Walking, complex CFT, or dilaton

Lattice Higgs Collaboration (L_{at}HC)

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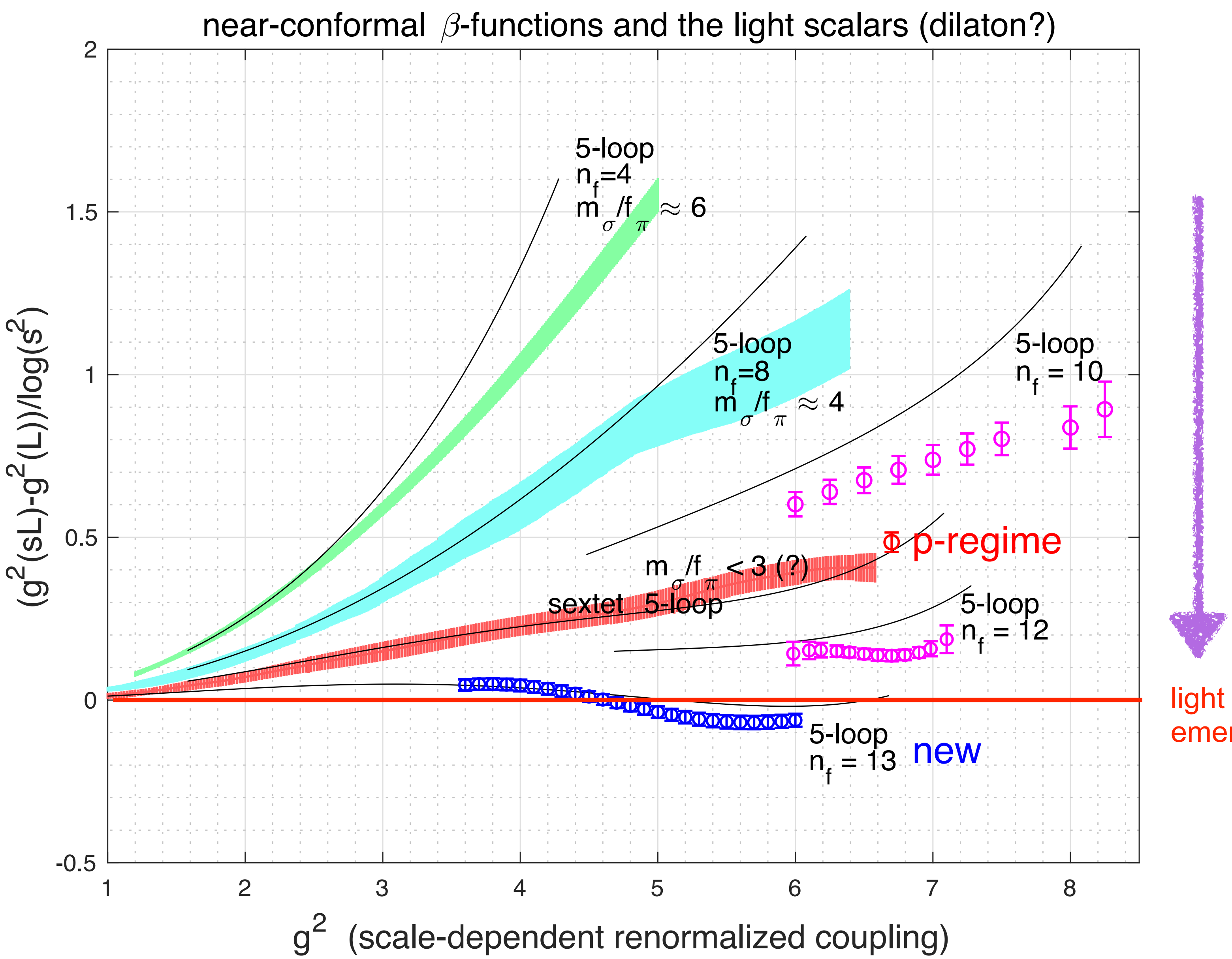
*Tantalizing questions on walking, complex CFT, or dilaton:

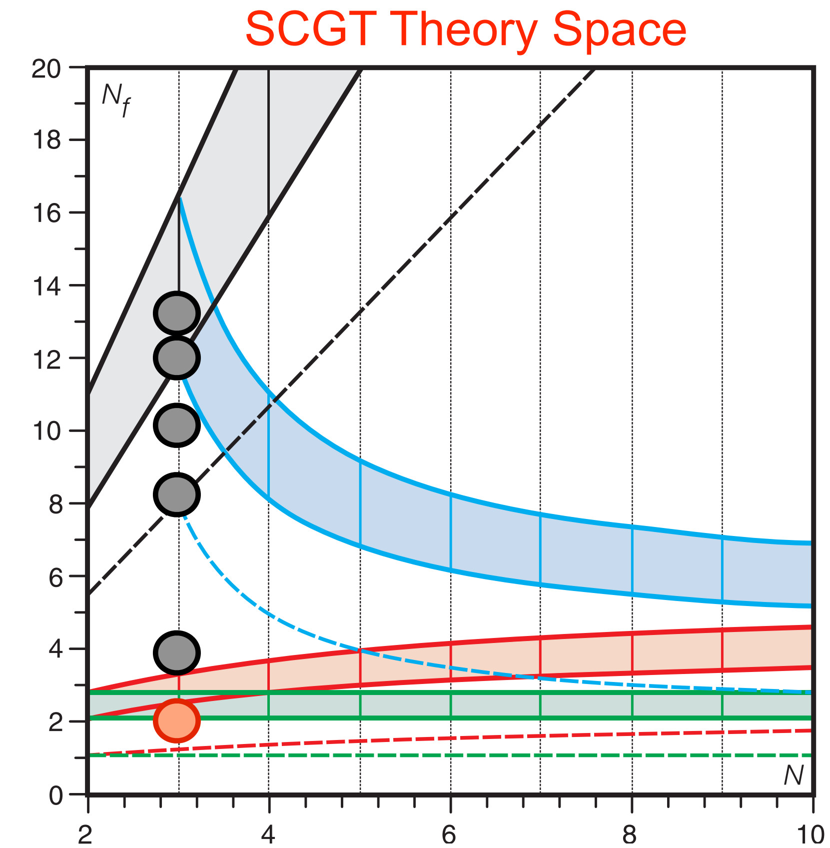
- Emergent light 0++ scalar when β-function is small (near-conformal)?
- Two different scenarios: complex CFT, or dilaton?
- IR EFT tests for complex CFT, or dilaton in p-regime $(1/M_\pi \ll L)$
- Strategy for EFT in the ε-regime $(1/M_\pi \gg L)$

*Tantalus, a king of ancient Phrygia in Greek mythology, made the mistake of gravely offending the gods. As a punishment, once dead the king was forced to stand in a pool of water, with fruit hanging just over his head. The water would recede every time the king tried to take a sip, and the fruit would lift away every time he reached to take a bite.

“walking” β-function in correlation with emergent light scalar



light 0++ scalar emerging



to illustrate: sextet SU(3) color rep

one massless fermion doublet $\begin{bmatrix} u \\ d \end{bmatrix}$

χSB on Λ~TeV scale

three Goldstone pions
become longitudinal
components of weak bosons

if Susskind and Weinberg only knew …

composite Higgs mechanism

- nf=10 β-function (new results, no IRFP) Kieran Holland’s talk
- nf=12 β-function (new LatHC results, no IRFP, no γ^*, walking?
- nf=13 β-function LatHC reports nf=13 conformal IRFP (?)
- 5-loop β-function below CW: a pair of complex conjugate zeros (avatar of complex CFT ?)
- our focus is on sextet model, also considering new nf=12 studies

“walking” β-function in correlation with emergent light scalar

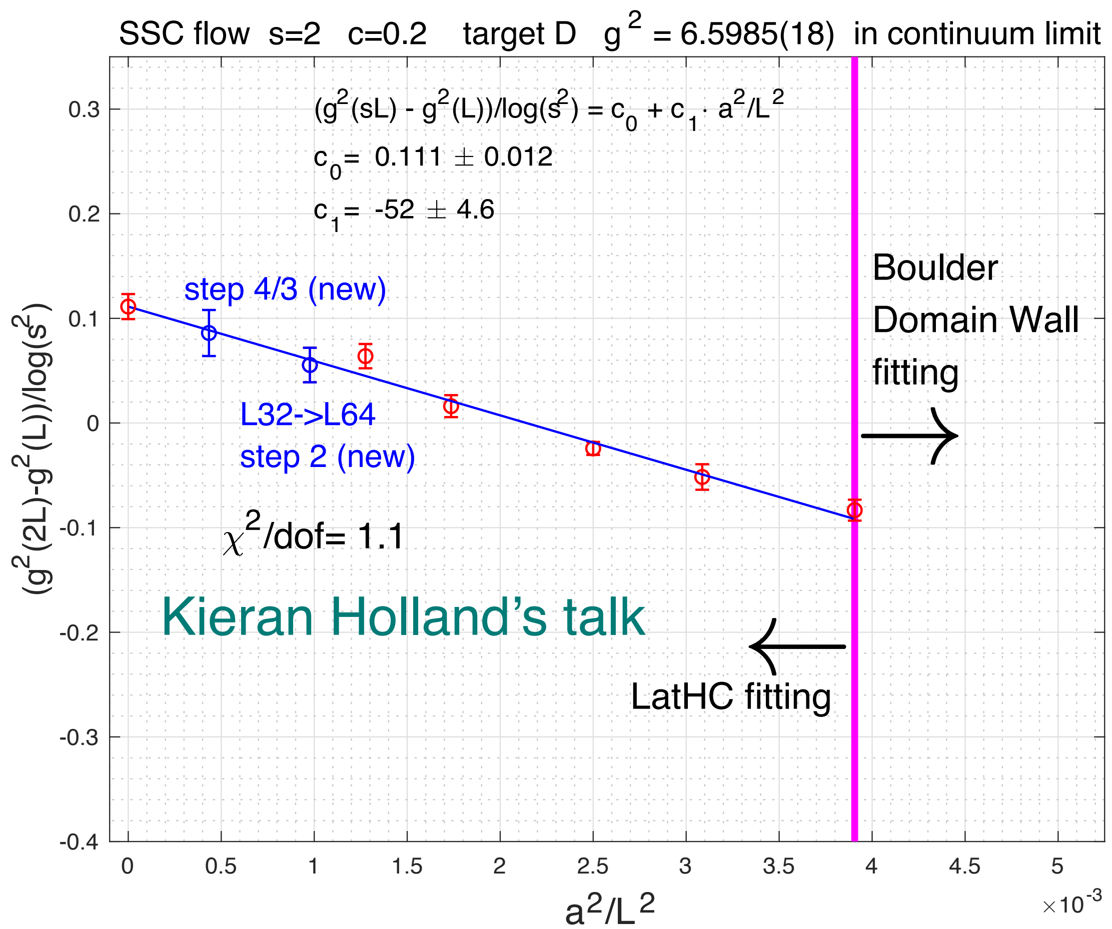
- for us two candidate theories remain tantalizing for field theory interest and BSM implications of the near-conformal walking in correlation with the emergent light scalar

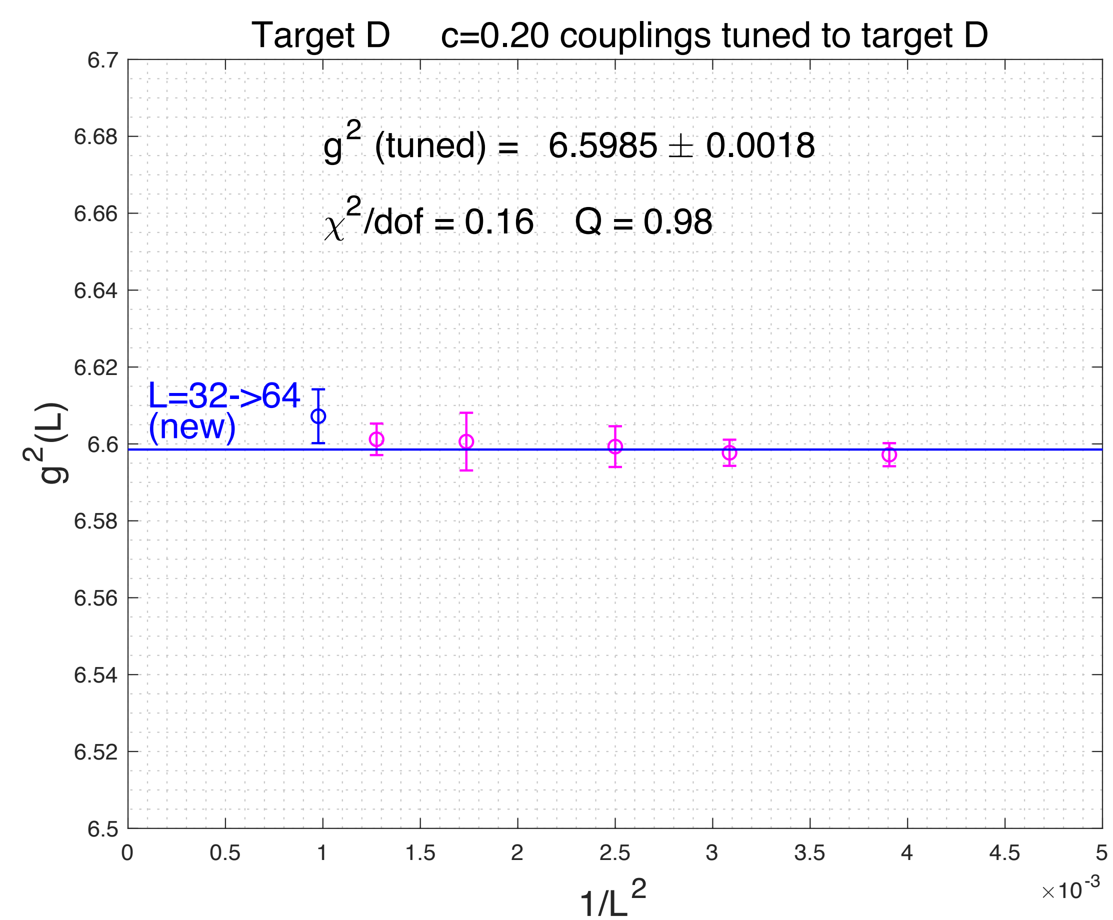
- sextet model: primary interest well motivated (case study)

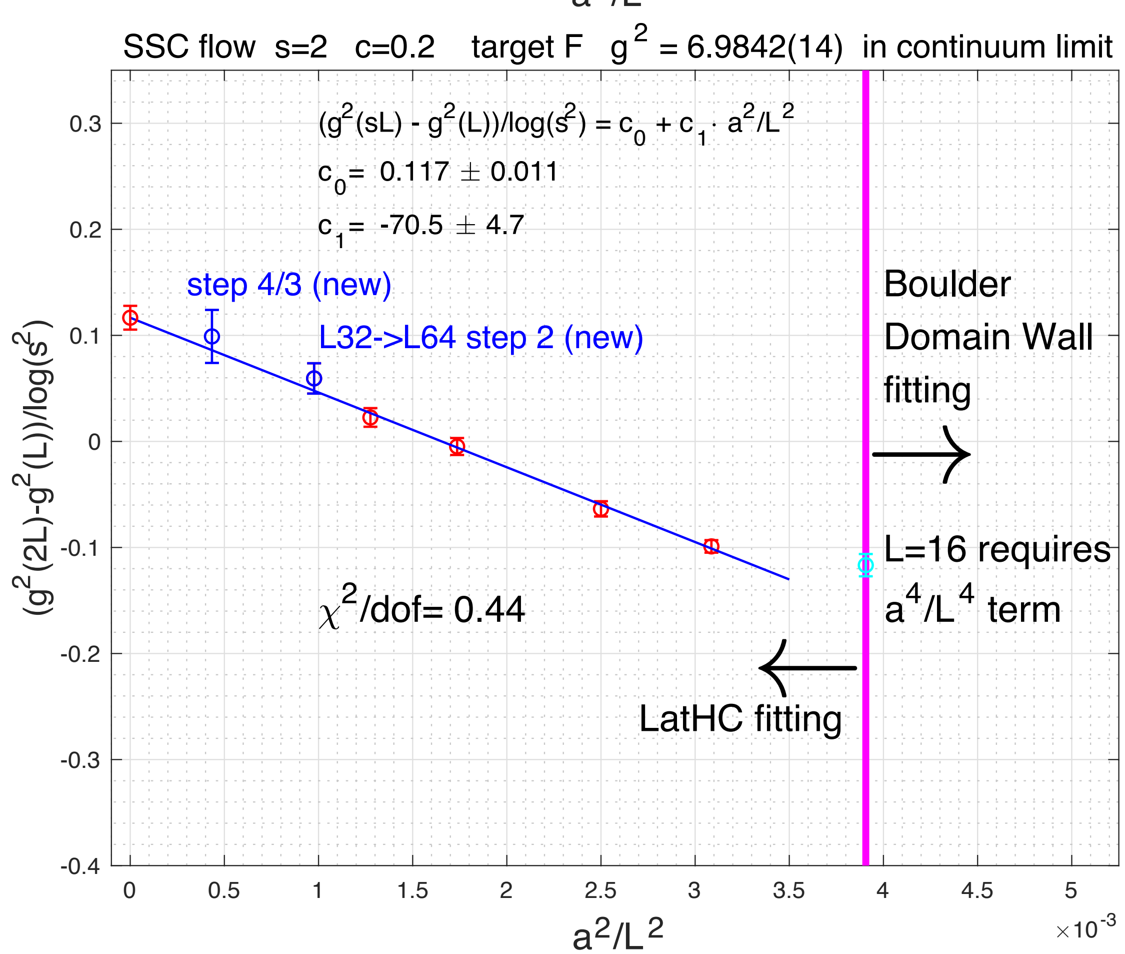
- nf=12 fundamental rep we continue to consider as perhaps the closest to a walking candidate

