Case studies of near-conformal and conformal beta functions

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on behalf of the Lattice Higgs Collaboration (LatHC):

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Overview

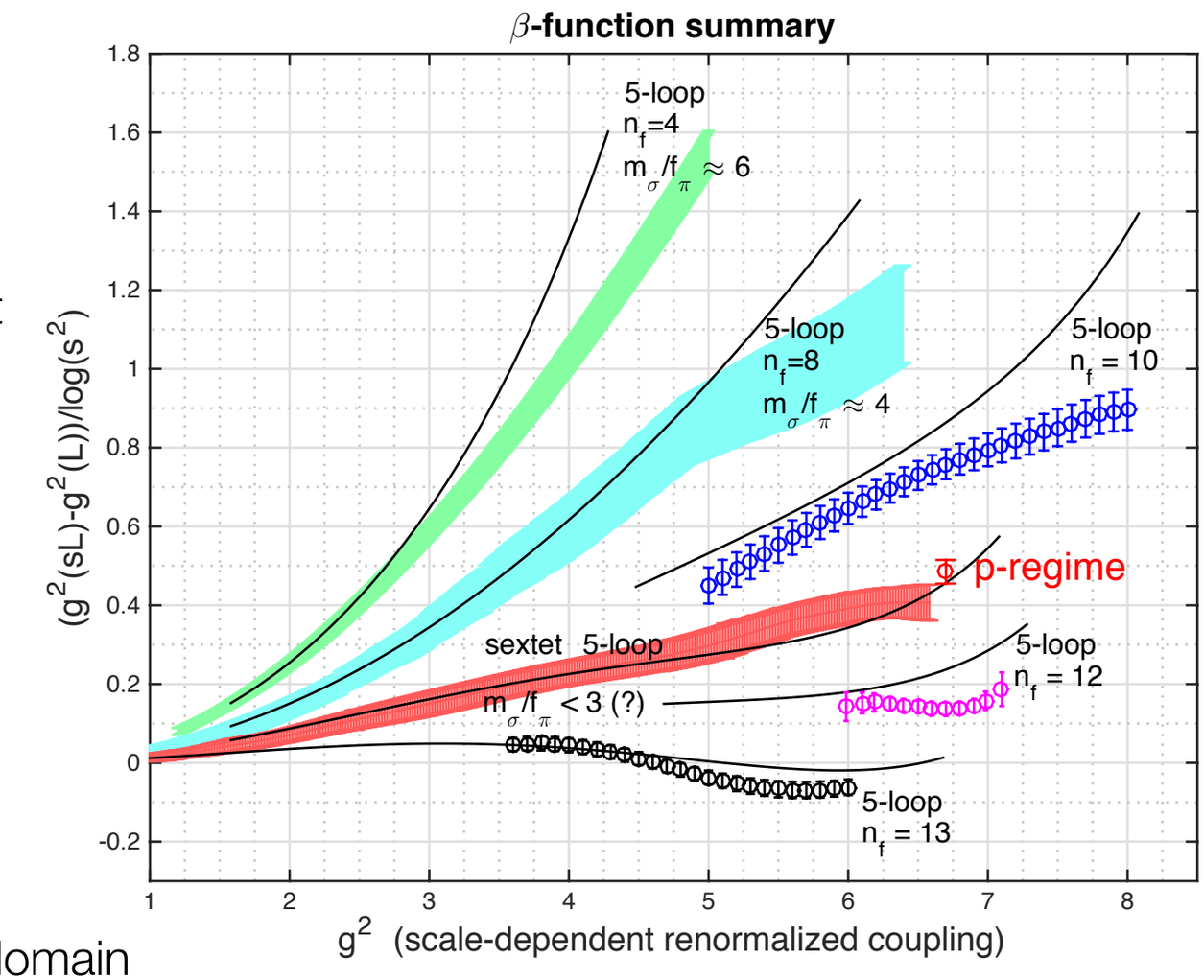
there have been many studies of the beta-function in the search for (near) conformal behavior by many groups — too many to summarize all

several years of **LatHC** studies shows a consistent trend of decreasing beta function with increasing flavor number Nf

combined with other work, a steadily lighter scalar emerges, the hoped for Higgs impostor

but how sure are we about all of this?

and what is new this year?



Nf refers to the number of massless flavors

1. there are recent discrepancies for Nf = 10 with domain wall studies - can we clear this up?

2. Nf = 12 is in a very delicate region - is the evidence in favor of the theory being near-conformal any stronger?

3. Nf = 2 sextet has been our flagship as a BSM candidate, what improvements can we show in the beta-function

Overall theme: are systematic effects under control?

Method

same technology is used across studies:

renormalized g^2(L) measured through the gradient flow on finite volume L with fixed $c = \sqrt{8t}/L$

beta function (g^2(sL) - g^2(L)) / log(s^2) from finite scale change L -> sL

step scaling with constant g^2(L) for many volumes to take continuum limit

example: $L = 16, 18, 20, 24, 28, 32$

$$\downarrow$$

$$2L = 32, 36, 40, 48, 56, 64$$

simulations needed on all these volumes, at various bare couplings

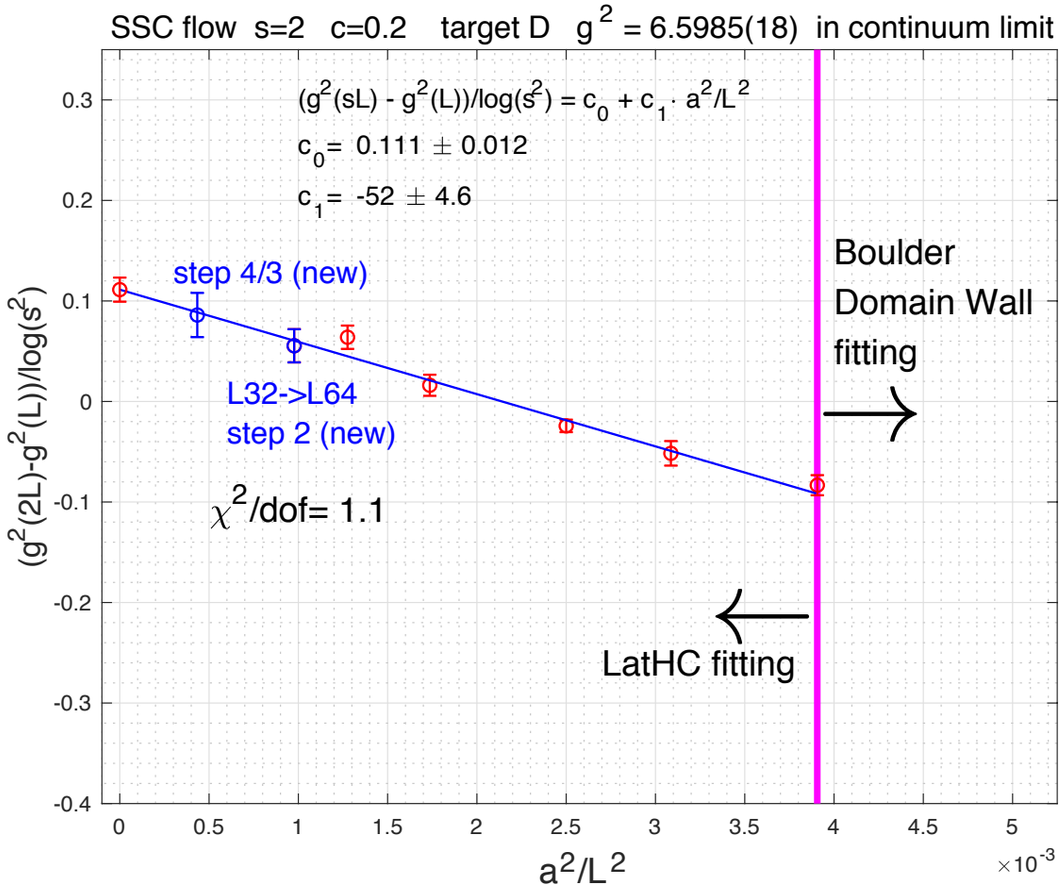
among the systematic effects:

