Phase diagram of Higgs-Yukawa model on lattice with reduced staggered fermions

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Abstract

We investigate the phase structure of a lattice Higgs-Yukawa model with four reduced staggered fermion coupled to a real scalar field. Fermions transform in the fundamental rep of a $SO(4)$ symmetry group while the scalar field belongs to a self-dual representation of the group. In the limit where the scalar kinetic term is set to zero the model (pure four fermi) possesses both massless and massive symmetric phases. A narrow symmetry broken phase separates these two phases. However after the introduction of a scalar kinetic term the broken phase can be eliminated leading to a single transition between massless and massive symmetric phases. Current evidence suggests the transition is continuous.

Lattice Action and Symmetries

Conventionally fermions acquire mass through spontaneous symmetry breaking. It is well known that certain lattice models can be constructed that gap fermions without breaking symmetries using four fermion interactions. However until now these symmetric massive phases have been regarded as lattice artifacts separated from the continuum by broken symmetry phases and 1st order phase transitions. The model described here seems to circumvent these constraints and appears to offer the possibility of building continuum massive fermions without breaking symmetries and without inducing a intermediate broken symmetry phase. The action for the model is

$$S = \sum_{x,\mu} \psi^a[\eta_\mu \Delta_\mu \delta_{ab} + G\phi^+_{ab}]\psi^b + \sum_x \frac{1}{4}\phi_+^2 - \frac{\kappa}{2}\sum_{x,\mu}[\phi^+_x\phi^+_{x+\mu} + \phi^+_x\phi^+_{x-\mu}] \quad (1)$$

We use the isomorphism $SO(4) = SU_+(2) \times SU_-(2)$ for introducing Yukawa field ϕ^{ab} where

$$\phi_+^{ab} = P_+\phi^{ab} = \frac{1}{2}(\phi^{ab} + \frac{1}{2}\epsilon_{abcd}\phi^{cd}) \quad (2)$$

ϕ_+^{ab} is self-dual and transforms under $SU_+(2)$ and is a singlet under $SU_-(2)$. With $\kappa = 0$ we can integrate out the Yukawa field resulting in a four-fermion model with the action

$$S = \sum_{x,\mu} \psi^a\eta_\mu\Delta_\mu\psi^a - \frac{G^2}{4}\sum_x \epsilon^{abcd}\psi^a\psi^b\psi^c\psi^d \quad (3)$$

$SO(4)$ flavor symmetry and lattice shift symmetries forbid any fermion bilinear term from being induced by quantum corrections. However, spontaneous breaking of symmetries is still possible. At $\kappa = 0$ a strong coupling expansion in $\frac{1}{G^2}$ leads to following momentum space propagator for fermions

$$F(p) = \frac{i\sqrt{6G^2}\sum_\mu \sin(p_\mu)}{\sum_\mu \sin^2(p_\mu) + m_F^2} \quad (4)$$

where $m_F^2 = 24G^2 - 2$. Fermions are massive at strong coupling. However simulations reveal an intermediate antiferromagnetic (AF) phase separating the massless (PMW) and massive (PMS) phases where $SO(4)$ symmetry is broken. Evidence for this AF phase comes from peak in staggered susceptibility (shown in Fig. 1) which scales with volume. To study spontaneous symmetry breaking and isolate AF phase from PMS phase we add source terms to action of the form

$$\delta S = \sum_x (m_1 + \epsilon(x)m_2)[\psi^a(x)\psi^b(x)]\Sigma_+^{ab} \quad (5)$$

where the $SO(4)$ symmetry breaking source Σ_+^{ab} is

$$\Sigma_+^{ab} = \begin{pmatrix} i\sigma_2 & 0 \\ 0 & i\sigma_2 \end{pmatrix}. \quad (6)$$

For $\kappa = 0$ we find evidence of anti-ferromagnetic bilinear(broken phase) in the limit $m_2 \to 0, V \to \infty$ as shown in Fig. 3. However, a natural extension of the model with non-zero κ leads to single transition between the PMW and PMS phase. There is no conventional order parameter to distinguish between the two phases. In Ref [2] we have argued using a related a continuum model that this transition is driven by unbinding of topological defects associated with the scalar field. These defects are labeled by a winding number corresponding to the Hopf map $S^3 \to S^2$ with pairs of defects experiencing a logarithmic interaction.