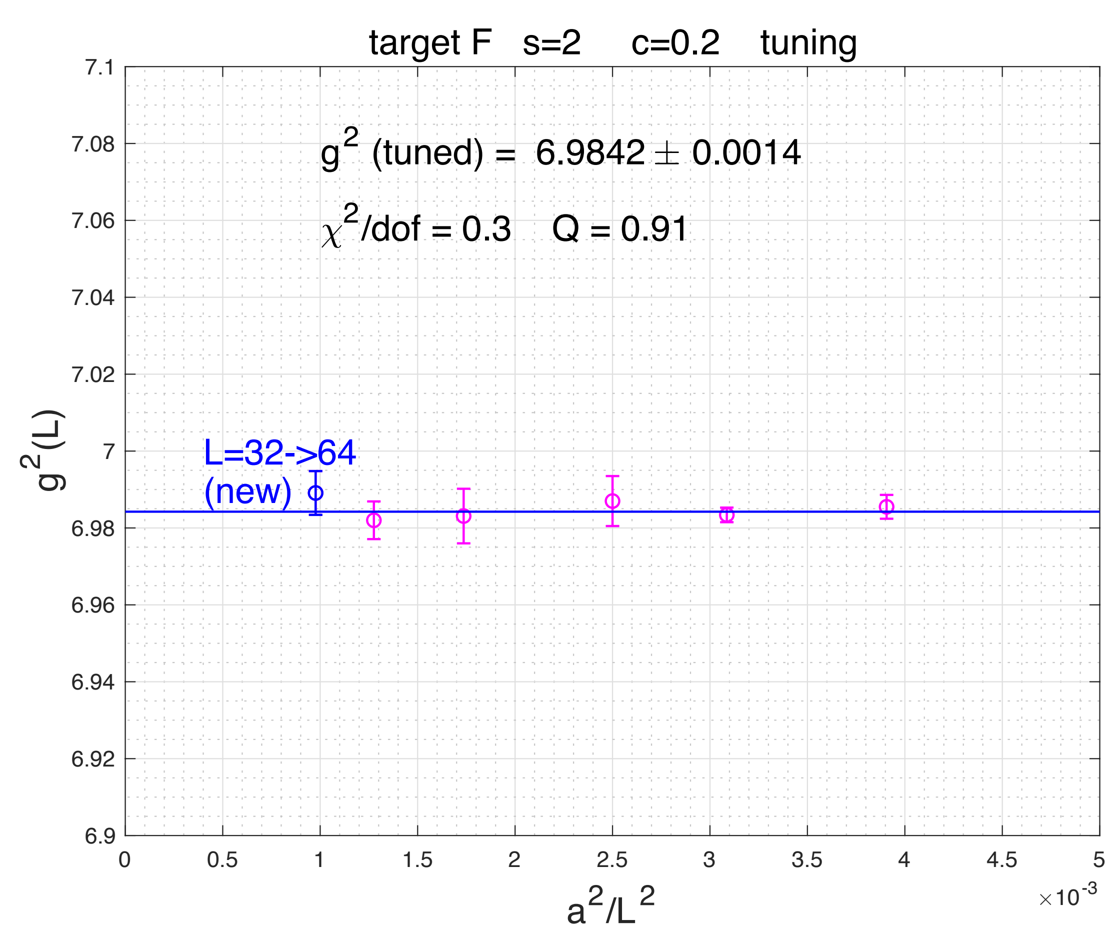
- we are curious about nf=8 and nf=10 but should be left for the LSD collaboration to understand

new at nf=12: tuned step β-function from 64^4 volumes









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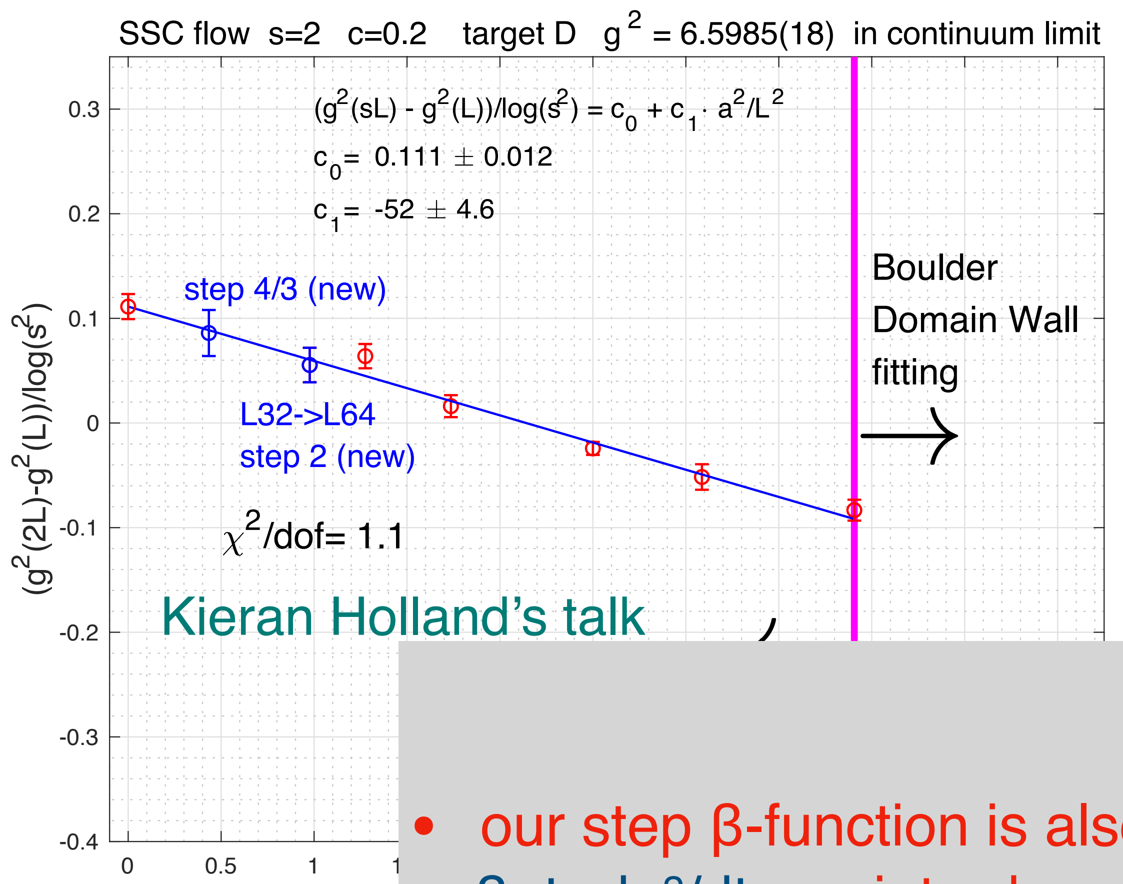
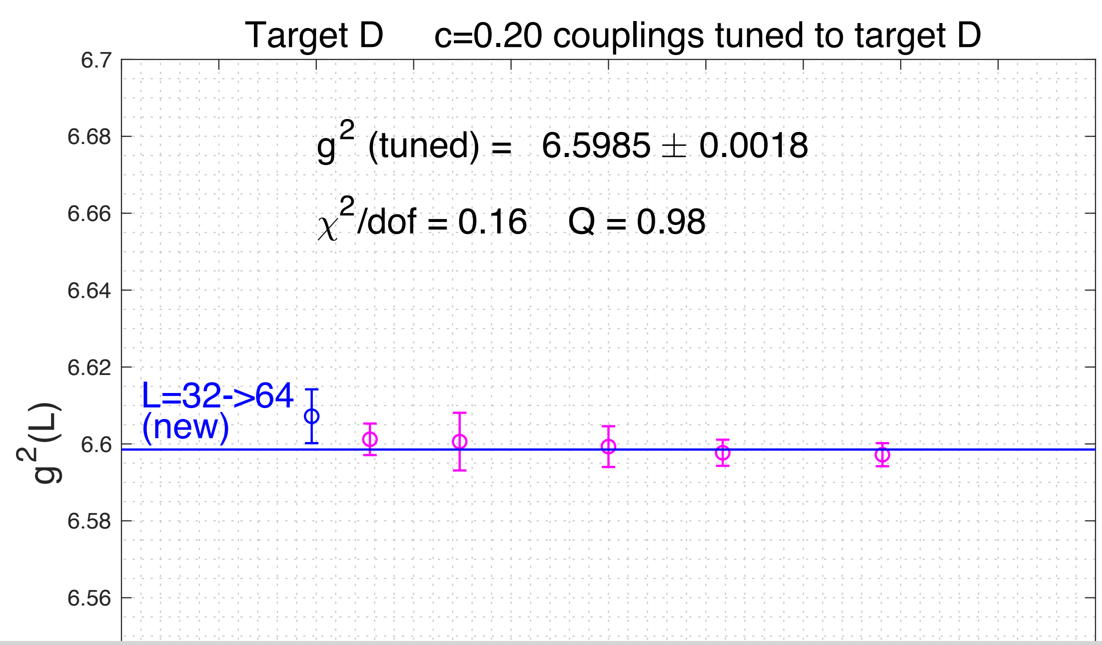
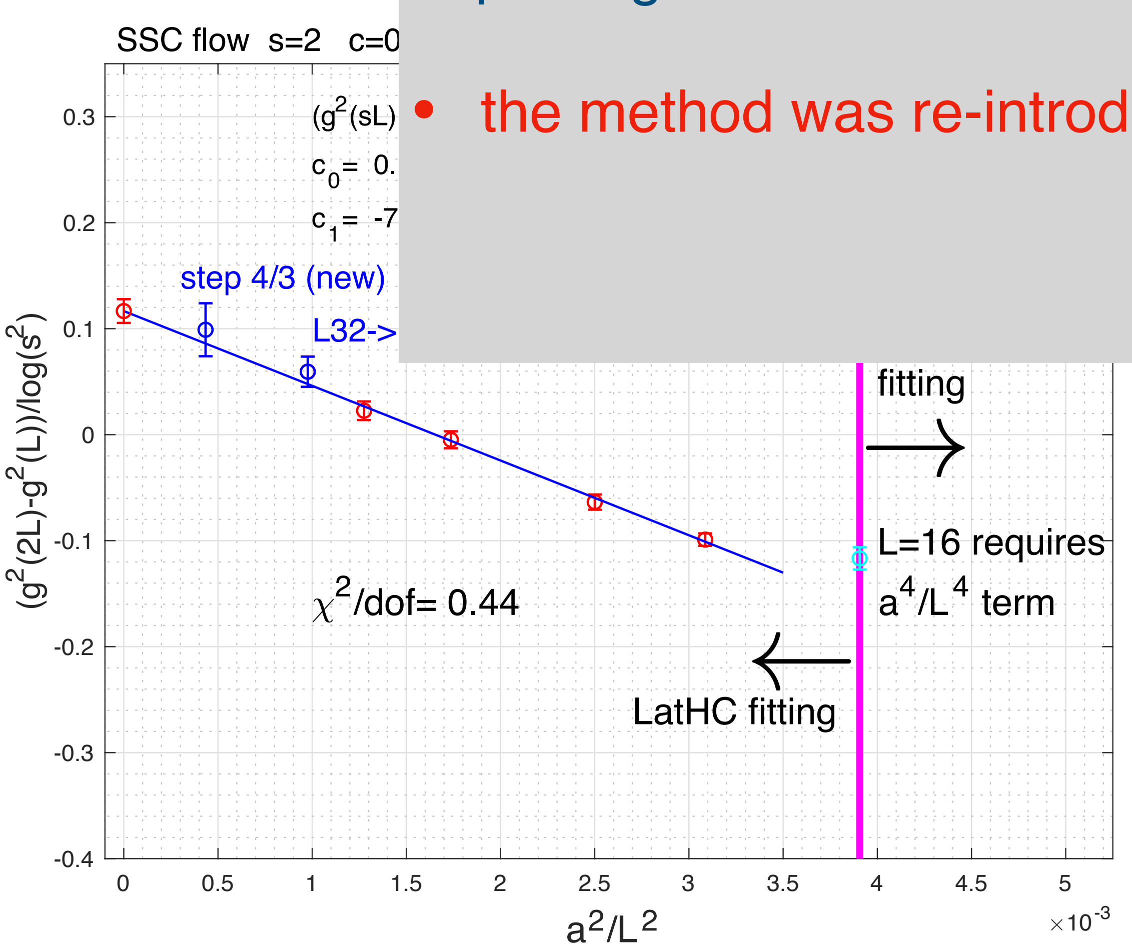
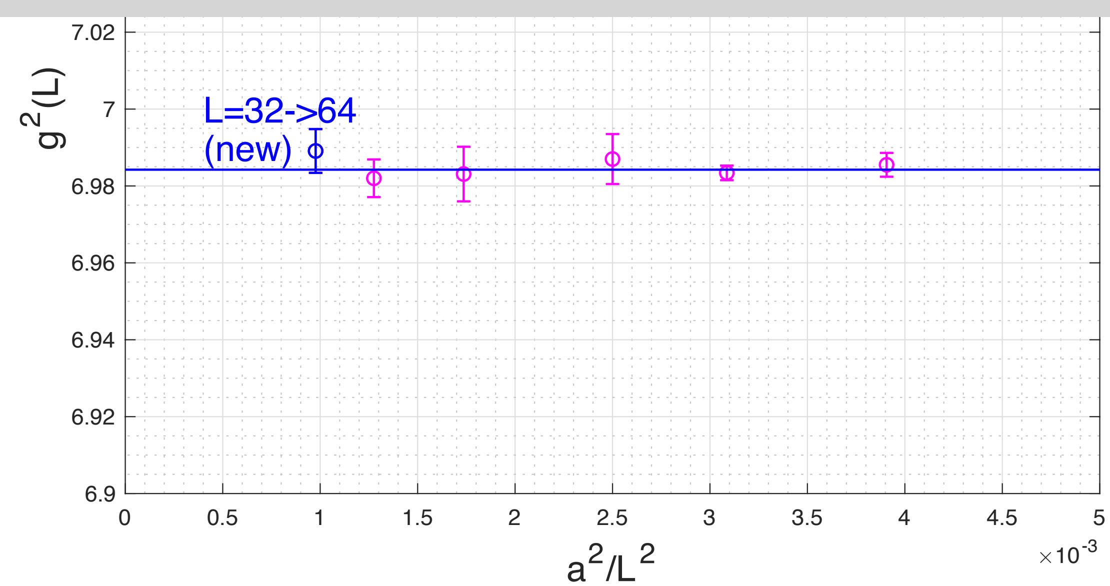


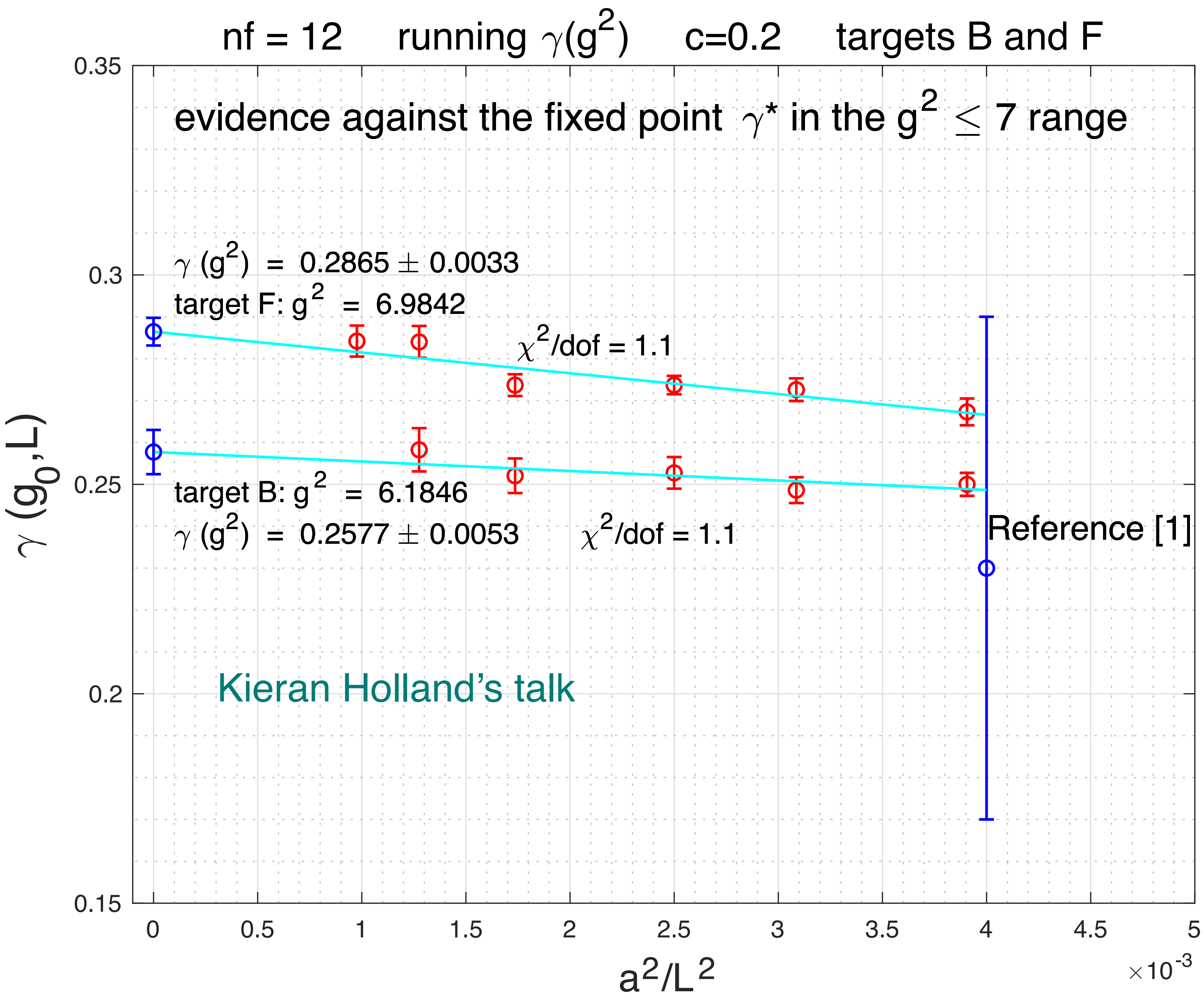


- our step β-function is also consistent with the continuous β-function on the GF $\beta = t \cdot dg^2/dt$ we introduced, tested, and used it on the lattice before
- the method was re-introduced at this conference





new at nf=12: anomalous dimension γ from Dirac spectrum



new: anomalous dimension γ from Dirac spectrum nf=12, sextet

γ drifting with scale

- staggered formulation is fine
- best choice for BSM explorations in large volumes
- and large volumes are needed!

