Lattice Analysis of SU(2) with 1 Adjoint Dirac Flavor

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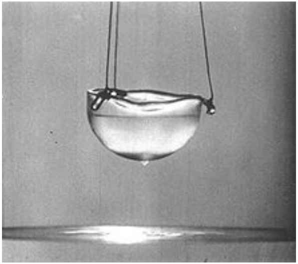
Outline

Motivation

- The critical point of phase transition in condensed matter systems often has interesting properties
- For some condensed matter systems, the critical point can be described by an effective theory of $SU(2)$ with 1 adjoint Dirac fermion
  - In particular, happens in type AIII topological superconductors (similar to superfluid helium)



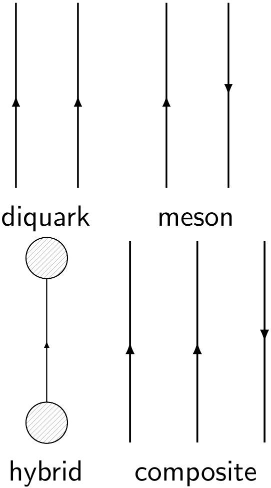
(Image credit: Wikipedia)

Duality

- Critical point of topological superconductor: $SU(2)$ with $N_f = 1$ adjoint Dirac flavor
- Critical point of topological insulator (same topology): single free fermion
- Conjecture: These theories are dual (in IR) (Z. Bi, T. Senthil, Phys. Rev. X 9, 021034, 2019)
  - May require spectator fields for anomaly matching
- Consequence: Free fermion emerges from $SU(2)$ with $N_f = 1$

Physics of $SU(2)$ Adjoint

- Adjoint representation of $su(2)$ has two key properties:
  - 3-dimensional representation $\Rightarrow$ invariant tensors are δ_{ab}, ϵ_{abc}
  - Representation is real
- This gives rise to a rich variety of particles:
  - glueballs (not studied here)
  - diquarks: $\psi_a^T C\Gamma\psi_a$
  - mesons: $\bar{\psi}_a\Gamma\psi_a$
  - hybrid: $\sigma_{\mu\nu}F_a^{\mu\nu}\psi_a$
  - composite: $[(\bar{\psi}_a\psi_b)\gamma_5 P\psi_c - (\bar{\psi}_a\gamma_5\psi_b)P\psi_c]\epsilon_{abc}$
- Conjecture: Composite particle acts as free fermion in IR limit (Z. Bi, T. Senthil, Phys. Rev. X 9, 021034, 2019)



Constraints

- 't Hooft anomaly matching constrains IR behavior
- If $m_q \to 0$, we have massless particles in UV theory $\Rightarrow$ must have massless particles in IR theory that match UV anomalies
- Possible scenarios:
  - Conformality: Sector of spectrum goes massless as $m_q \to 0$
  - Chiral symmetry breaking: Goldstone boson $(\psi^T C \gamma_5 \psi)$ goes massless (as $m_q^{1/2}$); other particles remain heavy
  - Bi-Senthil conjecture: Composite fermion goes light; other particles remain heavy

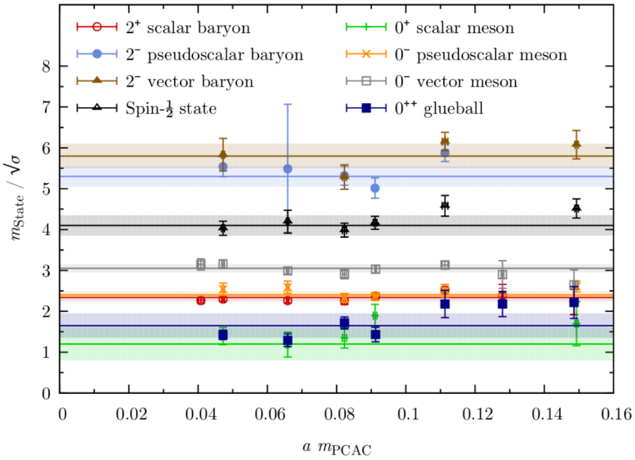
Hybrid-Composite Mixing

- For $m_q > 0$, hybrid and composite fermions have same quantum numbers
  - At $m_q = 0$, have different chiral properties
  - Composite fermion (but not hybrid) has correct chiral transformation properties for anomaly matching
- Unclear *a priori* whether hybrid and composite interpolators have significant overlap with same states (except that overlap vanishes as $m_q \to 0$)