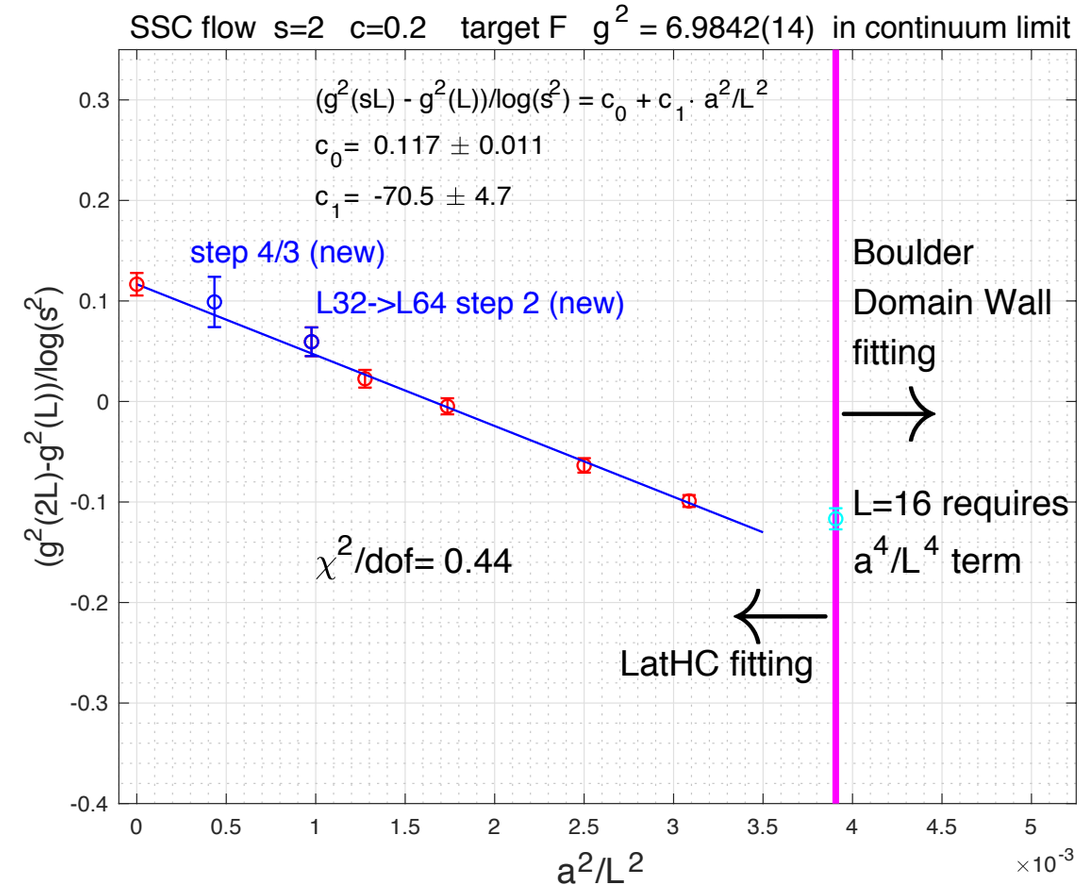
discretization of the gradient flow

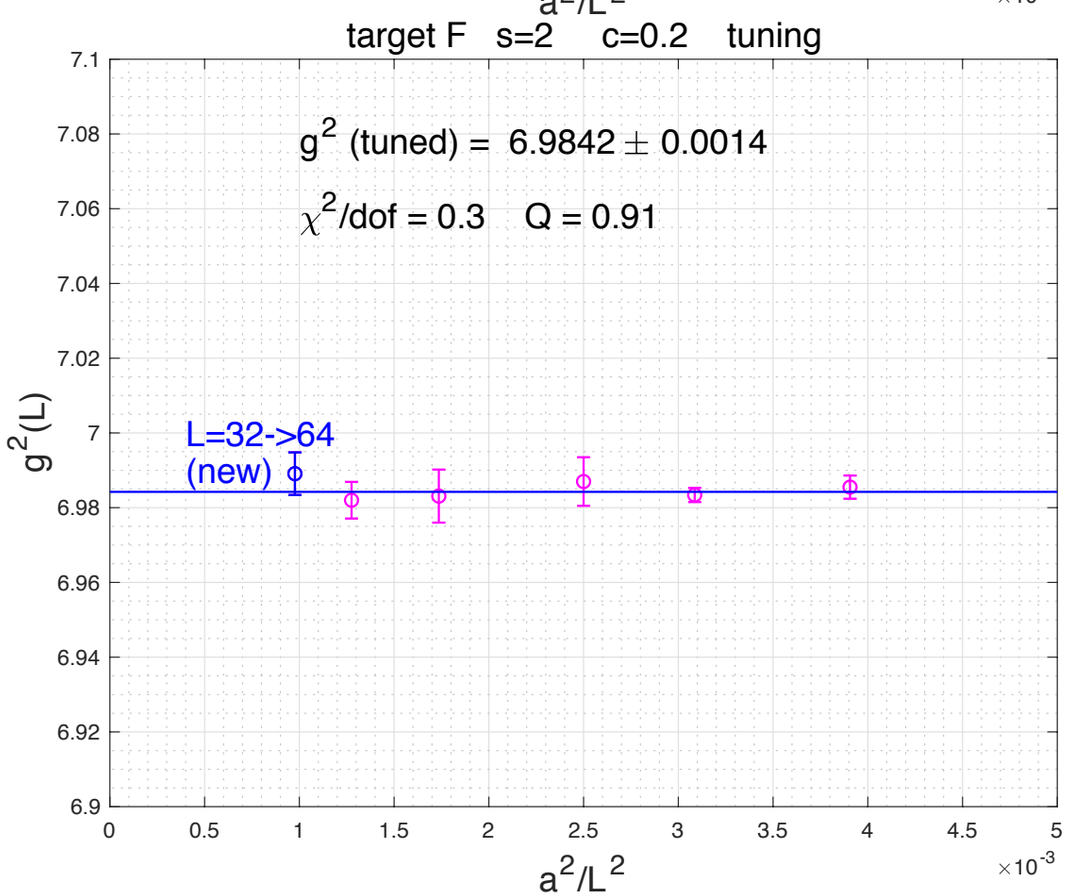
number of lattice spacings available for continuum extrapolation

choice of s value for scale change

choice of c value for renormalization scheme







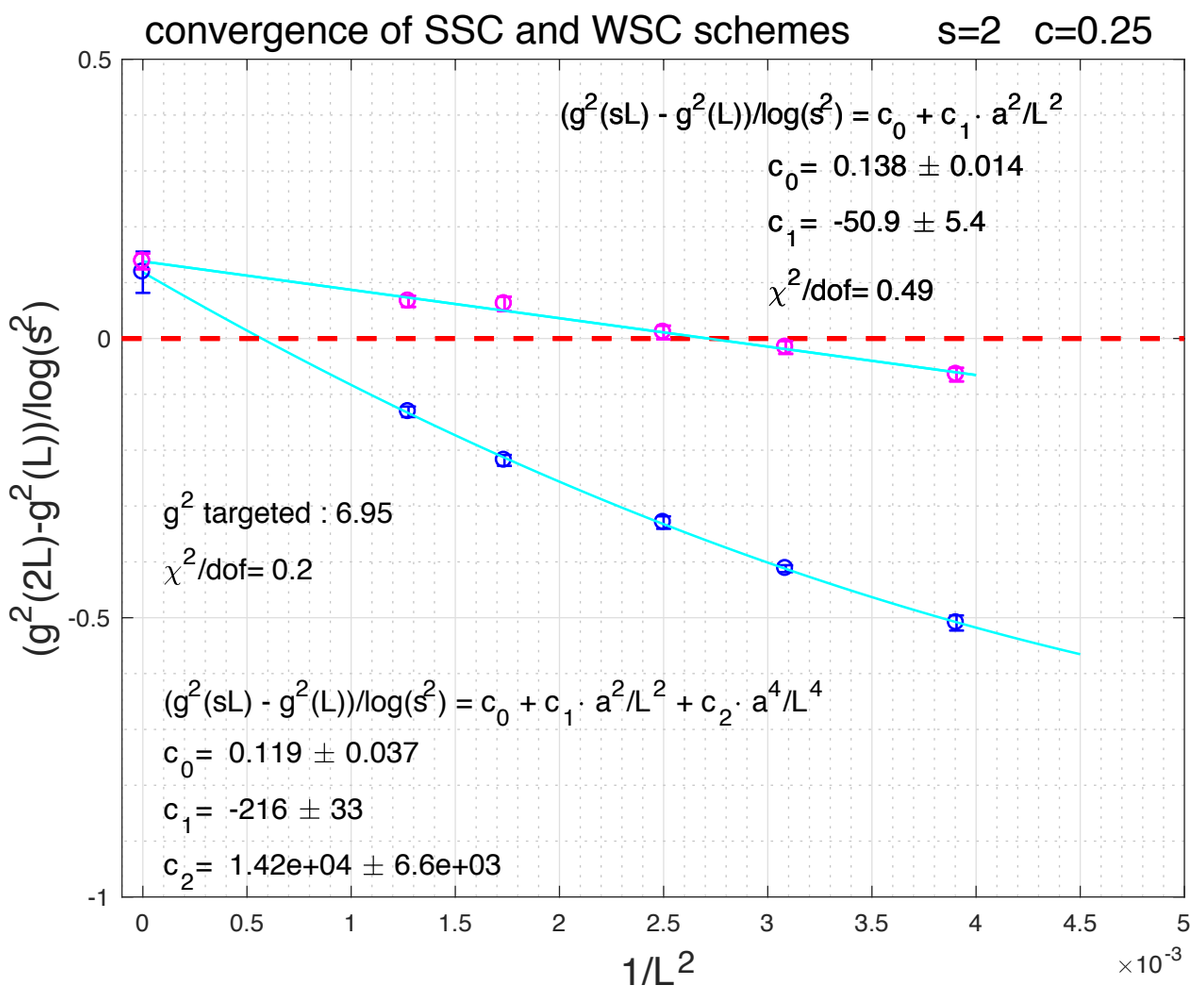
Nf = 12: previous claims of an infrared fixed point in a range where we simulate and found none

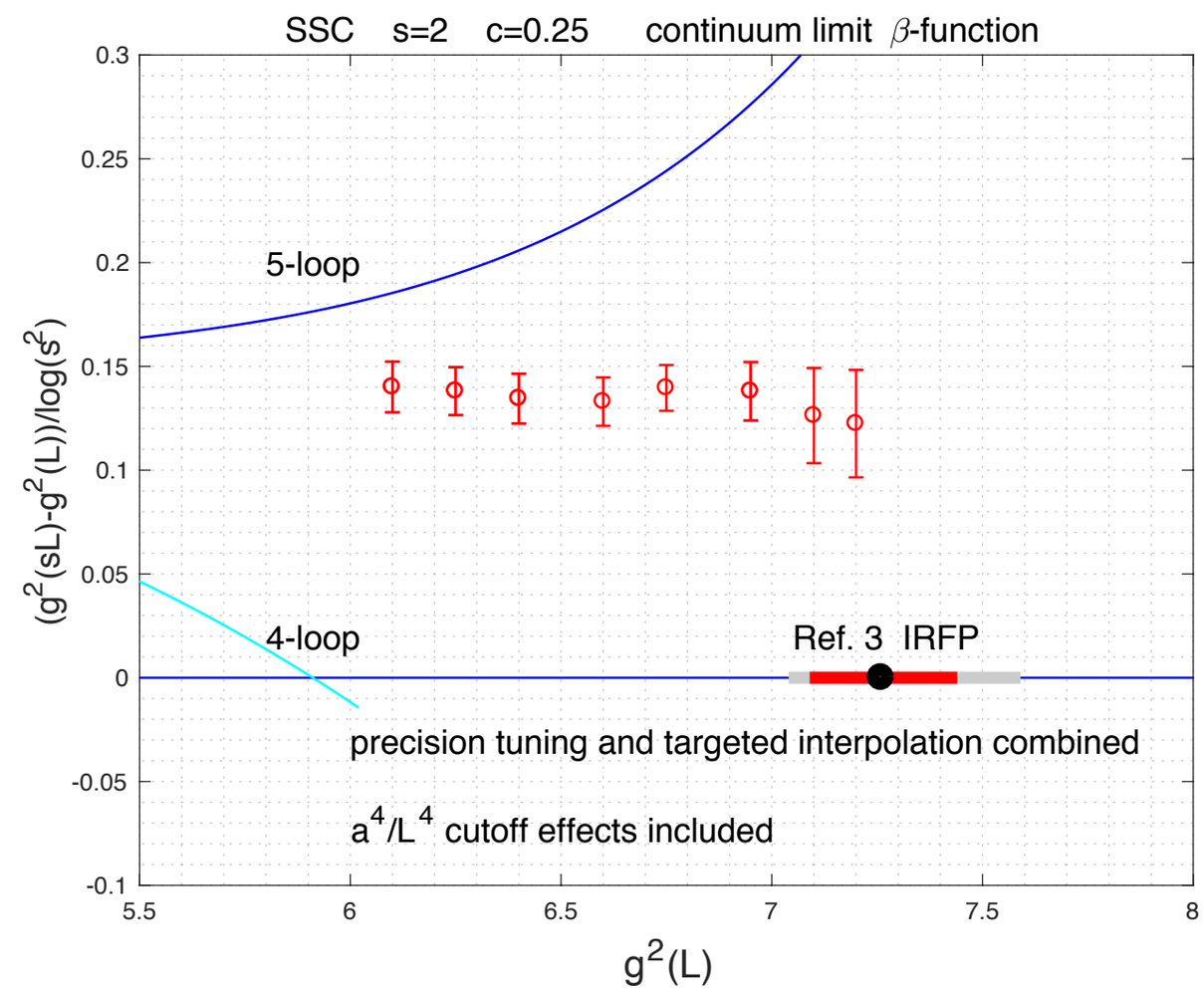
new this year: simulate even closer to the continuum with additional data for s = 2 with 32 -> 64

new data point is right in the location where previous fits would indicate and gives a consistency test with s = 4/3 with 48 -> 64, strengthening case against IRFP

g^2(L) tuned, not interpolated; O(a^2) good from L >= 18

consistent results: beta-function small but non-zero





Nf = 12

same behavior at c = 0.25 different renormalization scheme LatHC results from PLB 779 (2018) 230

very good agreement in continuum limit between two gradient flow discretizations

beta-function small but non-zero ***this is not a question about the value of c***

something else new for **Nf = 12**:

anomalous mass dimension gamma

method presented at Lattice 2018 for **Nf = 13**

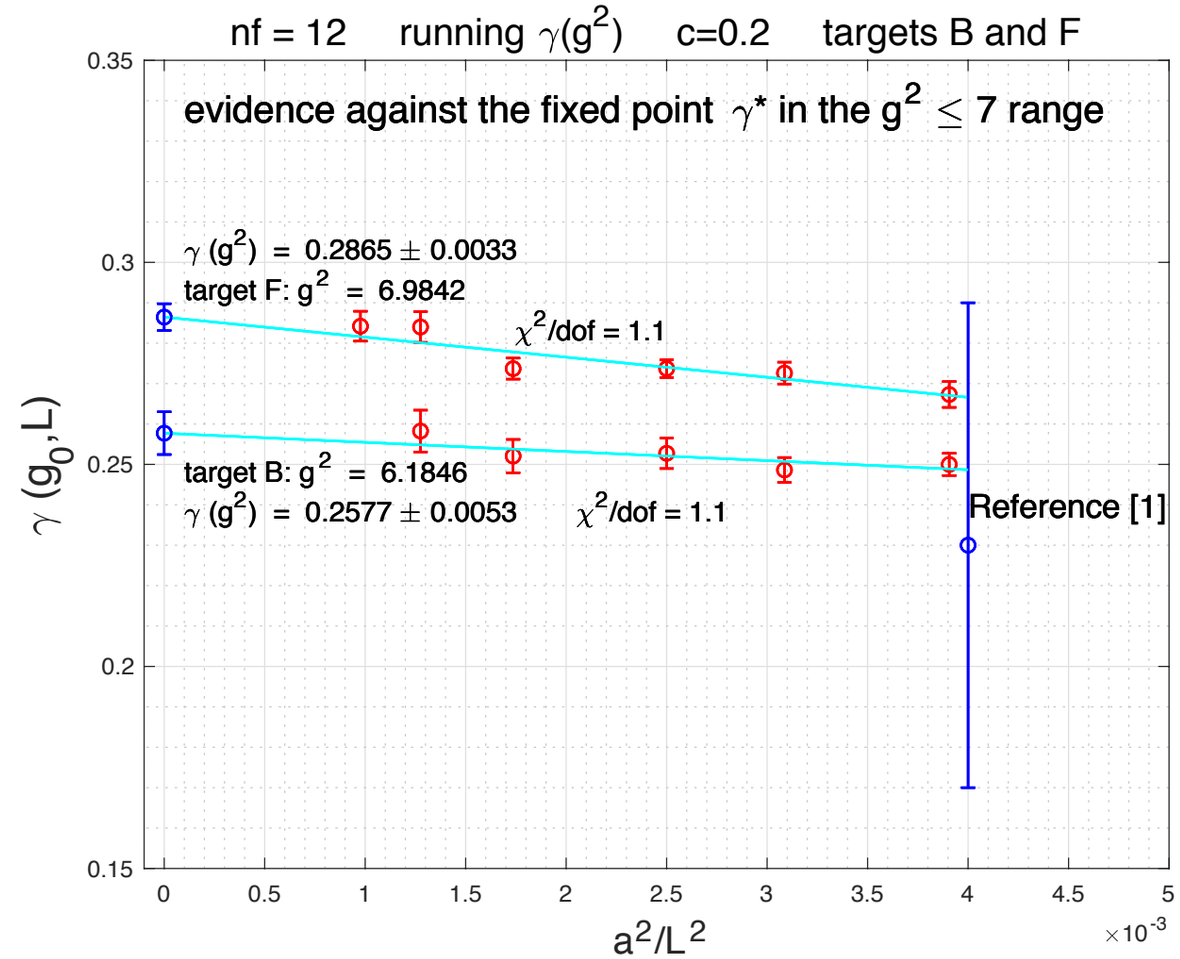
use the mode number via Chebyshev expansion

$$\rho(t) = \frac{1}{\sqrt{1-t^2}} \sum_{k=0}^{\infty} c_k T_k(t)$$

$$\nu(\lambda) = \int_0^{\lambda} \rho(\lambda') d\lambda'$$

$$\nu_R(\lambda_R) = \nu(\lambda), \quad \lambda_R = Z_p^{-1} \cdot \lambda$$

$$\lambda_L = \frac{c_\lambda}{L}$$



match mode numbers on different volumes

$$\frac{\lambda_L}{\lambda_{sL}} = \frac{s Z_p(g_0, L/a)}{Z_p(g_0, sL/a)} \qquad \gamma = \frac{\log Z_p(L)/Z_p(sL)}{\log(s)}$$

continuum limit through step-scaling

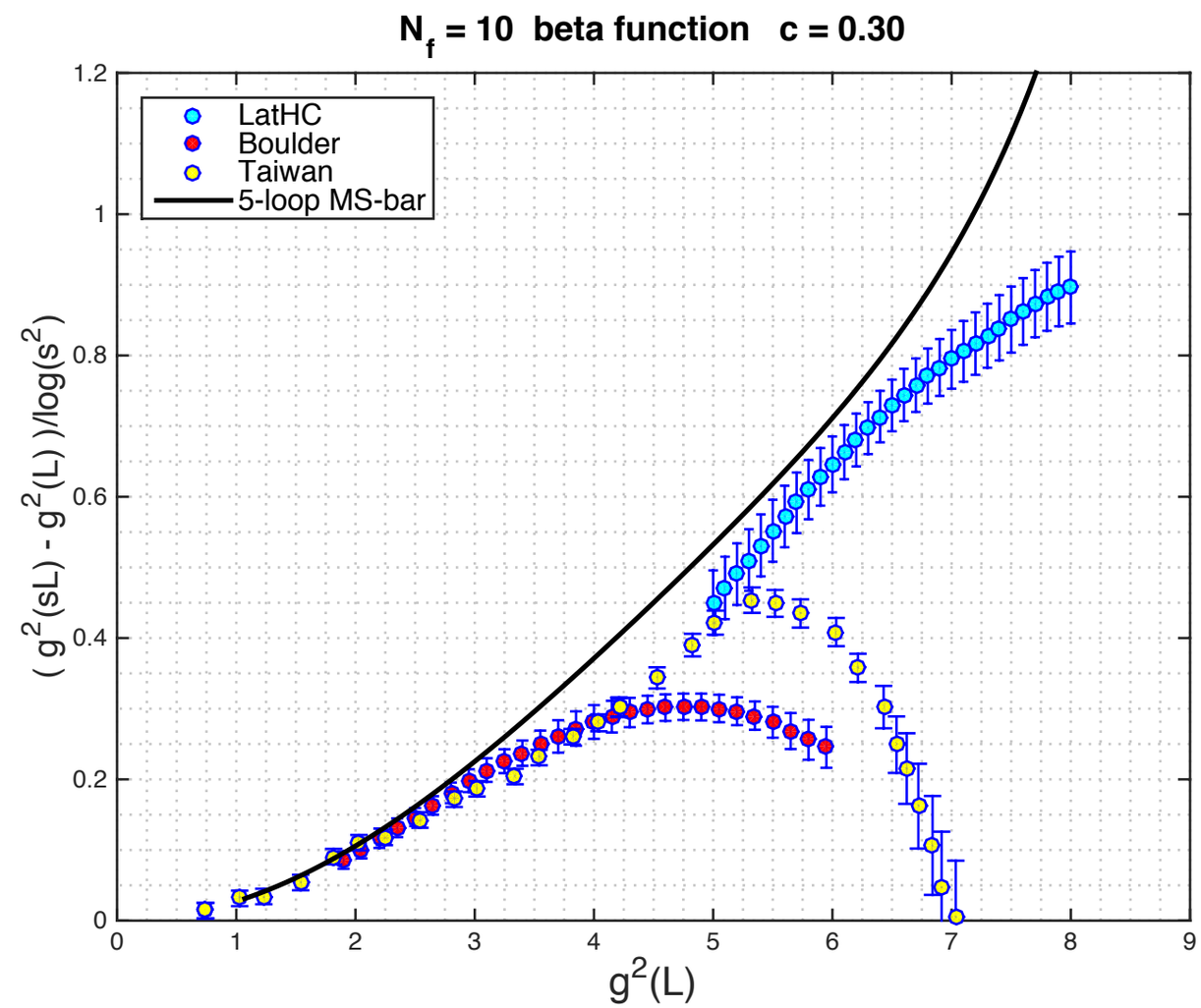
method works well: high accuracy, small cutoff effects

anomalous mass dimension gamma running with g^2 — no indication of flowing to a fixed point value

the **Nf = 10** model is an important anchor point:

the theory can be a BSM template with either mass splitting e.g. 4 + 6,
or with the Higgs as a pseudo-Nambu-Goldstone state — in either situation, would want to know if the Nf = 10 massless model is conformal or not

If lattice simulations can't be definitive about Nf = 10, it is hard to believe claims for larger values of Nf, or other models with a reportedly smaller beta function



This time last year, there were discrepancies between 3 groups reporting results, each with continuum extrapolation

LatHC: staggered

Boulder: Hasenfratz, Rebbi, Witzel ; domain wall

Taiwan: T.W. Chiu ; domain wall

There is striking difference between the 3 results in the region g^2 ~ 5 — 6, as well as huge inconsistency between a claimed infrared fixed point at g^2 ~ 7 as opposed to a far-from-zero beta function quite similar to the 5-loop perturbative prediction

Among the claims were that staggered fermions might be in the wrong universality class

is there anything new in the meantime?

updates on the domain wall studies have shown significant changes:

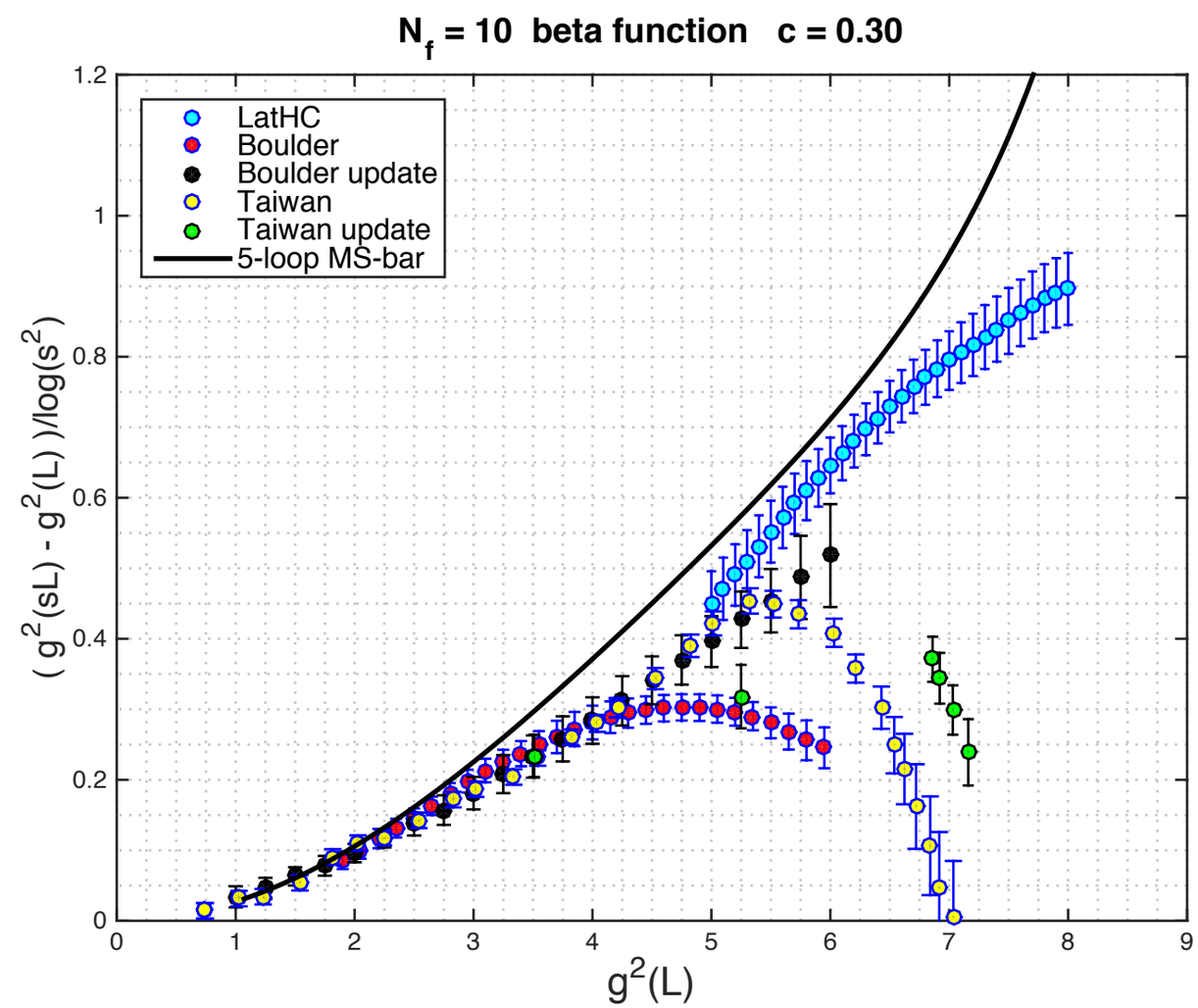
Boulder: an increase in the beta-function in the range g^2 ~ 4.5 — 6