$L \rightarrow 2L$ step function renormalization procedure:

$$\nu_R(\lambda_R) = \nu(\lambda) \qquad \lambda_R = Z_p^{-1} \cdot \lambda$$

$$\frac{\lambda_L}{\lambda_{2L}} = 2\frac{Z_p(g_0, L/a)}{Z_p(g_0, 2L/a)}$$ from the matching condition

walking and complex CFT

walking and complex CFT

Potts model Q potts spin ~ flavor
described by CFT

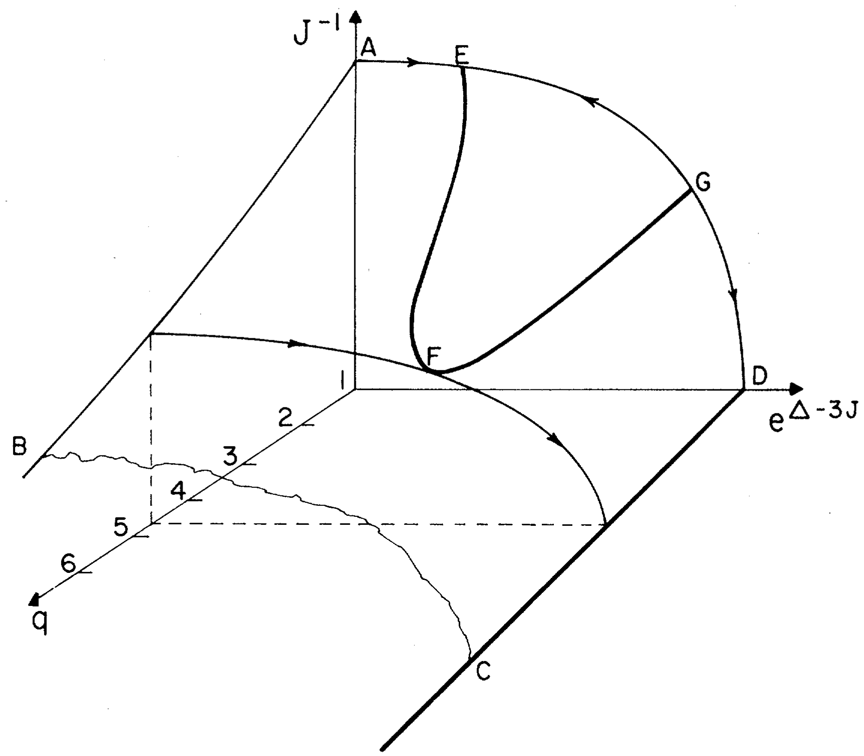
Q=2 - 4 pair of real CFT with pair of zeros of the beta function
works for continuous Q in cluster rep

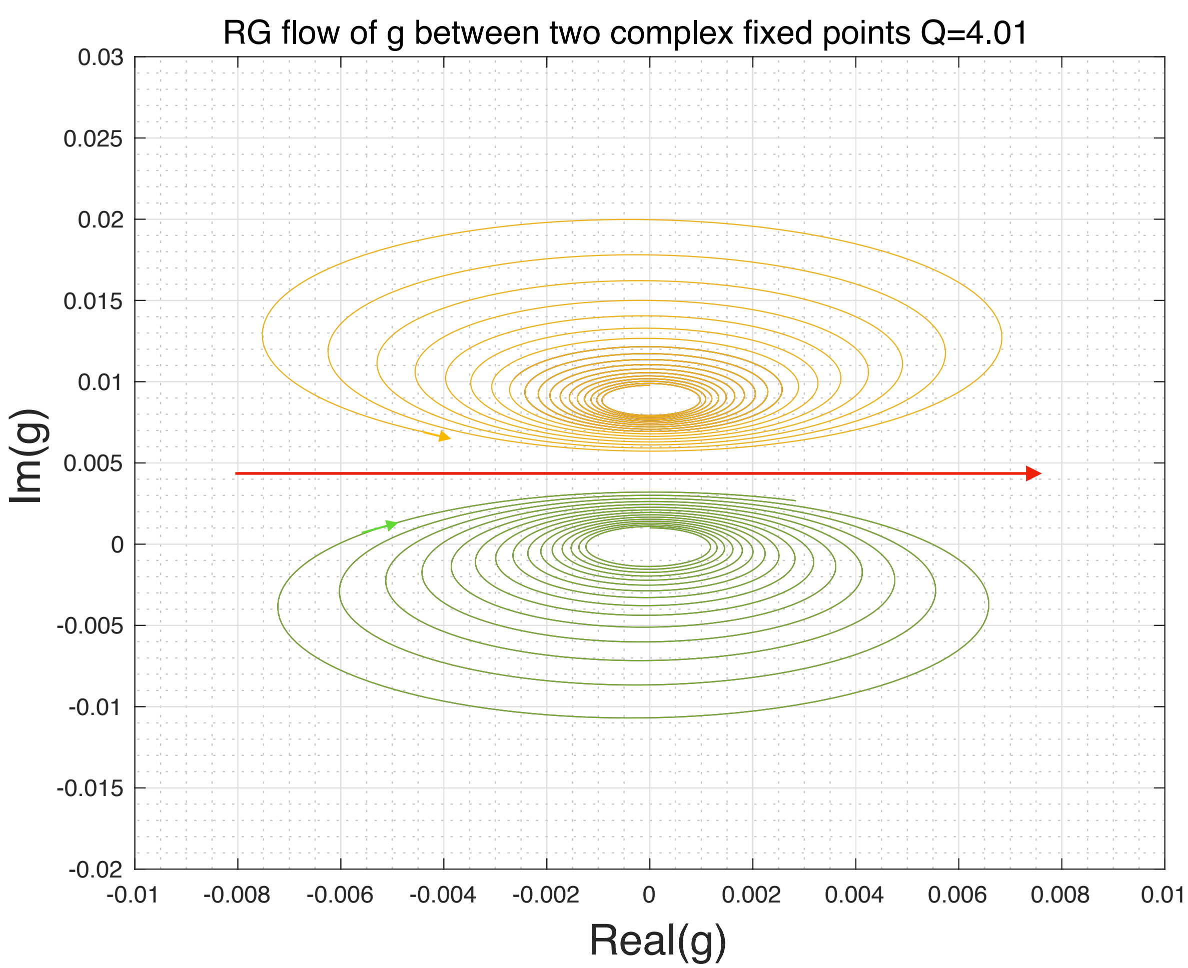
Q > 4 complex CFT, like Q=5, 6, 7 …

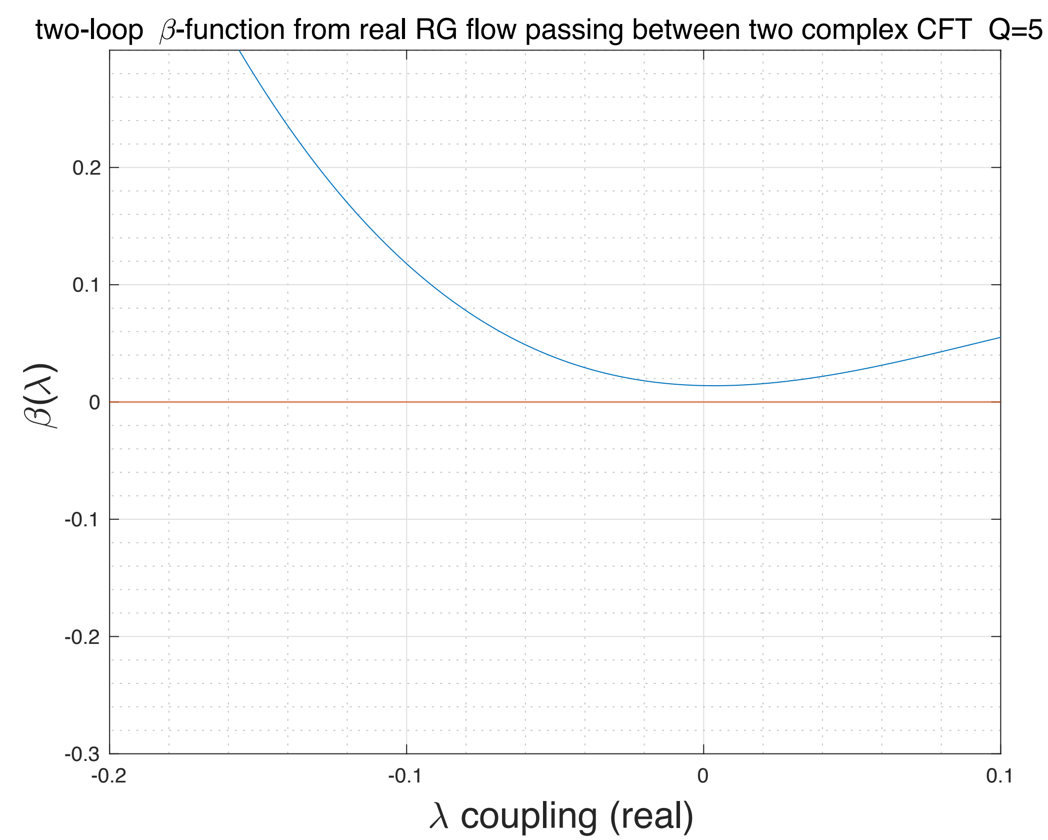
Q	5	6	7	8	9	10
ξ	2512.2	158.9	48.1	23.9	14.9	10.6

The Q=5 Potts model is interpreted now as near-conformal and walking, controlled by the complex IRFP pair Gorbenko et al.

- Q = 5 very large scale separation without tuning
- slowly drifting scale-dependence in critical exponents is calculable:
- central charge $c_{drift}(L) = c_R - \alpha \tan(\gamma \log \frac{L}{L_0}) + \ldots$
- flow in far infrared is the first order transition point (like χSB in gauge theories)







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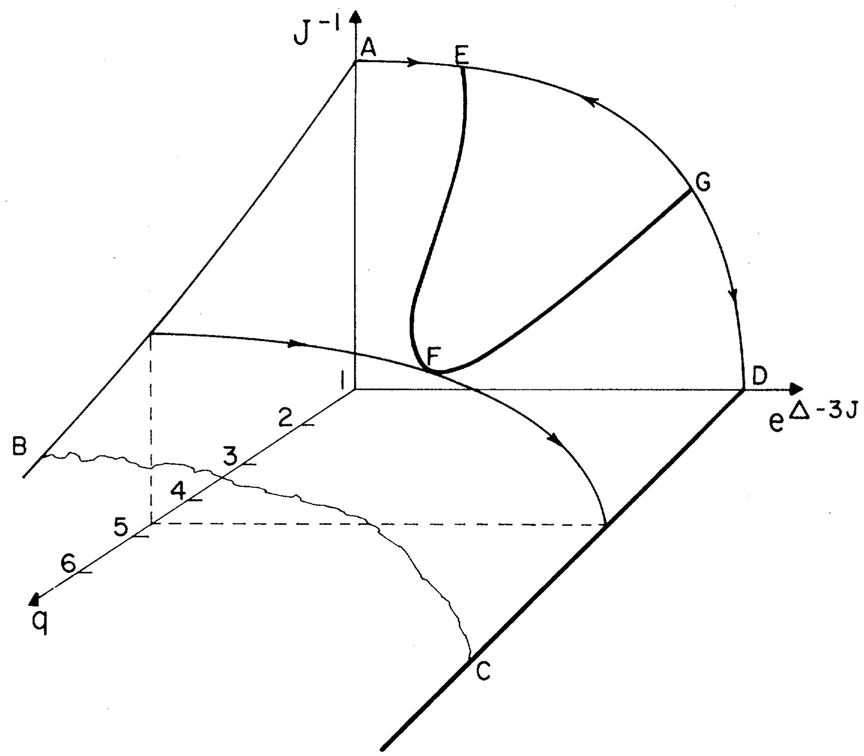
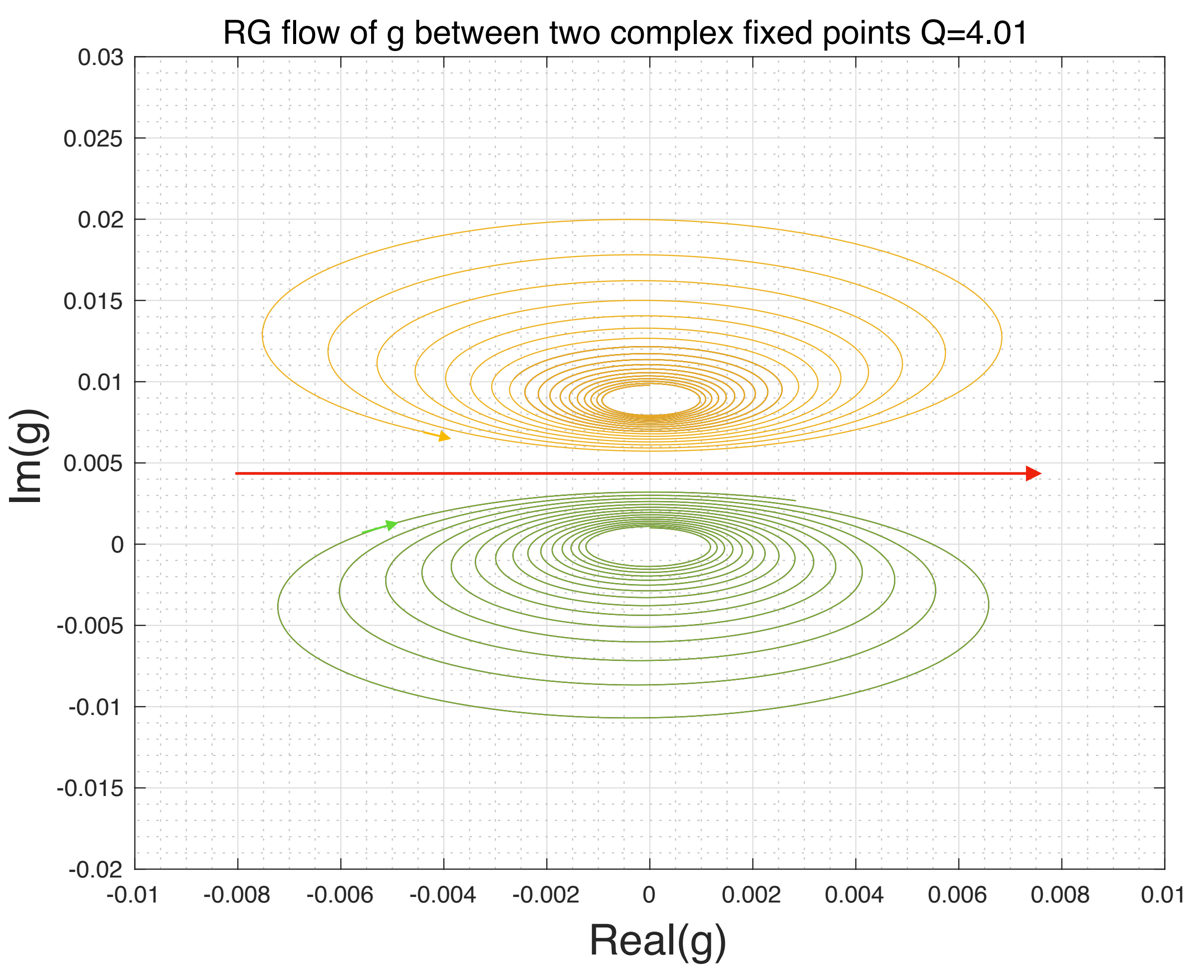
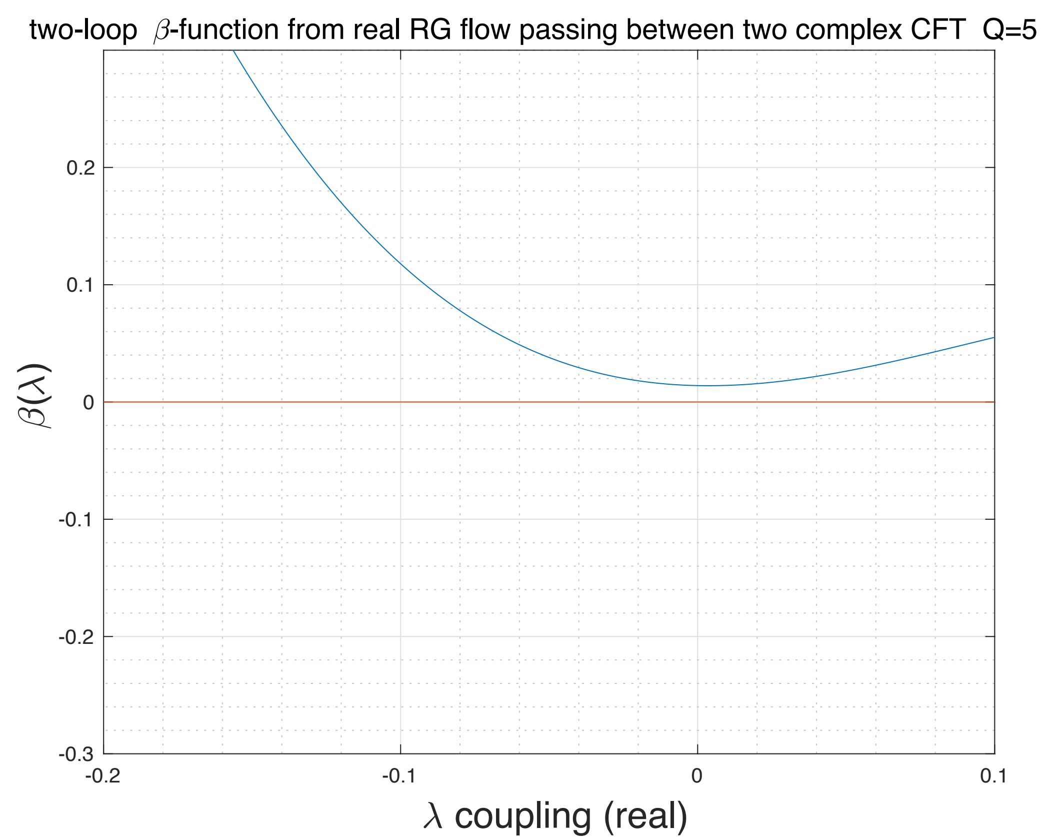
- efficient Potts CUDA code for integer Q
  Swendsen-Wang ~1 ns/spin
- CPU cluster code for arbitrary Q (2 complex couplings Q=4+ε)
  needed in lattice realization of the complex theory
- Corner transfer matrix combined with RG outperforms simulations at integer Q

