

# Computing Nucleon Electric Dipole Moments from Lattice QCD

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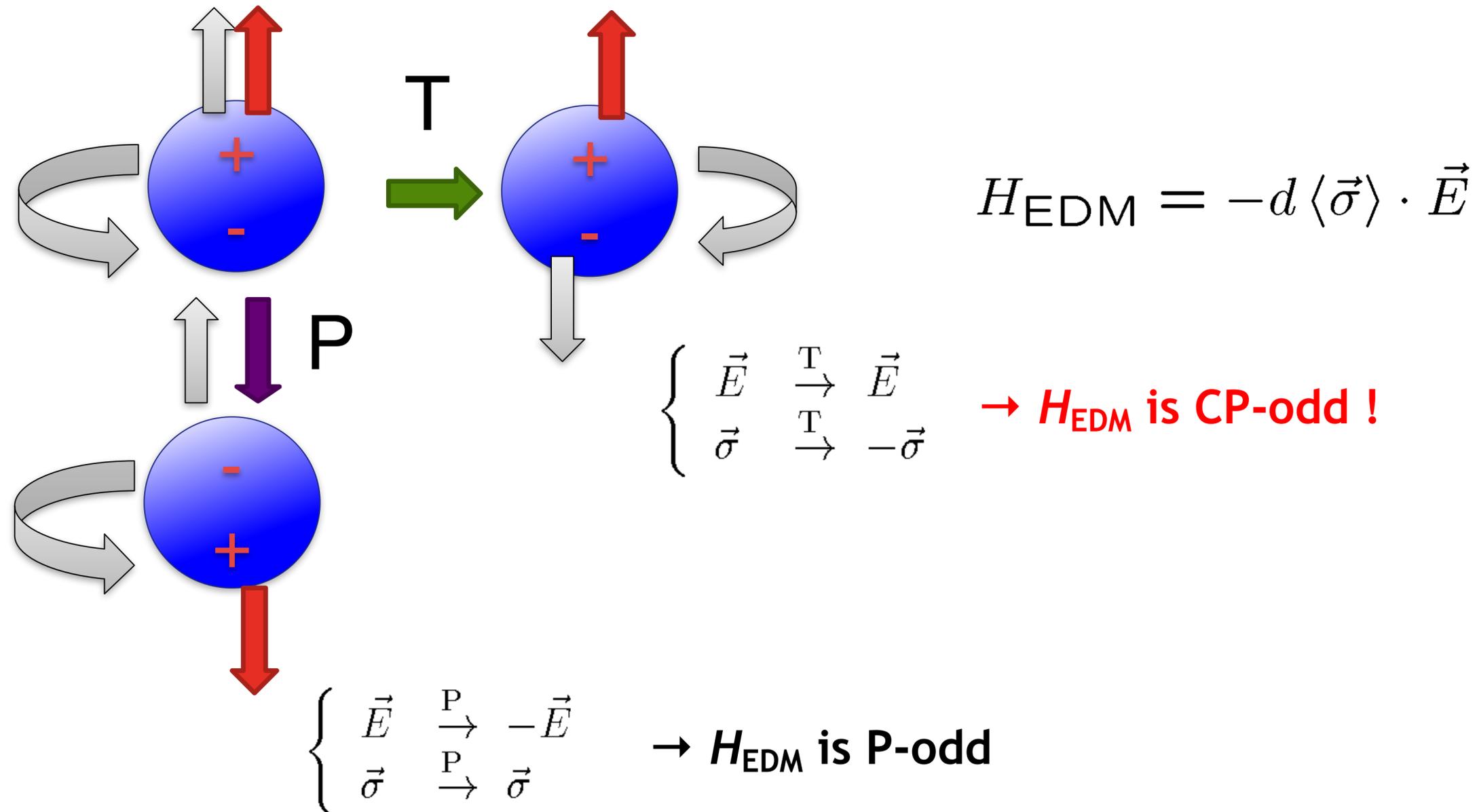
Thanks to M. Abramczyk, S. Aoki, T. Blum, T. Izubuchi and S. Syritsyn

The 37th International Symposium on Lattice Field Theory, June 20, 2019



# Introduction

- Electric Dipole Moment  $\mathbf{d}$   
Energy shift of a spin particle in an electric field
- Non-zero EDM : P&T (CP through CPT) violation



# Nucleon EDM Experiments

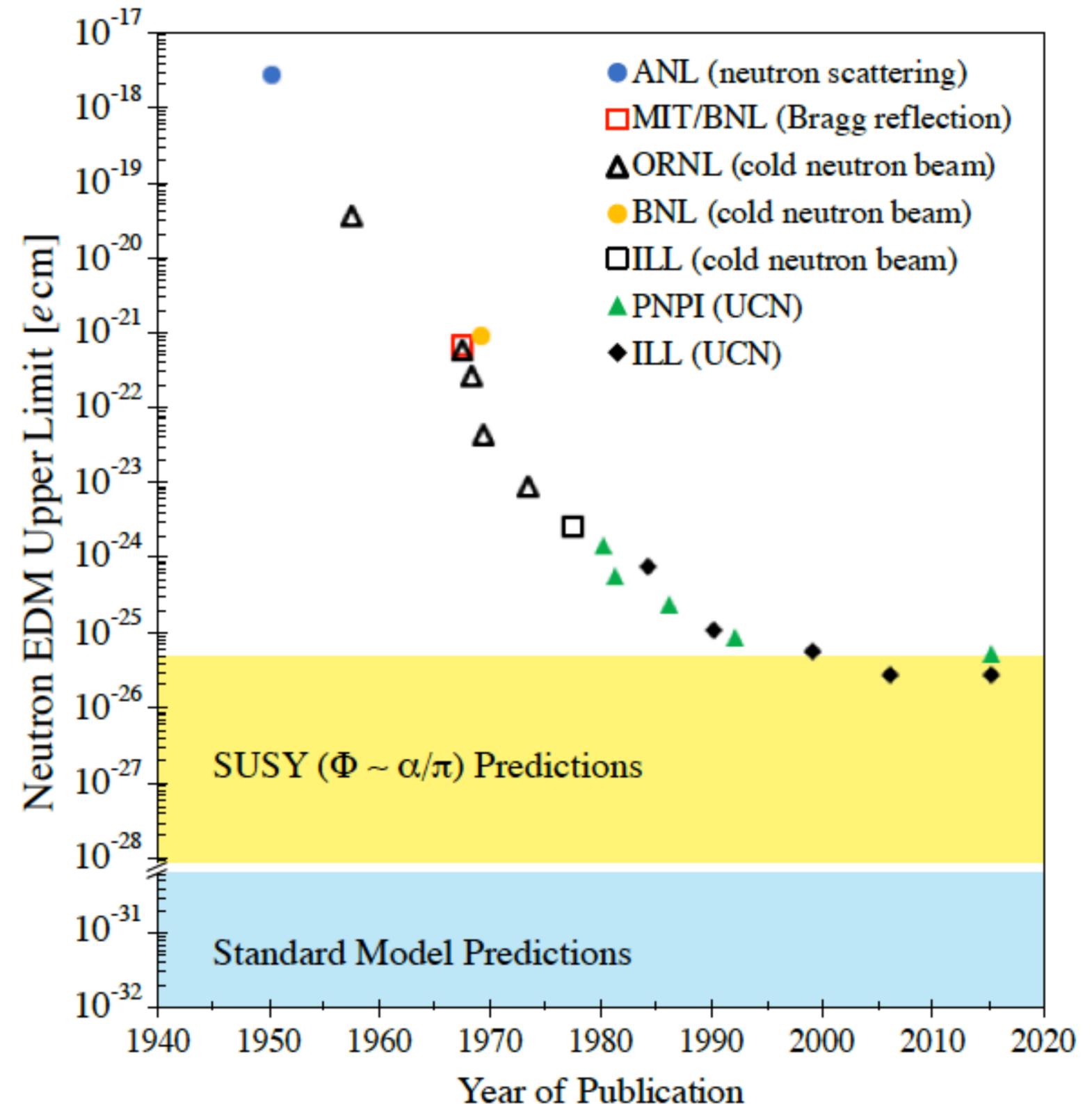
## Current nEDM limits:

$^{199}\text{Hg}$  spin precession (UW) [Graner et al, 2016]  
 Ultracold Neutrons in a trap (ILL) [Baker 2006]

$$|d_{Hg}| < 7.4 \times 10^{-30} \text{ e} \cdot \text{cm}$$

$$|d_n| < 2.6 \times 10^{-26} \text{ e} \cdot \text{cm}$$

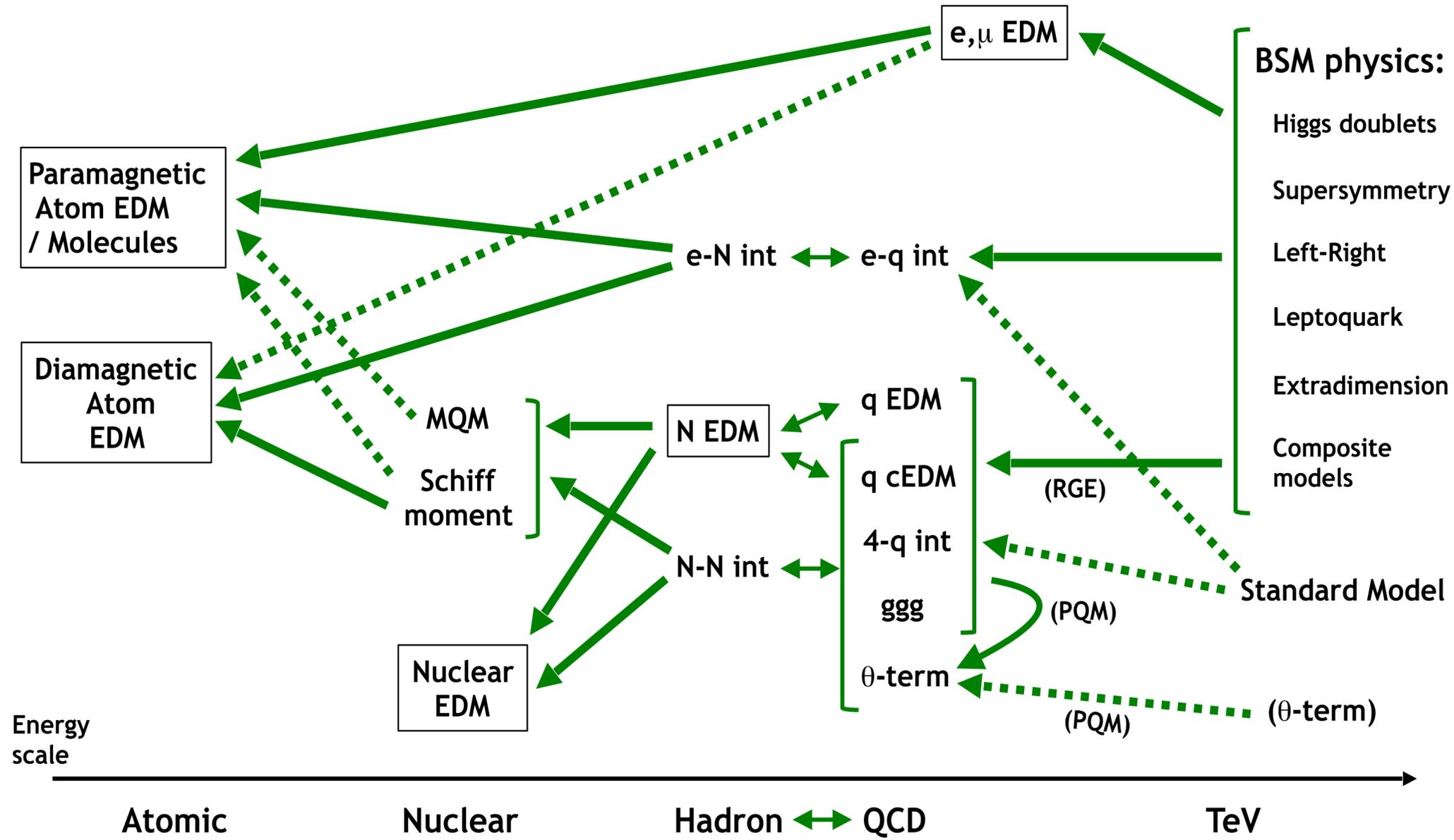
|                          | $10^{-28} \text{ e cm}$    |
|--------------------------|----------------------------|
| <b>CURRENT LIMIT</b>     | <b>&lt;300</b>             |
| Spallation Source @ORNL  | <b>&lt; 5</b>              |
| Ultracold Neutrons @LANL | ~30                        |
| PSI EDM                  | <50 (I), <b>&lt;5 (II)</b> |
| ILL PNPI                 | <10                        |
| Munich FRMII             | <b>&lt; 5</b>              |
| RCMP TRIUMF              | <50 (I), <b>&lt;5 (II)</b> |
| JPARC                    | <b>&lt; 5</b>              |
| Standard Model (CKM)     | < 0.001                    |



Figures from S.Kawasaki (KEK)