Taiwan: a decrease at g^2 ~ 5.2, and an increase around g^2 ~ 7

at this point there is no statistical difference between the LatHC and Boulder results, while the Taiwan result remains significantly smaller

The methodology of using the gradient flow to measure the beta-function is the same for each work e.g. step-scaling of finite volumes for the continuum extrapolation

Importantly, we can compare for exactly the same renormalization scheme set by the choice $c = \sqrt{8t}/L$ so there should be no quantitative difference between different studies

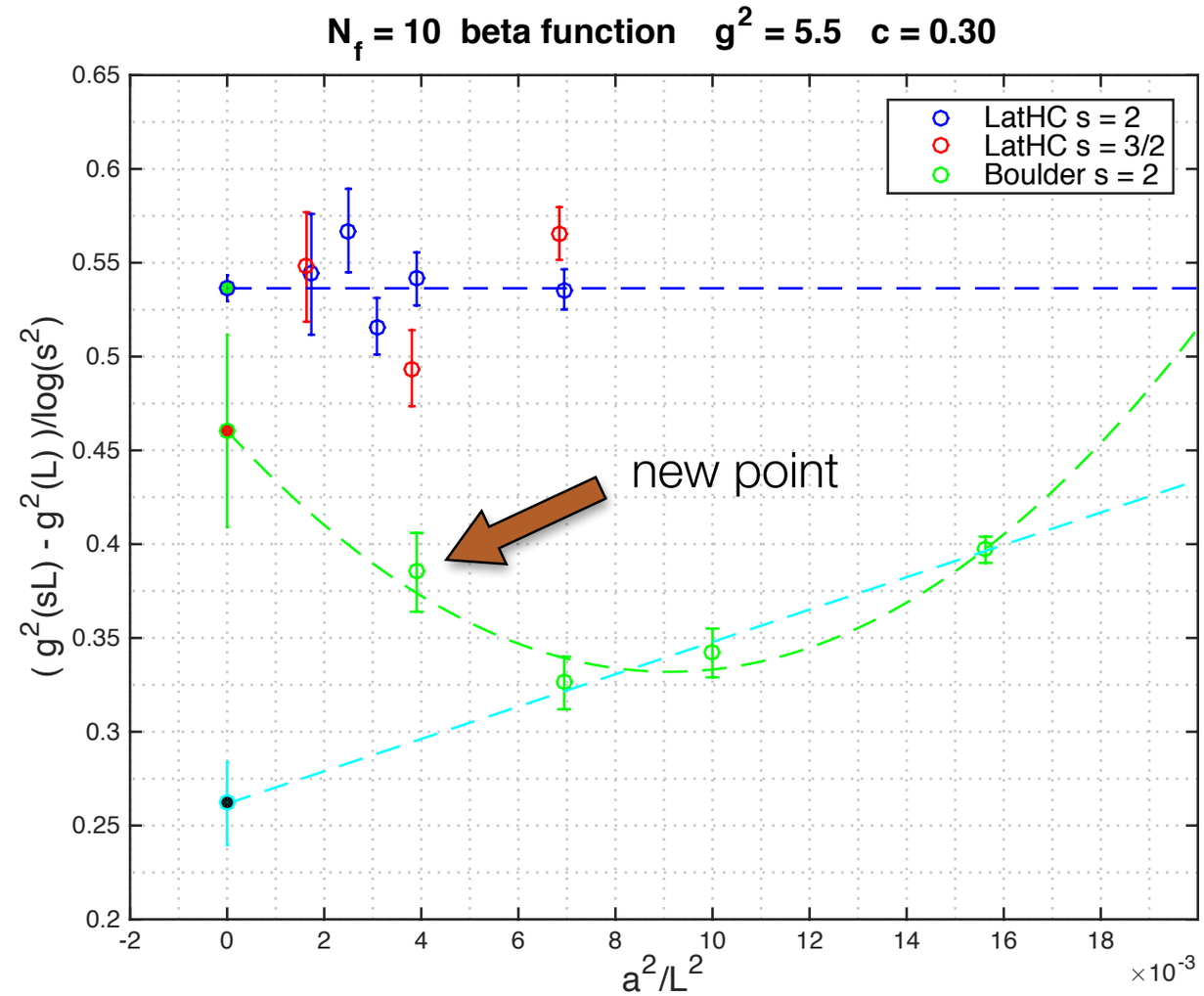


cutoff effects appear in the finite-step beta-function with L -> sL at O(a^2), and corrections of O(a^4) can be significant

lattice artifacts can be different in magnitude for different discretizations of the gradient flow — will return to this point later

LatHC data set: $12/16/18/20/24$
$\downarrow$
$24/32/36/40/48$

Boulder data set: $8/10/12$ $\downarrow$ $16/20/24$ **new point** 16 $\downarrow$ 32

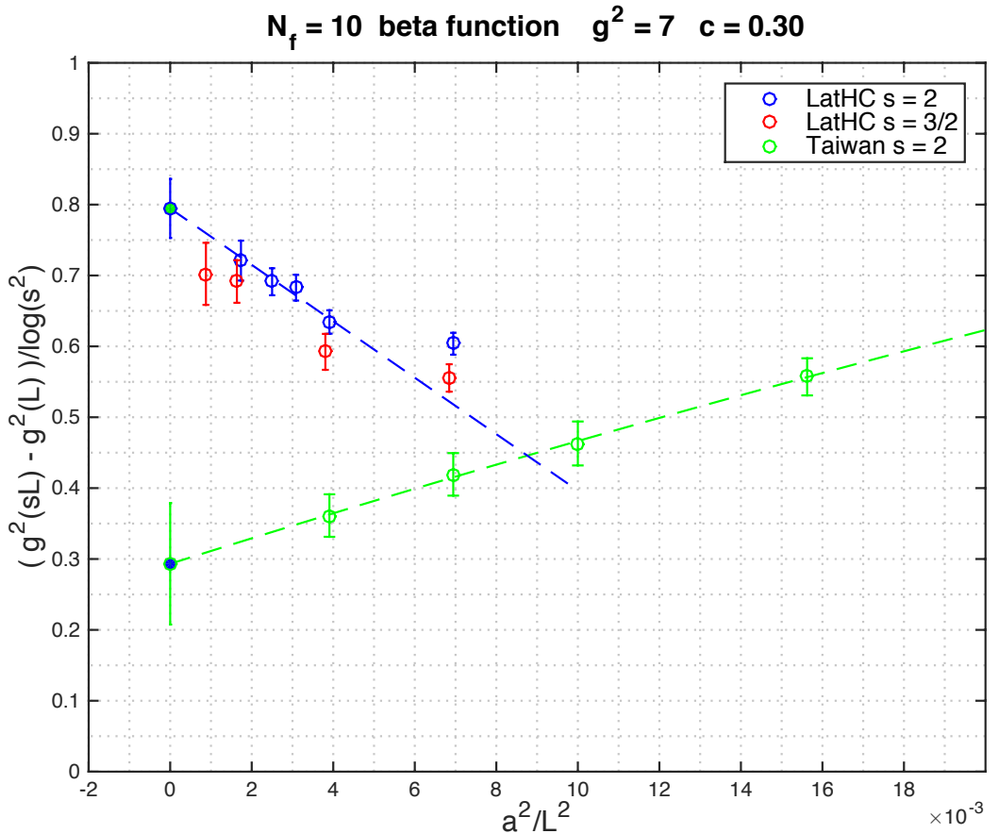


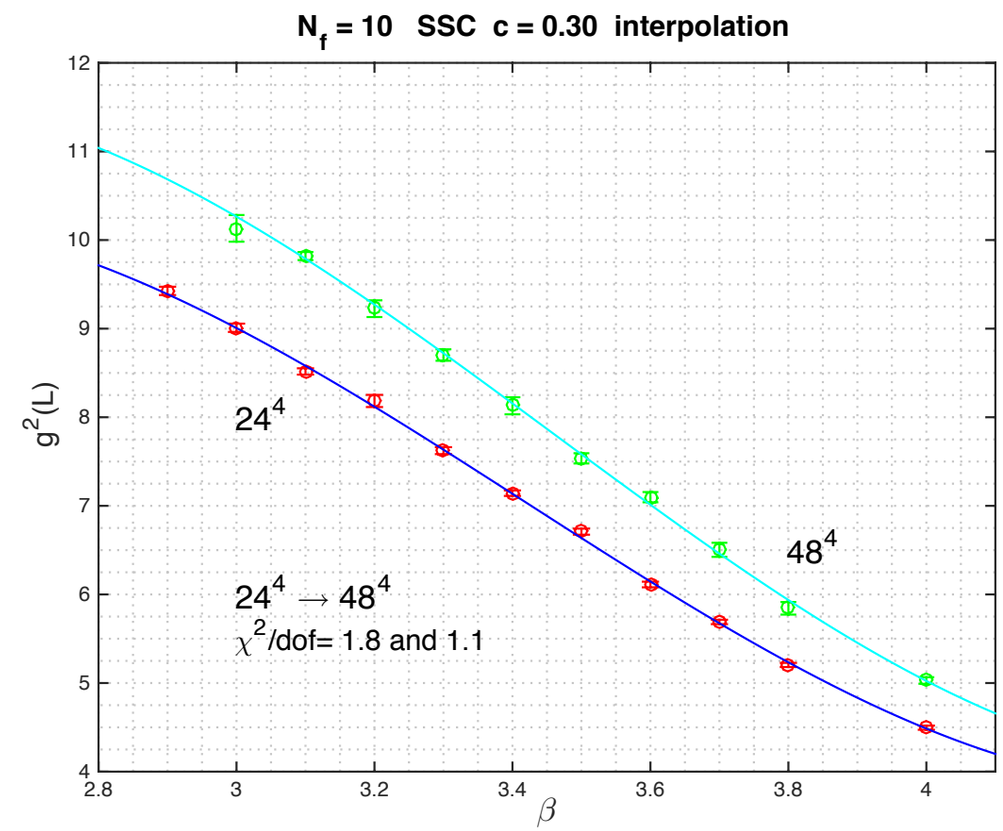
Typically across the range of g^2 we find O(a^2) describes the data for L >= 16 well, and L = 12 can be included with O(a^4). At this particular coupling g^2 = 5.5, the cutoff effects in our data appear to be accidentally small and the s = 2 data can be fit with a constant.

With the new point L = 16 for the Boulder work, O(a^2) is no longer a good fit starting from L = 8, and including an O(a^4) term gives a continuum result statistically in agreement with ours.

