Anomalous dimension in adjoint QCD from the gradient flow

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Phases of a QFT

- Given a general QFT, it is interesting to study its behaviour at different energy scales, i.e. its renormalisation group flows.
- IR phases:

  I. Gapped, e.g 4d Yang-Mills (YM)
  II. Massless, e.g massless QCD
  III. Conformal, e.g. theories with IR fixed point (FP)
  IV. Non-trivially gapped, i.e. topological QFT, BPS states...

- For a YM theory with fermions, one has different scenarios depending on N_f and N_c:
  1. Small N_f: chiral symmetry breaking (IR massless)
  2. $N_f^{\mathrm{l}} < N_f < N_f^{\mathrm{u}}$: Banks-Zaks (BZ) FP conformal window (IR conformal)
  3. $N_f > N_f^{\mathrm{u}}$: not asymptotically free

Conformal window

- Upper limit N_f^u of the conformal window can be computed perturbatively
- At smaller N_f the IR FP moves towards stronger couplings
- Finding the lower limit N_f^l is a nonperturbative problem
- For N_f below but near N_f^l: near conformal behaviour, walking coupling
- The values of N_f^u and N_f^l depend on the fermion representation. Lower values for adjoint rep.
- SYM ($N_f = 1/2$) is IR massless.
- Here we focus on the IR phase of $N_f = 2,\ 3/2$, i.e. 4 resp. 3 Majorana fermions.

IR conformal phase

- Gauge invariant operators obtain an anomalous scaling dimension γ as they flow
- γ freezes at the BZ fixed point
- At the fixed point:
  - ✓ Particle interpretation fails
  - ✓ Observables: correlation functions, operator dimensions
- Methods to compute observables: Lattice Monte Carlo (LMC), conformal bootstrap, ...
- Within LMC: take mass-deformed theory, i.e. away from the FP and compute the anomalous dimensions from
  - ✓ Mass spectrum of the theory
  - ✓ Monte Carlo renormalisation group techniques
  - ✓ Spectral density of Dirac operator (mode number)
  - ✓ Recently: Gradient flow and RG flow [Carosso, Hasenfratz and Neil, PRL 121 no.20, 201601]

IR conformal phase II

Why to study the IR phase of QFT (on a lattice)?

- In general, important to classify theories which become conformal at the IR
- It is hard to analitically study non-susy theories.
- Near conformal QFTs are important for phenomenology, e.g. technicolor models
- Being able to study RG flow through the GF opens up the possibility to compute conformal data on the lattice

Gradient flow vs RG flow

- GF similar to RG: smoothening of the fields $\leftrightarrow$ elimination of high energy modes
- YM GF is however not a complete RG transformation:
  - ✗ Lack of scale transformation (dilatation)
  - ✗ Lack of normalisation of the fields
- In lattice field theory:
  - ✓ Consider correlators at long distances
  - ✓ Include renormalisation of the fields by using an exact conserved current (e.g. vector)
- GF allows for blocked fields without having to know the blocked action

GF and RG flow

- GF: $\phi \to \phi_t$. High momentum modes over $\dfrac{1}{a\sqrt{t}}$ are suppressed.
- RG: changes lattice spacing $a \to a' = b\,a$ and couplings $g \to g'$, $m \to m'$

  Consider correlator of composite operators $\mathcal{O}(\phi; x)$. The two-point function at $x_0 >> a'$ transforms as:

$$\langle \mathcal{O}(0)\mathcal{O}(x_0)\rangle_{g,m} = b^{-2(d_{\mathcal{O}}+\gamma_{\mathcal{O}})}\langle \mathcal{O}(0)\mathcal{O}(x_0/b)\rangle_{g',m'}$$

- RHS: Monte Carlo RG (MCRG) $\Rightarrow$ First generate MC ensamebles from UV action and then RG transform

$$\langle \mathcal{O}(0)\mathcal{O}(x_0/b)\rangle_{g',m'} = \underbrace{\langle \mathcal{O}_b(0)\mathcal{O}_b(x_0/b)\rangle_{g,m}}_{\mathcal{O}_b \equiv \mathcal{O}(\phi_b)}$$

