Lattice study of the 2-flavor U(1) gauge Higgs model at topological angle $\theta = \pi$

Daniel Göschl

Work done in collaboration with
Christof Gattringer and **Tin Sulejmanpasic**

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UNI GRAZ

FWF Der Wissenschaftsfonds.

NAWI Graz
Natural Sciences

Motivation

The 2-flavor U(1) gauge Higgs model at topological angle $\theta = \pi$

- Novel discretization and dualization for topological term available:

  Generalized Villain action

- Effective field theory for a spin chain - interesting for condensed matter physics. E.g.:

$$\theta = 0 \quad \Rightarrow \quad \text{integer spin chain}$$
$$\theta = \pi \quad \Rightarrow \quad \text{half-integer spin chain}$$

- It has potential realizations (Spintronics, Quantum computers).

Continuum formulation

Matter fields $\phi^1(x),\ \phi^2(x) \in \mathbb{C}, \quad \Phi(x) = \begin{pmatrix} \phi^1(x) \\ \phi^2(x) \end{pmatrix}$

Vector potential $A_\mu(x)$

$$Z = \int D[A]D[\Phi]\ e^{-S_H[\phi^1,\phi^2,A]-S_G[A]}$$

$$S_H[\phi^1,\phi^2,A] = \int_{\mathbb{T}^2} d^2x \Big[(D_\mu\Phi(x))^\dagger (D_\mu\Phi(x)) + m^2\ \Phi(x)^\dagger\Phi(x) + \underbrace{\lambda\ \left(|\phi^1(x)|^4 + 2g|\phi^1(x)|^2|\phi^2(x)|^2 + |\phi^2(x)|^4\right)}_{\xrightarrow[g=1]{} \lambda(\Phi(x)^\dagger\Phi(x))^2} \Big]$$

$$S_G[A] = \int_{\mathbb{T}^2} d^2x \Big[\frac{1}{2e^2}\ F_{12}(x)^2 + \frac{i\theta}{2\pi}\ F_{12}(x) \Big], \quad F_{12}(x) = \partial_1 A_2(x) - \partial_2 A_1(x)$$

Symmetries

$$S_H[\phi^1,\phi^2,A] = \int_{\mathbb{T}^2} d^2x \Big[(D_\mu \Phi(x))^\dagger (D_\mu \Phi(x)) + m^2\ \Phi(x)^\dagger \Phi(x) + \underbrace{\lambda\ \left(|\phi^1(x)|^4 + 2g|\phi^1(x)|^2|\phi^2(x)|^2 + |\phi^2(x)|^4\right)}_{\xrightarrow[g=1]{} \lambda(\Phi(x)^\dagger\Phi(x))^2} \Big]$$

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- $U(1)$ Gauge symmetry:

  $\Phi(x) \longrightarrow e^{i\alpha(x)}\ \Phi(x)$,
  $A_\mu(x) \longrightarrow A_\mu(x) + \partial_\mu \alpha(x)$

- $\mathbb{Z}_2$ Flavor swapping symmetry:

  $$\begin{pmatrix}\phi^1\\ \phi^2\end{pmatrix} \longrightarrow \begin{pmatrix}\phi^2\\ \phi^1\end{pmatrix}$$

- $\mathbb{Z}_2$ Charge conjugation ($\theta = \pi$):

  $\Phi(x) \to \Phi(x)^*$, $A_\mu(x) \to -A_\mu(x)$

- For $g = 1$:

  Naively $SU(2)$ symmetry, but its center element $-\mathbb{1}$ is a gauge transformation!

  $\Rightarrow \frac{SU(2)}{\mathbb{Z}_2} = SO(3)$

- For $g \neq 1$:

  Broken to $O(2)$

Conventional lattice representation with generalized Villain action

Matter fields $\phi^1_x,\ \phi^2_x \in \mathbb{C}$, Non-compact gauge fields $A_{x,\mu} \in [-\pi, \pi]$

$$Z = \int D[A]\ B_G[A] \int D[\phi^1] D[\phi^2]\ e^{-S_H[\phi^1,\phi^2,A]}$$

$$S_H[\phi^1,\phi^2,A] = \sum_{x\in\Lambda} \Bigg[M(|\phi^1_x|^2 + |\phi^2_x|^2) + \lambda(|\phi^1_x|^4 + 2g|\phi^1_x|^2|\phi^2_x|^2 + |\phi^2_x|^4)$$
$$- \sum_{\mu=1}^{2} \left(\phi^{1\,*}_x e^{iA_{x,\mu}} \phi^1_{x+\hat{\mu}} + \phi^{2\,*}_x e^{iA_{x,\mu}} \phi^2_{x+\hat{\mu}} + c.c. \right) \Bigg]$$