Previous Calculations

- Athenodorou, Bennett, Bergner, and Lucini conducted a lattice simulation of $SU(2) + 1$ adjoint flavor motivated by composite Higgs models (Athenodorou *et al.*, Phys. Rev. D 91, 114508, 2015)
- Studied diquarks, mesons, glueballs, and hybrid but not composite
  - Study predates Bi-Senthil conjecture
- Found numerical support for conformality with $m^{1.95} \sim m_q$ for all particle masses m

Previous Results



Plot from Figure 4a (Athenodorou *et al.*, arXiv:1507.08892)

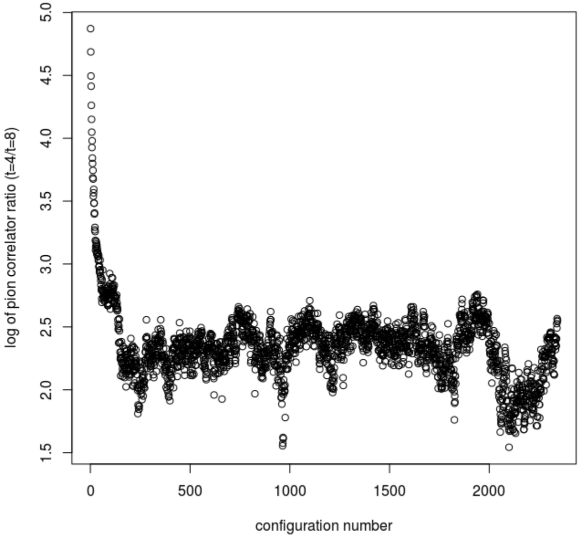
Simulation

- Many lattice codes (e.g. chroma) only handle fundamental fermions
- Qlua: USQCD software built on top of QDP and Lua scripting language (A. Pochinsky, J. Osborn, S. Syritsyn)
- Designed to combine flexible code development with high performance
- Can link to QUDA inverters for additional speed (K. Clark, Thurs. 11:15)
    - $SU(2)$ adjoint has 3 colors - can use standard $SU(3)$ inverters
- No built-in support for adjoint fermions or RHMC

RHMC in Qlua

- We have one Dirac flavor – need RHMC
- Also necessary for 3-flavor QCD simulations – useful to have in Qlua
- Implemented routines to evaluate rational expansion of power functions and related force terms
  - Built linkages to QUDA multishift inverter (needed for RHMC)
  - Implemented Ritz solver to determine spectrum (required for rational expansion)

Thermalization Plot

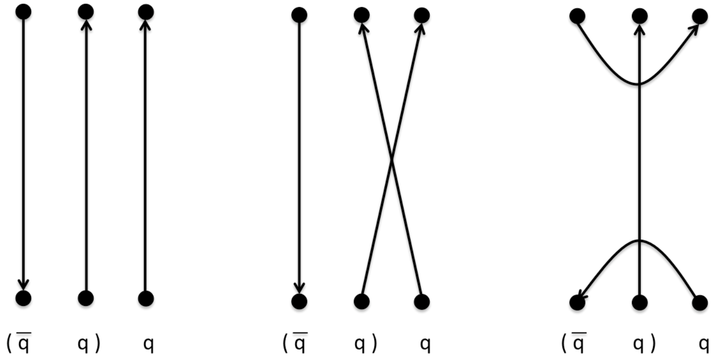


Simulation Parameters

- Matched simulation parameters of Athenodorou *et al.* in order to cross-check result
  - $\beta = 2.05$
  - $-am_q$ ranges from 1.475 to 1.524
- For now, lattices are $12^3 \times 24$ but $m_\pi L > 4.8$ throughout
- Observables measured using either point source/sink or combination of wavefunction and stout smearing
- Composite fermion computed using analog of baryon contraction algorithm on coarsened grid of propagators (Detmold and Originos, PRD 114512, 2013; Murphy, Friday 14:40)