Monte Carlo results

A useful observable to probe phase structure in (κ, G) plane is $\langle\phi_+^2\rangle$ which is plotted in Fig.2. $\langle\phi_+^2\rangle$ serves as a proxy for $\langle\epsilon_{abcd}\psi^a(x)\psi^b(x)\psi^c(x)\psi^d(x)\rangle$. The staggered susceptibility χ_{stag} at $\kappa = 0.05$ (fig. 5) lacks volume scaling in comparison to χ_{stag} at $\kappa = 0$ (fig. 1). However the system is still critical due to diverging number of CG iterations in this region – see Fig. 4. Absence of a broken phase in the critical region at $(\kappa, G) = (0.05, 1.05)$ is confirmed through a vanishing anti-ferromagnetic bilinear plotted vs $m = m_2 = m_1$ in Fig. 6 (compare the $\kappa = 0$ result in fig. 3). Putting all this together we have sketched the phase diagram in Fig.7.

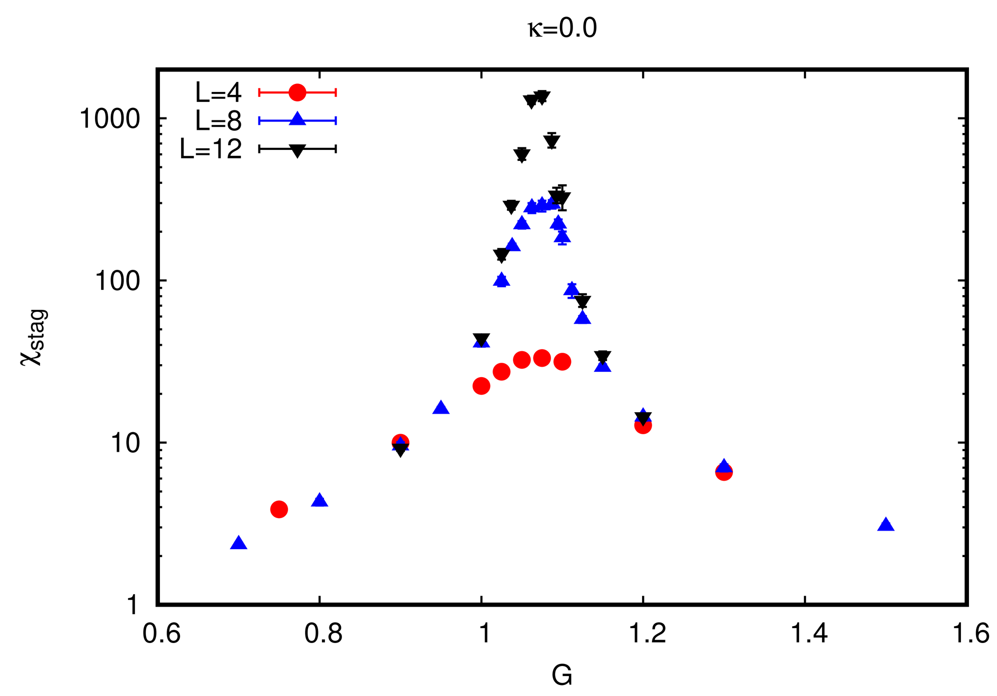

Figure 1: $\frac{1}{V}\sum_{x,y,a,b}\langle\epsilon(x)\psi^a(x)\psi^b(x)\epsilon(y)\psi^a(y)\psi^b(y)\rangle$