# Nucleon EDM

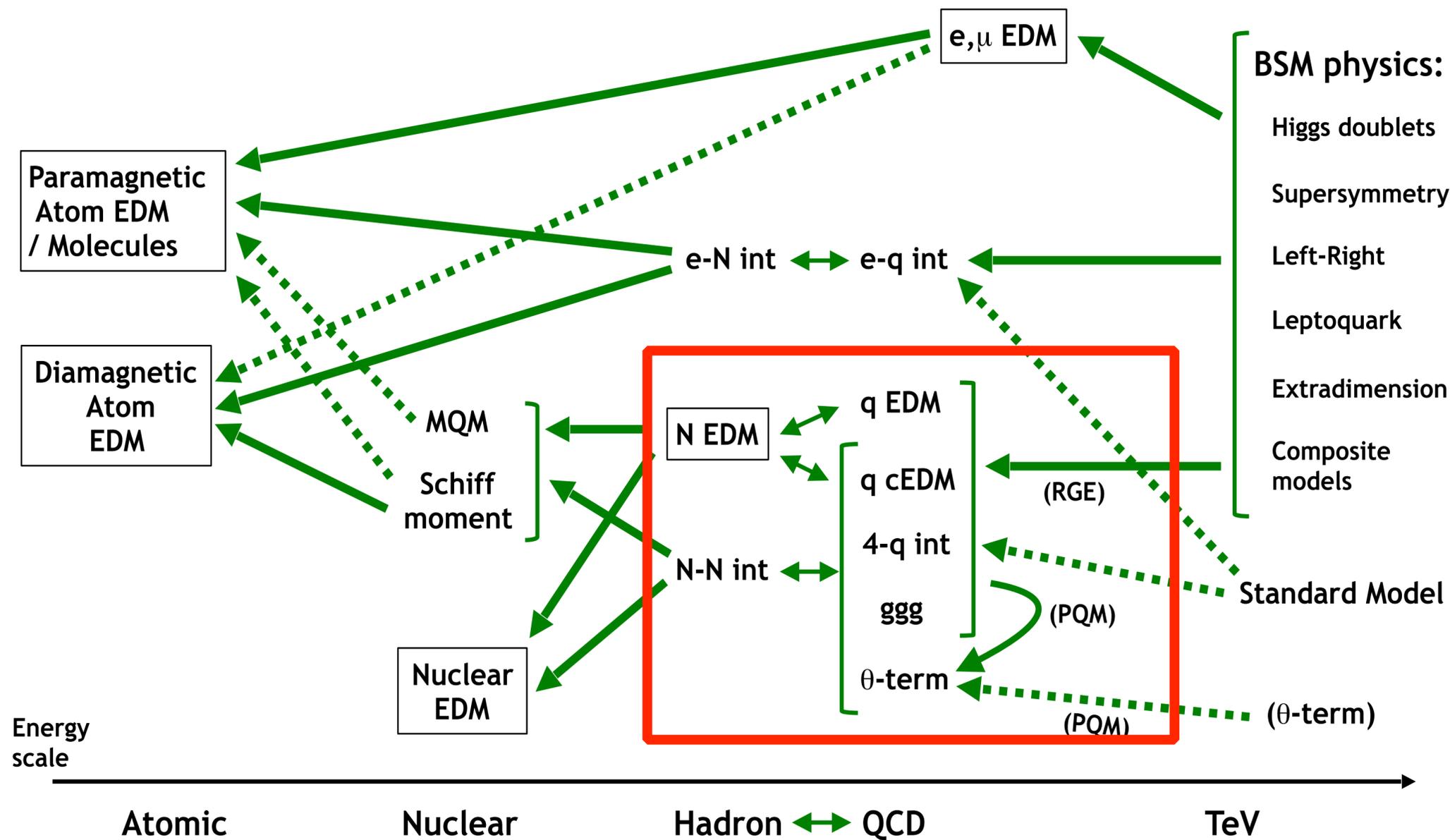
[N. Yamanaka, et al. Eur. Phys. J. A53 (2017) 54, Ginges and Flambaum Phys. Rep. 397, 63, 2004]



|                   |                                      |
|-------------------|--------------------------------------|
| <b>observable</b> | : Observable available at experiment |
|                   | : Sizable dependence                 |
|                   | : Weak dependence                    |
|                   | : Matching                           |

# Nucleon EDM

[N. Yamanaka, et al. Eur. Phys. J. A53 (2017) 54, Ginges and Flambaum Phys. Rep. 397, 63, 2004]



**Important bottleneck of the EDM calculation!**

- observable : Observable available at experiment
- ← : Sizable dependence
- ⋯ : Weak dependence
- ↔ : Matching

**Role of (lattice) QCD** : connect quark/gluon-level (effective) operators to hadron/nuclei matrix elements and interactions (Form factor,  $d_n$ )

**Non-perturbative determination is important → Lattice QCD calculation!**

# Effective CPV operators

$$\begin{aligned}\mathcal{L}_{eff}^{CP} &= \frac{g_s^2}{32\pi^2} \bar{\theta} G_{\mu\nu} \tilde{G}^{\mu\nu} && \text{dim}=4, \theta_{QCD} \\ &- \frac{i}{2} \sum_{i=u,d,s} \tilde{d}_i \bar{\psi}_i G \cdot \sigma \gamma_5 \psi_i && \text{dim}=5, \text{chromo EDM} \\ &- \frac{i}{2} \sum_{i=e,u,d,s} d_i \bar{\psi}_i F \cdot \sigma \gamma_5 \psi_i && \text{dim}=5, e, \text{quark EDM} \\ &+ \omega f^{abc} G_{\mu\nu,a} G^{\mu\beta,b} G_{\beta}^{\nu,c} && \text{dim}=6, \text{Weinberg three gluon} \\ &+ \sum C_i^{(4q)} \mathcal{O}_i^{(4q)} && \text{dim}=6, \text{Four-quark operators}\end{aligned}$$

$\bar{\theta} \leq \mathcal{O}(10^{-10})$  : Strong CP problem

→RG flow suggests that QCD is Coulomb phase due to non-zero vacuum angle  $\theta$ ? [G. Schierholz, talk on Friday]

quark-chromo EDM Dim=5 operators suppressed by  $m_q/\Lambda^2 \rightarrow$  effectively dim=6,

quark EDM ... the most accurate lattice data for EDM (~5% for u,d)

cEDM and Weinberg ops. are ongoing. [T. Bhattacharya, plenary talk]

Lattice QCD calculations are important to constrain  $\theta$ , cEDM etc.

# Calculating CP-odd interaction on the lattice

## CP-violating interaction on lattice

■ Linearization of CP-odd interaction (e.g. :  $\theta$ -EDM) [S. Aoki et al, (2005); F. Berruto et al (2005); A. Shindler et al, (2015), C. Alexandrou et al (2015); E. Shintani et al (2016)]

$$e^{-S_{QCD} - i\theta Q} = e^{-S_{QCD}} [1 - i\theta Q + \mathcal{O}(\theta^2)]$$

$$\langle \mathcal{O} \rangle_{CP} = \underbrace{\langle \mathcal{O} \rangle_{CP-even}}_{\text{(CP-even)}} - i\theta \underbrace{\langle Q \cdot \mathcal{O} \rangle_{CP-even}}_{\text{(CP-odd)}} + \mathcal{O}(\theta^2)$$

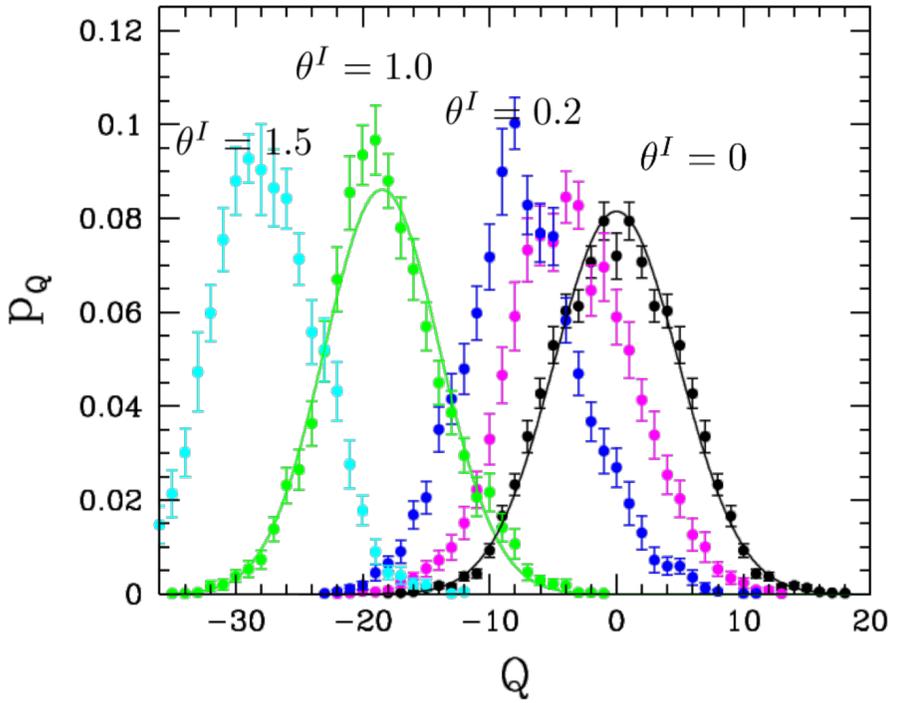
CPV operator : Q, cEDM, etc...,  $\theta \ll 1$

Original (CP-even) gauge configurations can be used. No sign problem.

## ■ Dynamical simulation including CP-odd interactions

$$\langle \mathcal{O} \rangle_{\theta} \sim \int \mathcal{D}U(\mathcal{O}) e^{-S_{QCD} - \theta_{imag} Q} \quad [\text{R. Horsley et al. (2008); H. K. Guo, et al., 2015)]$$

Need additional simulation for ensemble generations to get non-zero topological sector.  
 Better sampling of non-zero topological charge sector  
 Check linearity of  $\theta$  (ensemble generation for various imaginary  $\theta$ )



[R. Horsley, et al (2008)]

# Extraction of $d_N$

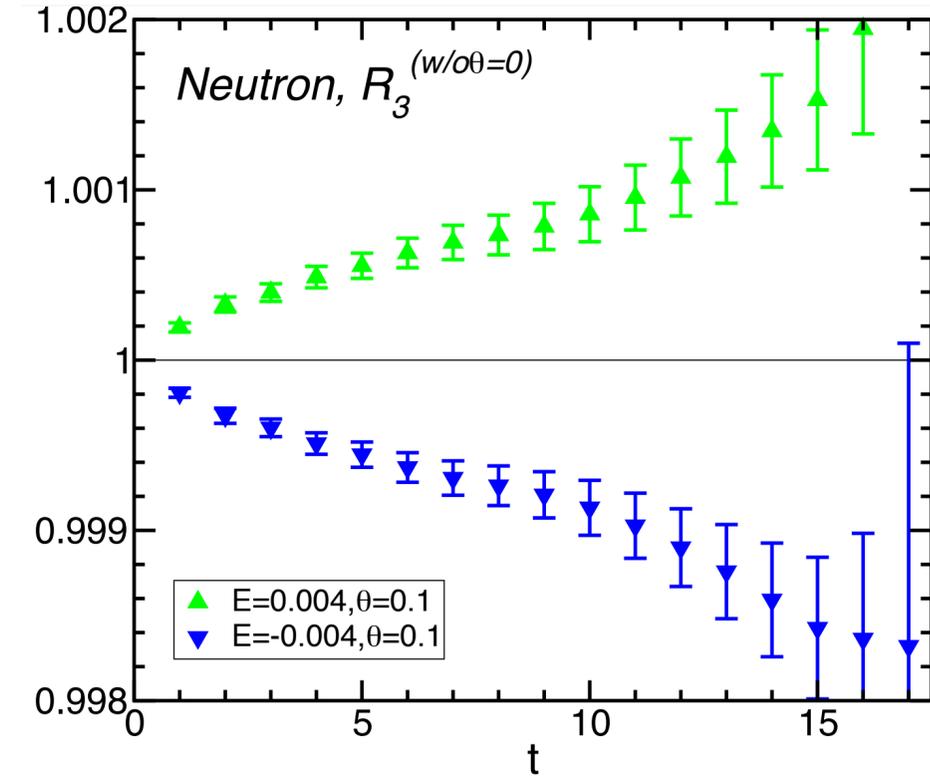
## ■ Nucleon spectrum in the background electric field

[E. Shintani et al, 2006, 2007]

$$\langle N(t) \bar{N}(0) \rangle_{\theta, \vec{E}} \sim e^{-(m_N + d_N \vec{\Sigma} \cdot \vec{E})t}$$

EDM : Spin-dependent energy shift of the nucleon

Need to check linearity in bg. electric field



## ■ P, T-odd form factor

[E. Shintani et al 2005, F. Berruto et al 2015, A. Schindler et al, 2015, C. Alexandra et al, 2015, J. Dragos et al, 2019]

$$\langle p', \sigma' | J^\mu | p, \sigma \rangle = \bar{u}_{p', \sigma'} \left[ \underbrace{F_1(Q^2) \gamma^\mu + F_2(Q^2) \frac{i\sigma^{\mu\nu} q_\nu}{2m_N}}_{\text{P, T even}} - \underbrace{F_3(Q^2) \frac{\gamma_5 \sigma^{\mu\nu} q_\nu}{2m_N}}_{\text{P, T odd}} \right] u_{p, \sigma}$$