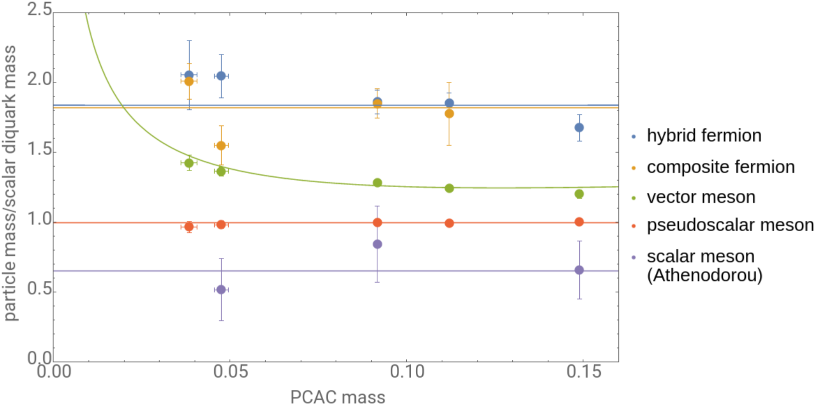
Composite Fermion Diagrams

- 3 different quark contractions possible

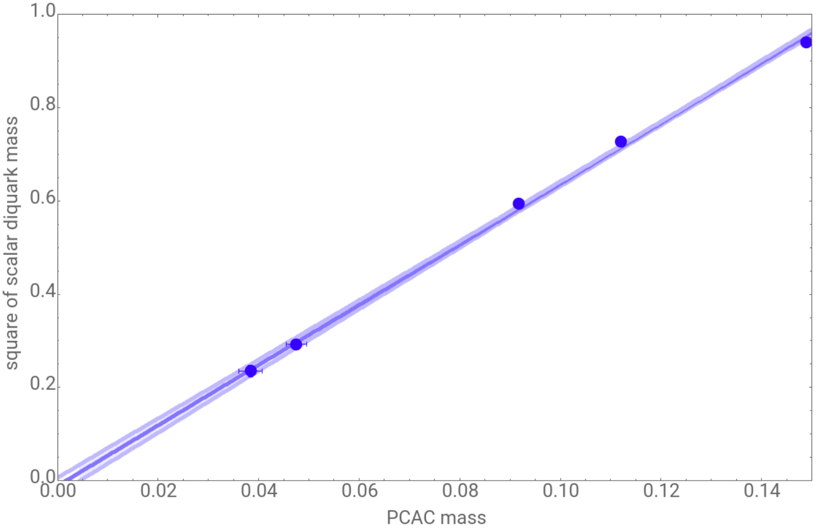


$$\Psi = \epsilon_{abc}\left[(\bar{\psi}_a\psi_b)\gamma_5 P\psi_c - (\bar{\psi}_a\gamma_5\psi_b)P\psi_c\right]$$

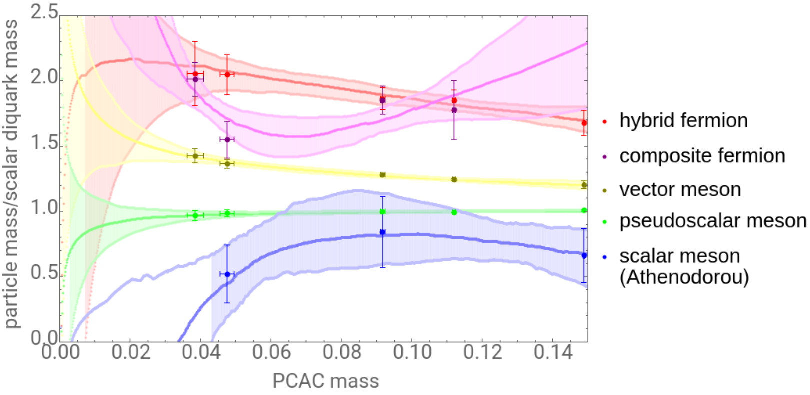
Results


2.5
2.0
1.5
1.0
0.5
0.0
0.00
0.05
0.10
0.15
particle mass/scalar diquark mass
PCAC mass
hybrid fermion
composite fermion
vector meson
pseudoscalar meson
scalar meson
(Athenodorou)

Results


1.0
0.8
0.6
0.4
0.2
0.0
square of scalar diquark mass
0.00
0.02
0.04
0.06
0.08
0.10
0.12
0.14
PCAC mass

Results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vector meson
pseudoscalar meson
scalar meson (Athenodorou)

Future Work

- Simulations at lighter masses
- This requires finer lattice spacings, larger volumes
- Improvements to action (clover term)

References

- Bi, Z., and Senthil T., "An Adventure in Topological Phase Transitions in 3 + 1-D: Non-abelian Deconfined Quantum Criticalities and a Possible Duality" (2018).
- Athenodorou A. et al., "The Infrared Regime of $SU(2)$ with One Adjoint Dirac Flavour" (2014).
- Athenodorou A. et al., "Recent results from $SU(2)$ with One Adjoint Dirac Fermion" (2015).
- Detmold, W., and K. Originos, "Nuclear correlation functions in lattice QCD" (2013).
- Pochinsky A., J. Osborn, S. Syritsyn, Qlua software, https://usqcd.lns.mit.edu/redmine/projects/qlua.