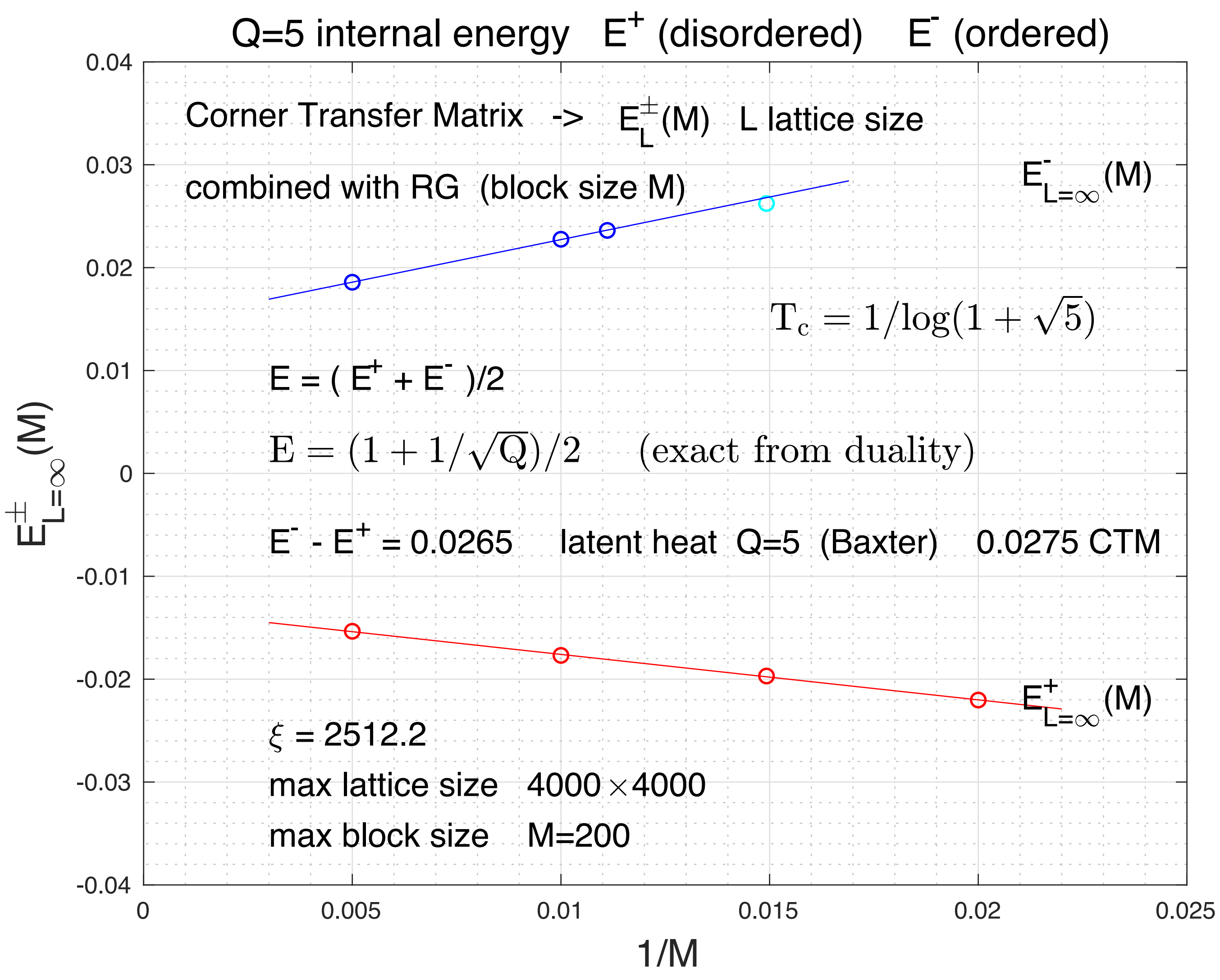


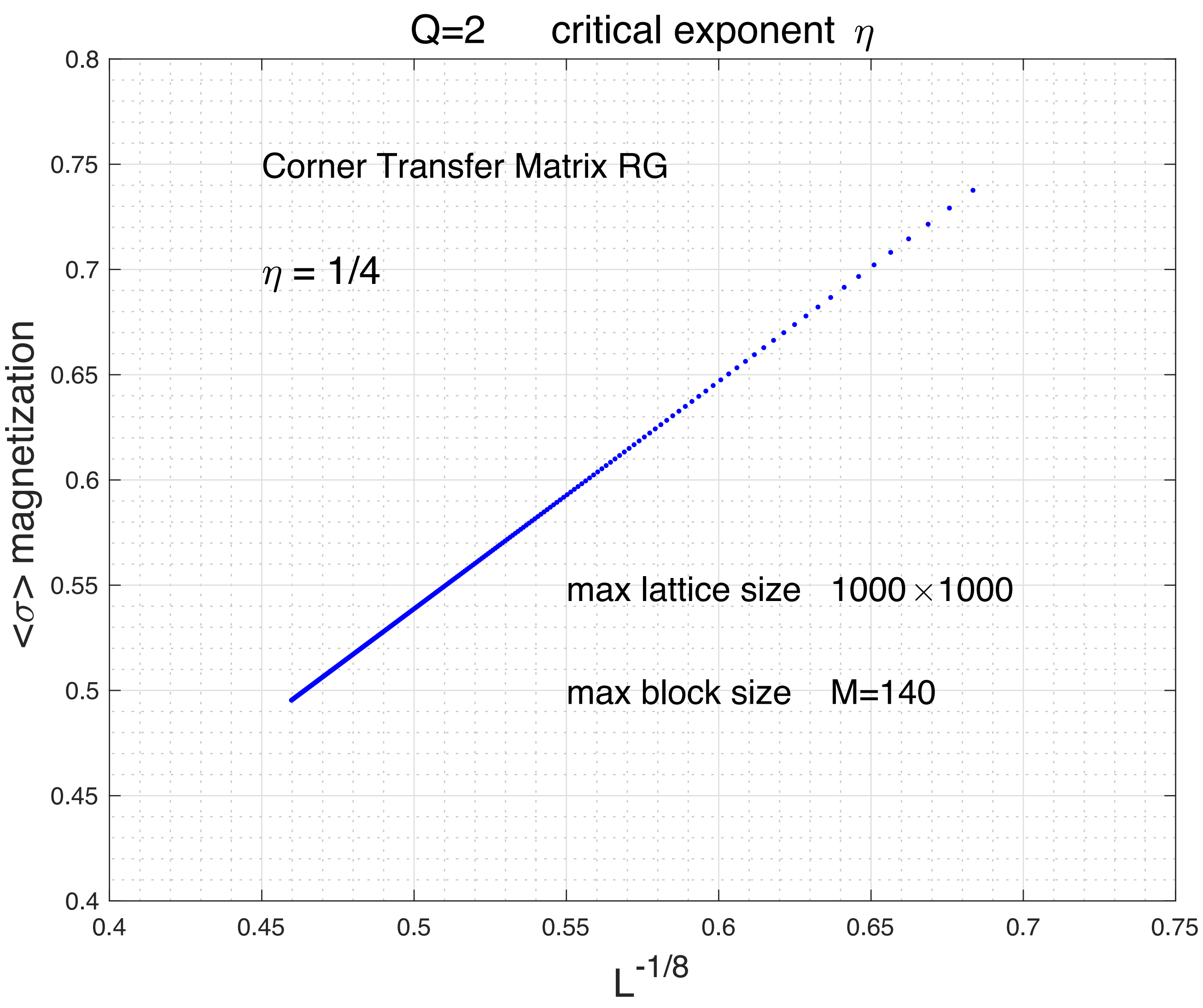




walking and complex CFT

sample results from Corner Transfer Matrix (Potts model)





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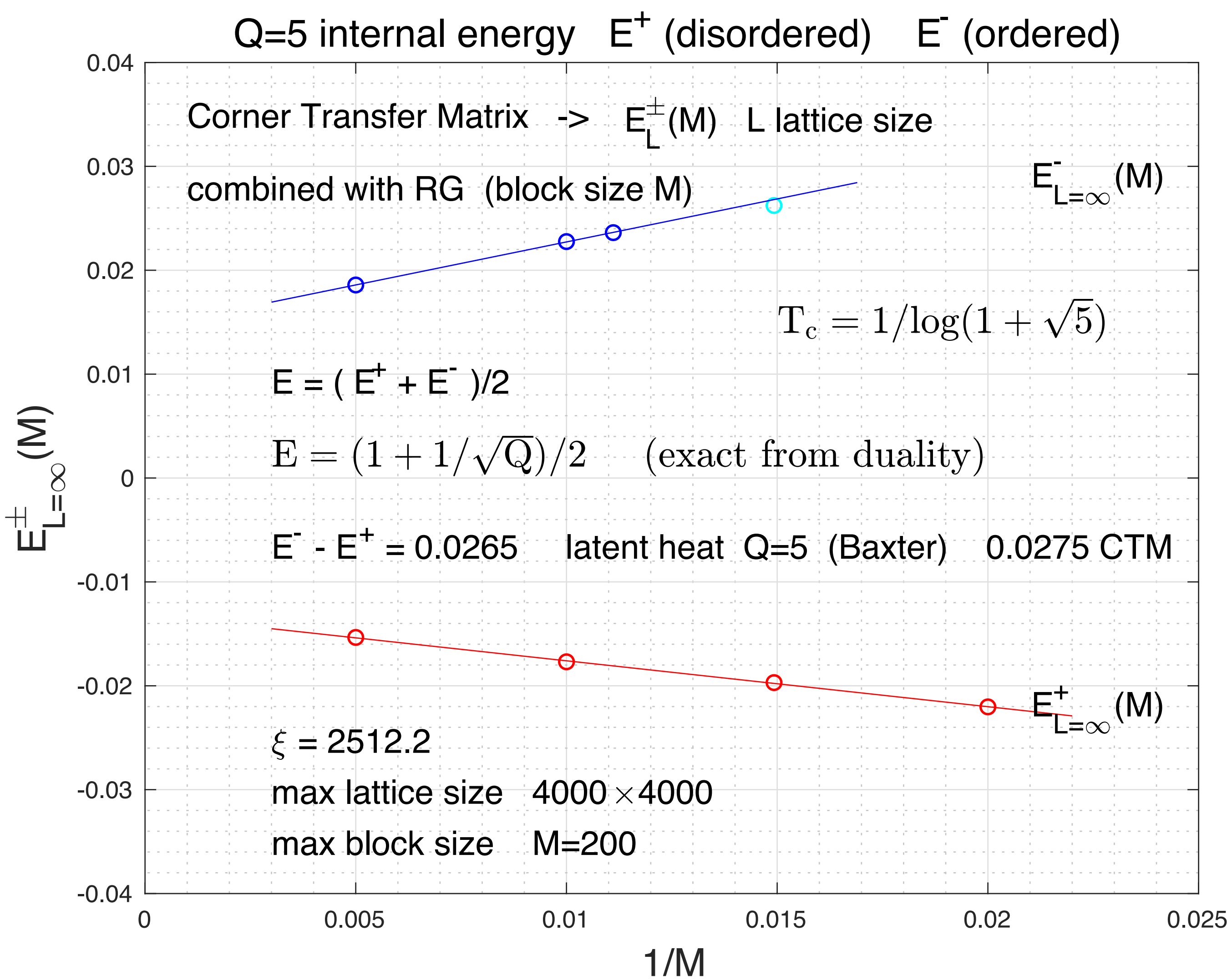

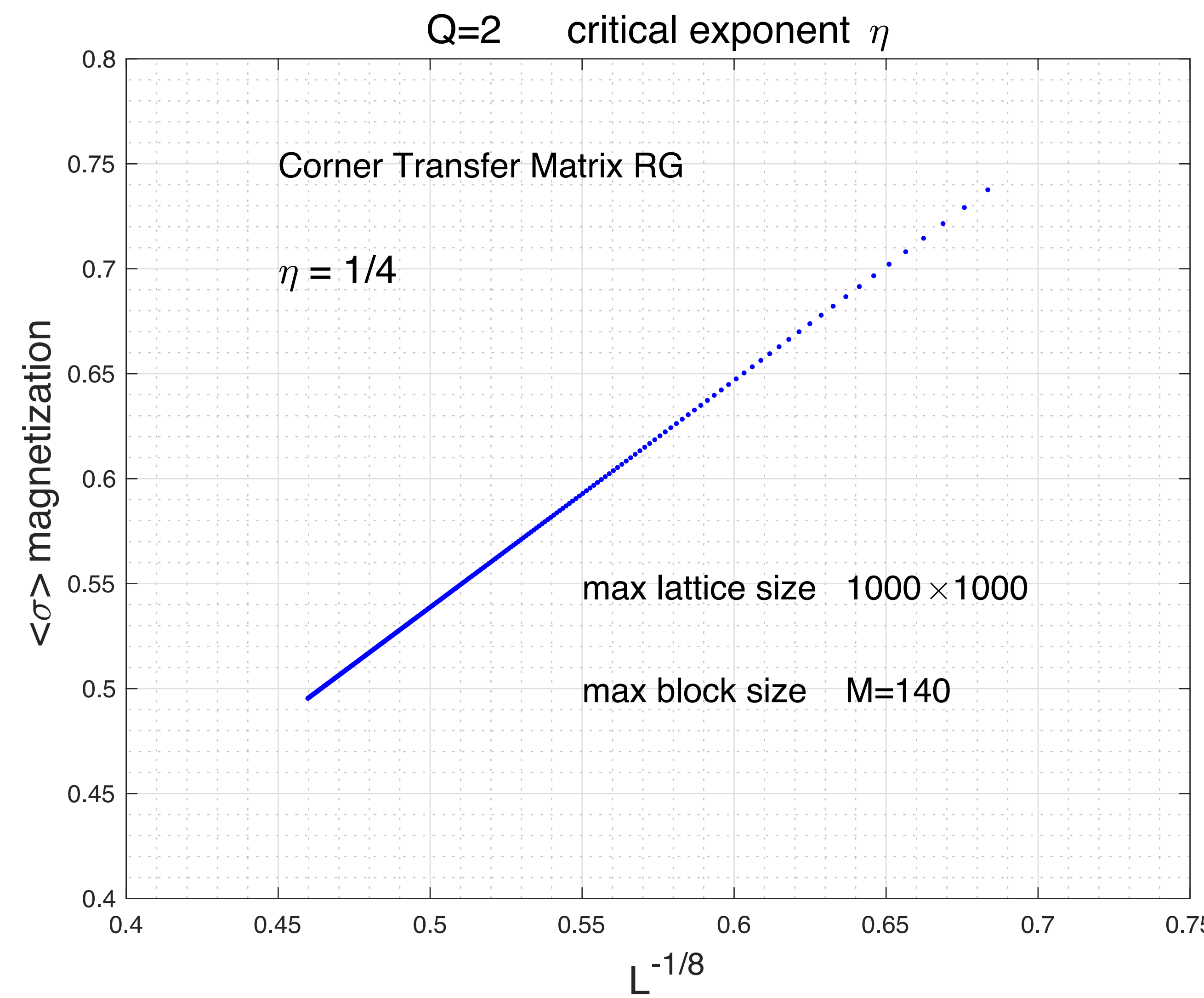

but walking from complex CFT emerged first in 4d gauge theories

walking and complex CFT

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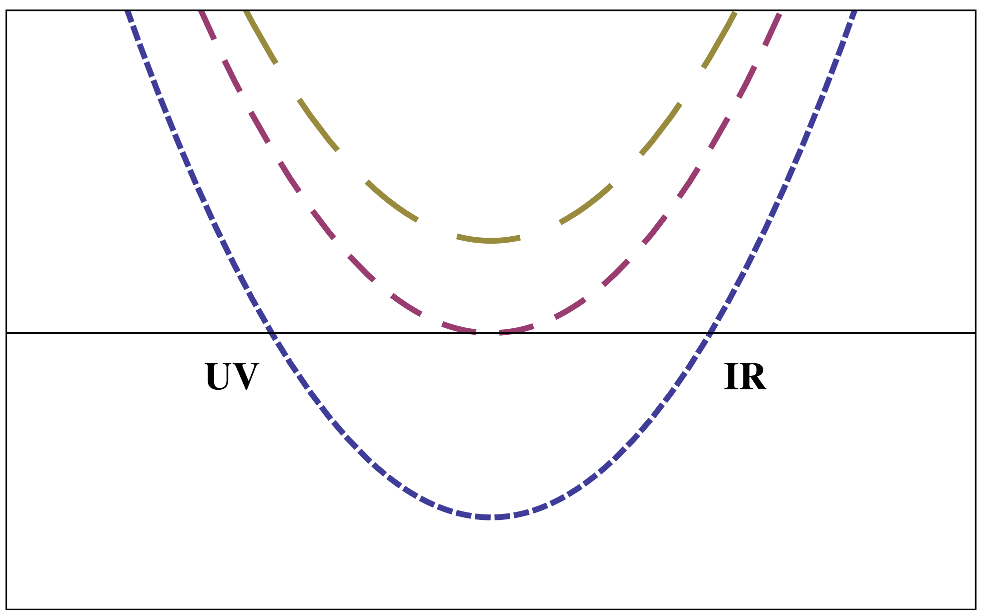
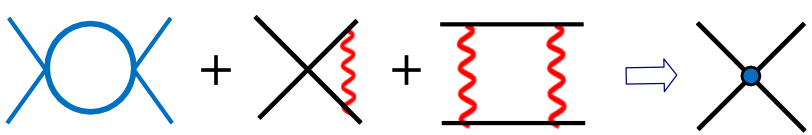




$$\mathcal{L}_{\text{CFT}} + \frac{f}{2}\mathcal{O}^{\dagger}_{ij}\mathcal{O}^{ij}$$ four-fermion deformation

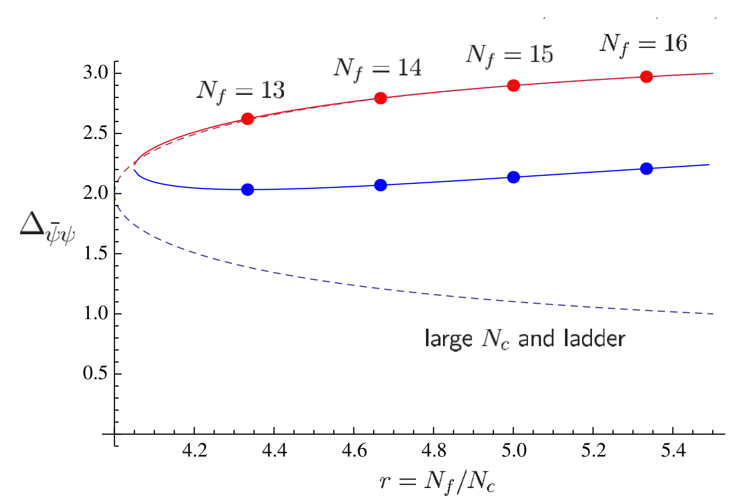
$$\Lambda \frac{d\bar{f}}{d\Lambda} = v\bar{f}^2 + (2\Delta - d)\bar{f} + a$$

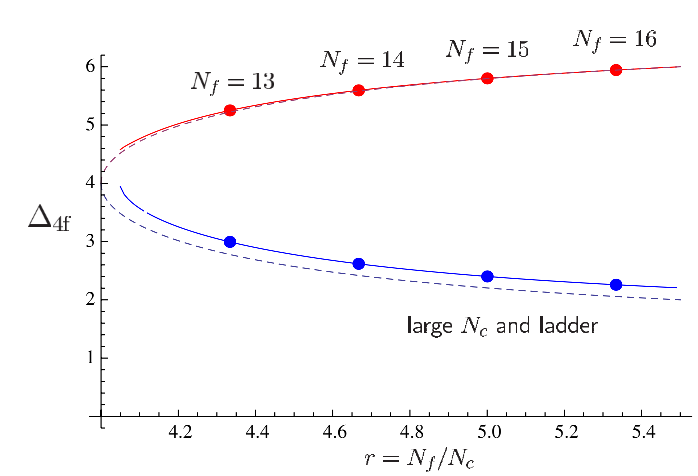
$$\beta'_{\bar{f}}|_{\pm} = \pm 2\sqrt{D}$$

$$\Delta_{\pm} = \frac{d}{2} \pm \sqrt{D}$$

Luca Vecchi 2010 talks about complex CFT
built on Gies et al., Terao et al., Kaplan et al.…

walking and complex CFT new paradigm?
flavor symmetry group is the same for walker and the CFT! paradigm change from dilaton?