$$d_n = \lim_{Q^2 \rightarrow 0} \frac{F_3(Q^2)}{2m_N}$$

Need  $Q^2 \rightarrow 0$  extrapolation

# $\theta_{QCD}$ induced Nucleon EDMs

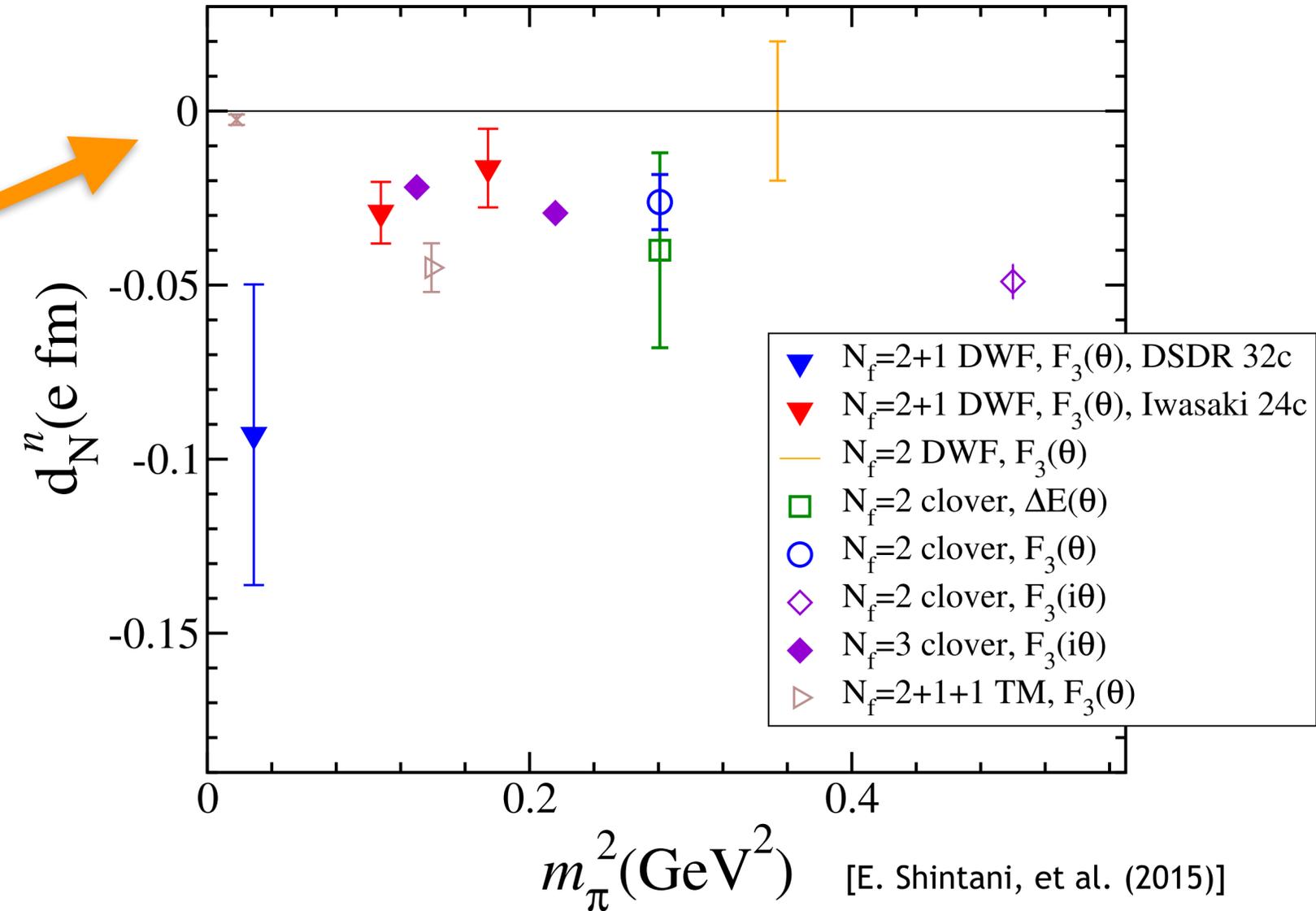
## Phenomenological estimates

| method              | value                             |
|---------------------|-----------------------------------|
| ChPT/NDA            | $\sim 0.002$ e fm                 |
| QCD sum rules [1,2] | $0.0025 \pm 0.0013$ e fm          |
| QCD sum rules [3]   | $0.0004^{+0.0003}_{-0.0002}$ e fm |

- [1] M. Pospelov, A. Ritz, Nuclear Phys. B 573 (2000) 177,  
 [2] M. Pospelov, A. Ritz, Phys. Rev. Lett. 83 (1999) 2526,  
 [3] J. Hisano, J.Y. Lee, N. Nagata, Y. Shimizu, Phys. Rev. D 85 (2012) 114044.

## Lattice calculations

### Neutron



Phenomenology:  $|dn| \sim \theta_{QCD} 10^{-3}$  e fm  $\rightarrow |\theta_{QCD}| < 10^{-10}$

Lattice :  $|dn| \sim \theta_{QCD} 10^{-2}$  e fm  $\rightarrow$  severer constraint on  $|\theta_{QCD}|$

# $\theta_{QCD}$ induced Nucleon EDMs

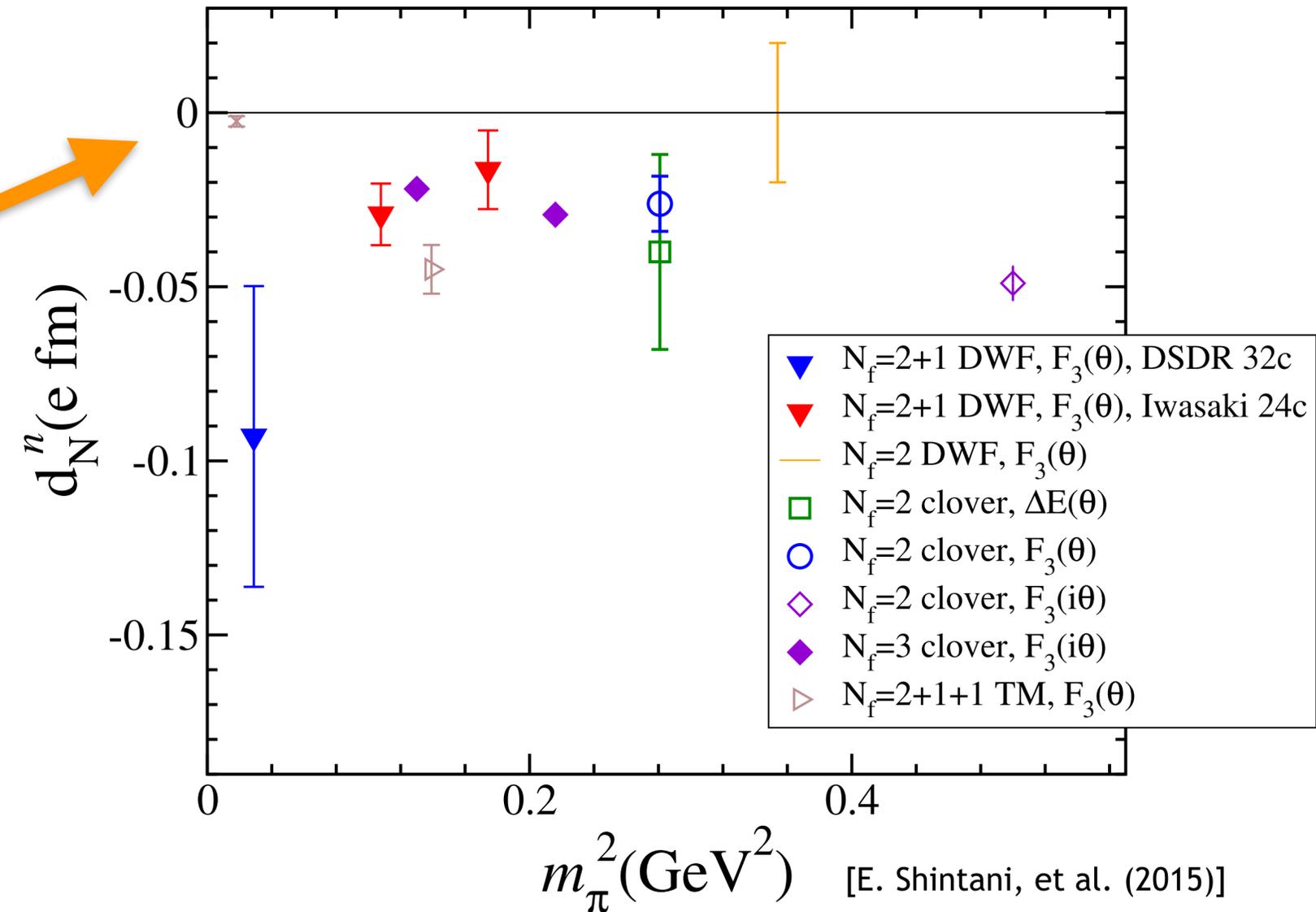
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**Problem: a spurious mixing between EDM and magnetic moments in all previous lattice computations of nucleon form factor.**

# Spurious mixing problem in lattice form factor method

[M. Abramczyk, et al, 2017]

■ CP violating interaction induces a chiral phase :  $\langle 0|N|p, \sigma\rangle_{\mathcal{CP}} = e^{i\alpha\gamma_5} u_{p,\sigma} = \tilde{u}_{p,\sigma}$

■ This mixing angle  $\alpha$  has to be calculated, and rotated away to get “net” CP-violation effect.

$$\bar{\tilde{u}}_{p',\sigma'} \left[ \tilde{F}_1 \gamma^\mu + (\tilde{F}_2 + i\tilde{F}_3 \gamma_5) \frac{i\sigma^{\mu\nu} q_\nu}{2m_N} \right] \tilde{u}_{p,\sigma} \equiv \bar{u}_{p',\sigma'} \left[ F_1 \gamma^\mu + (F_2 + iF_3 \gamma_5) \frac{i\sigma^{\mu\nu} q_\nu}{2m_N} \right] u_{p,\sigma}$$

[Previous “lattice” parametrization prior to 2017]

$$(F_2 + iF_3 \gamma_5) = e^{2i\alpha\gamma_5} (\tilde{F}_2 + i\tilde{F}_3 \gamma_5)$$



$$[F_2]_{\text{correct}} = \tilde{F}_2 + \mathcal{O}(\alpha^2)$$

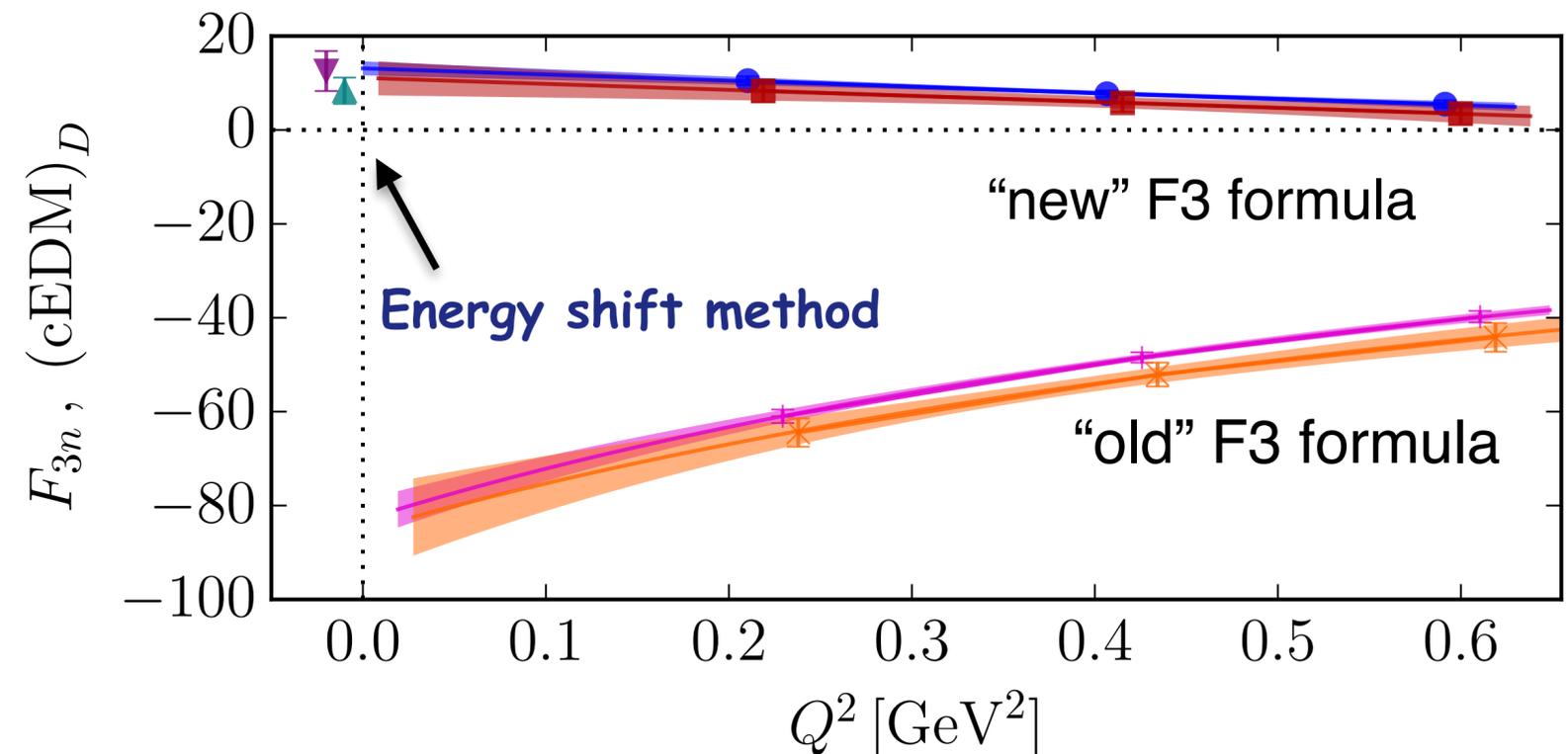
$$[F_3]_{\text{correct}} = \tilde{F}_3 + 2\alpha F_2$$

■ Numerical check: F3 form factor vs. energy shift in background electric field

Neutron, d-cEDM, large spurious mixing.

$$\alpha_d \sim 30$$

Previous lattice EDM results (prior to 2017) were subject to large contamination from F2,3 mixing.