$$B_G[A] = \prod_{x\in\Lambda} \sum_{n_x\in\mathbb{Z}} e^{-\frac{\beta}{2}(F_x+2\pi n_x)^2 - i\frac{\theta}{2\pi}(F_x+2\pi n_x)}\ , \qquad F_x = A_{x,1} + A_{x+\hat{1},2} - A_{x+\hat{2},1} - A_{x,2}$$

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Topological charge:

$$Q = \frac{1}{2\pi} \sum_x (F_x + 2\pi n_x) = \sum_x n_x \in \mathbb{Z}$$

Q depends only on Villain variables, with $Q \in \mathbb{Z}$ and $\mathcal{C}$ is exactly implemented.

Worldline representation solves complex action problem

$$Z = \sum_{\{j,k,p\}} W_H[j,k]\; W_G[p] \prod_x \delta(\vec{\nabla}\vec{j_x})\; \delta(\vec{\nabla}\vec{k_x})$$

[Nucl.Phys B935 (2018), 344-364]

$$\prod_x \delta(j_{x,1} + k_{x,1} + p_x - p_{x-\hat{2}})\; \delta(j_{x,2} + k_{x,2} - p_x + p_{x-\hat{1}})$$

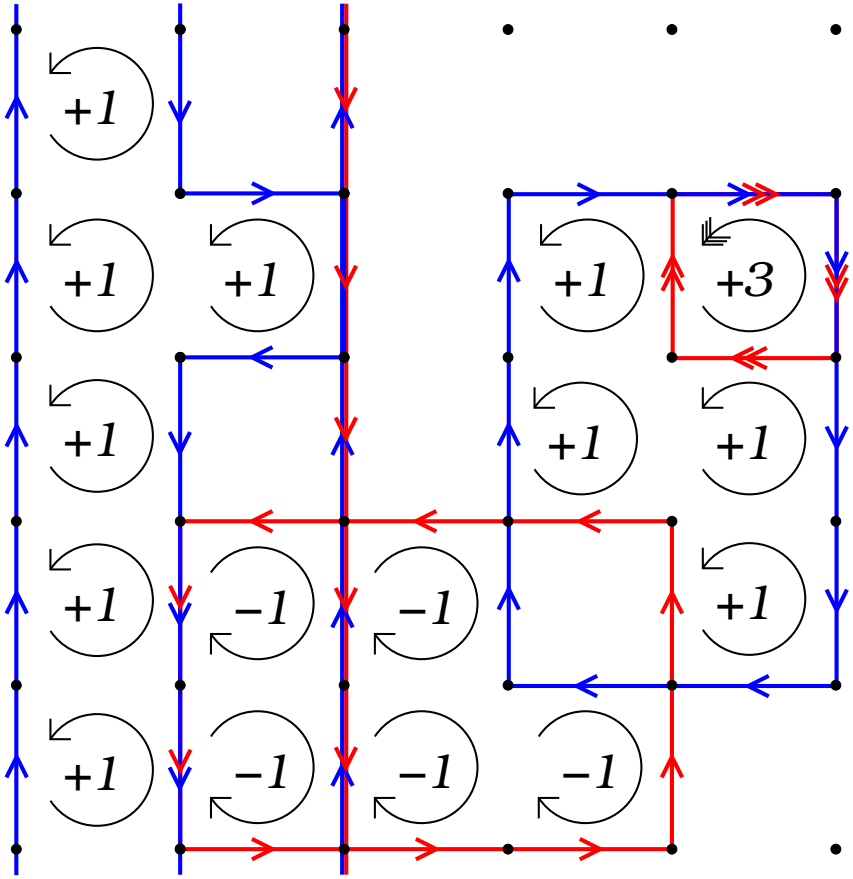
Dual variables:

- $p_x \in \mathbb{Z} \sim$ gauge d.o.f.
- $j_{x,\mu}, k_{x,\mu} \in \mathbb{Z} \sim$ matter d.o.f.