- Relate blocked and flowed fields through $\phi_b(x_b) = b^{d_\phi+\eta/2}\phi_t(bx_b)$ and $\sqrt{t} \propto b$

$$\frac{\langle \mathcal{O}_t(0)\mathcal{O}_t(x_0)\rangle}{\langle \mathcal{O}(0)\mathcal{O}(x_0)\rangle} = b^{2\Delta_{\mathcal{O}}-2n_{\mathcal{O}}\Delta_\phi}, \quad \Delta_i = d_i + \gamma_i \text{ (canonical + anomalous dim)}$$

GF and RG flow II

- It is numerically easier to only flow one of the operators i.e. $\mathcal{O}_t(0) \to \mathcal{O}(0)$. The cost is to have errors $O(a\sqrt{t}/x_0)$.
- One can get rid of Δ_ϕ by using some conserved operator $\mathcal{V}$ as $\gamma_\mathcal{V} = 0$

$$\mathcal{R}_\mathcal{O}(t, x_0) = \frac{\langle \mathcal{O}(0)\mathcal{O}_t(x_0)\rangle}{\langle \mathcal{O}(0)\mathcal{O}(x_0)\rangle} \left(\frac{\langle \mathcal{V}(0)\mathcal{V}(x_0)\rangle}{\langle \mathcal{V}(0)\mathcal{V}_t(x_0)\rangle} \right)^{n_\mathcal{O}/n_\mathcal{V}} = b^{\Delta_\mathcal{O} - (n_\mathcal{O}/n_\mathcal{V})d_\mathcal{V}}$$

$$\propto t^{\gamma_\mathcal{O}/2 + d_\mathcal{O}/2 - (n_\mathcal{O}/n_\mathcal{V})d_\mathcal{V}/2}$$

- From there, the mass anomalous dimension of the operator $\mathcal{O}$ can be defined as

$$\gamma_\mathcal{O}(\bar{t}) = \frac{\log(\mathcal{R}_\mathcal{O}(t_1)/\mathcal{R}_\mathcal{O}(t_2))}{\log\left(\sqrt{t_1}/\sqrt{t_2}\right)}$$

Simulations

- We simulated YM theory + 3 and 4 Majorana Wilson fermions in the adjoint rep.
- Tree-level Symanzik improved gauge action and stout smearing for the link fields in the Wilson-Dirac operator
- The stout smearing is iterated 3 times with parameter $\rho = 0.12$
- Fermion path integral:

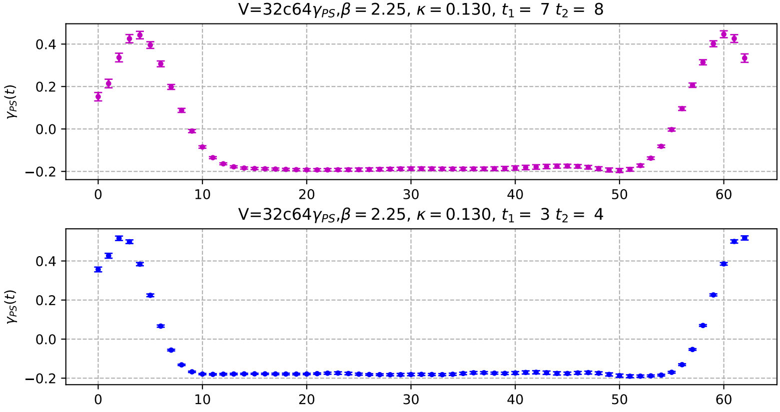
$$\int [d\psi]\, \mathrm{e}^{-\frac{1}{2}\bar{\psi} D_w \psi} = \mathrm{Pf}(C D_w) = \pm\sqrt{\det D_w}$$