Our data also allow an analysis with s = 3/2 and we find very good consistency with the s = 2 results

Being able to reach to finer lattice spacing with L = 18, 20, 24 with staggered fermions is crucial





Taiwan update: interpolation in bare coupling across a range of g^2 replaced by tuning of bare coupling at a few target values of g^2; same set of lattice spacings as Boulder i.e. L = 8/10/12/16 on the smaller volumes for L -> sL. Switch from interpolation to tuning gives increase in beta function at stronger coupling.

Somewhat surprisingly an O(a^2) fit of the Taiwan data works down to L = 8, we see O(a^4) becoming relevant already at L = 12.

gradient flow discretization: **Flow-Simulation-Observable**
Taiwan: Wilson-Wilson-Clover **LatHC**: Symanzik-Symanzik-Clover

smallness of cutoff effects at tree-level for any discretization not relevant at Nf = 10 where dynamical fermion effects are essential — otherwise we're simply studying pure Yang-Mills

Our interpolation O(beta^3) across a narrow beta range, not extended to cover weak coupling as well

remaining systematic?: domain wall residual mass, which grows quickly with renormalized g^2

Nf = 10

concerns about systematic errors for staggered fermions?
non-degeneracy of Dirac eigenvalues:
colored lines: very close pairs, quartets are split

calculate reference Dirac determinant from arithmetic average of eigenvalue quartets

difference between staggered and quartet-averaged determinants should vanish in the continuum limit to generate gauge ensemble with correct weight

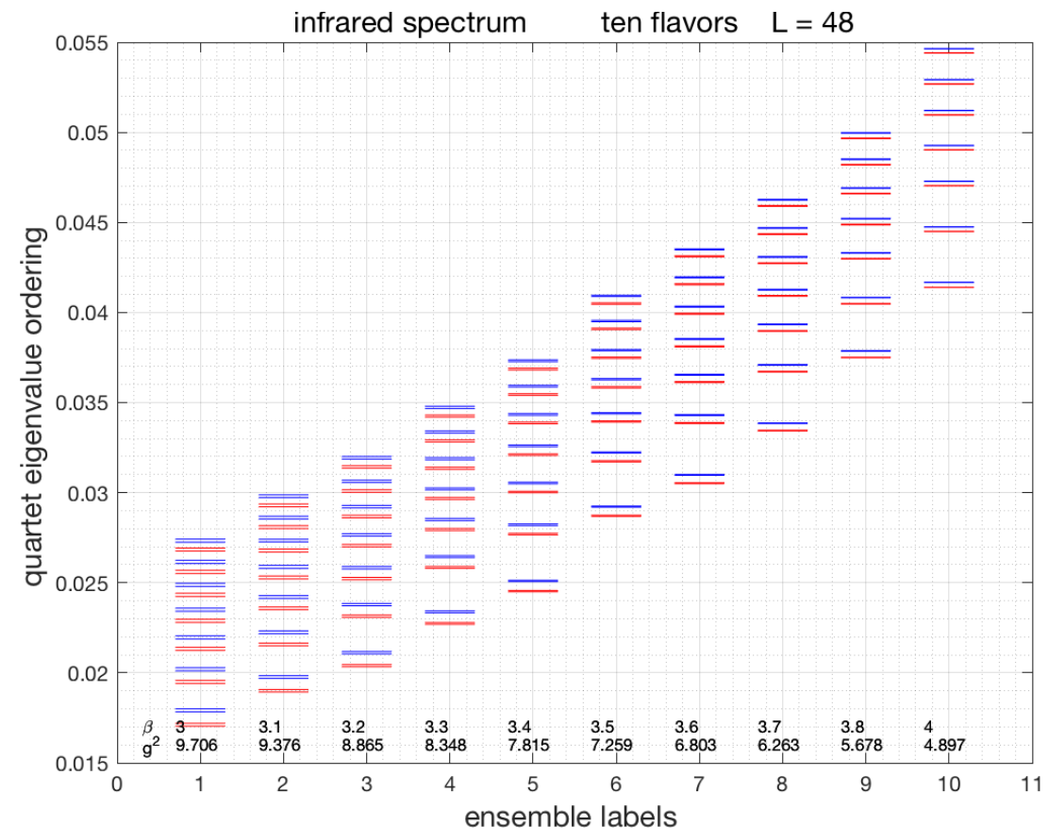
not feasible to calculate the entire Dirac spectrum numerically, compare determinants up to an eigenvalue scale lambda_max set by the lattice volume

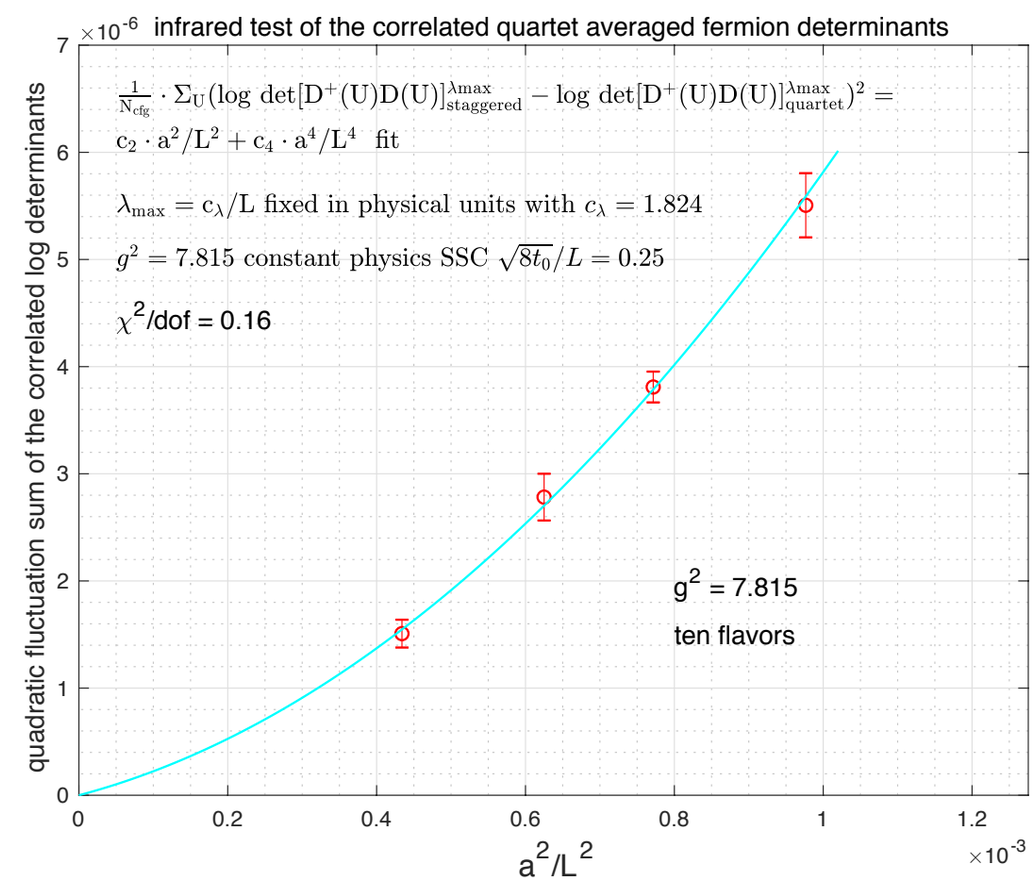
$$\lambda_{\max} = c_\lambda / L$$

at fixed renormalized g^2

data at L = 32, 36, 40, 48 consistent with O(a^2,a^4) behavior, difference vanishes as a -> 0, as hoped for

determinant test showing that taste breaking of staggered fermions appears under control



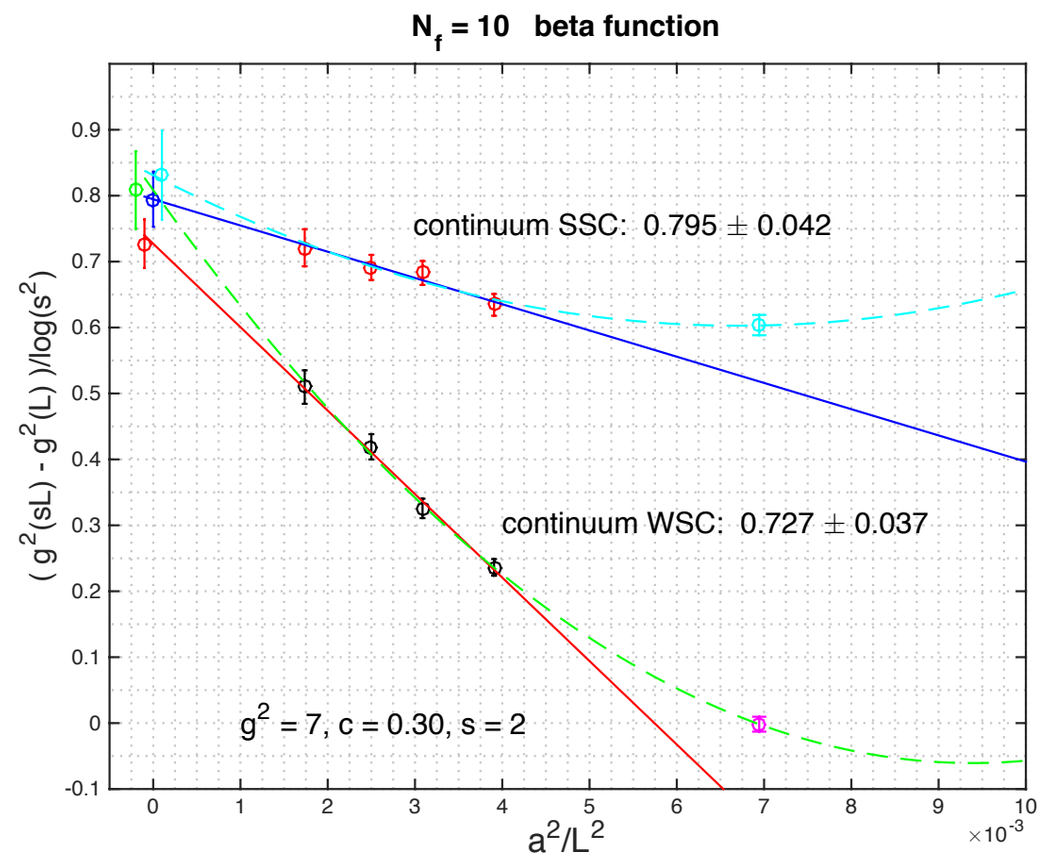


test consistency of continuum extrapolations by varying gradient flow discretization