Constraints:

- Vanishing divergence for matter-flux at each lattice point
- Combination of j-, k- and p-flux has to cancel at each link

Real and positive weights
$W_H[j,k]$, $W_G[p]$



Observables

Topological charge, susceptibility and Binder cumulant:

$$\langle q \rangle = -\frac{1}{V}\frac{\partial}{\partial\theta}\ln(Z)\ , \qquad \chi_t = \frac{1}{V}\frac{\partial^2}{\partial\theta^2}\ln(Z)\ , \qquad U_t = 1 - \frac{\langle q^4 \rangle}{3\langle q^2 \rangle^2}$$

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Flavor symmetry breaking parameter, susceptibility and Binder cumulant:

$$\langle c \rangle = -\frac{1}{V}\langle |\phi_x^1|^2 - |\phi_x^2|^2 \rangle\ , \qquad \chi_c = \frac{1}{V}\left[\langle c^2 \rangle - \langle c \rangle^2\right]\ , \qquad U_c = 1 - \frac{\langle c^4 \rangle}{3\langle c^2 \rangle^2}$$

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Spin stiffness:

$$\langle s \rangle = \langle W_s^2 \rangle \qquad W_s \ \ldots \text{ spatial net winding number of worldlines}$$

$\langle s \rangle = 0 \ \ldots \ \exists$ mass gap

$\langle s \rangle > 0 \ \ldots$ gapless model

Global symmetries and order parameters

- **Charge conjugation symmetry $\mathcal{C}$ at $\theta = \pi$:**

$$p_x \xrightarrow{\mathcal{C}} -p_x - 1$$

$$j_{x,\mu} \xrightarrow{\mathcal{C}} -j_{x,\mu}$$

$$k_{x,\mu} \xrightarrow{\mathcal{C}} -k_{x,\mu}$$

Order Parameter:

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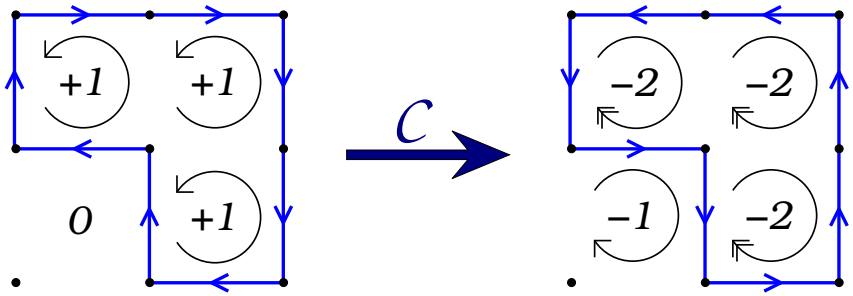

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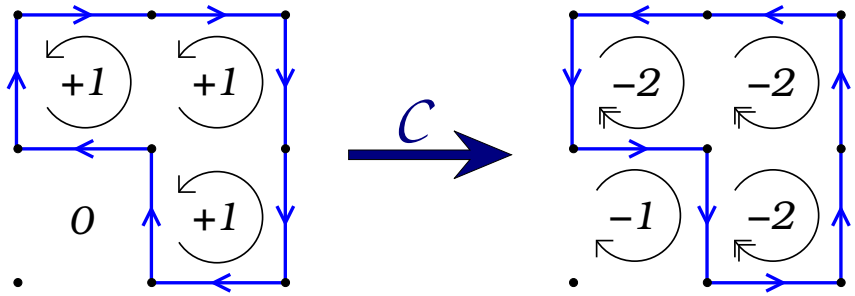


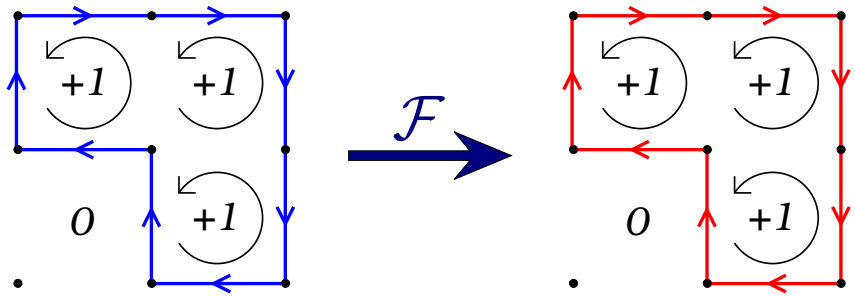
- **Flavor swapping symmetry $\mathcal{F}$:**