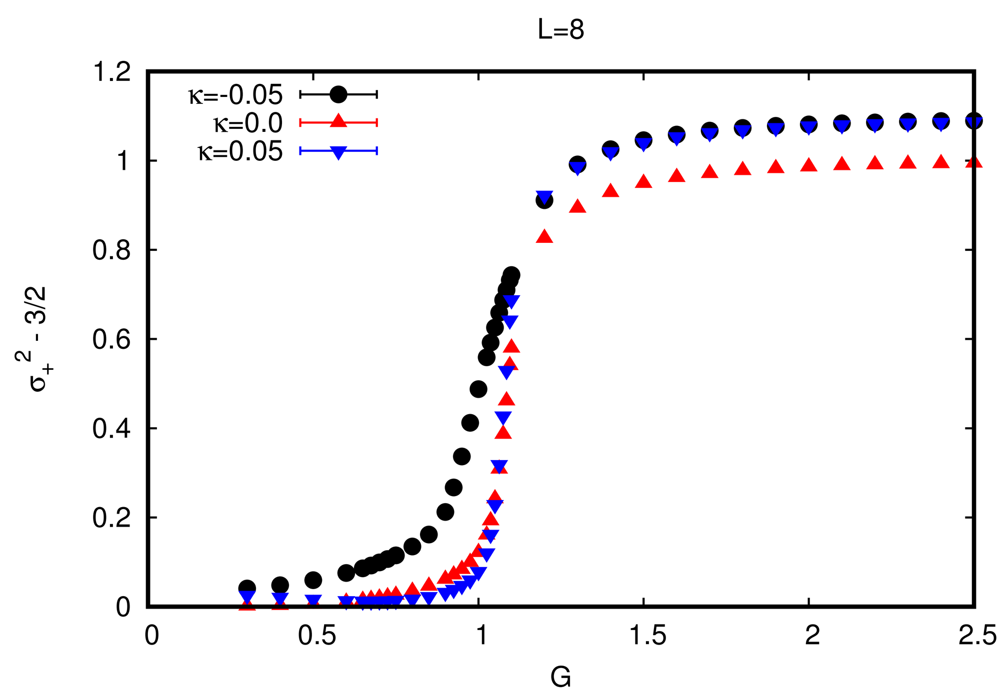

Figure 2: $\langle(\phi_+)^2\rangle$

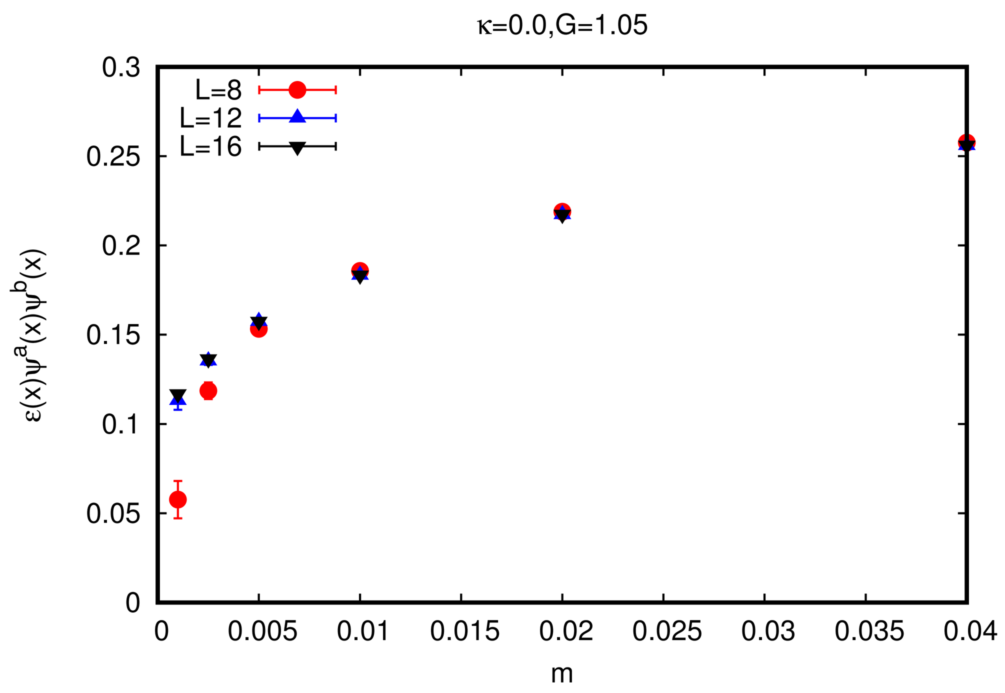

Figure 3: $\langle\epsilon(x)\psi^a(x)\psi^b(x)\rangle$