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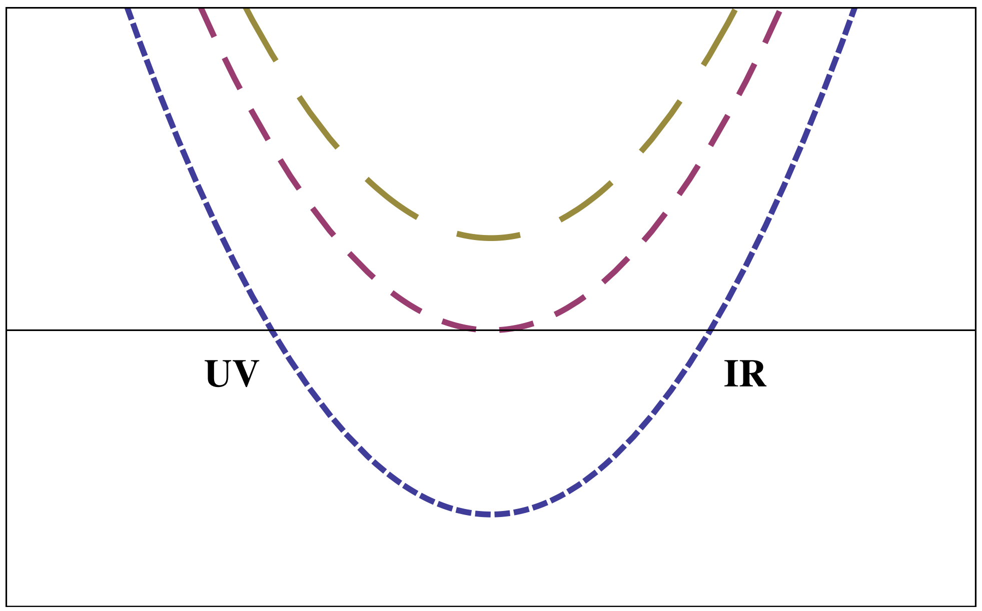

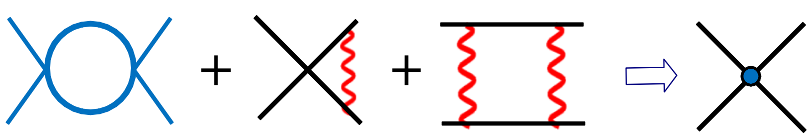

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walking and complex CFT new paradigm?
flavor symmetry group is the same for walker and the CFT! paradigm change from dilaton?

we started work earlier on the realization of walking based on this idea

to distinguish near-conformal and conformal finite volume correlators (drifting scaling exponents distinguished from fixed conformal exponents).
not knowing the conformal exponents of the complex theory makes the analysis challenging

Gorbenko et al. turned to a two-dimensional example (Potts model) for detailed realization of walking without apparently knowing about Vecchi

Light scalar is expected to be described by EFT of linear σ-model in deep infrared limit m ➛ 0

Data is high above linear σ-model regime

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simplest: light Higgs-like particle of linear σ-model in PT:

$SU(2)\otimes SU(2)\sim O(4)$ for sextet model

$$L=\frac{1}{2}\left(\partial_\mu\vec{\pi}\right)^2+\frac{1}{2}\left(\partial_\mu\sigma\right)^2-\frac{1}{2}\mu^2\left(\sigma^2+\vec{\pi}^2\right)+\frac{1}{4}g\left(\sigma^2+\vec{\pi}^2\right)^2-\varepsilon\sigma$$

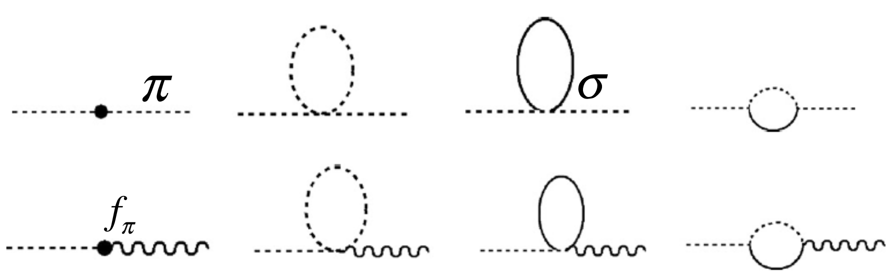
triviality analysis in $m_\sigma/f_\pi < 3$ range
$m_\pi = 0$ circa 1987-1988

$m_\sigma^2 \geq 3m_\pi^2$ tree level relation

$m_\sigma^2 \geq 2m_\pi^2$ with loop corrections

high above it non-linear σ-model or dilaton EFT

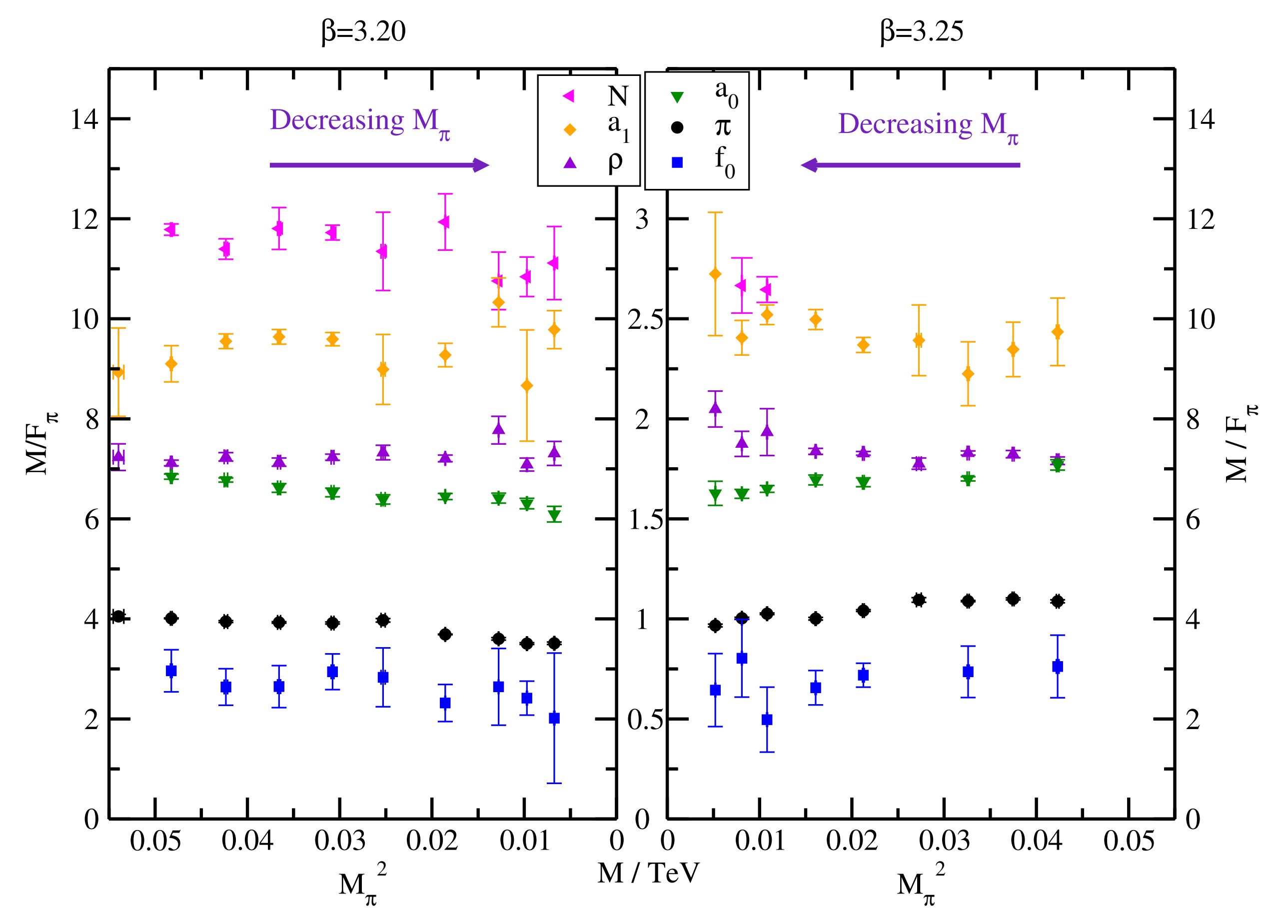
p-regime data: 0++ is tracking the Goldstone pion with $m_\pi^2 \geq m_\sigma^2$, not like linear σ-model
New EFT is needed to extrapolate data to massless chiral limit



$$L=\frac{1}{2}\partial_\mu\sigma\,\partial_\mu\sigma-V(\sigma)+\frac{f_\pi^{\,2}}{4}\left(D_\mu\Sigma^\dagger D_\mu\Sigma\right)\cdot\left(1+2a\frac{\sigma}{f_\pi}+b\frac{\sigma^2}{f_\pi^{\,2}}+b_3\frac{\sigma^{\,3}}{f_\pi^{\,3}}+\ldots\right)$$

$\Sigma=e^{i\pi^a\tau^a/f_\pi}$ with τ^a Pauli matrices

$$V(\sigma)=\frac{1}{2}m_\sigma^2\cdot\sigma^{\,2}+d_3\left(\frac{m_\sigma^2}{2f_\pi}\right)\cdot\sigma^{\,3}+d_4\left(\frac{m_\sigma^2}{8f_\pi^{\,2}}\right)\cdot\sigma^{\,4}+\ldots$$



linear σ-model limit (SM): $a=b=d_3=d_4=1$

dilaton EFT will require $a=b^2,\ b_3=0$ and special y(μ) in L

we will test the elegant Golterman-Shamir formulation of the dilaton theory

Data is high above linear σ-model regime

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we will test the Golterman-Shamir formulation of the dilaton theory high above the linear σ-regime

high above it non-linear σ-model or dilaton EFT

p-regime data: 0++ is tracking the Goldstone pion with $m_\pi^2 \ge m_\sigma^2$, not like linear σ-model
New EFT is needed to extrapolate data to massless chiral limit

alternative to pinched walking from complex CFT which does not require dilaton of spontaneous scale symmetry breaking

$\Sigma = e^{i\pi^a\tau^a/f_\pi}$ with τ^a Pauli matrices

$$V(\sigma)=\frac{1}{2}m_\sigma^2\cdot\sigma^2+d_3\left(\frac{m_\sigma^2}{2f_\pi}\right)\cdot\sigma^3+d_4\left(\frac{m_\sigma^2}{8f_\pi^2}\right)\cdot\sigma^4+\ldots$$

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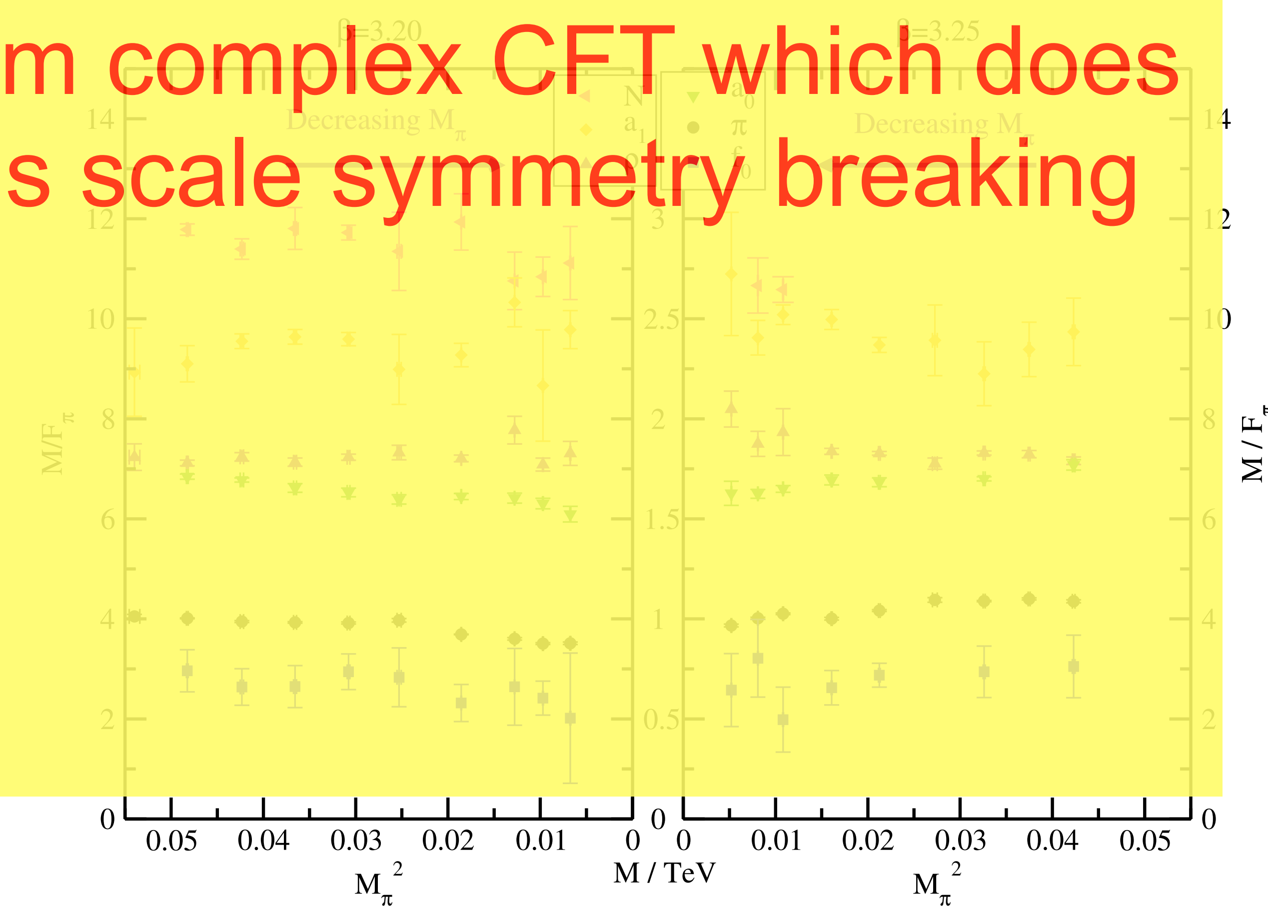


dilaton EFT with $\sigma(x)$ dilaton field and $\pi^a(x)$ Goldstone bosons

$$L = \frac{1}{2}\partial_\mu \chi \partial_\mu \chi - V_d(\chi) + \frac{f_\pi^2}{4}\left(\frac{\chi}{f_d}\right)^2 tr\left[\partial_\mu \Sigma^\dagger \partial_\mu \Sigma\right] - \frac{f_\pi^2 m_\pi^2}{4}\left(\frac{\chi}{f_d}\right)^y tr\left(\Sigma + \Sigma^\dagger\right)$$

Golterman-Shamir

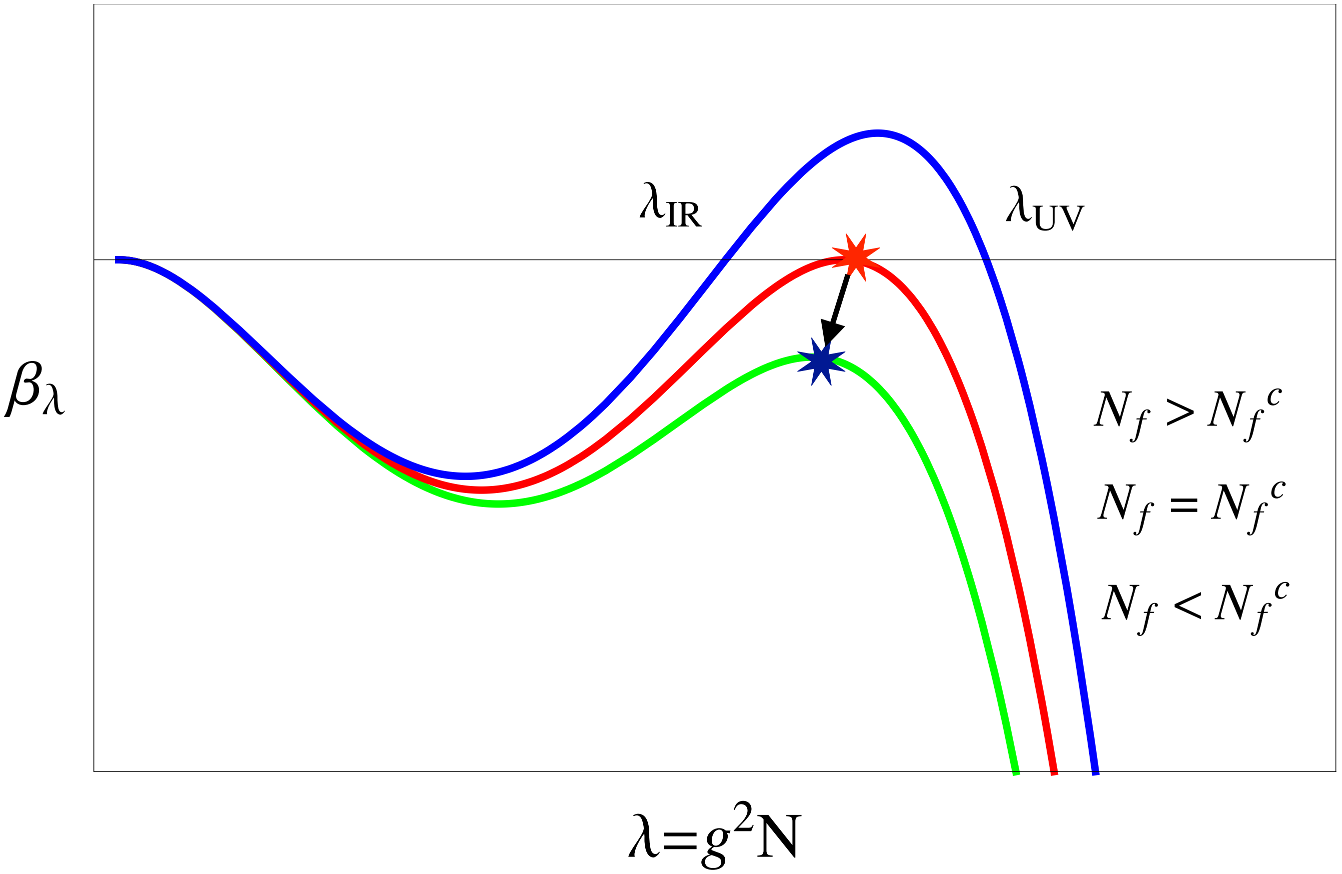
Appelquist et al. notation

but we do our own IML analysis which is required for any conclusion!