$$j_{x,\mu} \xrightarrow{\mathcal{F}} k_{x,\mu}$$

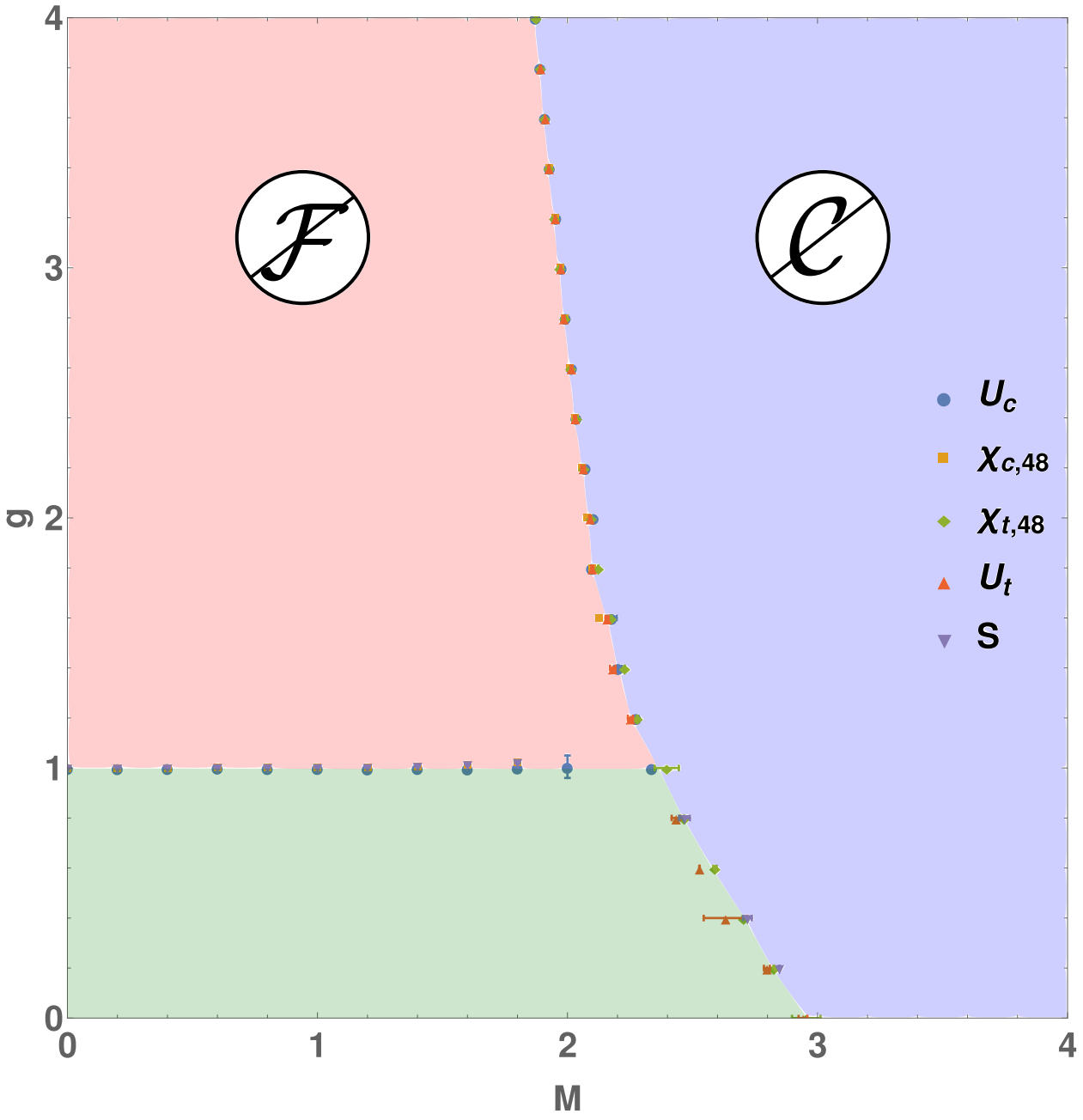
$$k_{x,\mu} \xrightarrow{\mathcal{F}} j_{x,\mu}$$

