\Gamma_W[U]} \equiv \int \mathcal{D}[\psi]\mathcal{D}[\bar{\psi}] \prod_x F[T_+(x)] \prod_x F[\bar{T}_+(x)]\, \mathrm{e}^{-S_W[\psi_-,\bar{\psi}_-]}$$

$$S_{\mathrm{Ov}}[\psi,\bar{\psi},E^a,\bar{E}^a] = \sum_{x\in\Lambda} \bar{\psi}_-(x) D\psi_-(x) - \sum_{x\in\Lambda}\{E^a(x)\psi_+^{\mathrm{T}}(x)i\gamma_5 C_D \mathrm{T}^a\psi_+(x) + \bar{E}^a(x)\bar{\psi}_+(x)i\gamma_5 C_D \mathrm{T}^{a\dagger}\bar{\psi}_+(x)^T\}$$

Decoupling limit of the mirror (right-handed) Overlap Weyl fermions

$$y=\bar{y}, \quad \frac{z_+}{\sqrt{y\bar{y}}} \to 0, \quad v=\bar{v}=1, \quad \lambda'=\bar{\lambda}' \to \infty, \quad \kappa=\bar{\kappa}\to 0.$$

$$E^a(x)E^a(x) = 1$$

(PMS phase)

$$S_{\mathrm{Ov/Mi}}[\psi,\bar{\psi},X^a,\bar{X}^a] = \sum_{x\in\Lambda}\{\bar{\psi}_-(x)D\psi_-(x) + z_+\bar{\psi}_+(x)D\psi_+(x)\} - \sum_{x\in\Lambda}\{y\,X^a(x)\psi_+^{\mathrm{T}}(x)i\gamma_5 C_D\mathrm{T}^a\psi_+(x) + \bar{y}\,\bar{X}^a(x)\bar{\psi}_+(x)i\gamma_5 C_D\mathrm{T}^{a\dagger}\bar{\psi}_+(x)^T\} + S_X[X^a]$$

cf. $\hat{P}_+^T i\gamma_5 C_D P_+ \mathrm{T}^a E^a(x)\hat{P}_+ = (1-D)^T i\gamma_5 C_D P_+ \mathrm{T}^a E^a(x)(1-D)$

$$S_{\mathrm{DW/Mi}} = \sum_{t=1}^{L_5}\sum_{x\in\Lambda}\bar{\psi}(x,t)\{[1+a_5'(D_{4\mathrm{w}}-m_0)]\delta_{tt'} - P_-\delta_{t+1,t'} - P_+\delta_{t,t'+1}\}\psi(x,t') + \sum_{x\in\Lambda}(z_+-1)\,\bar{\psi}(x,L_5)P_-[1+a_5'(D_{4\mathrm{w}}-m_0)]\psi(x,L_5) - \sum_{x\in\Lambda}\{y\,X^a(x)\psi^{\mathrm{T}}(x,L_5)i\gamma_5 C_D\mathrm{T}^a\psi(x,L_5) + \bar{y}\,\bar{X}^a(x)\bar{\psi}(x,L_5)P_-i\gamma_5 C_D\mathrm{T}^{a\dagger}\bar{\psi}(x,L_5)^T\} + S_X[X^a],$$

cf. $q(x) = \psi_-(x,1) + \psi_+(x,L_5)$

$\bar{q}(x) = \bar{\psi}_-(x,1) + \bar{\psi}_+(x,L_5)$

applications of lattice Standard Model/ SO(10) CGTs

1) Phase transitions, Phase structures in EW theory & GUT theories

2) Realizations of gauge and flavors symmetries in EW theory & GUT theories

3) Baryon & Lepton number generations

   a. B symmetry violation/chiral anomaly, CP violation, non-equilibrium process

   b. Chern num. diffusion process, Sphaleron process

4) Phase transitions in the Early Universe, Dynamics of Inflation

and so on

- Schwinger-Keldysh formalism for lattice gauge theories

  real-time, non-equilibrium dynamics / finite-temperature・density

  [*Fujii, Hoshina & YK (2019)*]

- Lefschetz-Thimble methods

  sign problem

  generalized method(GLTM), tempered method(TLTM)

  [*Alexandru et al.(2016)*] [*Fukuma & Umeda(2017)*]