$y = 3 - \gamma$ where γ is the mass anomalous dimension

$\chi(x) = f_d\, e^{\sigma(x)/f_d}$ describes the dilaton field $\sigma(x)$

pion field $\Sigma = e^{i\pi^a\tau^a/f_\pi}$ with τ^a Pauli matrices, tree level pion mass $m_\pi^2 = 2Bm$



- dilaton EFT has long history
- Golterman-Shamir expansion in $x = N_f/N$ variable
- Veneziano limit N ➛ ∞
- predicts walking around ✷ in p-regime (tree level)

  from expanding around CFT ✷
- based on scheme-dependent β-function ?
- flavor symmetry at ✷ is different from symmetry at ✷ ?

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$$V_\sigma = \frac{m_d^2}{2f_d^2}\left(\frac{\chi^2}{2} - \frac{f_d^2}{2}\right)^2$$ relevant deformation of IRFP theory

$$V_d = \frac{m_d^2}{16 f_d^2}\chi^4\left(4\ln\frac{\chi}{f_d} - 1\right)$$ nearly marginal deformation

Golterman-Shamir form

f_π	Goldstone decay constant
$m_\pi = 2mB$	Goldstone pions
f_d	dilaton decay constant
m_d	dilaton mass

$F_\pi,\ M_\pi,\ F_d,\ M_d$ with mass deformation

covered in Ricky Wong's talk:

$M_\pi^2 \cdot F_\pi^{2-y} - 2B_\pi \cdot f_\pi^{(2-y)} \cdot m = 0$ general V indep. scaling law

$F_\pi^{(4-y)} \cdot (1 - f_\pi^2/F_\pi^2) - 2y \cdot n_f f_\pi^{(6-y)} B_\pi / m_d^2 f_d^2 \cdot m = 0$ $\boxed{V'_\sigma(\chi = F_d)}$

$3F_\pi^2/M_\pi^2 - f_\pi^2/M_\pi^2 - 2M_d^2/m_d^2 \cdot f_\pi^2/M_\pi^2 - y(y-1) n_f f_\pi^4 / m_d^2 f_d^2 = 0$ $\boxed{V''_\sigma(\chi = M_d)}$

$F_\pi^{(4-y)} \cdot \log(F_\pi/f_\pi) - y \cdot n_f f_\pi^{(6-y)} B_\pi \cdot m / m_d^2 f_d^2 = 0$ $\boxed{V'_d(\chi = F_d)}$

$(F_\pi^2/M_\pi^2) \cdot (3\log(F_\pi/f_\pi) + 1) - (M_d^2/m_d^2) \cdot (f_\pi^2/M_\pi^2) - y(y-1) n_f f_\pi^4 / 2m_d^2 f_d^2 = 0$ $\boxed{V''_d(\chi = M_d)}$

M_π, F_π, M_d input data at each m

$f_\pi,\ B,\ f_d,\ m_d,\ y$ fitted for all m

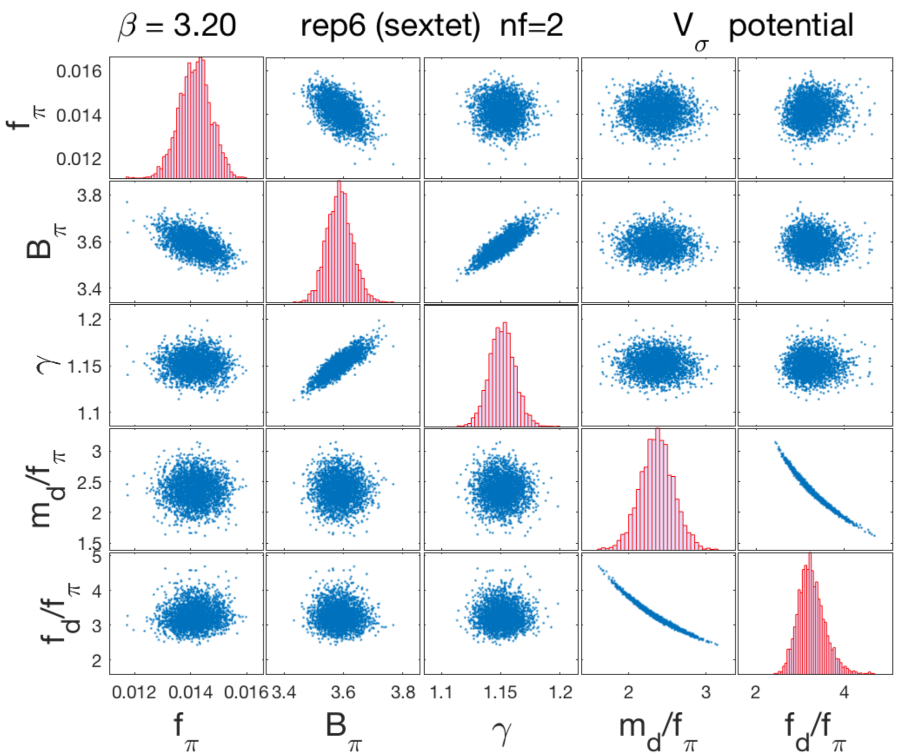
IML: Implicit Maximum Likelihood test

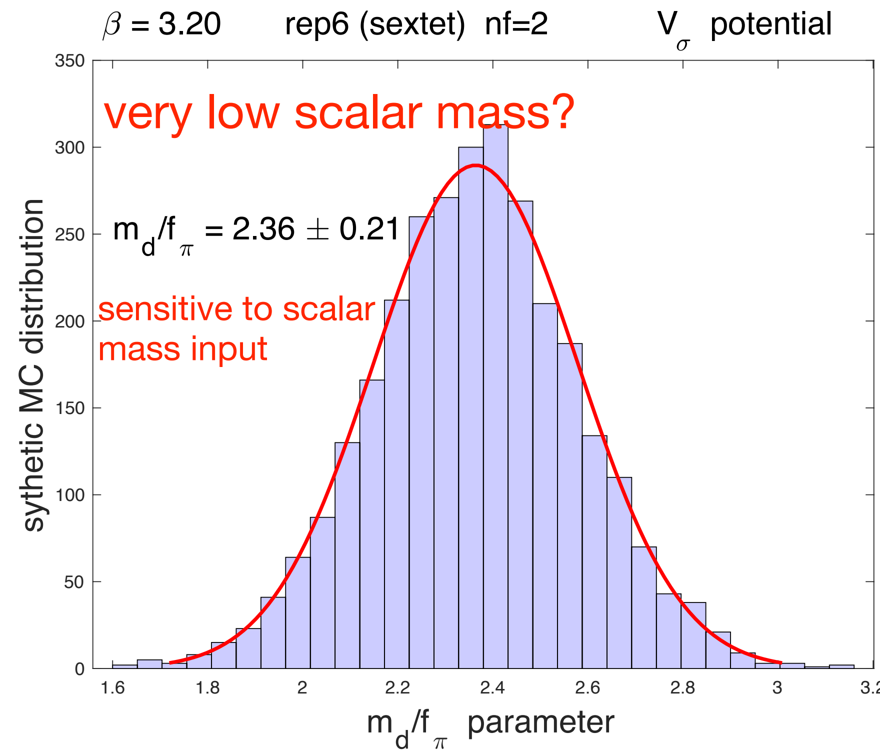
IML is very different from ML fitting

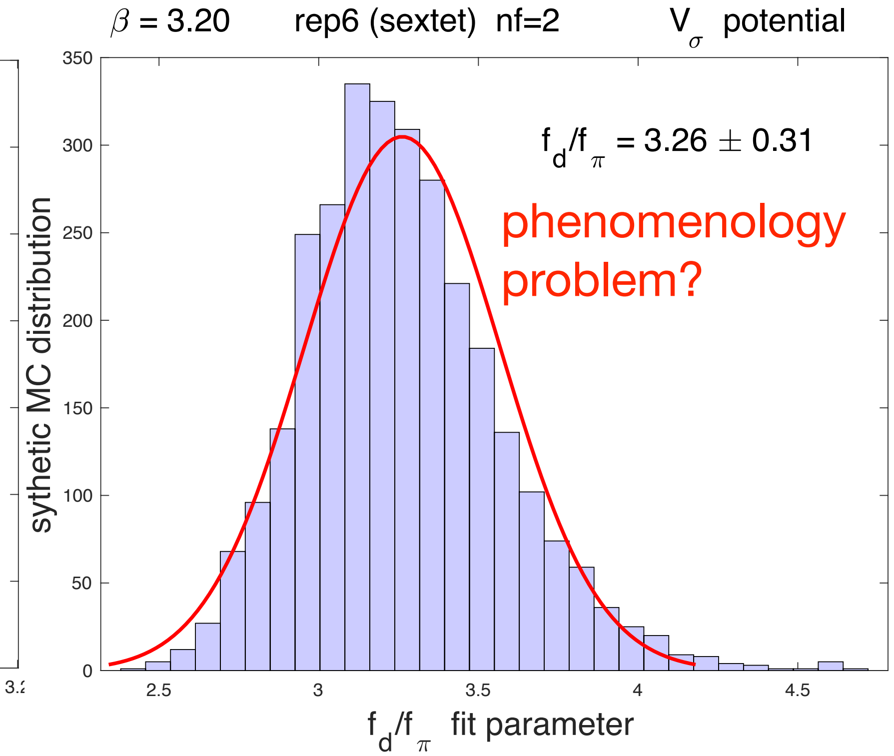
Perfect fits for V_σ!

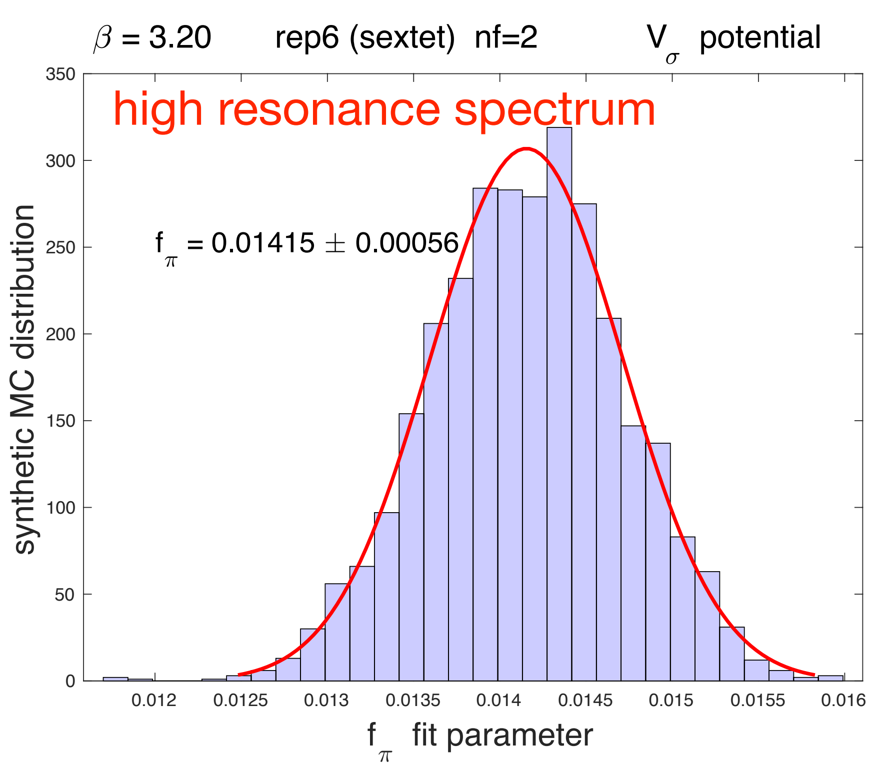
V_d fails!

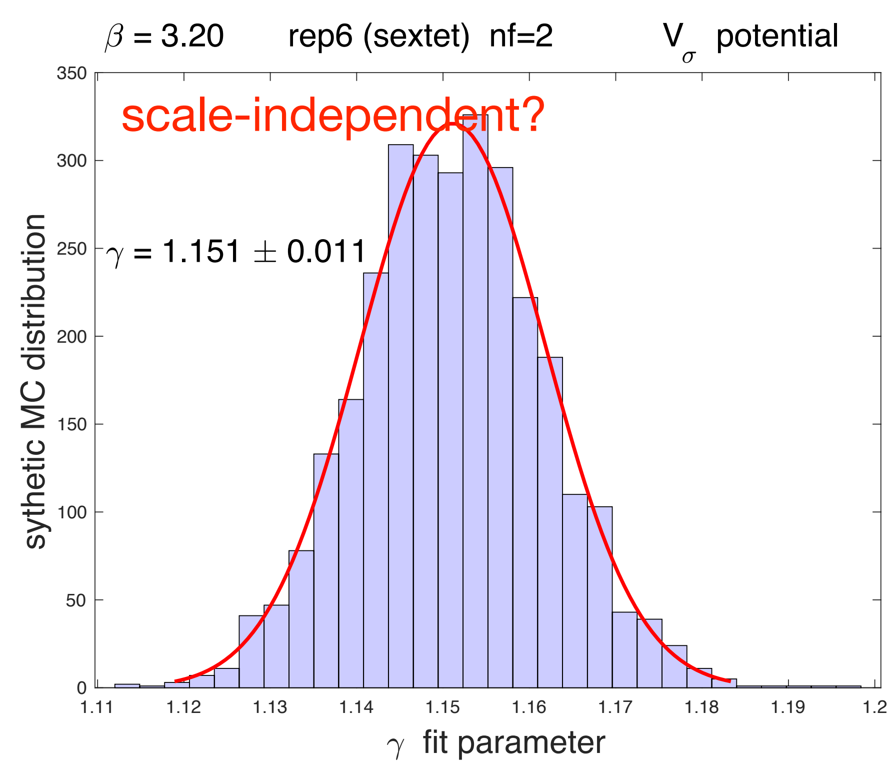
dilaton EFT with $\sigma(x)$ dilaton field and $\pi^a(x)$ Goldstone bosons







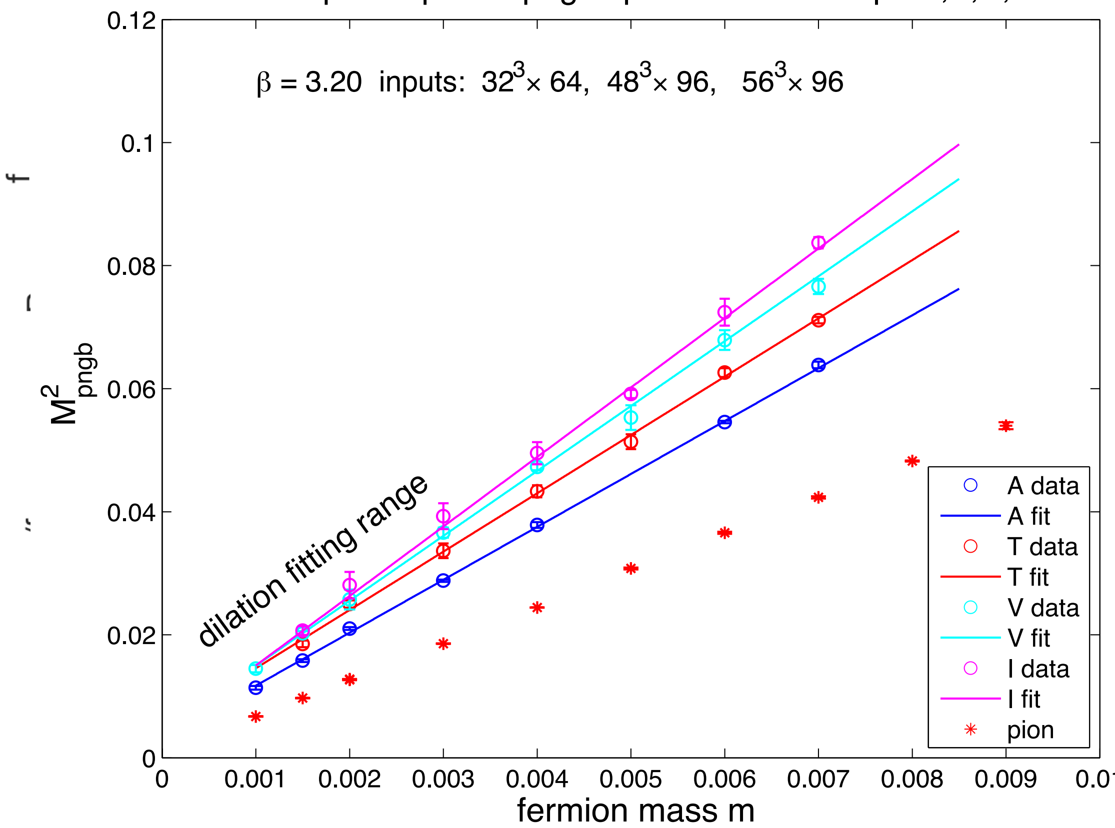


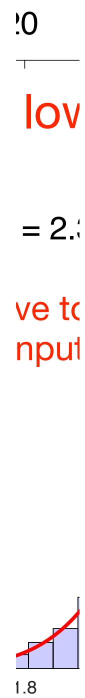


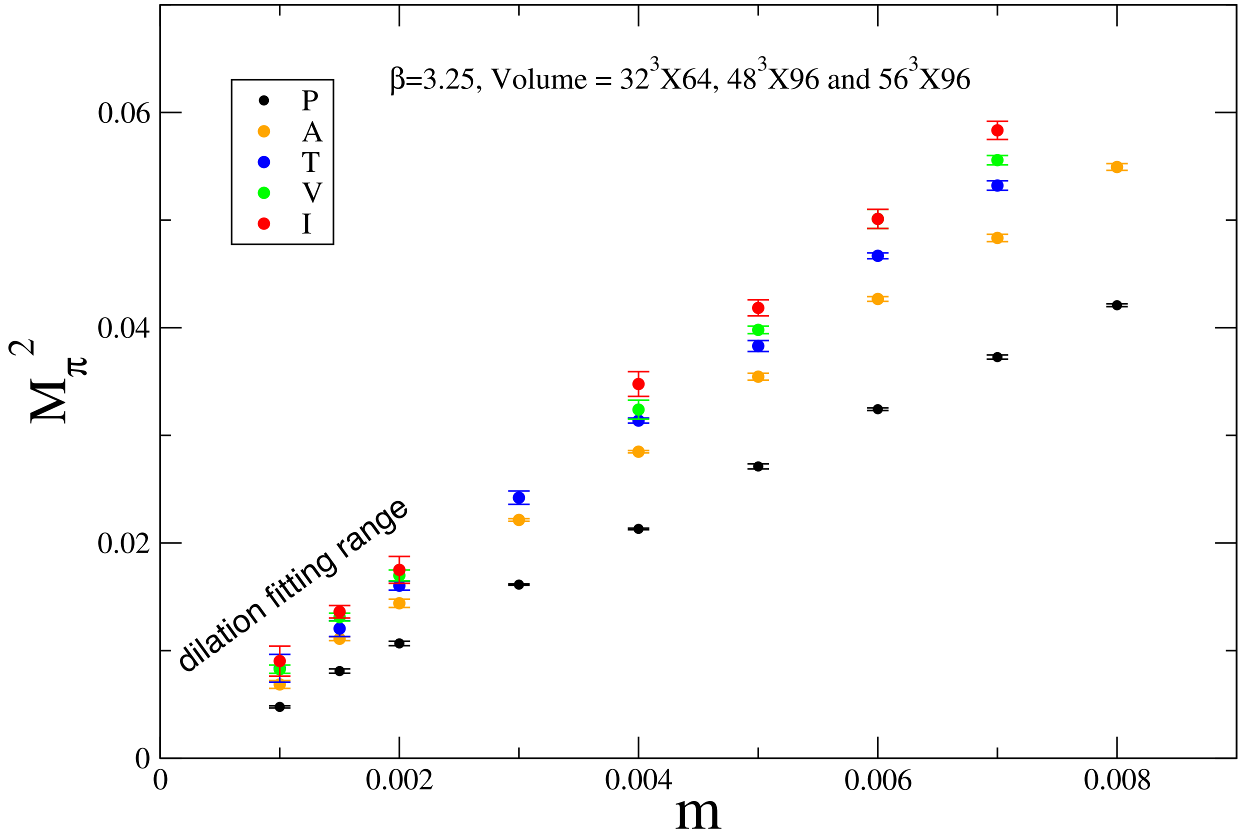
- the dilaton potential of Shamir-Golterman as tree level theory expanding around IRFP is not working well for sextet data
- drifting γ ignored?
- consistent LatHC analysis for two lattice spacing, with third under construction
- taste breaking issue?