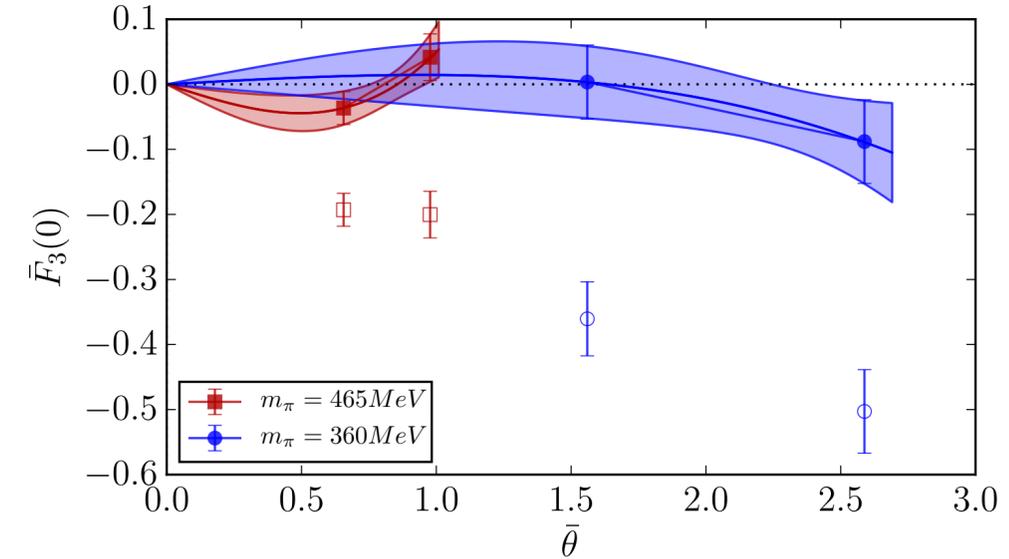


# Reanalysis of “lattice” $\theta$ induced EDM

[M. Abramczyk, et al, 2017]

Correction is simple:  $[F_3]_{\text{correct}} = \tilde{F}_3 + 2\alpha F_2$

Dynamical calculations with finite imaginary  $\theta$  angle,  
Form factor method [F. Guo et al (2015)]



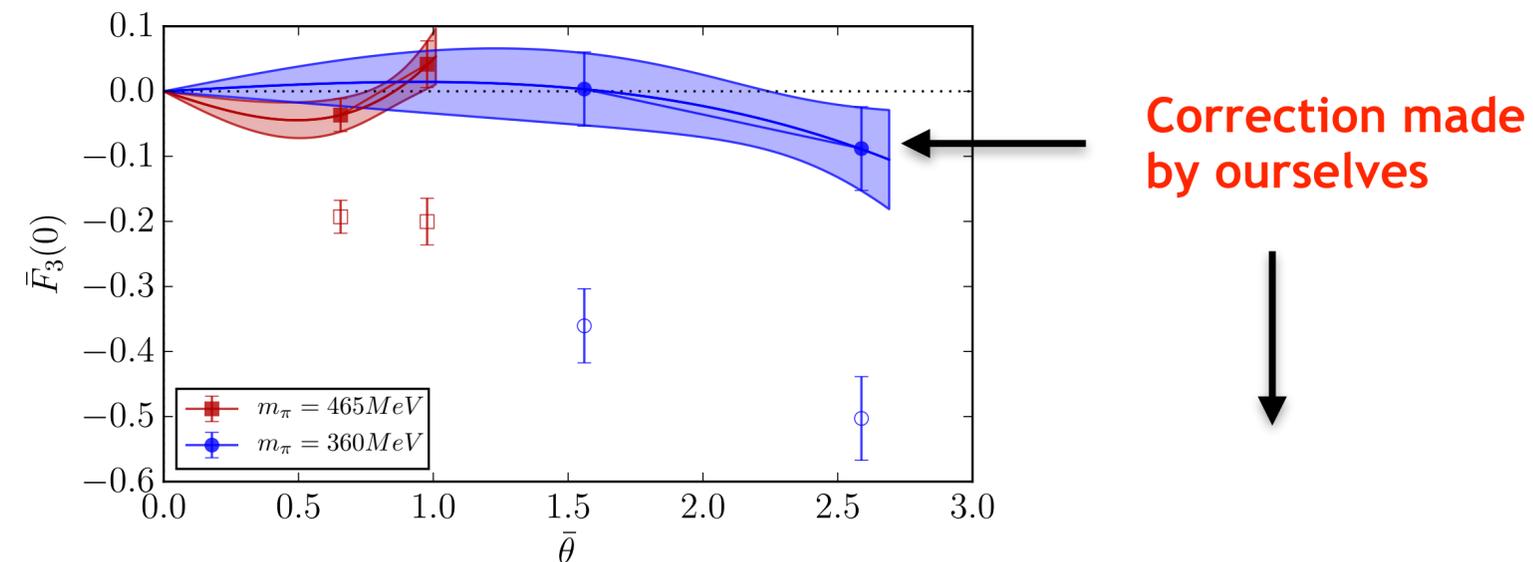
|                       | $m_\pi$ [MeV] | $m_N$ [GeV] | $F_2$     | $\alpha$   | $\tilde{F}_3$ |             |
|-----------------------|---------------|-------------|-----------|------------|---------------|-------------|
| [ETMC 2016]           | $n$           | 373         | 1.216(4)  | -1.50(16)  | -0.217(18)    | -0.555(74)  |
| [Shintani et al 2005] | $n$           | 530         | 1.334(8)  | -0.560(40) | -0.247(17)    | -0.325(68)  |
|                       | $p$           | 530         | 1.334(8)  | 0.399(37)  | -0.247(17)    | 0.284(81)   |
| [Berruto et al 2006]  | $n$           | 690         | 1.575(9)  | -1.715(46) | -0.070(20)    | -1.39(1.52) |
|                       | $n$           | 605         | 1.470(9)  | -1.698(68) | -0.160(20)    | 0.60(2.98)  |
| [Guo et al 2015]      | $n$           | 465         | 1.246(7)  | -1.491(22) | -0.079(27)    | -0.375(48)  |
|                       | $n$           | 360         | 1.138(13) | -1.473(37) | -0.092(14)    | -0.248(29)  |

# Reanalysis of “lattice” $\theta$ induced EDM

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|                       | $m_\pi$ [MeV] | $m_N$ [GeV] | $F_2$     | $\alpha$   | $\tilde{F}_3$ | $F_3$       |             |
|-----------------------|---------------|-------------|-----------|------------|---------------|-------------|-------------|
| [ETMC 2016]           | $n$           | 373         | 1.216(4)  | -1.50(16)  | -0.217(18)    | -0.555(74)  | 0.094(74)   |
| [Shintani et al 2005] | $n$           | 530         | 1.334(8)  | -0.560(40) | -0.247(17)    | -0.325(68)  | -0.048(68)  |
|                       | $p$           | 530         | 1.334(8)  | 0.399(37)  | -0.247(17)    | 0.284(81)   | 0.087(81)   |
| [Berruto et al 2006]  | $n$           | 690         | 1.575(9)  | -1.715(46) | -0.070(20)    | -1.39(1.52) | -1.15(1.52) |
|                       | $n$           | 605         | 1.470(9)  | -1.698(68) | -0.160(20)    | 0.60(2.98)  | 1.14(2.98)  |
| [Guo et al 2015]      | $n$           | 465         | 1.246(7)  | -1.491(22) | -0.079(27)    | -0.375(48)  | -0.130(76)  |
|                       | $n$           | 360         | 1.138(13) | -1.473(37) | -0.092(14)    | -0.248(29)  | 0.020(58)   |

Removing spurious contributions : no signal of EDM → consistent with phenomenological estimates

How to improve the signal?

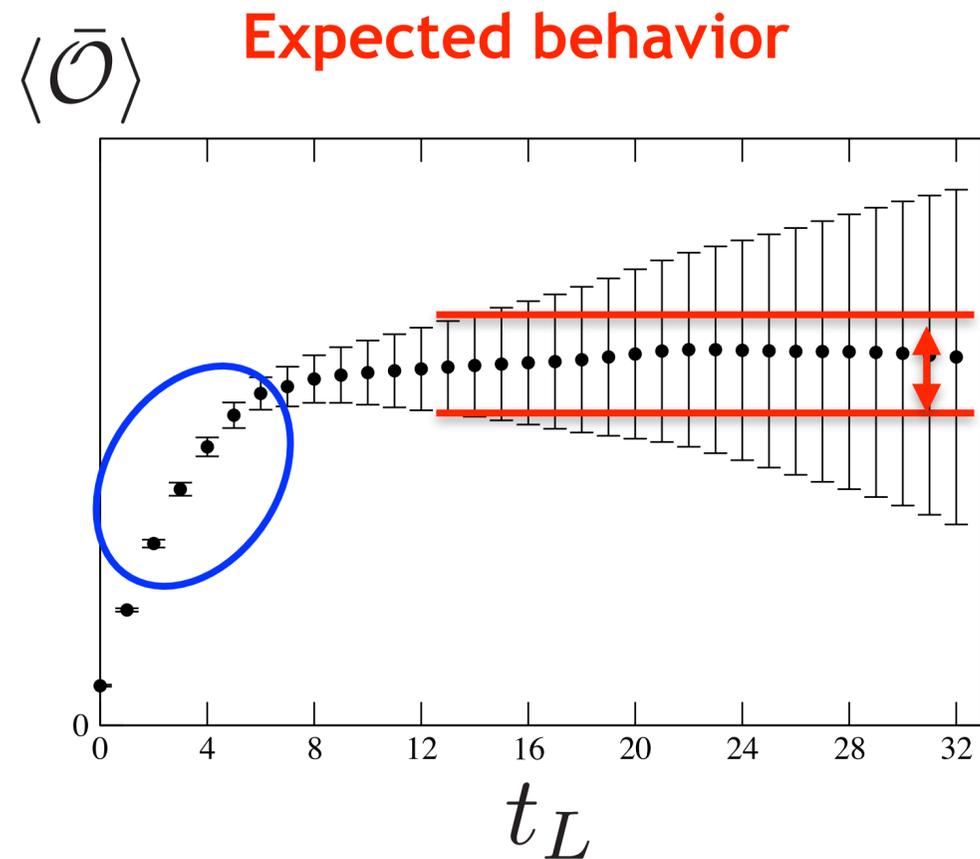
## Noise reduction technique for $\theta$ -induced EDM

# Generic strategy for noise reduction

$$\langle \bar{\mathcal{O}}(T) \rangle = \lim_{t_L \rightarrow \infty} \int_{-t_L}^{t_L} dt \int_{V_3} d^3x \langle \rho(x, t) \mathcal{O}(T) \rangle \quad (T \equiv t_{\text{sink}} - t_{\text{src}})$$

$\rho(x, t)$  is a density quantity which is independent of source, sink positions.

$\theta$ -EDM case :  $\rho$  is the topological charge density operator  $\rho(x, t) = G_{\mu\nu} \tilde{G}^{\mu\nu}(x, t)$



$$t_L \sim t_{\text{src}}, t_{\text{sink}} \rightarrow \mathcal{O}(1)$$

$$t_L \gg t_{\text{src}}, t_{\text{sink}} \rightarrow \mathcal{O}(e^{-\Delta m t_L})$$

**Noise truncation introduces systematic error which must be estimated correctly.**

**Important to understand the behavior for earlier time region.**

# Various noise reduction technique on the lattice

Noise reduction technique used in various fields of lattice computations

■ point-source method with noise → Hadronic light-by-light

■ Constrain Q sum to fiducial volume for  $\theta$ -EDM : Topological charge:  $Q \sim \int_{V_4} G\tilde{G}$ ,  $\langle Q^2 \rangle \sim V_4$

$$Q \sim \int_{V_Q} d^4x q(x)$$

(Statistical error<sup>2</sup>  $\sim V_4$ )

\* in time around current

$$|t_Q - t_J| < \Delta t \quad [\text{E. Shintani et al (2015), B. Yoon et al (2019)}]$$

\* 4d spherical around sink

$$|x_Q - x_{sink}| < R \quad [\text{K. -F. Liu et al (2017)}]$$

\* 4d “cylinder”

$$V_Q : |\vec{x}| < r_Q, \quad -\Delta t_Q < t_0 < T + \Delta t_Q \quad [\text{S. Syritsyn et al (2018)}]$$

\* in time around source

$$|t_Q - t_{src}| < \Delta t \quad [\text{J. Dragos et al (2019)}]$$

■ Background fields and/or Feynman-Hellman theorem:

4pt → 3pt (cEDM), 3pt → 2pt (nucleon matrix elements, e.g.  $g_A$ )

■ Gradient flow: noise reduction of the topological charge, etc...

**Selected recent progresses for  $\theta$ -EDM will be shown.**

## Related recent studies

- 1: J. Dragos et al [arXiv:1902.03254]
- 2: S. Syritsyn et al ('18)
- 3: B. Yoon (talk on Friday)

|                                      | 1  | 2                                | 3  |
|--------------------------------------|--|----------------------------------|--|
| $m_\pi$ [MeV]                        | 410-700  | 139-420                          | 130-310  |
| $S_F$                                | Wilson   | DWF                              | Clover   |
| $a$ [fm]                             | 0.07, 0.09, 0.10   | 0.11                             | 0.09, 0.12   |
| measurements                         | 15-30k   | 30-80k                           | 123-165k   |
| truncation method                    | $\alpha$ -improvement<br>$ t_Q - t_{\text{src}}  < \Delta t$ | $t$ -direction<br>(4-d cylinder) | $t$ -direction<br>$ t_Q - t_J  < \Delta t$ for $F_3$ |
| $Q^2, a \rightarrow 0$ extrapolation | ✓  | N/A                              | N/A  |

**All use the gradient flow for topological charge density.**

**Method: Form factor method**

# 1: $\alpha$ -improvement

[J. Dragos et al, arXiv:1902.03254]

- Consider 3-pt functions with topological charge density

$$\Delta C_{3pt}(\tau) \equiv \langle T\{N(T)\bar{Q}(\tau)\bar{N}(0)\}\rangle, \quad \bar{Q}(\tau) \equiv \int d^3x G\tilde{G}(x, \tau)$$

- Performing the spectral decomposition

(1)  $0 < \tau < T$

$$\Delta C_{3pt}(\tau) = \langle N(T)\bar{Q}(\tau)\bar{N}(0) \rangle \sim \sum_{n,m} e^{-E_n(T-\tau) - E_m\tau} \langle 0|N|n\rangle \langle n|\bar{Q}|m\rangle \langle m|\bar{N}|0\rangle \sim \sum_{m \neq n} \cosh(\Delta m_{mn}(\tau - T/2))$$

$$\langle n|\bar{Q}|n\rangle = 0 \text{ due to } P \text{ sym.}$$

(2)  $T < \tau$

$$\Delta C_{3pt}(\tau) = \langle \bar{Q}(\tau)N(T)\bar{N}(0) \rangle \sim \sum_{n,m} e^{-E_n\tau - E_mT} \langle 0|\bar{Q}|n\rangle \langle n|N|m\rangle \langle m|\bar{N}|0\rangle \sim \sum_n e^{-E_n\tau}, \quad (E_0 \sim m_{\eta'})$$