- We used polynomial hybrid Monte Carlo to generate field configurations
- We analysed the mass anomalous dimension of the pseudoscalar operator with the GF
- To compute $\mathcal{R}_{\mathcal{O}}$, we set $\mathcal{V}$ to be the vector current
- Results compared to previous computations using the mass spectrum and the mode number [Desy-Münster collaboration, JHEP01(2018)119,Phys. Rev. D 96, 034504]

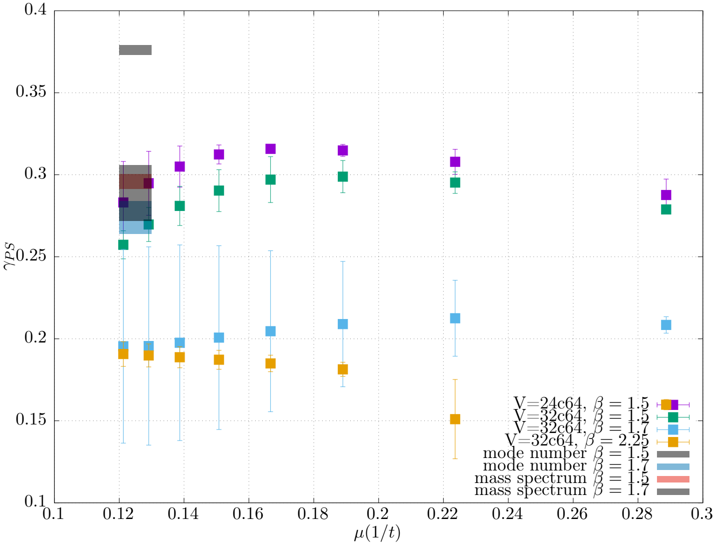
Lattice parameters

N_f	L_S	L_T	β	κ	am_{PCAC}
2	24	64	1.5	0.1315	0.16775(25)
2	24	64	1.5	0.1325	0.128730(46)
2	24	64	1.5	0.1350	0.03136(15)
2	32	64	1.5	0.1325	0.128840(55)
2	32	64	1.5	0.1335	0.089619(74)
2	32	64	1.5	0.1350	0.030414(45)
2	32	64	1.7	0.1285	0.147091(22)
2	32	64	1.7	0.1290	0.131717(22)
2	32	64	1.7	0.1300	0.100878(47)
3/2	24	48	1.7	0.134	-0.00097(22)
3/2	32	64	1.7	0.134	-0.00052(11)

$\gamma_{PS}(x_0)$, $N_f = 2$



$N_f = 2$, chiral extrapolation $\beta = 1.5,\ 1.7$

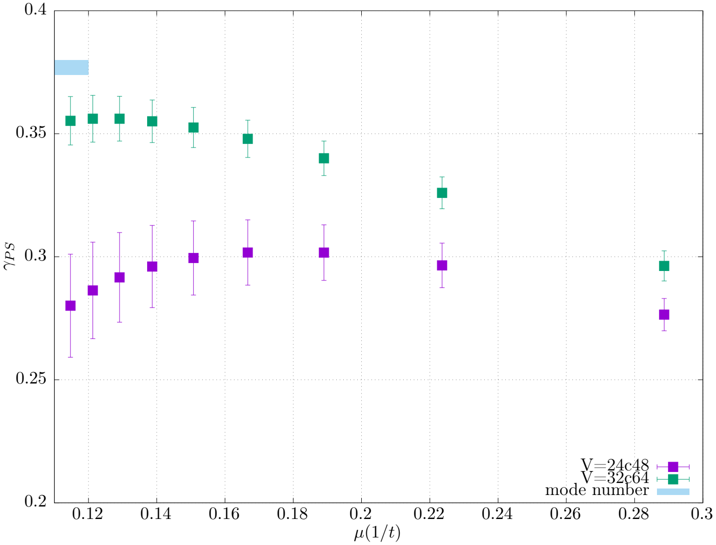


Other previous results for $N_f = 2$

Paper	β	γ^*
Ref 1	1.5	0.376(3)
Ref 1	1.7	0.274(10)
Ref 2	-	0.371(20)
Ref 3	-	0.269(2)(5)
Ref 4	-	0.20(3)
Ref 5	-	0.31(6)
Ref 6	-	0.22(6)
Ref 7	-	0.50(26)