Flow-Simulation-Observable

e.g. Wilson-Symanzik-Clover **WSC**

4 versions: **SSC**, **WSC**, **SSS**, **WSS**



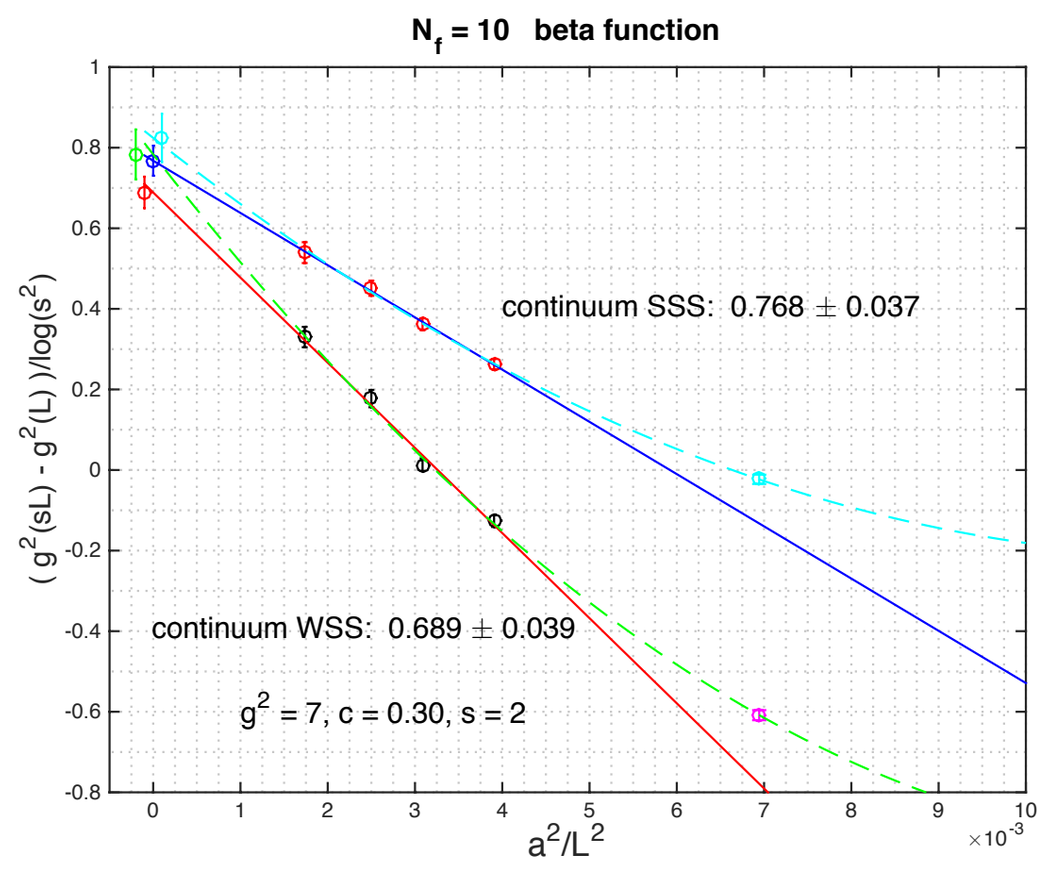
continuum extrapolation:

O(a^4) with L = 12, 16, 18, 20, 24

O(a^2) excluding L = 12

very nice agreement among the various combinations

cutoff effects in **SSC** are mild — use as reference

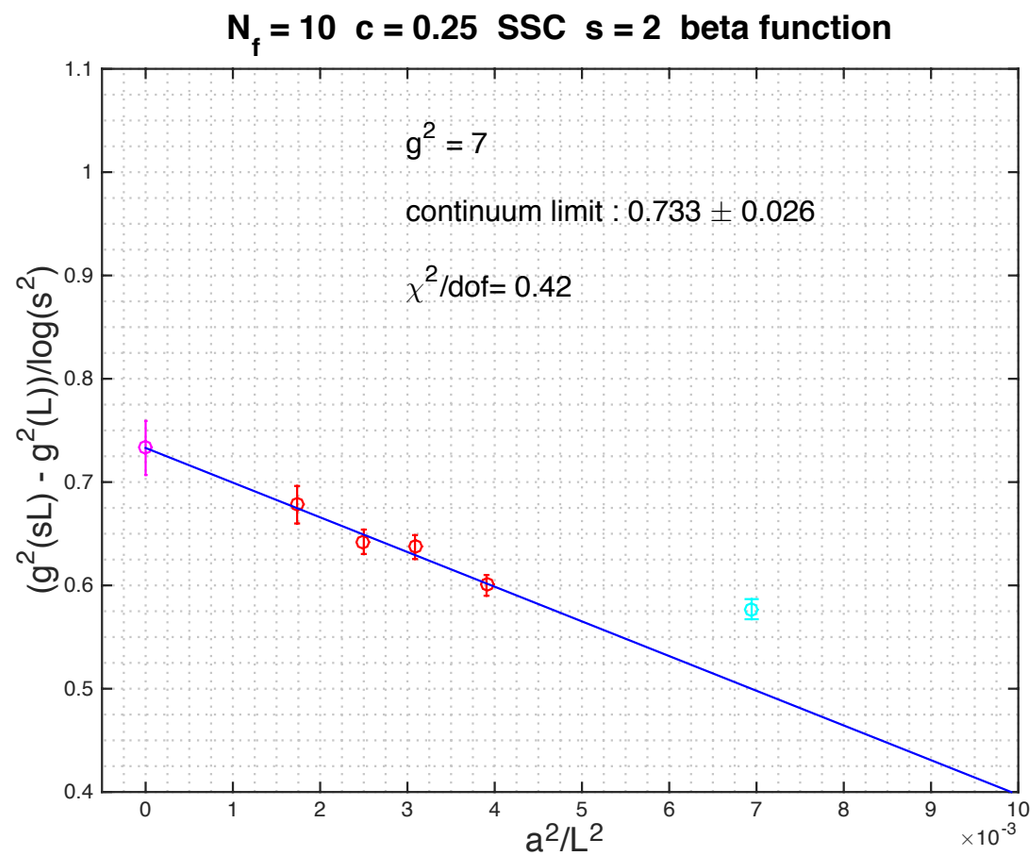


vary the renormalization scheme through choice of c value

smaller: c = 0.25

lattice artifacts are actually not larger than at c = 0.3, contrary to some expectations

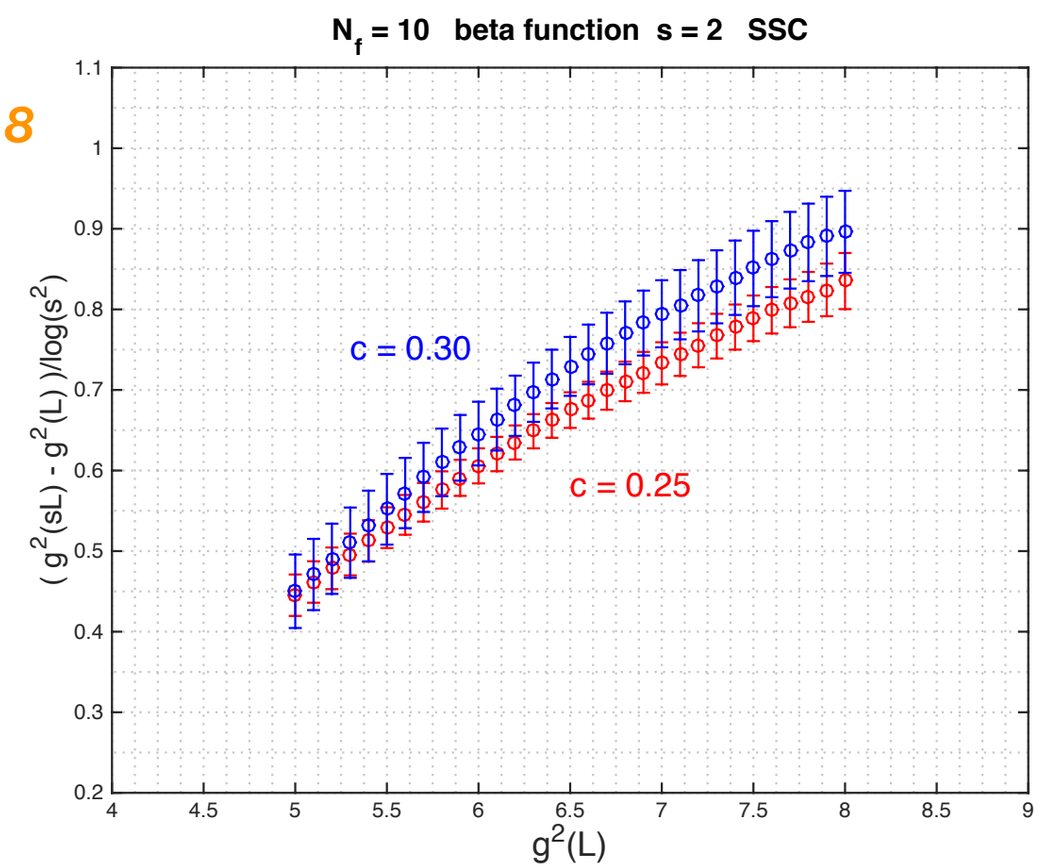
beta-functions for different schemes do not have to be quantitatively the same, the difference does not appear to be large