- mixing angle  $\alpha$  is obtained by fit analysis

**exponentially suppressed**

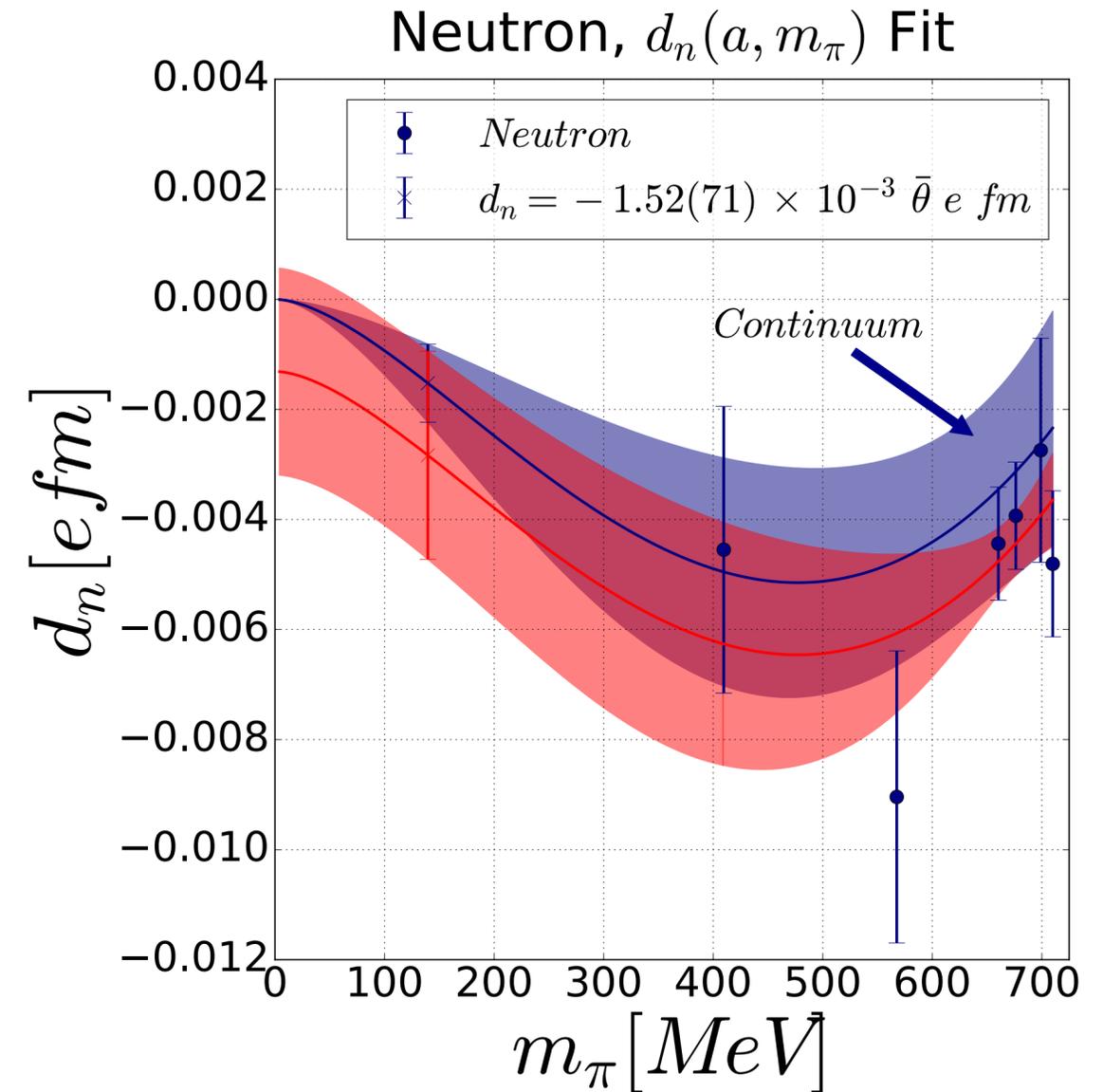
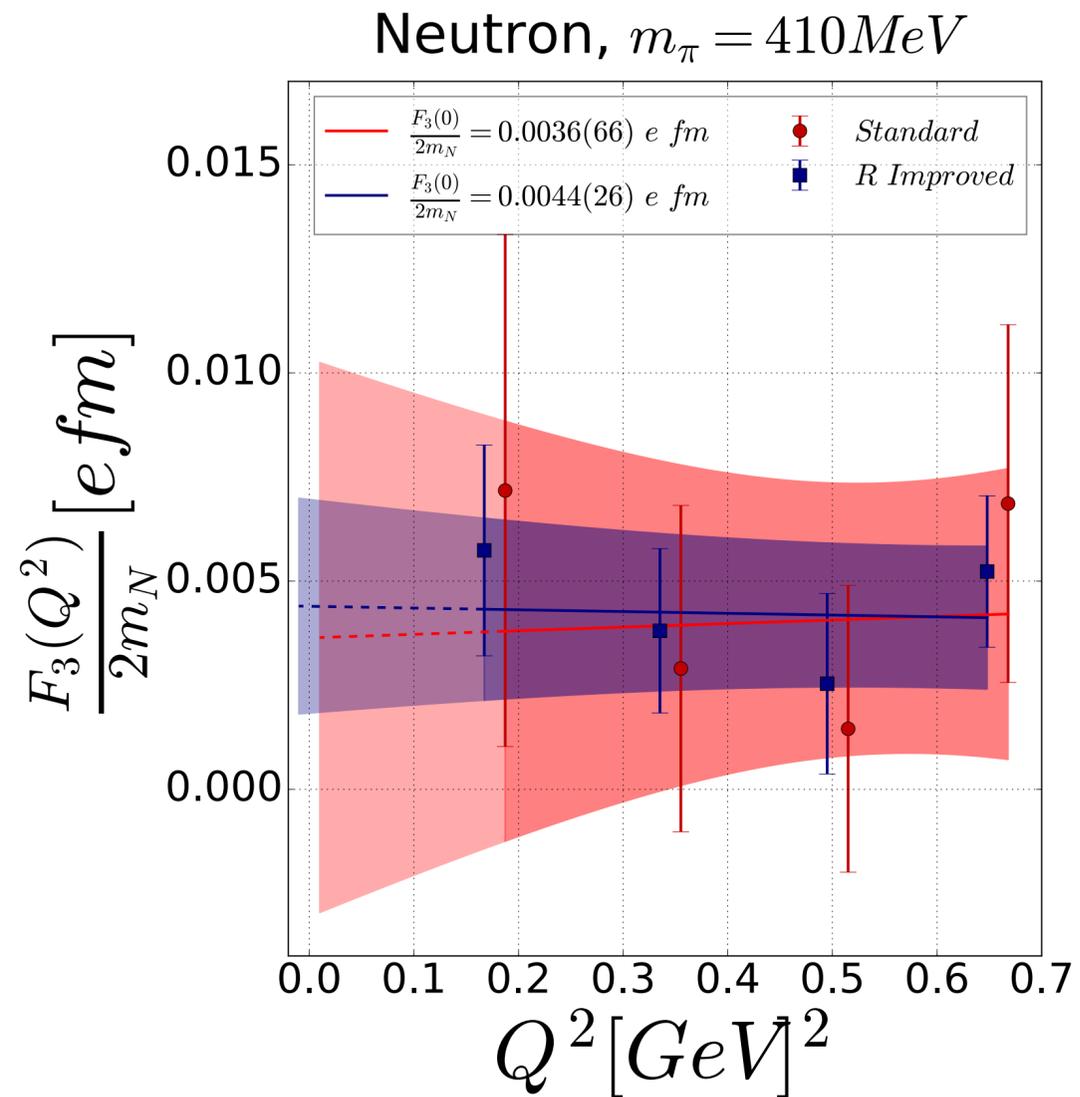
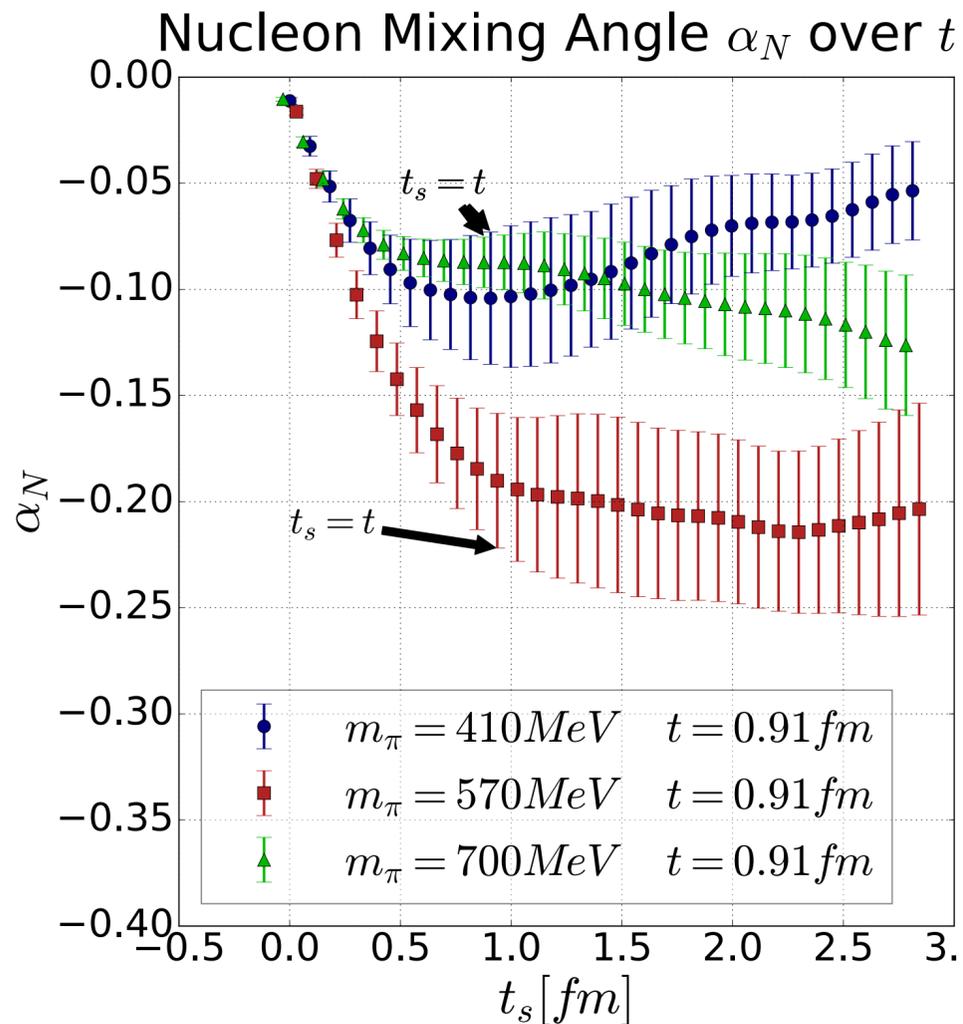
$$C_{3pt}(t_s) = \sum_{\tau=-t_s}^{\tau=t_s} \Delta C_{3pt}(\tau) = A + Be^{-Et_s} \quad (t_s \gg T)$$

- Extension to the 4-pt function (form factor  $\langle NJQN \rangle$ ) is straightforward, and the formula becomes complicated.

# 1: $\alpha$ -improvement and ChPT fit for $F_3$

[J. Dragos et al, arXiv:1902.03254]

$$t = T = t_{\text{sink}} - t_{\text{src}}$$



**Fit ansatz:**  $d_{p/n}(a, m_\pi) = C_1 m_\pi^2 + C_2 m_\pi^2 \log\left(\frac{m_\pi^2}{m_{N,phys}^2}\right) + C_3 a^2$

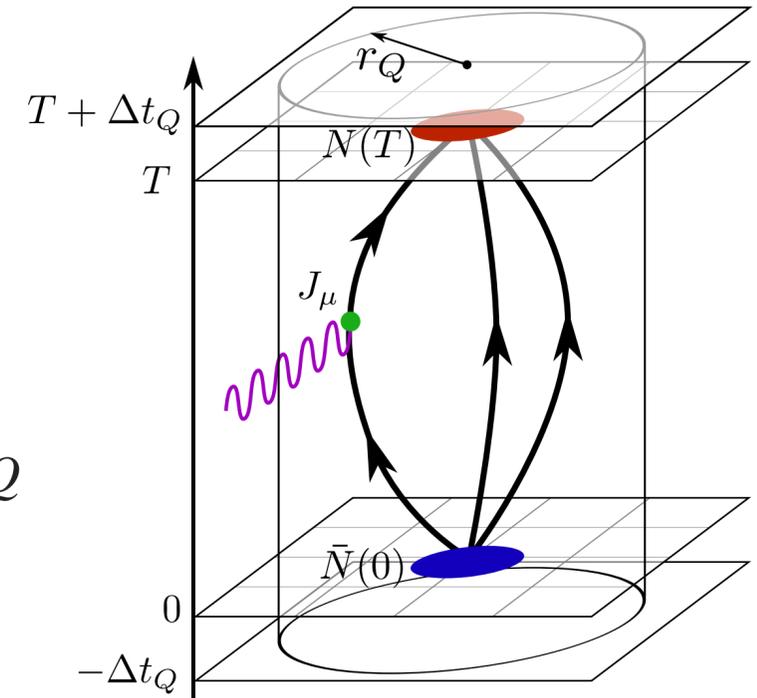
**Non-zero signal at physical point for  $\theta$ -EDM by extrapolation.**