Order Parameter:

$$\langle c \rangle = -\frac{1}{V}\langle |\phi_x^1|^2 - |\phi_x^2|^2 \rangle$$

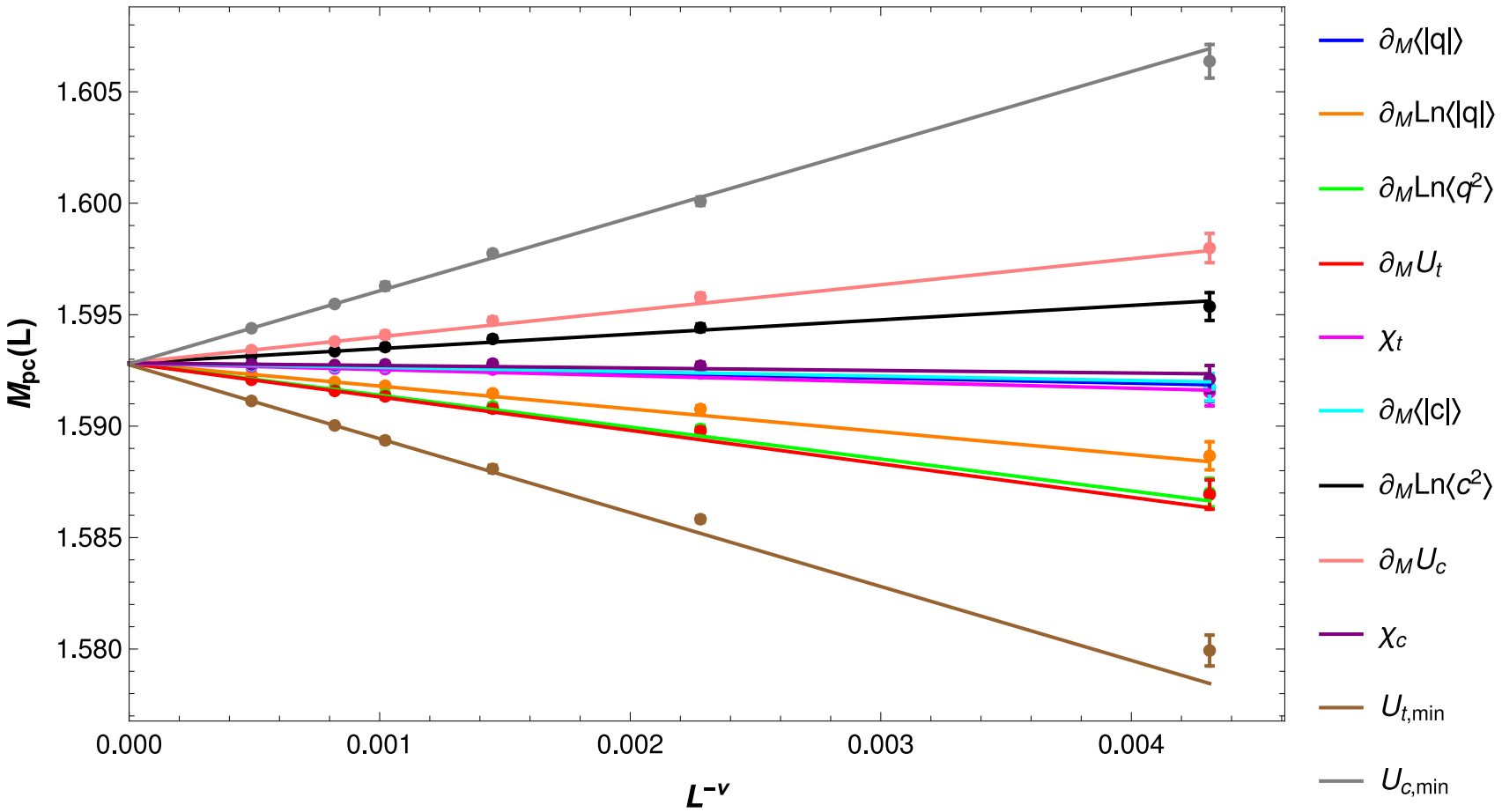


Phase diagram: $(\theta = \pi,\ \lambda = 0.5,\ \beta = 3)$, $M = 4 + m^2$

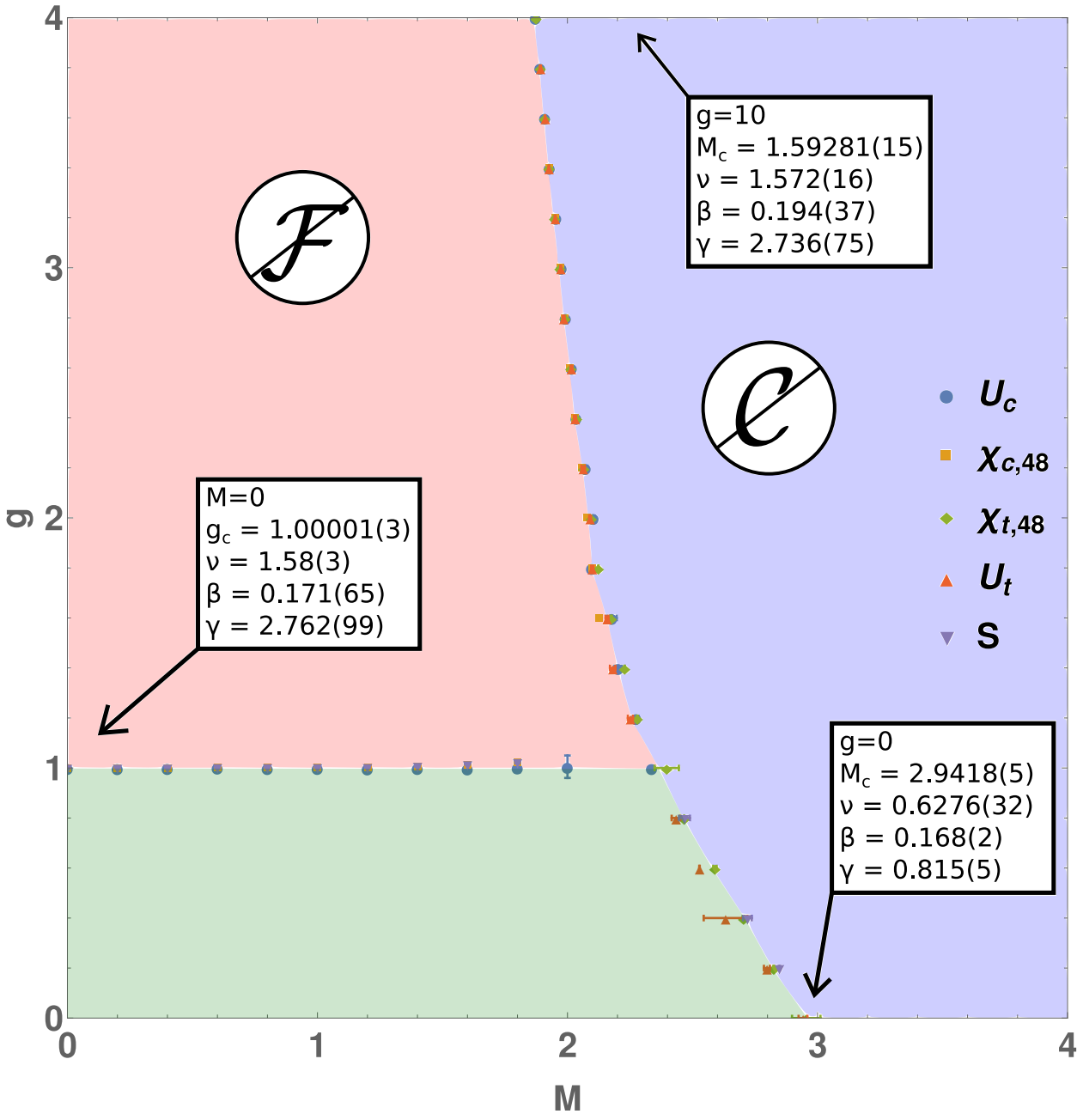


Transition at g=10: $(\theta = \pi,\ \lambda = 0.5,\ \beta = 3)$, $M = 4 + m^2$

$$M_c = \mathbf{1.59281(15)},\ \nu = 1.572(16),\ \beta = 0.194(37),\ \gamma = 2.736(75)$$



Critical lines



Summary

- We study the phase diagram of the 2-flavor U(1) gauge-Higgs model at topological angle $\theta = \pi$.
- The generalized Villain action implements the charge conjugation symmetry at $\theta = \pi$ as an exact Z_2 symmetry.
- Complex action problem is solved by simulating in the world line representation.
- We locate the critical lines and determine their universality class.