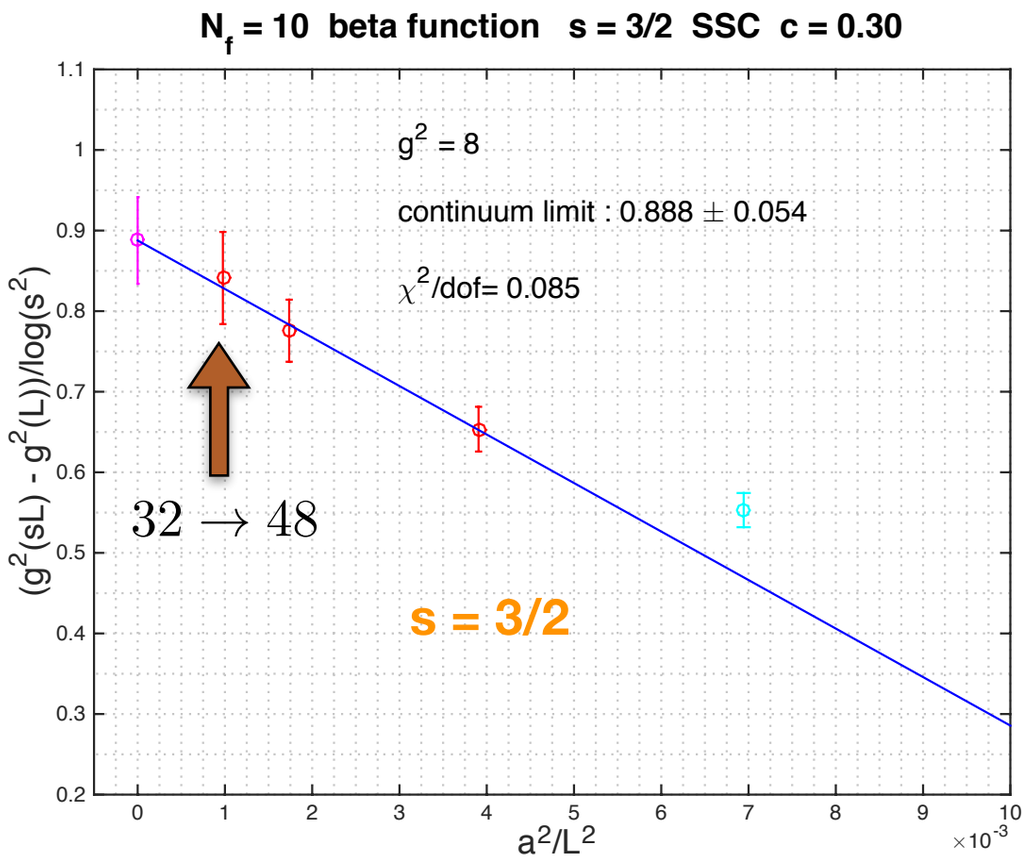


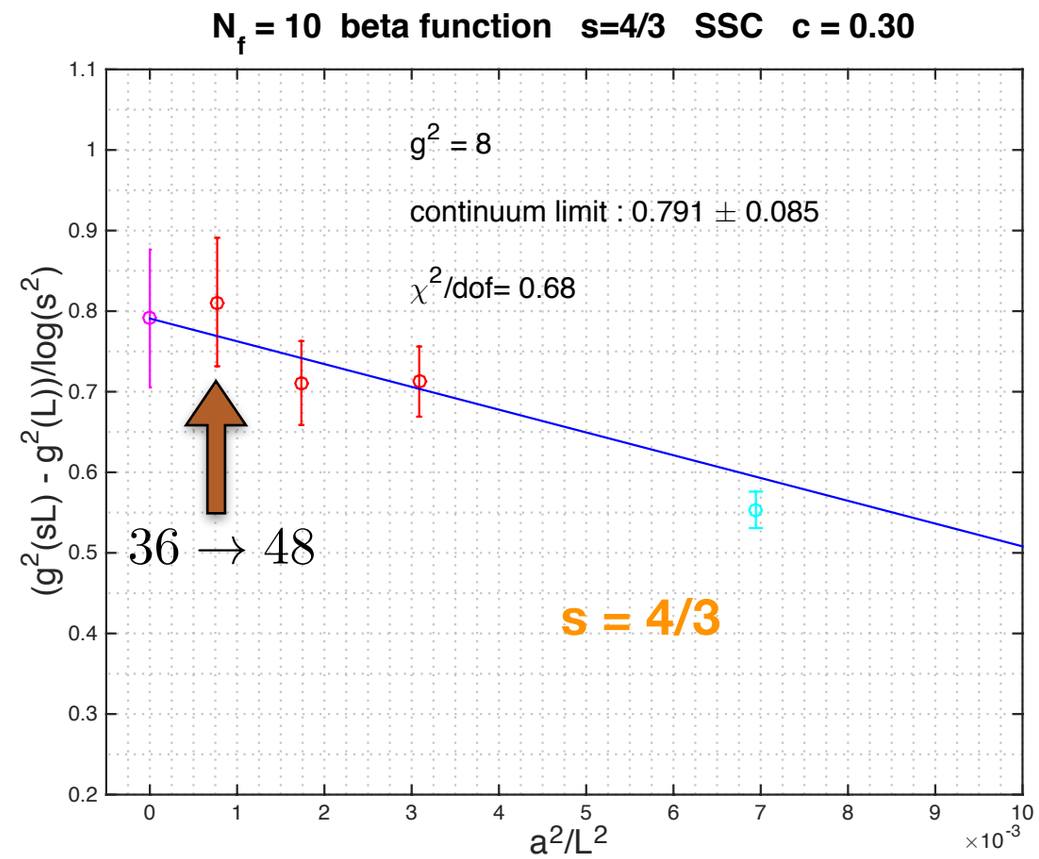
same picture emerges:

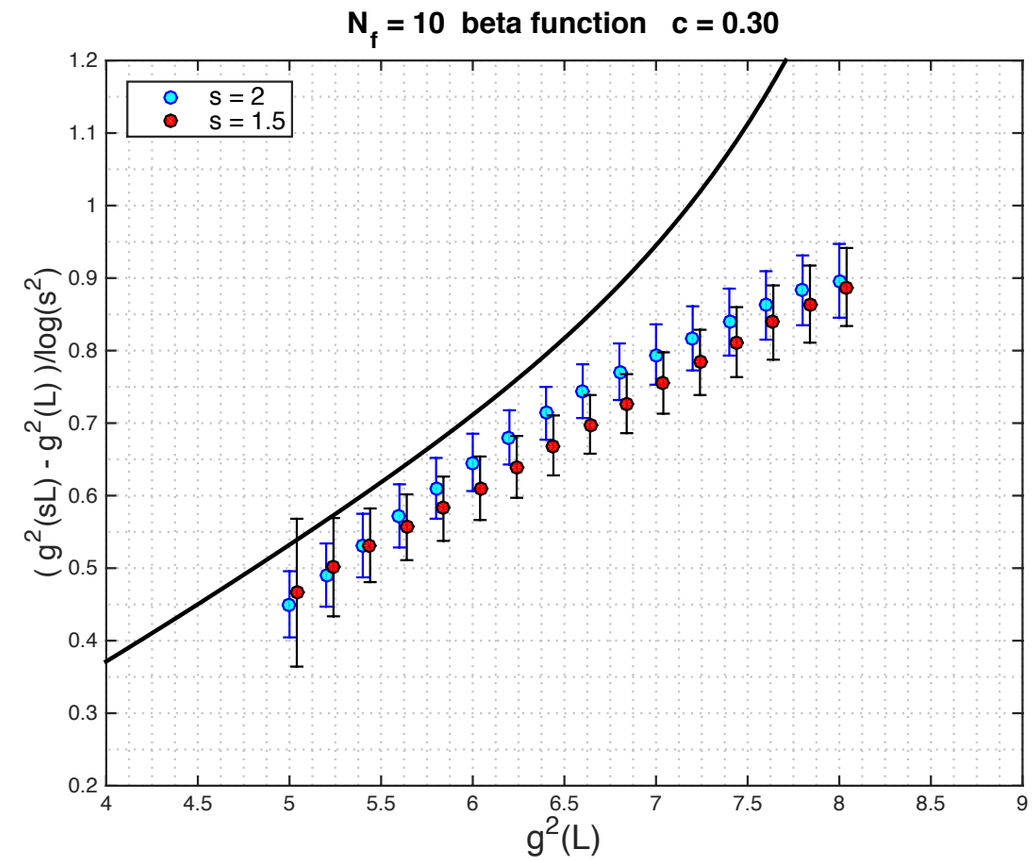
beta-function far above zero out to strong coupling g^2 = 8

the value c = 0.3 is useful to compare work by different groups, but is not the only possibility









data set allows other step-scales: s = 3/2, 4/3

pushes closer to the continuum limit

e.g. s = 3/2 32 -> 48 , s = 4/3 36 -> 48

very good agreement with s = 2 beta-function results

practically no room left for the beta-function to turn down between finest lattice spacing and a = 0

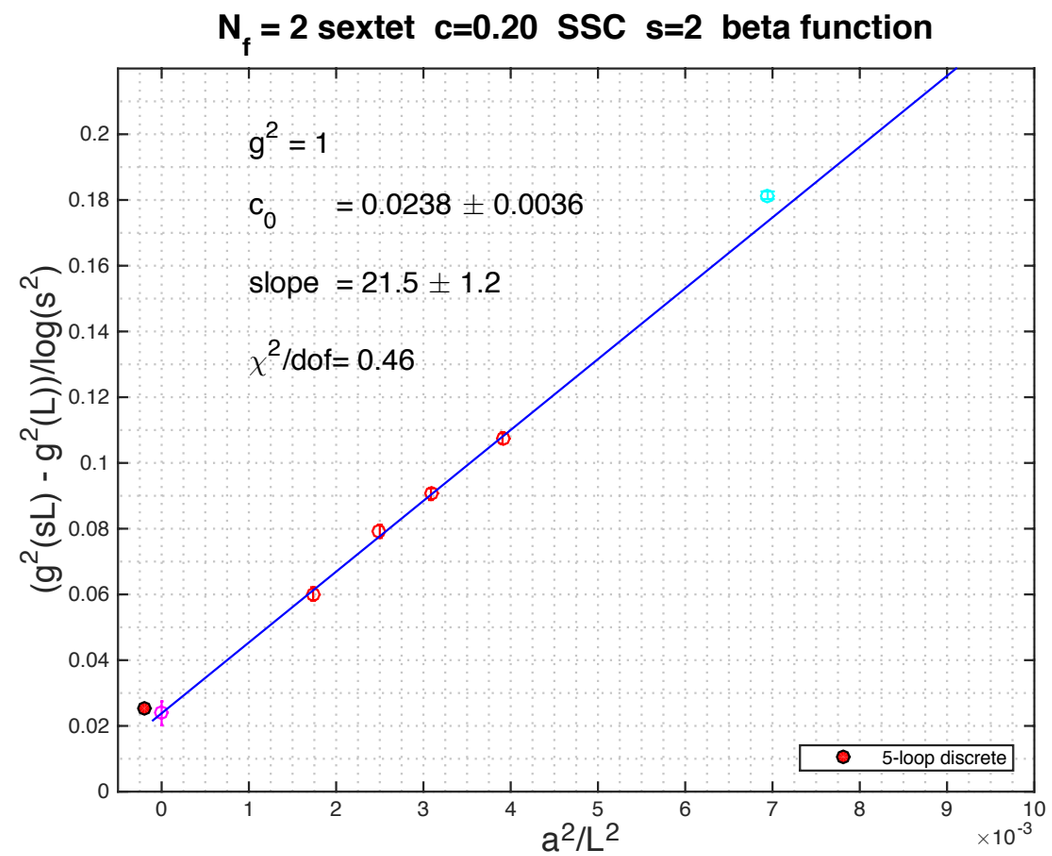
what's new for the **Nf = 2 sextet** model: has been our focus as flagship minimal BSM theory

previous results:

beta-function small but non-zero out to g^2 ~ 6.5, appears near-conformal

consistent with chiral SB in the particle spectrum, Random Matrix Theory match to Dirac eigenvalue distributions

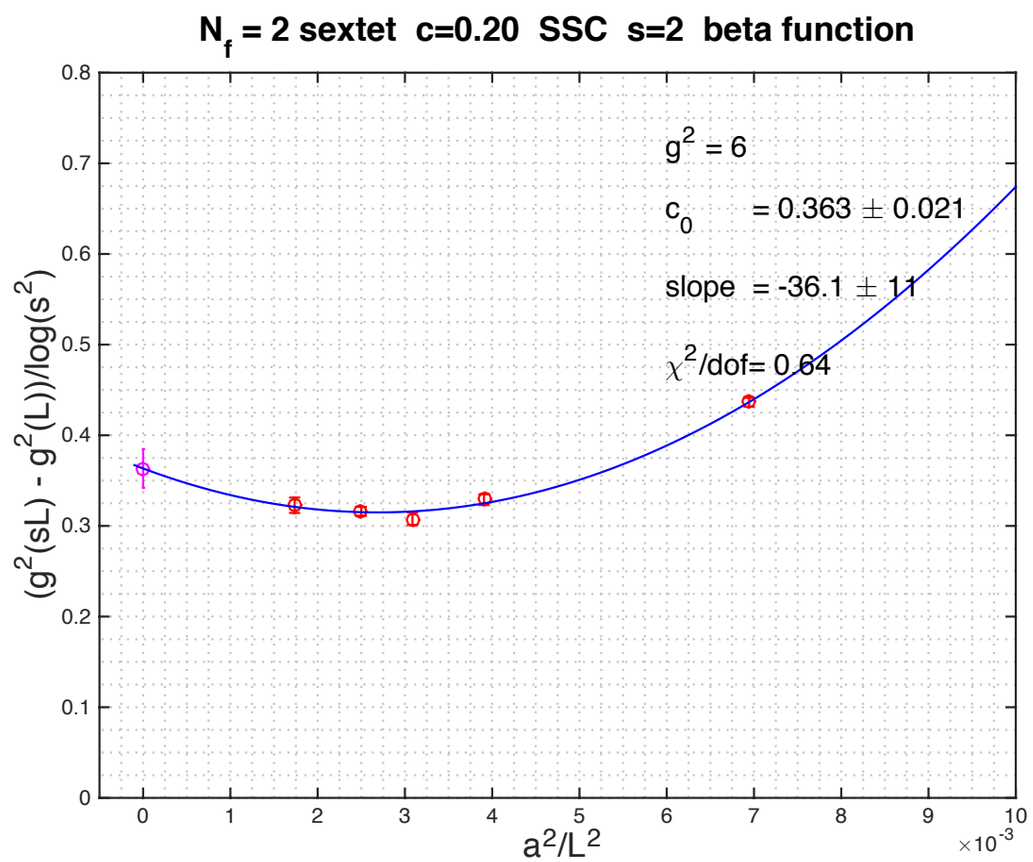
we are generating new data and going through the same systematic tests as for Nf = 10