**Far from chiral regime?**

## 2. Our work

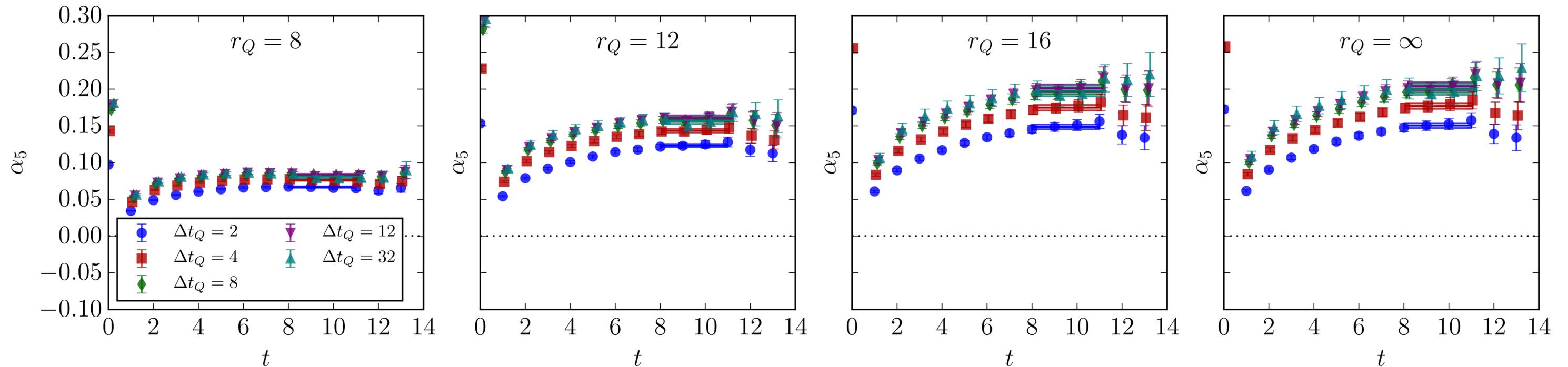
- Nf=2+1 (Mobius) Domain wall fermion, Iwasaki gauge action
- Reduced topological charge density

$$\tilde{Q}(\Delta t_Q, r_Q) = \frac{1}{16\pi^2} \sum_{x \in V_Q} \text{Tr}[\hat{G}_{\mu\nu} \tilde{G}_{\mu\nu}]_x, \quad (\vec{x}, t) \in V_Q : \begin{cases} |\vec{x} - \vec{x}_0| \leq r_Q, \\ t_0 - \Delta t_Q < t < t_0 + t_{\text{sep}} + \Delta t_Q \end{cases}$$



Convergence test of the parity-mixing angle from the reduced topological charge

$m_\pi = 340 \text{ MeV}$

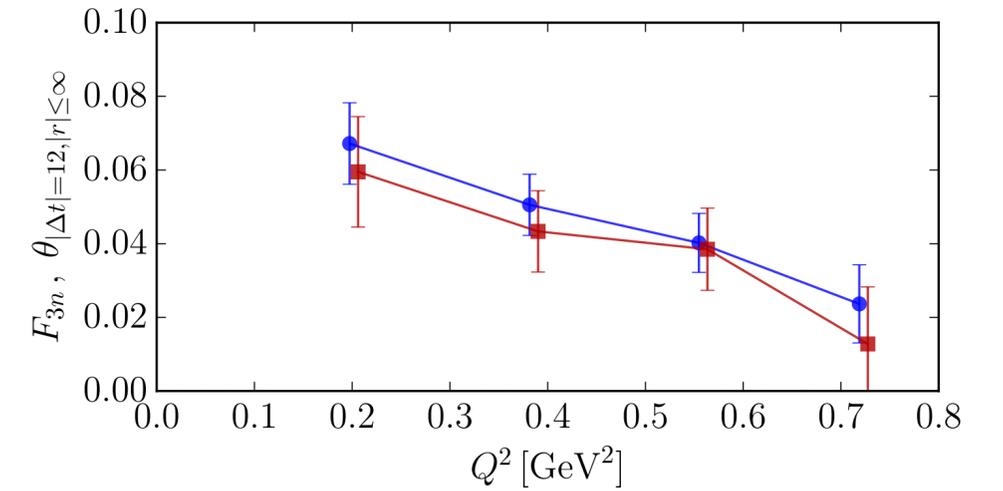
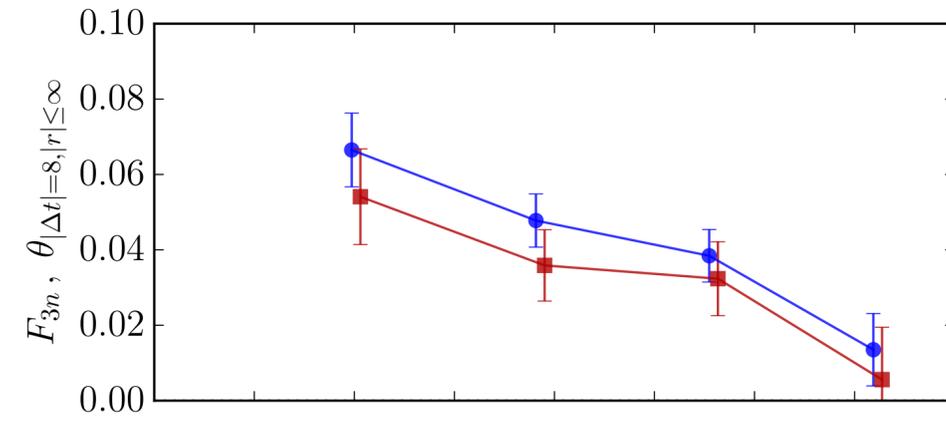
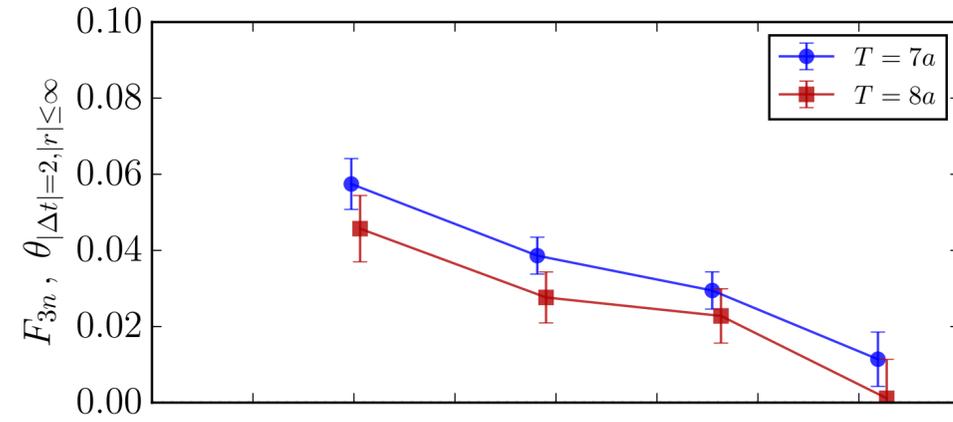


**convergence :**  $r_Q \geq 16a, \Delta t_Q \geq 8a$

# $F_{3n}$ form factor

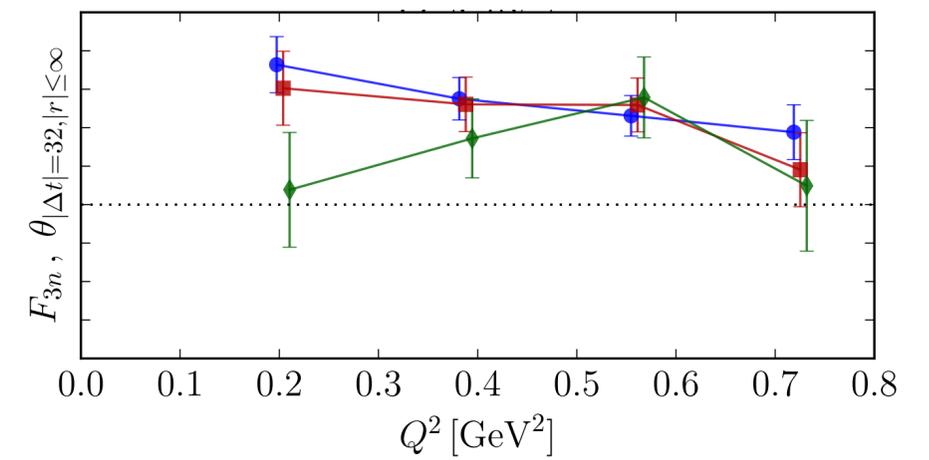
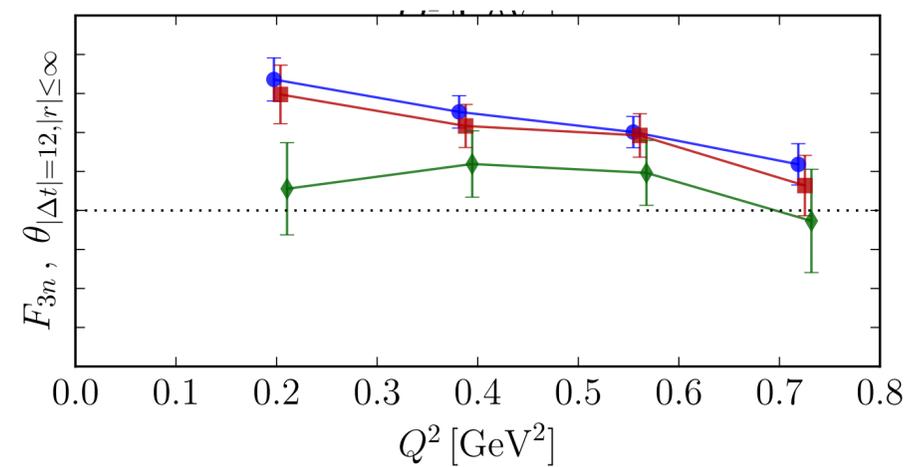
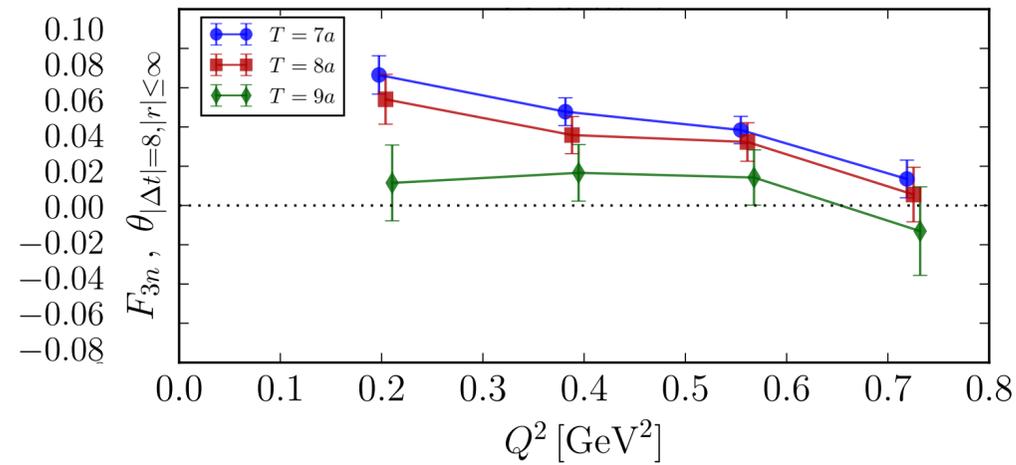
$m_\pi = 410$  MeV

1500 configs x 64 (AMA) samples = 96000 stat.



$m_\pi = 340$  MeV

1400 configs x 64 (AMA) samples = 89600 stat.