- Ref 1: Desy-Münster collaboration, JHEP01(2018)119,Phys. Rev. D 96, 034504
- Ref 2: A. Patella, Phys. Rev. D86(2012) 025006
- Ref 3: M. García Pérez, A. González-Arroyo, L. Keegan and M. Okawa JHEP1508(2015)
- Ref 4: J. Rantaharju et. al Phys. Rev. D93(2016) 094509
- Ref 5: T. DeGrand, Y. Shamir and B. Svetitsky, Phys. Rev. D83(2011) 074507
- Ref 6: L. Del Debbio, B. Lucini, A. Patella, C. Pica and A. Rago,Phys. Rev. D82(2010)
- Ref 7: J. Giedt, Int. J. Mod. Phys. A31(2016) 1630011

$N_f = 3/2$, mass ~ 0, $\beta = 1.7$



Summary

- Study of the IR phases of a QFT important from the theoretical and phenomenological point of view
- More specifically the conformal and near conformal regions
- It is possible to directly study the RG flow through the GF on the lattice
- This could also represent a new way to compute CFT data on the lattice
- We computed γ for $\mathrm{N}_f = 2$ and $N_f = 3/2$ adjoint QCD:
  * Results of $\mathrm{N}_f = 2$ seem to be more compatible with smaller values $0.2 < \gamma^* < 0.3$.
  * For $\mathrm{N}_f = 3/2$ there is less data to compare to. However, the results show possible IR conformality
- We still have to better control the errors when approaching the IR, i.e the lattice artifacts which explicitely drive the theory away from the FP.

Thank you for your attention!

Minimal SYM in four dimensions

- Only supersymmetric theory without scalars and thus similar to QCD
- Vector supermultiplet with one Yang-Mills field A and one Majorana spinor λ in the adjoint representation

$$\mathcal{L}_{\mathrm{E}} = \frac{1}{4}F^2 + \frac{1}{2}\bar{\lambda}(\not{D} + m_{\tilde{g}})\lambda + \frac{\theta}{32\pi^2}\tilde{F}F$$

- Lagrangian invariant under SUSY-transformations $\delta A_\mu = 2i\bar{\epsilon}\gamma_\mu\lambda$, $\delta\lambda^a = -\sigma_{\mu\nu}F^a_{\mu\nu}\epsilon$
- Expected to have mass gap, confinement and spontaneous breaking of chiral symmetry
- Low energy degrees of freedom: glueballs, meson-like states, baryon-like (being currently investigated)

Quark confinement: centre symmetry

- SU(N) SYM vacuum is a confining medium for external quarks
- Medium probed through Polyakov loops (PL) $< \Phi >= \exp(-\beta F)$
- Centre symmetry unbroken at zero temperature, vanishing PL VEV
- Quark deconfinement at high temperatures, non vanishing PL VEV : broken centre symmetry
- Unlike QCD, the adjoint spinors do not break the centre symmetry explicitely. Deconfinement is true phase transition for all gaugino masses
- Determine deconfinement temperature : Compute behaviour of PLs at finite temperatures

Chiral symmetry

- Chiral symmetry in $\mathcal{N} = 1$ SYM is an U(1)-symmetry
  $\lambda \to \lambda' = \exp(-i\omega\gamma_5)\lambda$
- U(1)-symmetry broken by instantons $\to \partial_\mu J_5^\mu \sim N_c g^2 \tilde{F} F$