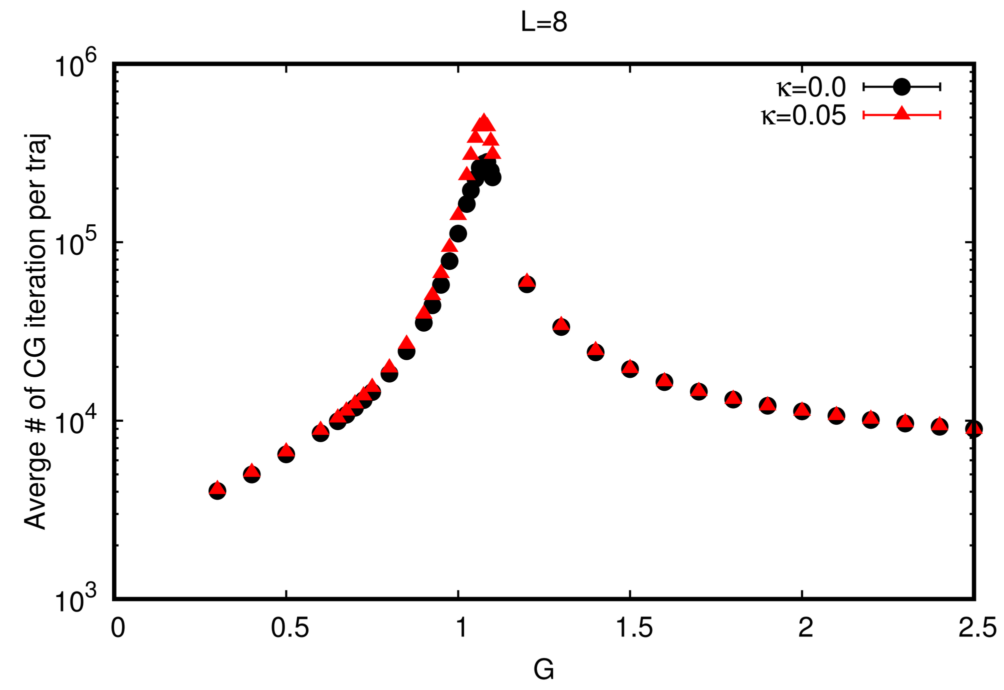

Figure 4: Average number of CG iterations for Dirac operator inversions on $L = 8$ lattice, plotted vs G for $\kappa = 0$ and 0.05