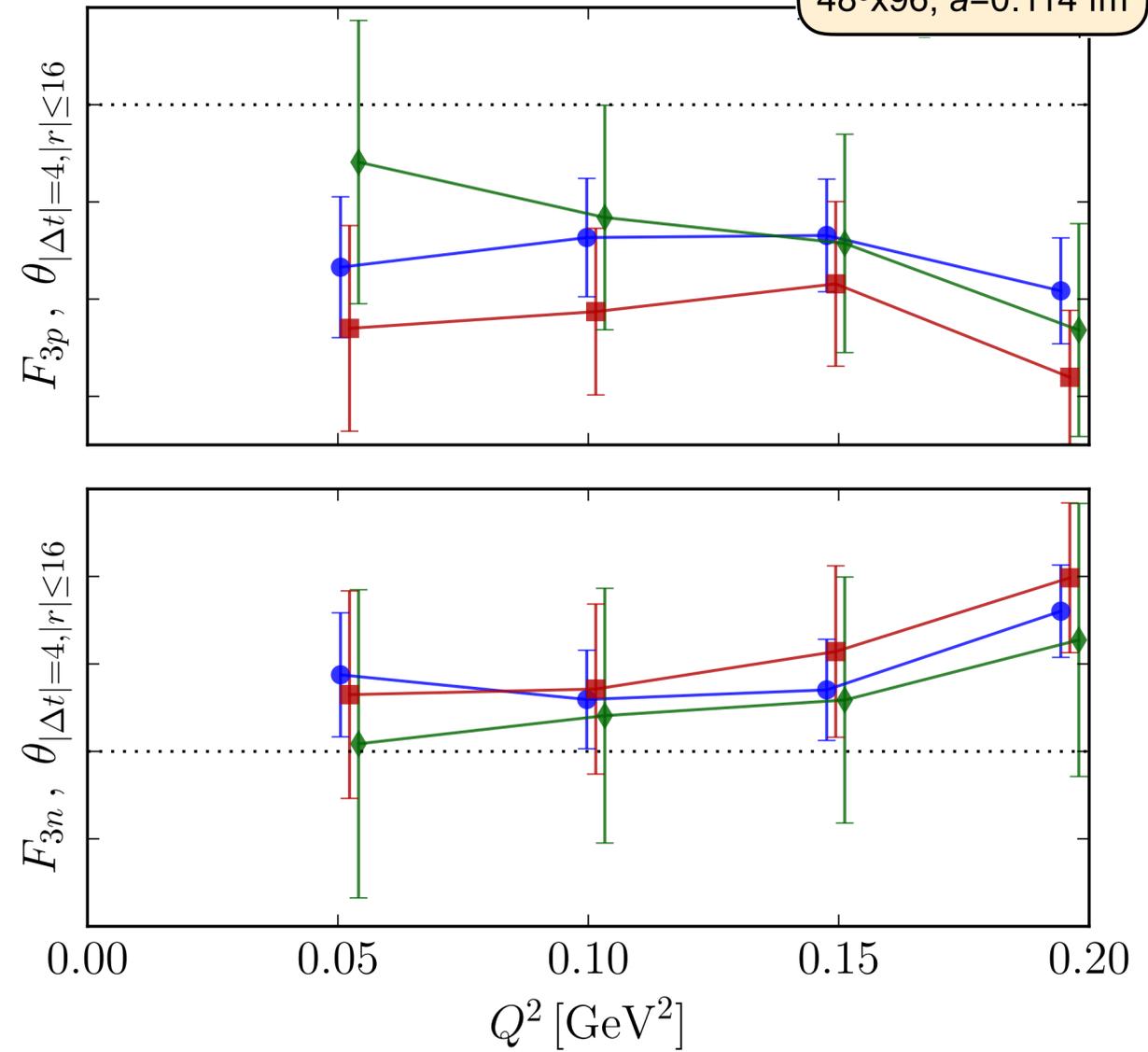
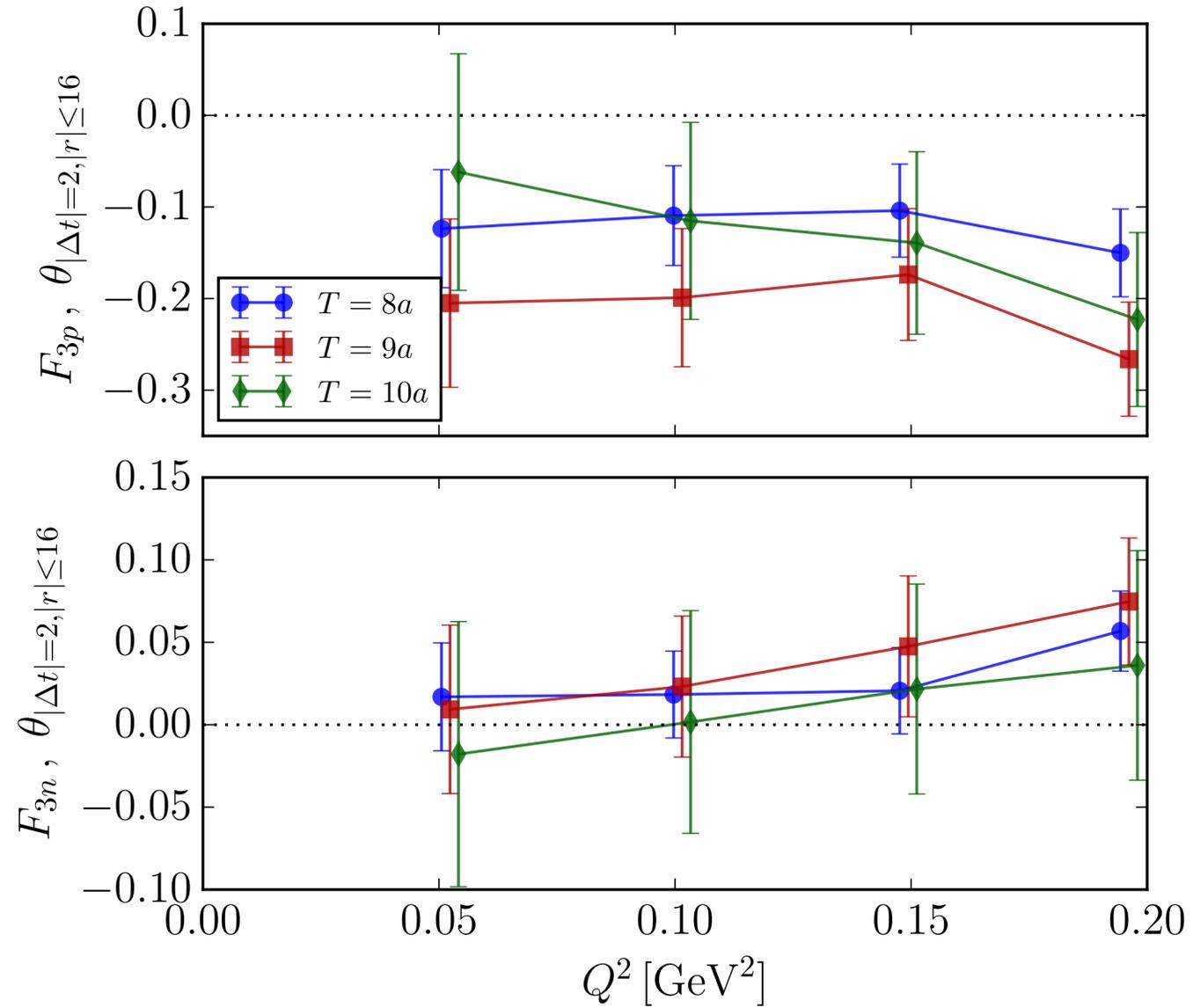


convergence:  $\Delta t_Q \geq 8a$

$$\lim_{Q^2 \rightarrow 0} F_{3n}(Q^2) \sim 0.1$$

# Preliminary result: $\theta$ -EDM on physical point

Physical point  
DWF  $N_f=2+1$   
 $48^3 \times 96$ ,  $a=0.114$  fm



**33000 stat.**

$F_{3n}(Q^2)$  : **consistent with zero.**

## Our naive estimate of $\theta$ -nEDM at the physical point

- Chiral fermion,  $m_\pi = 330$  MeV (our result) :  $2m_N |d_n| = F_{3n}(0) \simeq 0.1$
- scaling based on leading order ChPT:  $d_n \sim m_q \sim m_\pi^2$



$$F_{3n}(0) \sim 0.02 \cdot \theta, \quad |d_n| \sim 0.002 e \text{ fm} \cdot \theta \quad (\text{physical point})$$

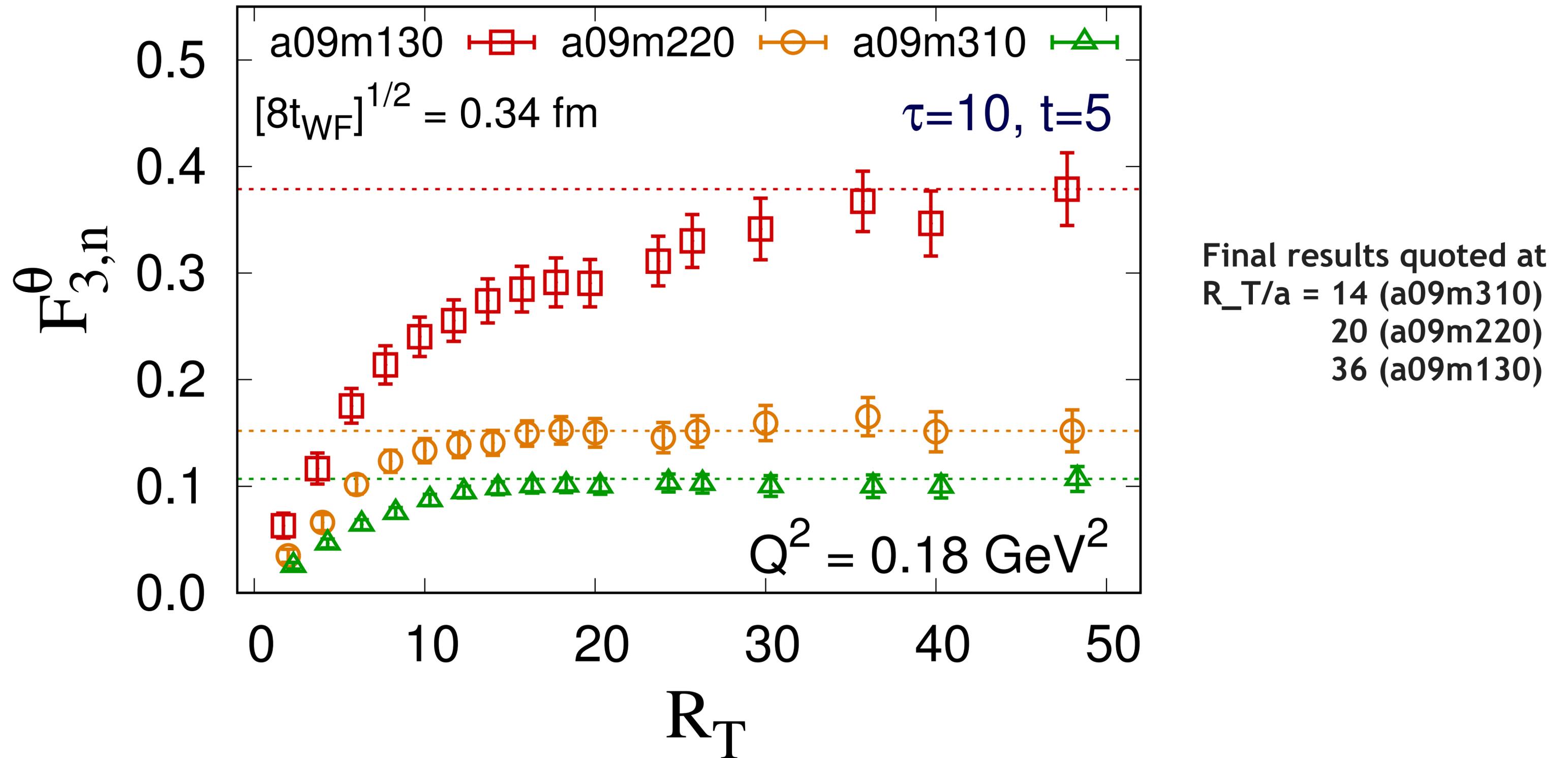
- Consistent with the results from QCD sum rule and the lattice result with ChPT fits.

$$d_n = -0.00152(71)e \text{ fm} \cdot \theta \quad [\text{J. Dragos et al, arXiv:1902.03254}]$$

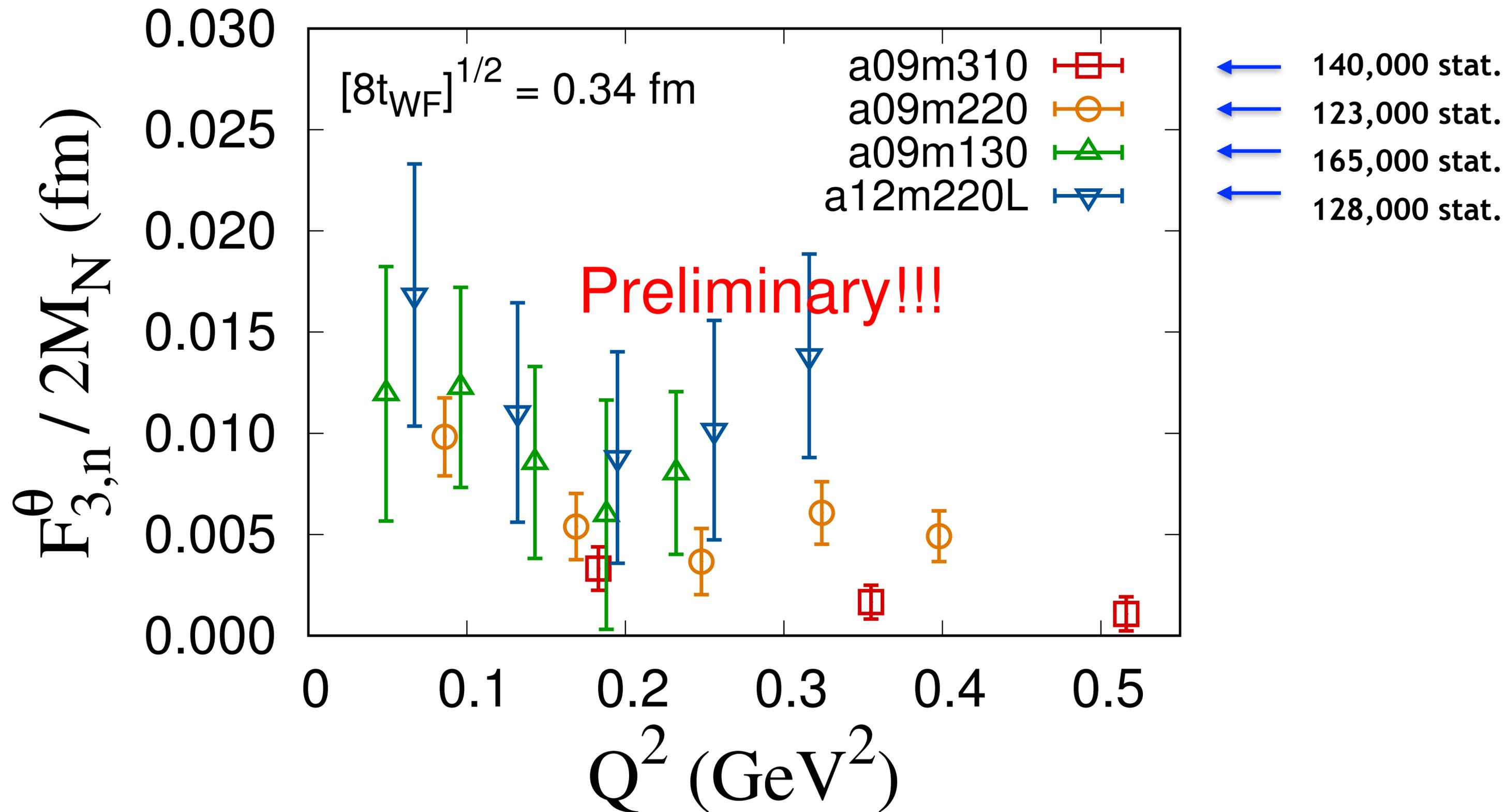
- To constrain  $|F_{3n}| < 0.02$  at  $m_{\text{phys}}$ , we need 25 ~ 100 times statistics  
( $\delta F_{3n}/F_{3n} \sim 5$  at physical point)