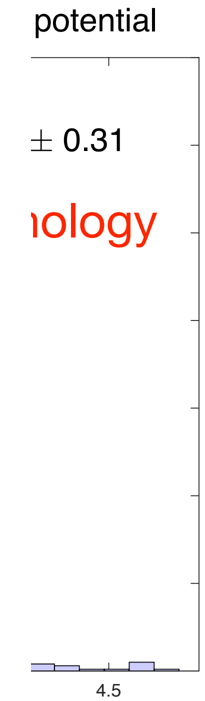
covered in Ricky Wong’s talk

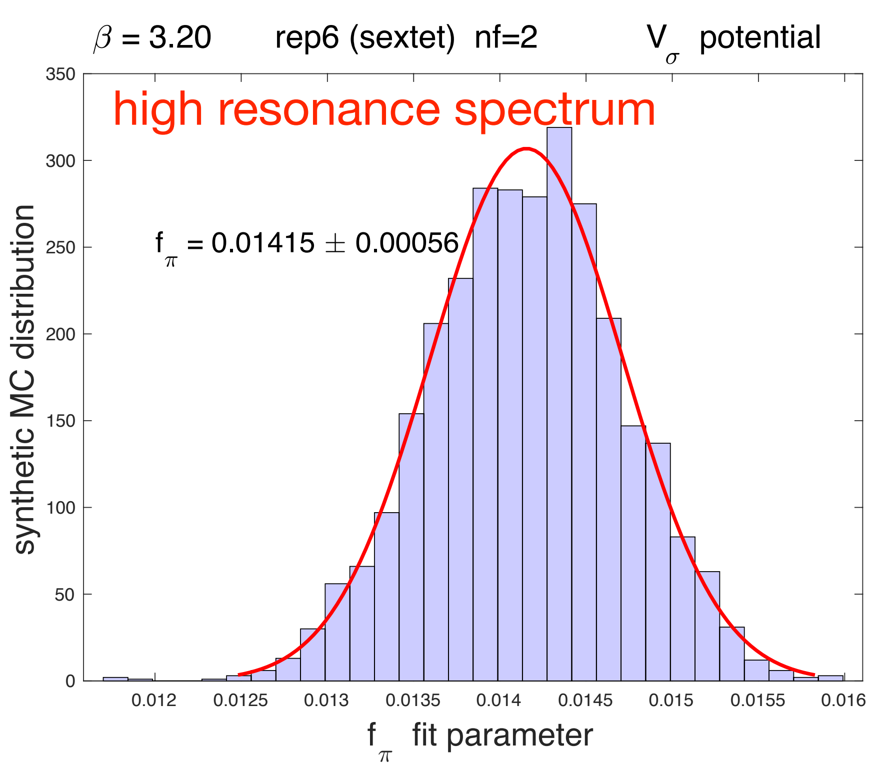
'd and $\pi^a(x)$ Goldstone bosons

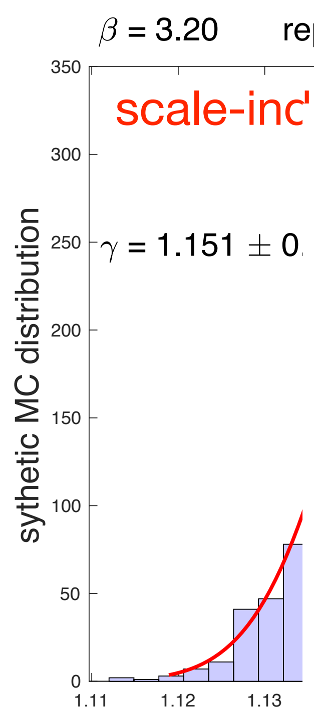












- the dilaton potential of Shamir-Golterman as tree level theory
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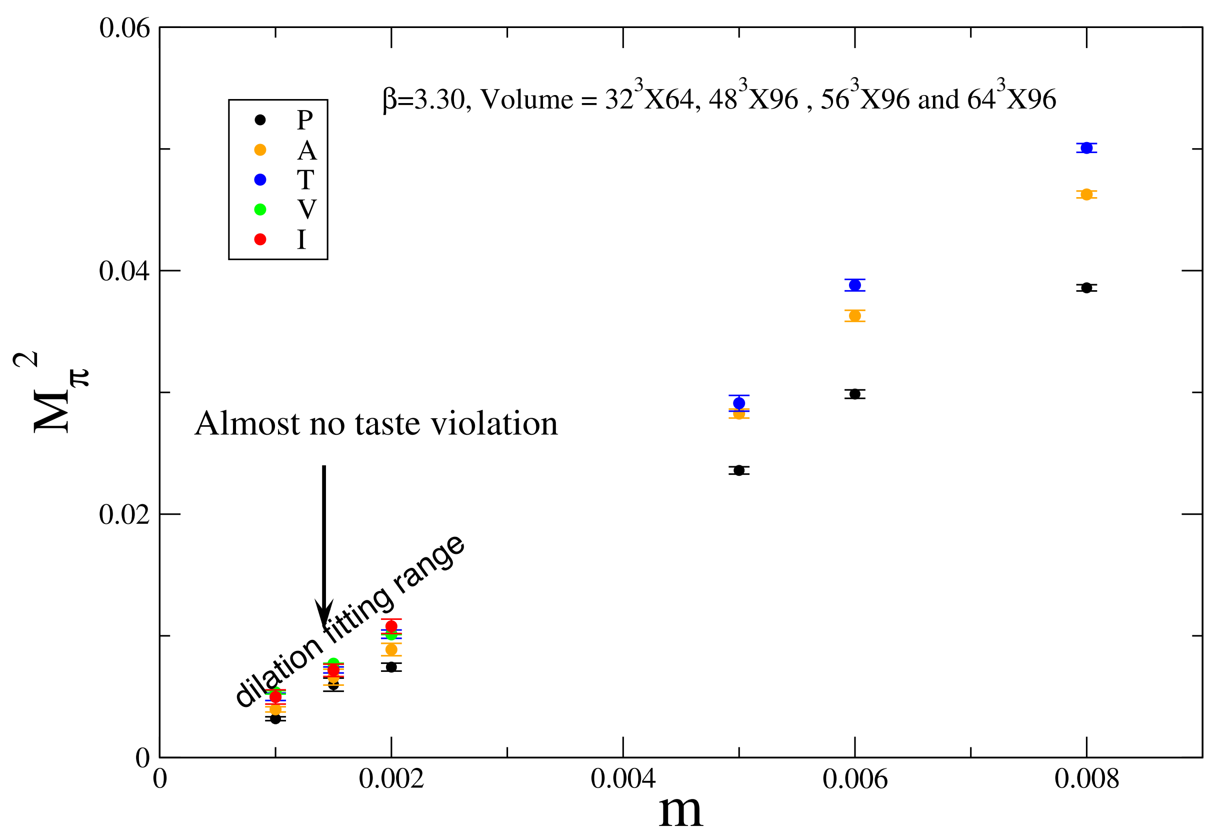
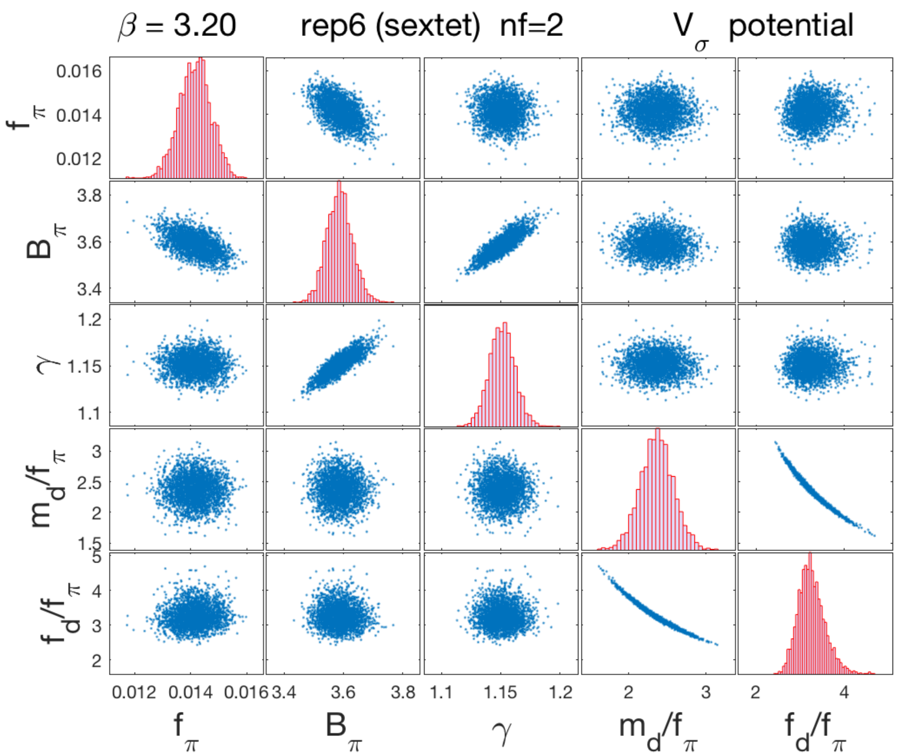
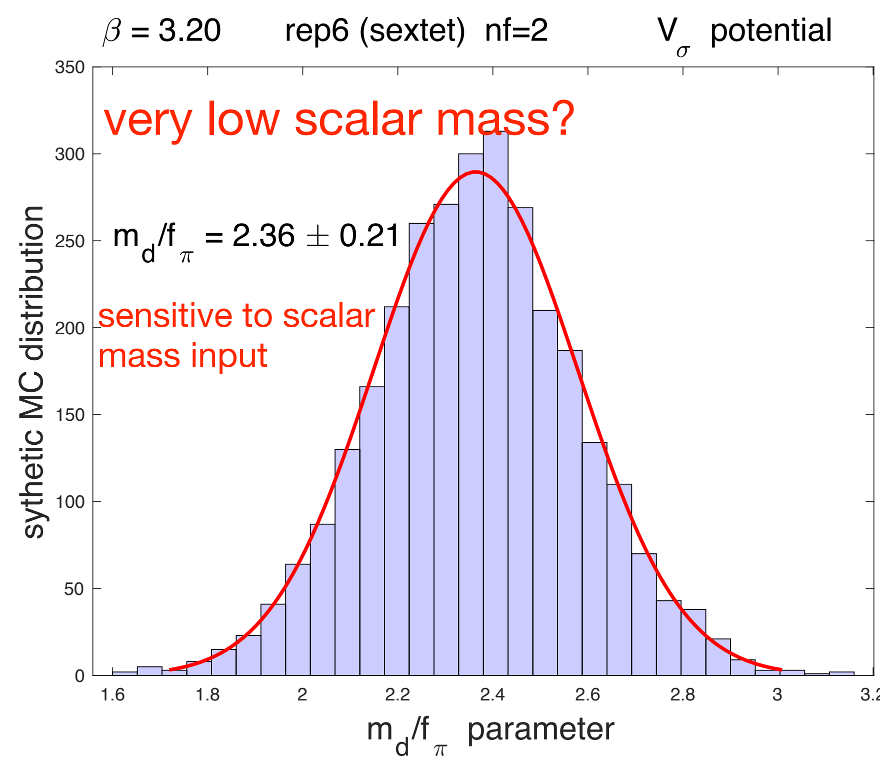
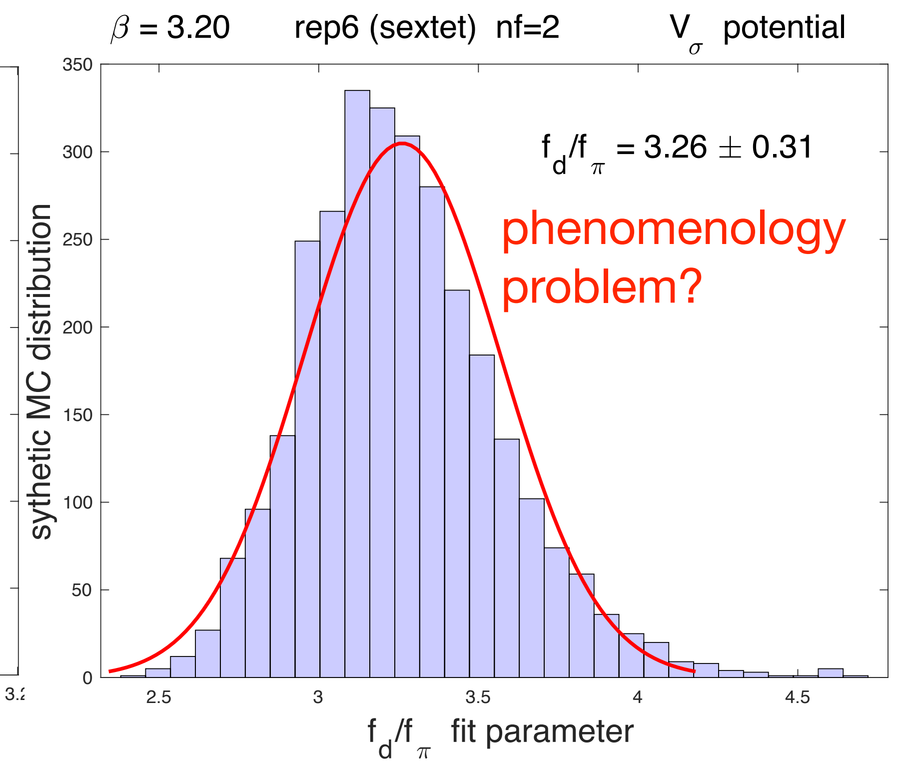
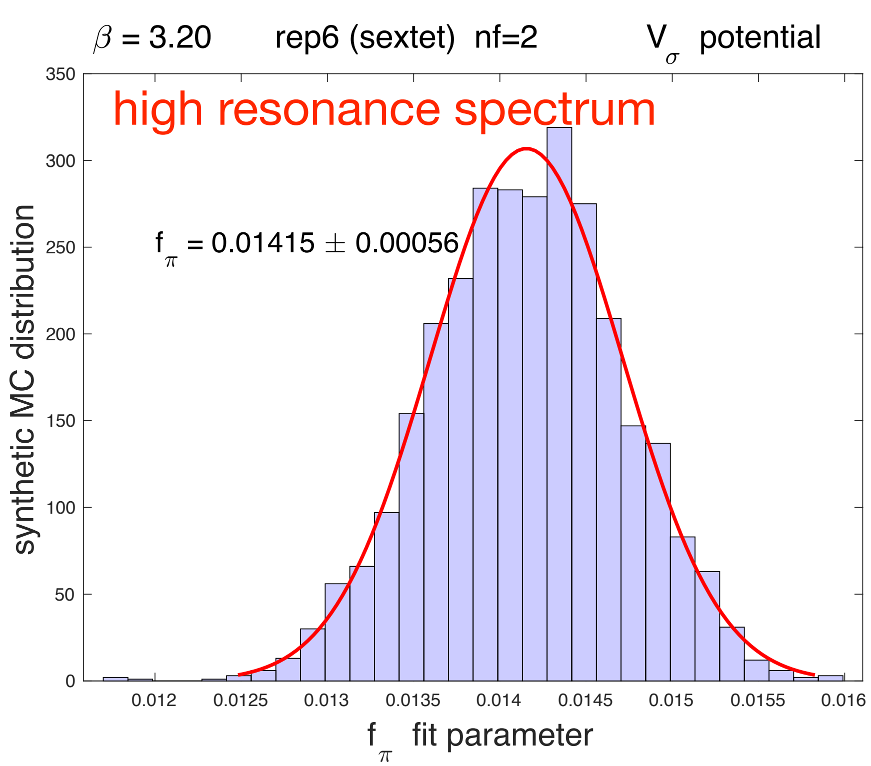