  $\hookrightarrow$ the actual chiral symmetry is the discrete subgroup Z_{2N_c}
- Z_{2N_c} spontaneously broken down to Z_2 at zero temperature by a non-vanishing bi-gaugino condensate $< \bar{\lambda}\lambda > \sim \Lambda^3$
- confinement $\to$ chiral symmetry breaking: It can be seen from non-renormalisation and holomorphicity of the superpotential, assuming that low-energy EFT is massive with colour singlets d.o.f
- At high temperatures Z_{2N_c} symmetry is expected to be restored
- Phase transition to vanishing condensate
- Existence of N_c degenerate vacua
- Breaking of discrete symmetry $\to$ domain wall interpolating between vacua
- Domain wall is a BPS-saturated state (Dvali and Shifman, 96)

Testing the phase structure of $\mathcal{N} = 1$ SYM

- At $T = 0$ the theory is confining and chiral symmetry is spontaneously broken
- *Expectation* at high temperatures: deconfinement and restoration of Z_{2N_c}chiral symmetry

Do both phase transitions occour at the same critical temperature?

Measure order parameters on the lattice varying N$_t$ $\left(T = \frac{1}{N_t a(\beta)}\right)$

Polyakov loop: phase transition when $< |P_{\mathrm{L}}| > \neq 0$

gluino condensate vanishes when chiral symmetry is restored $\sim < \mathrm{tr}(\mathrm{D}_{\mathrm{w}}^{-1}) >$

It looks simple but...

- Measurement of Polyakov loops is not problematic as it only depends on the gauge fields
- Measuring the gaugino condensate is however not straightforward: On the lattice an additive renormalisation constant is needed because of the Wilsonian fermion discretisation

$$\langle\lambda\lambda\rangle_{\mathrm{R}} = Z_{\lambda\lambda}(\beta)(\langle\lambda\lambda\rangle_{\mathrm{B}} - \mathbf{b_0}).$$

- In some previous finite temperature studies b_0 is chosen so that the condensate vanishes at $T = 0$
- Thus it is not possible to get a reliable zero temperature value for Wilson fermions
- One way out is to use chiral lattice fermions...
- Another way out is by means of a gradient flow

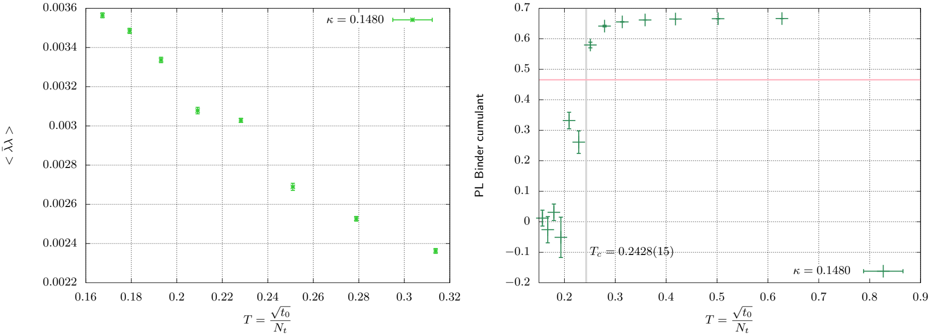
Gaugino condensate from the gradient flow

- Currents and densities which are explicitly broken by the lattice discretization should be more accessible within this method
- No additive renormalisation constant necessary for the flowed condensate, even with Wilson fermions
- The flowed condensate is measured on the lattice through

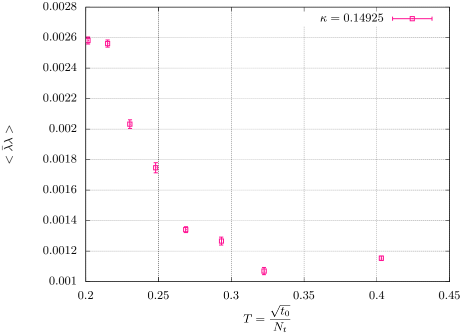
$$\langle \chi_t(x) \rangle = -\sum_{v,w} \left\langle \mathrm{tr} \left\{ \underbrace{K(t,x;0,v)}_{\text{diff eq kernel}} \overbrace{S(v,w)}^{\text{Dirac propagator}} K(t,x;0,w)^\dagger \right\} \right\rangle$$