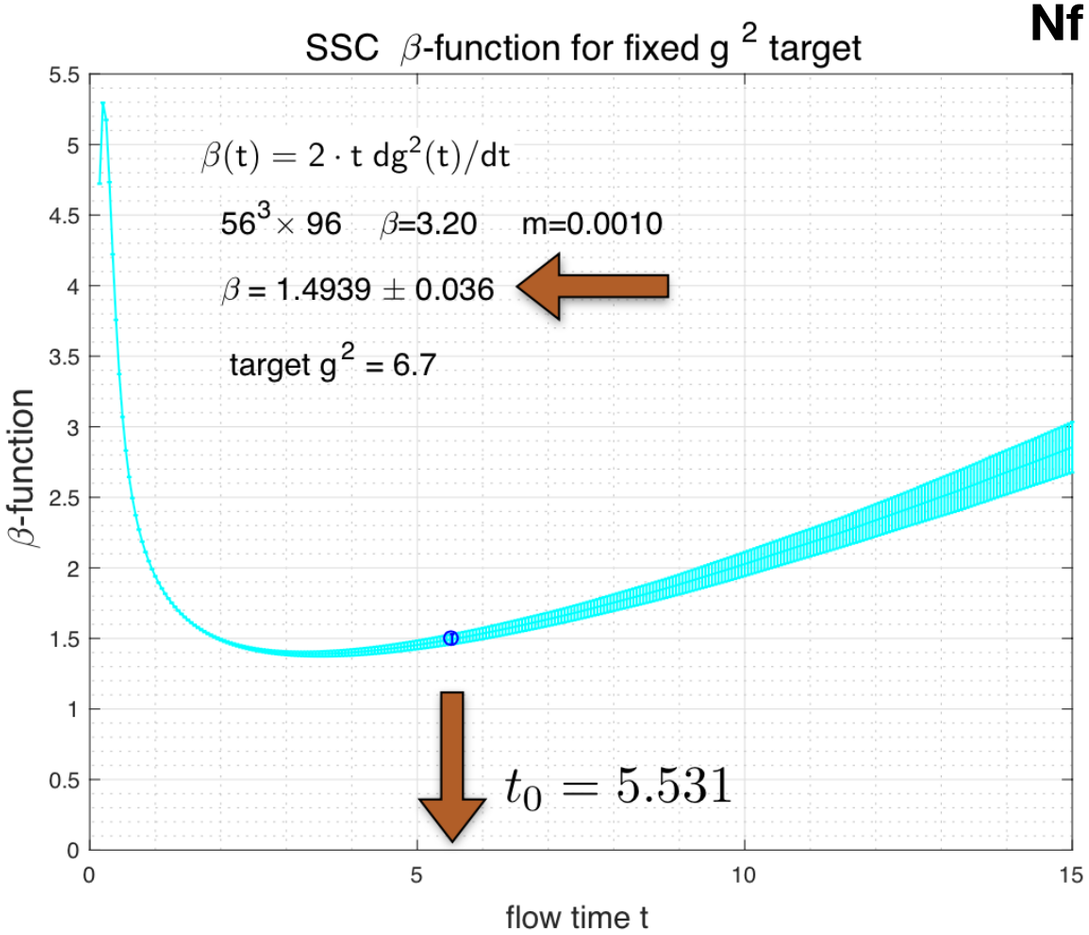
at the weak coupling end, looks like it's getting into perturbative region

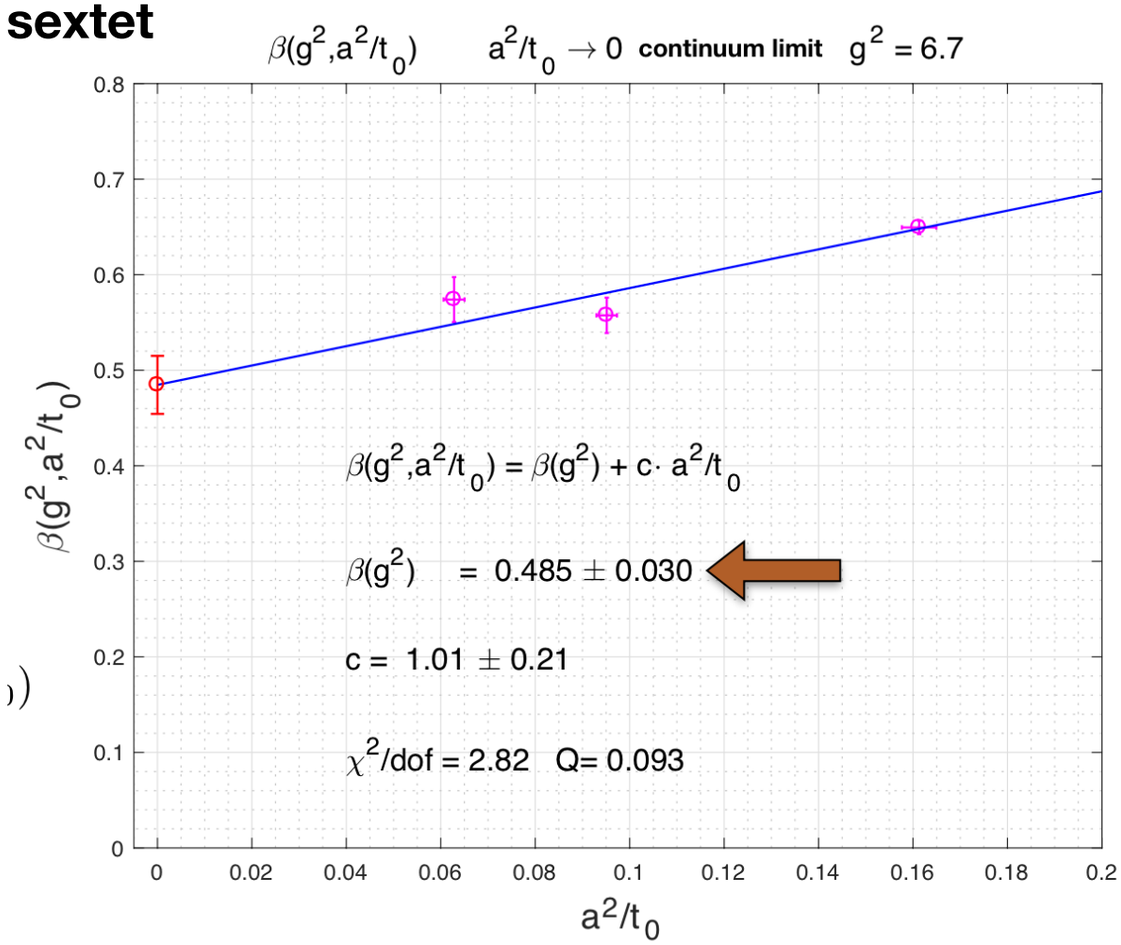
at stronger coupling, cutoff effects with SSC variant look relatively mild

continuum limit consistent with our previous results



Nf = 2 sextet





presented by LatHC at Lattice 2017, Granada: **infinitesimal beta-function through the gradient flow**

reminder from Anna Hasenfratz's talk on Monday

can vary the RG scale smoothly with flow time t

p-regime simulations, order of limits: infinite volume, zero mass, continuum limit

connects the chirally broken phase at strong coupling with the beta-function from step-scaling at m = 0

once you reach the chirally broken phase, no space left for an infrared fixed point to emerge

Status

the picture is holding up after additional simulations and various tests to see if systematic effects are under control

previous discrepancies e.g. Nf = 10 may be clearing up, removing concern about the use of staggered fermions

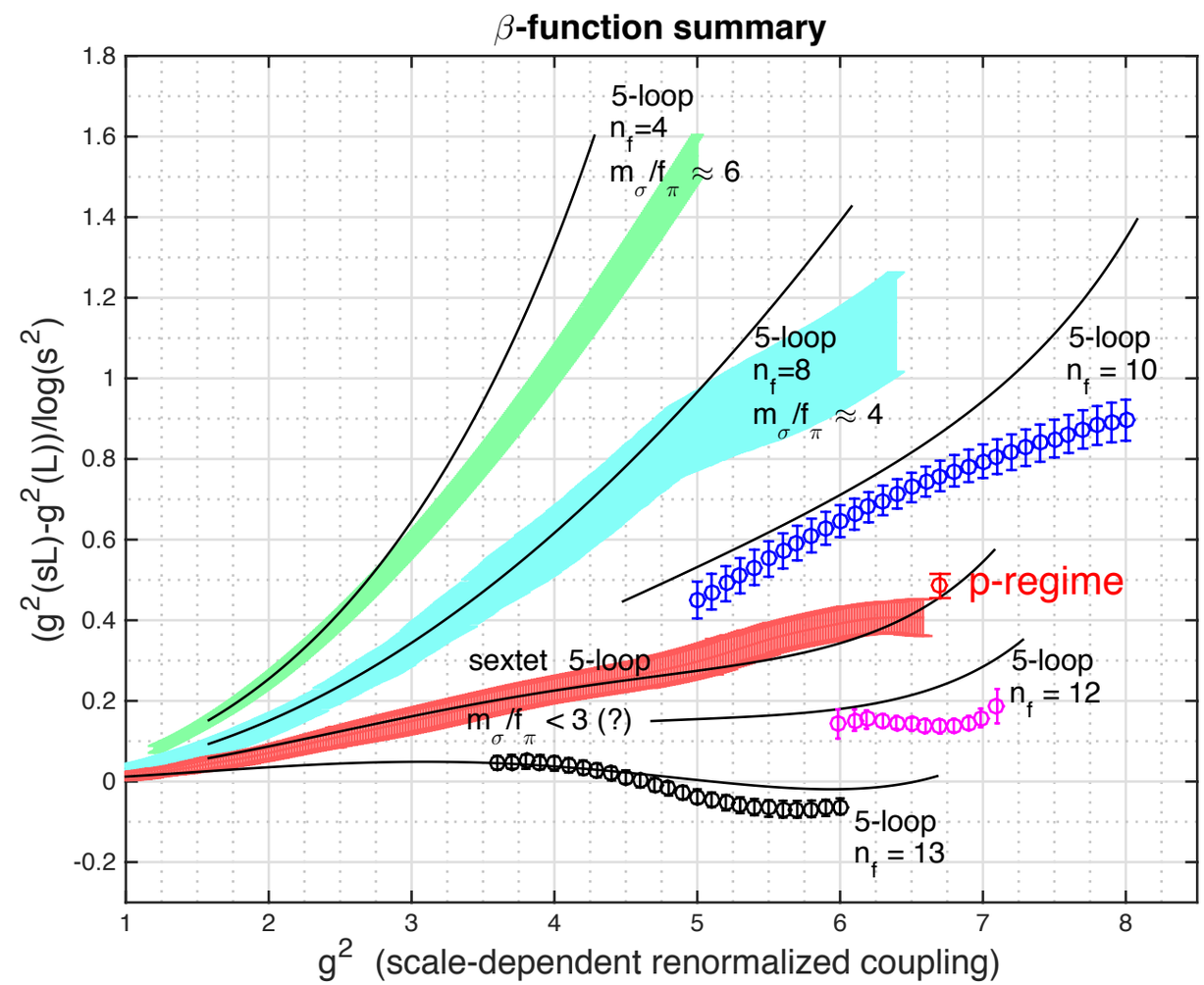
the beta-function is the beginning, not the end, of the story, complemented by newer work:

dilaton analysis

complex fixed points as a “walking” signal

Dirac operator eigenvalues and RMT

…



talks by Julius Kuti and Ricky Wong