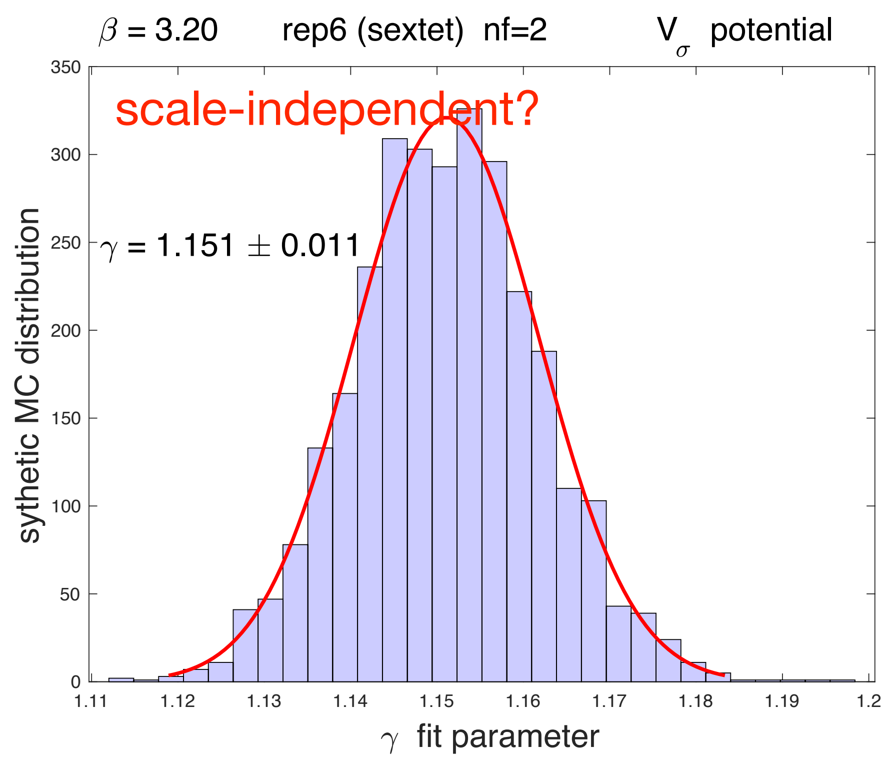
dilaton EFT with $\sigma(x)$ dilaton field and $\pi^a(x)$ Goldstone bosons











- the dilaton potential of Shamir-Golterman as tree level theory expanding around IRFP is not working well for sextet data
- drifting γ ignored?
- consistent LatHC analysis for two lattice spacing, with third under construction
- taste breaking issue?

to reach the chiral regime requires two orders of magnitude drop in fermion mass: switch from p-regime to epsilon regime and related RMT

epsilon regime and RMT

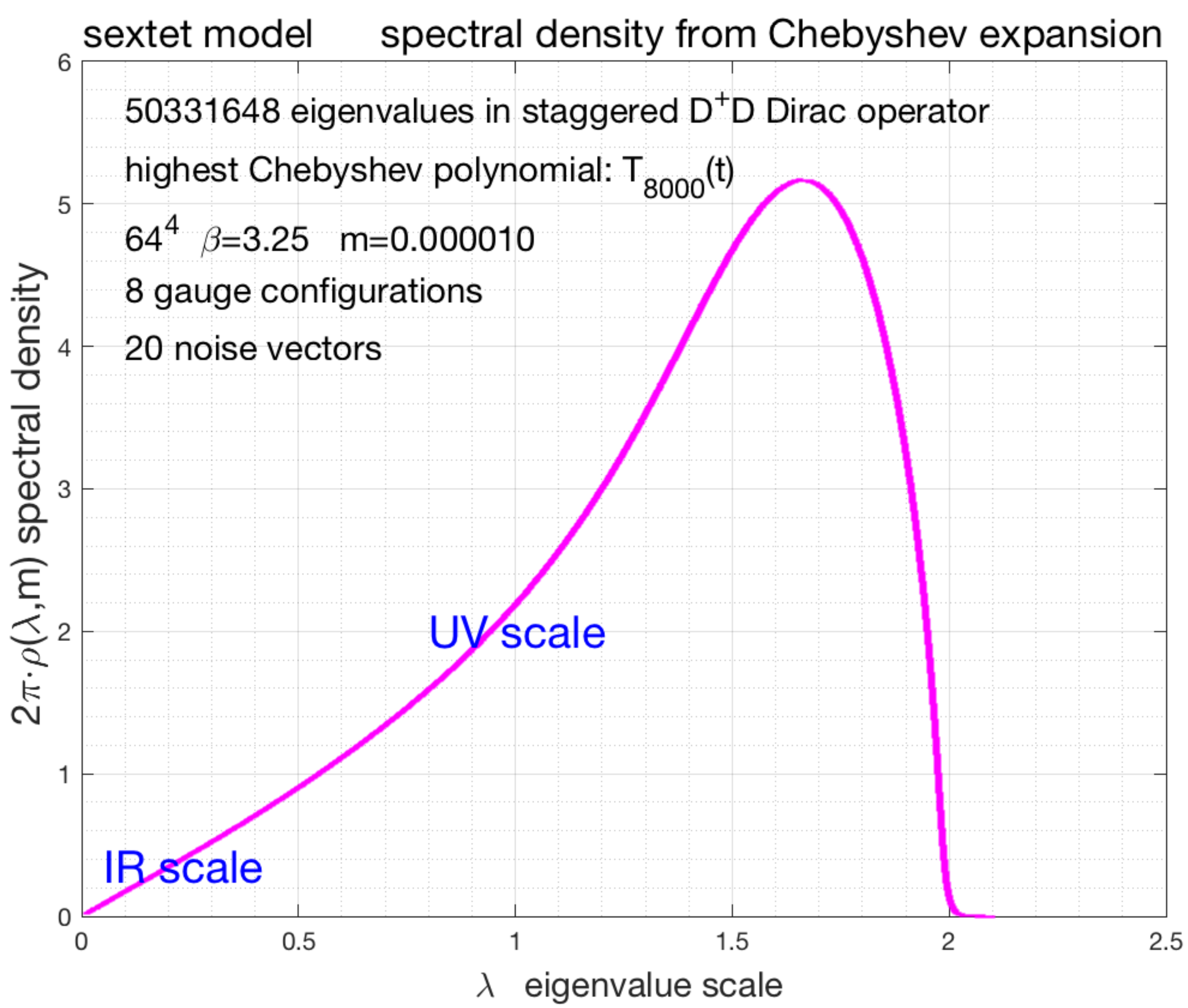
$$\mathscr{L}_\varepsilon = \frac{1}{2}\partial_\mu\chi\partial_\mu\chi - V_d(\chi) + \frac{m_\pi^2 f_\pi^2}{4}\left(\frac{\chi}{f_d}\right)^y \mathrm{tr}\left[\Sigma_0 + \Sigma_0^\dagger\right]$$

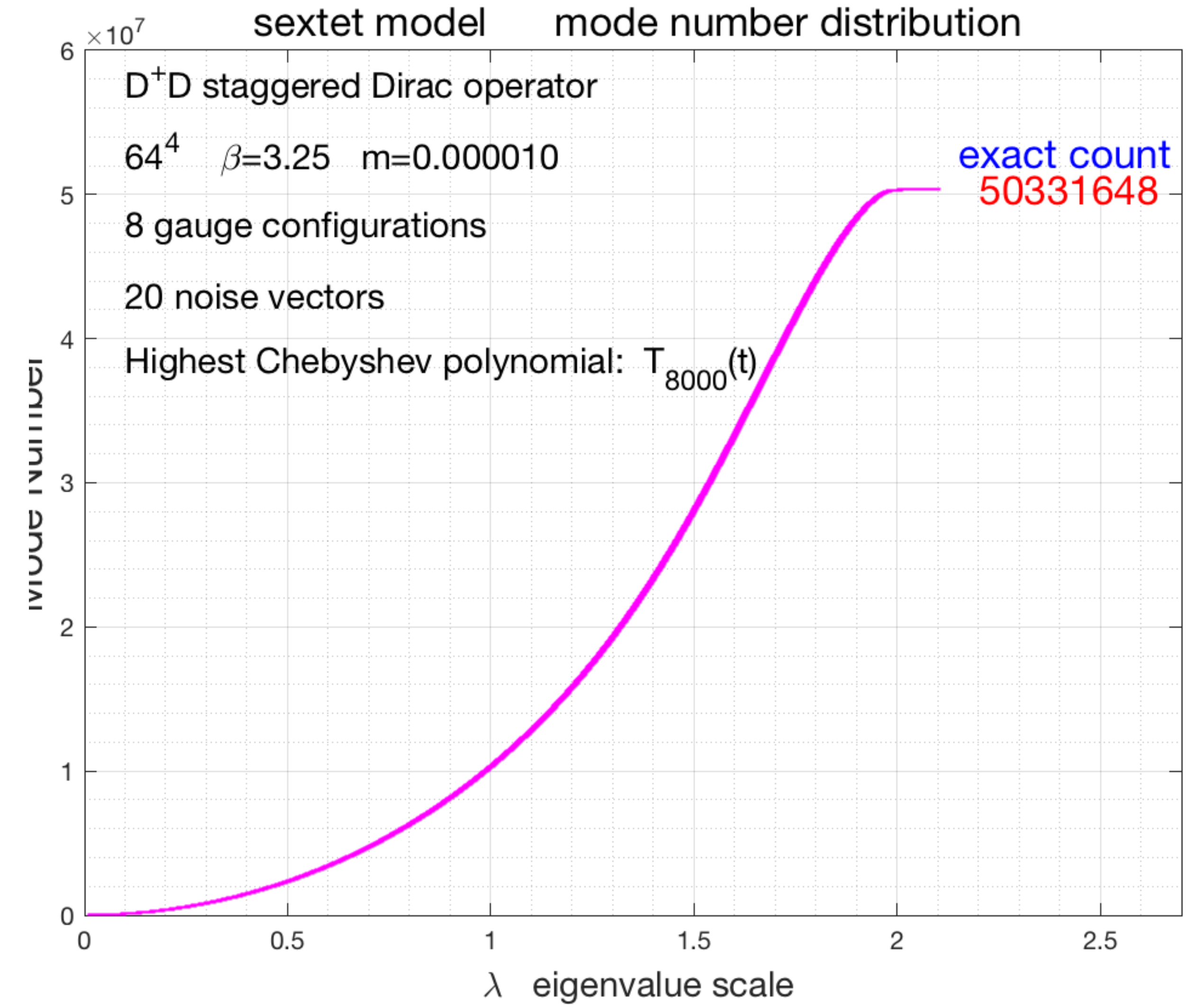
epsilon regime with very small fermion mass deformation

$$\mathscr{L}_\delta = \frac{1}{2}\partial_\mu\chi\partial_\mu\chi - V(\chi) + \frac{f_\pi^2}{4}\left(\frac{\chi}{f_d}\right)^2 \mathrm{tr}\left[\partial_t\Sigma_0\,\partial_t\Sigma_0^\dagger\right]$$

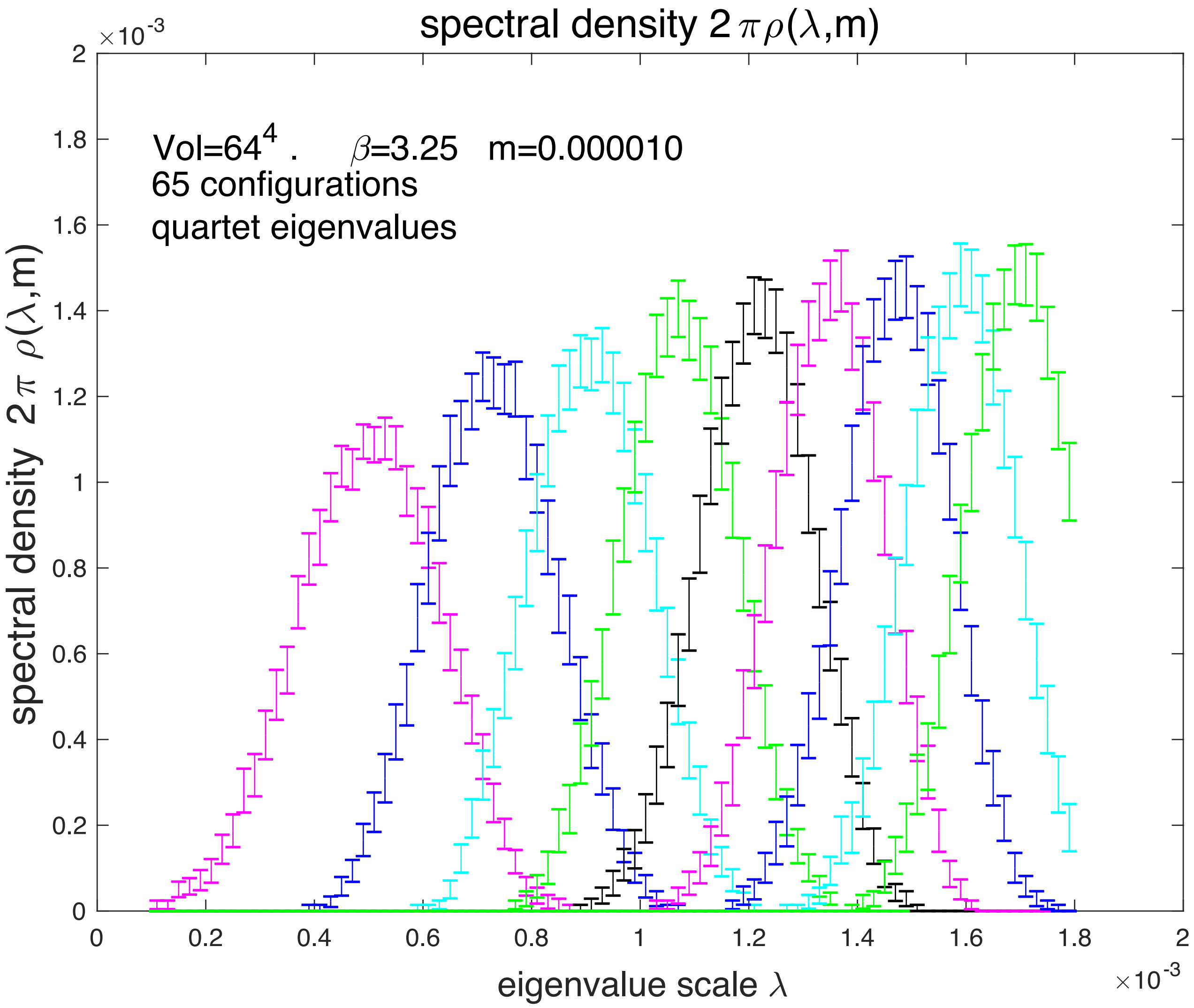
delta regime m=0
very small fermion mass deformation
can be added

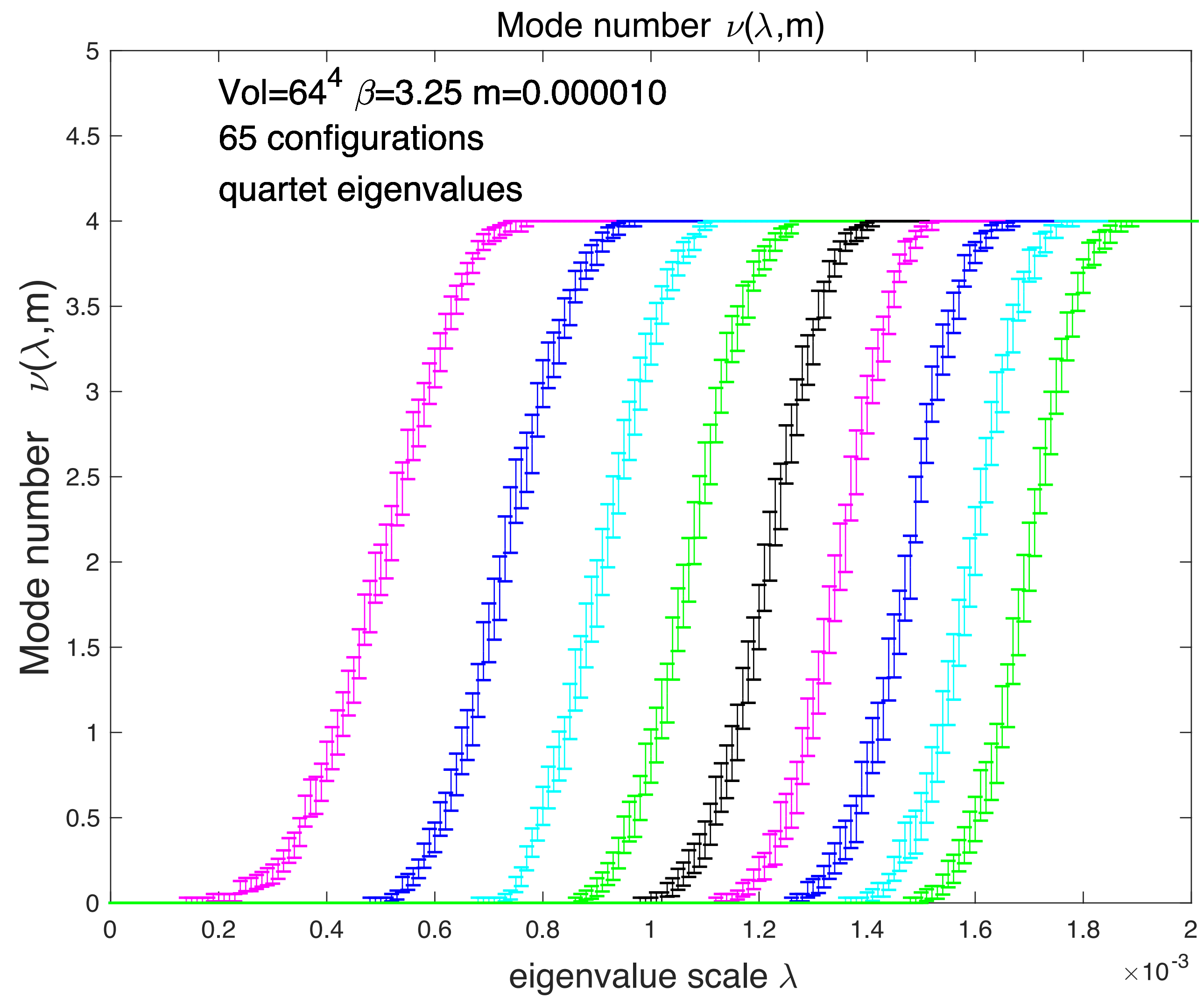
new ensembles at equivalent p-regime pion mass Mpi ~ 100 and volume size 64^4





epsilon regime and RMT





successful testing

ongoing analysis (preliminary results not shown)

Conclusions and outlook

- Idea of walking from complex CFT is attractive, rep independent
- Needs EFT description on several scales, far IR is σ-model
- Does not imply the existence of the dilaton
- EFT of the dilaton remains attractive possibility
- Its tests above the linear σ-regime has issues
- Unified framework for complex CFT and the dilaton?