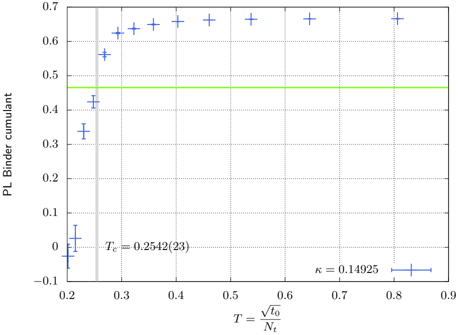
- Numerically, one *only* has to flow the inverse Dirac operator...
- ...The inversion and the fermion flow are *however* the most expensive part of the numerics

Numerical results for SU(2) SYM

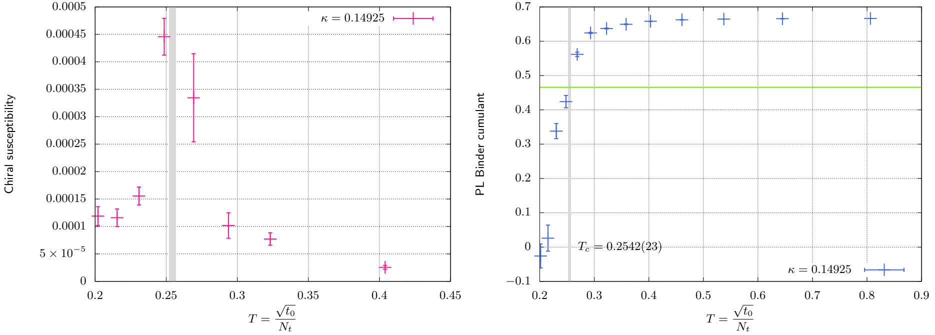


- small $\kappa \Rightarrow$ big gaugino mass: Chiral restoration crossover very flattened out





- Our biggest $\kappa \Rightarrow$ gaugino mass close to zero: jump of order parameter more pronounced
- Similar results for different lattice parameters $\rightarrow$ there should be a real phase transition



- Deconfinement critical temperature coincides with peak of chiral susceptibility

> Deconfinement and chiral restoration phase transitions
>
> occur at the same critical temperature $T \sim 0.25$

How can we understand/explain this observation?

- Witten, 97: semiclassically studied configuration of branes in M-theory, which is in universality class of $N = 1$ SYM
- He formally showed that (QCD strings $\leftrightarrow$ fundamental strings) can end in (domain walls $\leftrightarrow$ D-branes)
- Qualitatively (Rey):
  - Domain wall connects different θ-vacua
  - Confinement: Monopole cond. $(\theta = 0)$, dyons $(\theta \neq 0)$
  - Domain wall colour charged when dyons pass through $\rightarrow$ confining string can end there
- Wiese, Holland, Campos; 98:
  - EFT of PL and condensate with SU(3)
  - Low temperatures: domain wall with chiral symmetry and deconfinement in the core
  - Near phase transition: deconfined layer appears and expands towards infinity
  - Witten observation holds only if chiral restoration and deconfinement occur simultaneously

How can we understand/explain this observation?

- Our results seem to support such a nice semiclassical picture... by solving the full theory numerically!
- Recent developments on higher form discrete symmetries and anomaly matching $\rightarrow$ analytical study of phase structure in thermal QFT
- From there: predictions for a bound $T_{\text{deconfinement}} \leq T_{\text{chiral}}$ in SYM [Komargodski,Sulejmanpasic, Ünsal; Shimizu and Yonekura]
- Our results imply that the bound must be saturated.
- Same observation for adjoint QCD on $\mathbb{R}^3 \times S^1$ [Ünsal, Poppitz, Anber]