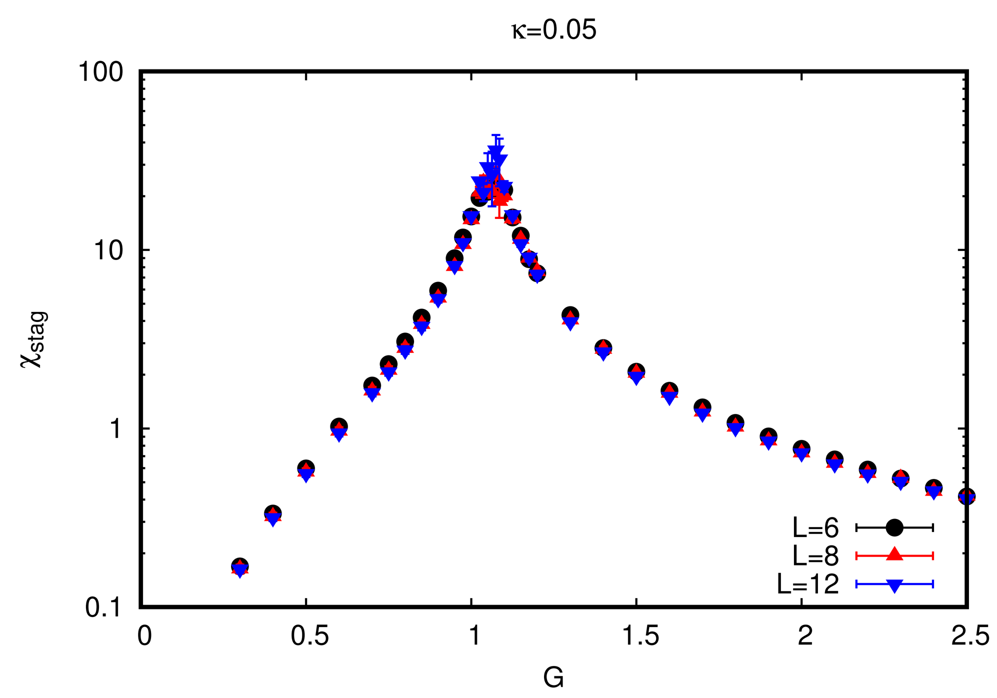

Figure 5: χ_{stag} at $\kappa = 0.05$ for $L = 6, 8, 12$

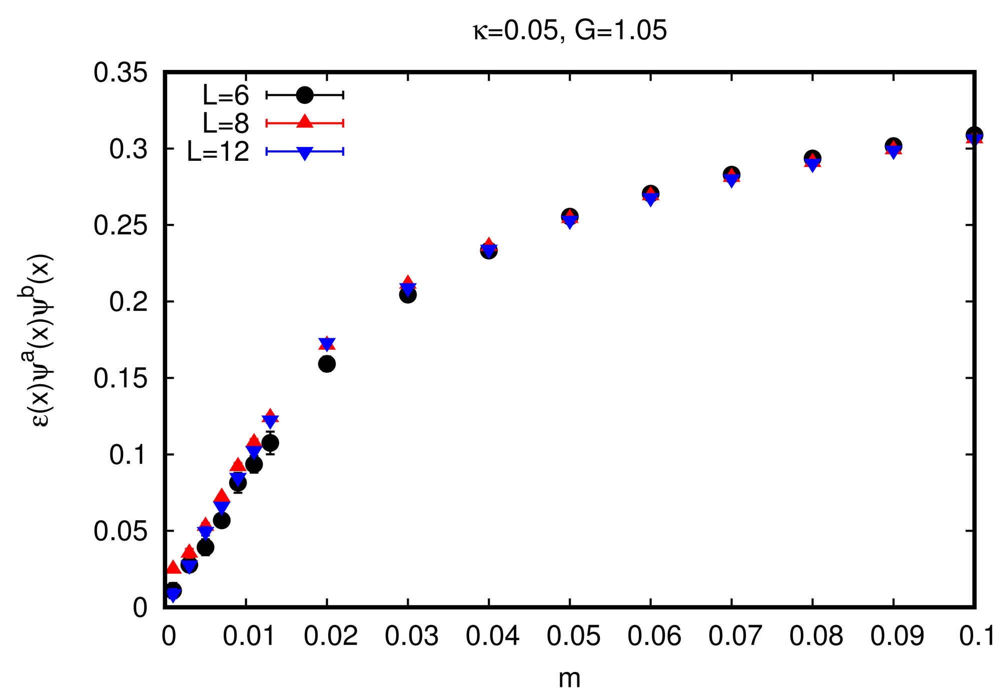

Figure 6: $\langle\epsilon(x)\psi^a(x)\psi^b(x)\rangle$

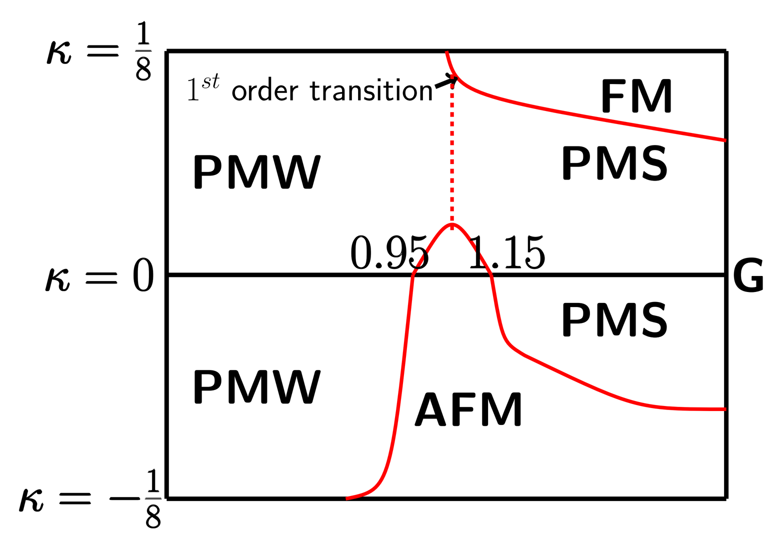

Figure 7: Sketch of the phase diagram in (κ, G) plane

Summary and Future Prospects

Our main result is that the broken phase can be removed in the expanded phase diagram by tuning κ such that it suppresses the formation of an anti-ferromagnetic bilinear. Instead there is range $\kappa_1 < \kappa < \kappa_2$ where there appears to be a direct phase transition between the massless and massive fermion phases. A possible future direction for this work would be to add Wilson-like Yukawa interactions to the action to try to gap out doubler modes. This offers the hope that in the (κ, G, r) phase diagram we can find a region where all the doubler modes are massive while the primary fermion mode remains massless thus realizing the old Eiichten-Preskill proposal for chiral lattice gauge theories.

References

[1] $SO(4)$ invariant Higgs-Yukawa model with reduced staggered fermions, Phys.Rev. D.98.114514

[2] Topology and strong four fermion interactions in four dimensions, Phys. Rev. D 97, 094502