### 3. B. Yoon et al

convergence test: t-dependence of F3 [more detailed study → B. Yoon, talk on Friday]



### 3. B. Yoon et al

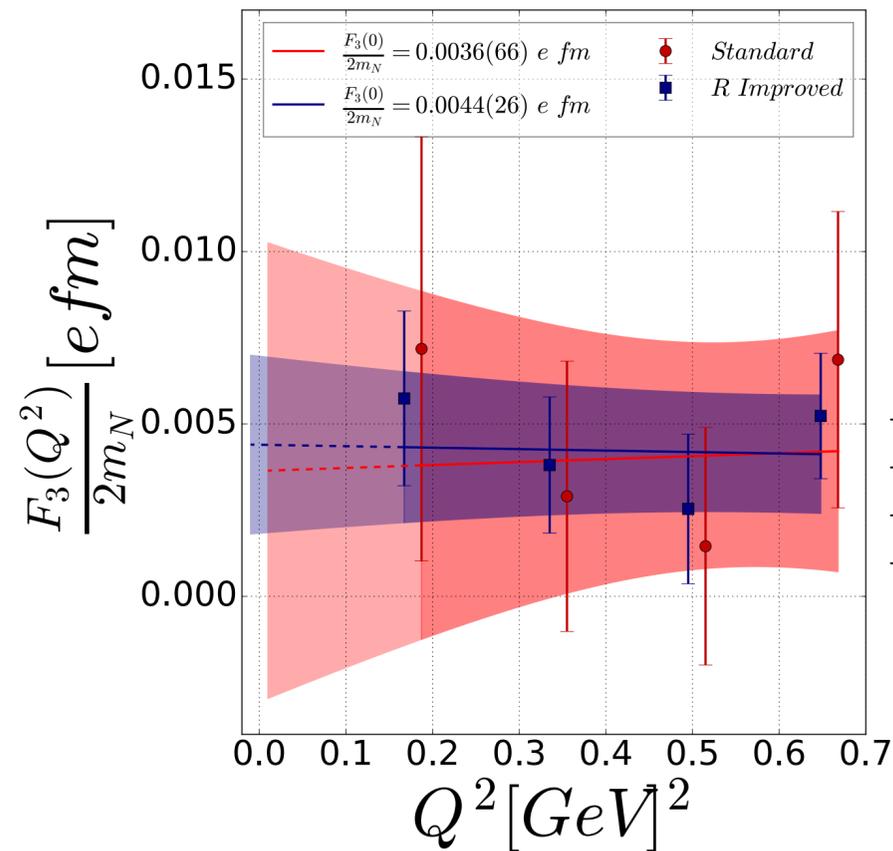


# Comparison of three groups

- 1: J. Dragos et al [arXiv:1902.03254]
- 2: S. Syritsyn, et al ('18)
- 3: B. Yoon (talk on Friday)

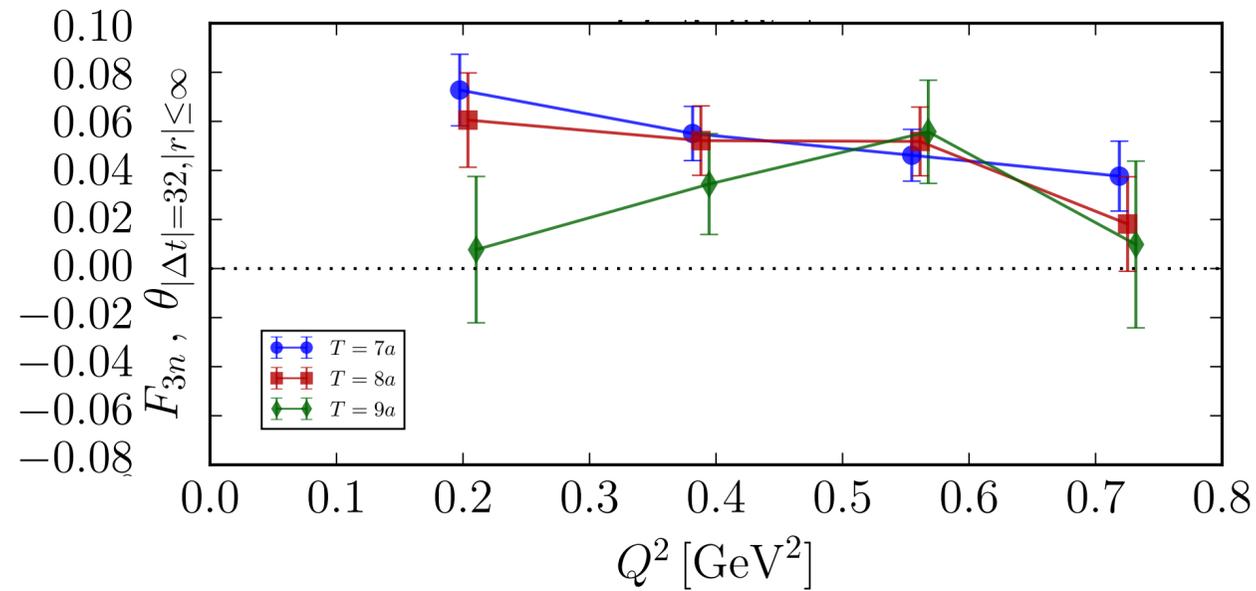
1.

Neutron,  $m_\pi = 410 \text{ MeV}$

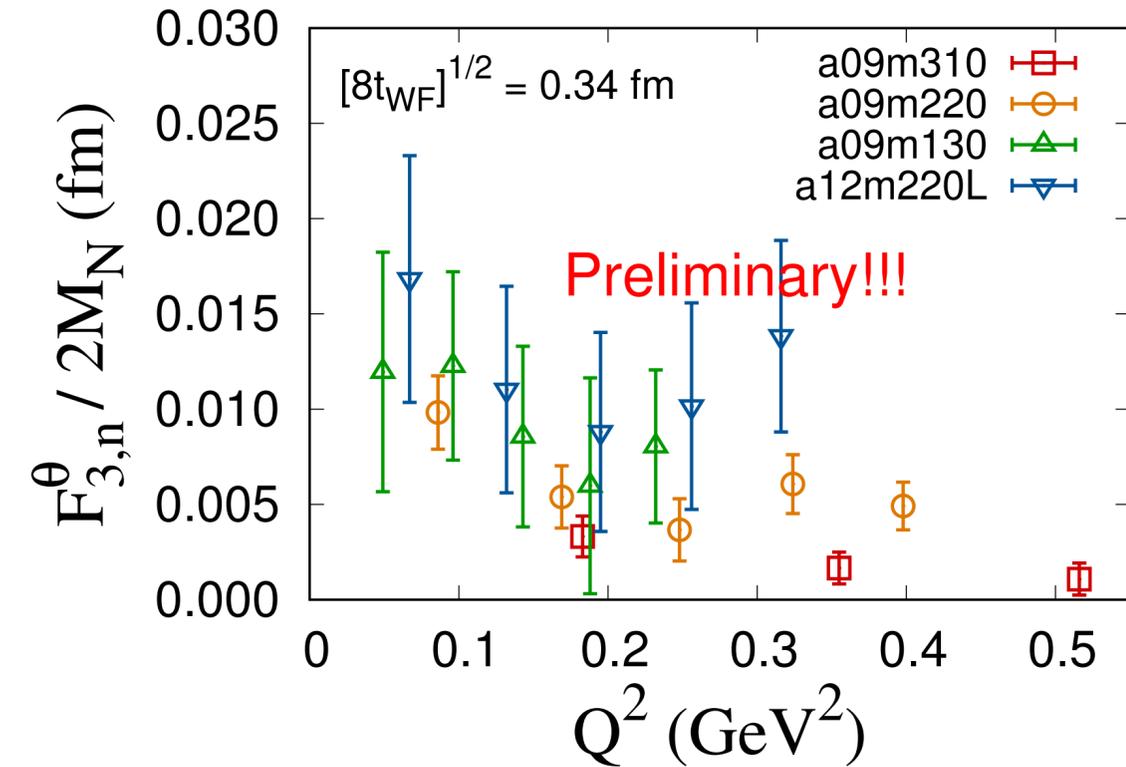


2.

$m_\pi = 340 \text{ MeV}$



3.



**Consistent result at heavier mass with non-zero  $Q^2$**   
**Difficult to see chiral behavior.**

## Short summary : lattice $\theta$ -EDM calculations

- Various noise reduction techniques have been used, which in fact improve the signal-to-noise ratio in the form factor calculations.
- Clear signal at heavier mass with non-zero  $Q^2$
- Result at the physical point has 50-100% error.
- There may be a tension between chiral (and  $Q^2$ ) extrapolated value and a direct result at physical point.

$$1 \ \& \ 2: |d_n| = \mathcal{O}(10^{-3}), \ 3: |d_n| = \mathcal{O}(10^{-2}) \text{ [fm]}$$

- Need to understand  $\pi$  mass and  $Q^2$  dependence of  $d_n$ .
- Constrain  $\theta$ -induced nEDM at physical point is still challenging.

**A new method**

**Matrix element approach with background electric field**

## 3pt function with topological charge density in the presence of the background electric field

- Consider 3-pt functions with topological charge density

$$\Delta C_{3pt}(T, \tau) = \langle N(T) \bar{Q}(\tau) \bar{N}(0) \rangle_{\vec{\mathcal{E}}}, \quad (0 < \tau < T)$$

- Performing the spectral decomposition

$$\begin{aligned} \Delta C_{3pt}(\tau, \vec{\mathcal{E}}) &= \langle N(T) \bar{Q}(\tau) \bar{N}(0) \rangle_{\vec{\mathcal{E}}} \sim \sum_{n,m} e^{-E_n(T-\tau) - E_m \tau} \langle 0|N|n\rangle \langle n|\bar{Q}|m\rangle \langle m|\bar{N}|0\rangle_{\vec{\mathcal{E}}} \\ &= |Z_N|^2 e^{-m_N T} \langle N|\bar{Q}|N\rangle_{\vec{\mathcal{E}}} + (\text{excited states}) \end{aligned}$$

This matrix element can be non-zero due to non-zero electric field, which corresponds to the energy shift ( $\delta E$ )

$$\langle N|\bar{Q}|N\rangle_{\vec{\mathcal{E}}} = \delta E = d_n \times \vec{\Sigma} \cdot \vec{\mathcal{E}},$$

c.f. 1st order energy correction in the perturbation theory of quantum mechanics

$$\hat{H} = \hat{H}_0 + \delta \hat{H}, \quad \delta E_n = \langle n | (\delta \hat{H}) | n \rangle$$

# Lattice QCD with background constant electric field

- \*Uniform electric field preserving translational invariance and periodic boundary conditions on a lattice (Euclidean imaginary electric field)
- \*used for the nucleon polarizability [W. Detmold, Tiburzi, and Walker-Loud, (2009)]
- \*No sign problem: Analytic continuation of CP-odd interaction
- \*consistency check of energy shift method and form factor method via cEDM operator.

$$U_\mu \rightarrow e^{iQ_q A_\mu} U_\mu$$

$$A_t(z, t) = \mathcal{E}_n z$$

$$A_z(z, t) = -\mathcal{E}_n L_z t \delta_{z=L_z-1}$$

strength of E field  $\mathcal{E}_n = n \frac{6\pi}{L_z L_t}, \quad (n = \pm 1, \pm 2, \dots)$

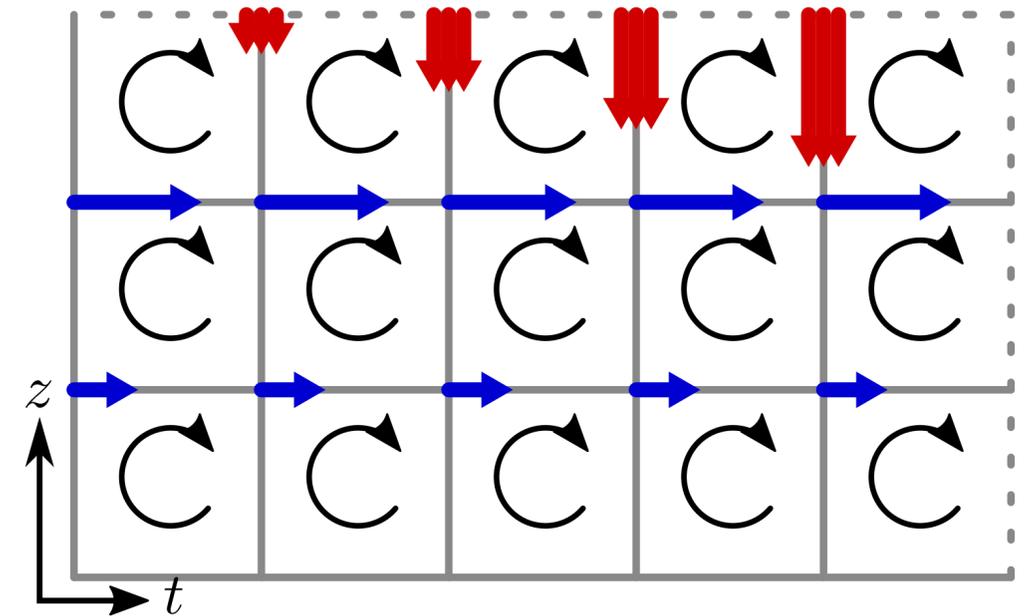
charge quanta  $Q_q \mathcal{E}_n L_z L_t = 2\pi m, \quad (m : \text{integer})$   
 $(Q_u = 2/3, \quad Q_d = -1/3)$

24<sup>3</sup> x 64 lattice minimal value of E ( $|n|=1$ )

$$\mathcal{E}_0 = \frac{6\pi}{L_z L_t} \sim 0.037 \text{ GeV}^2$$

$$\sim 186 \text{ MV/fm}$$

Charge quantization due to finite volume.



# EDM : difference between spin up (positive $E$ ) and spin down (negative $E$ ) energy shift

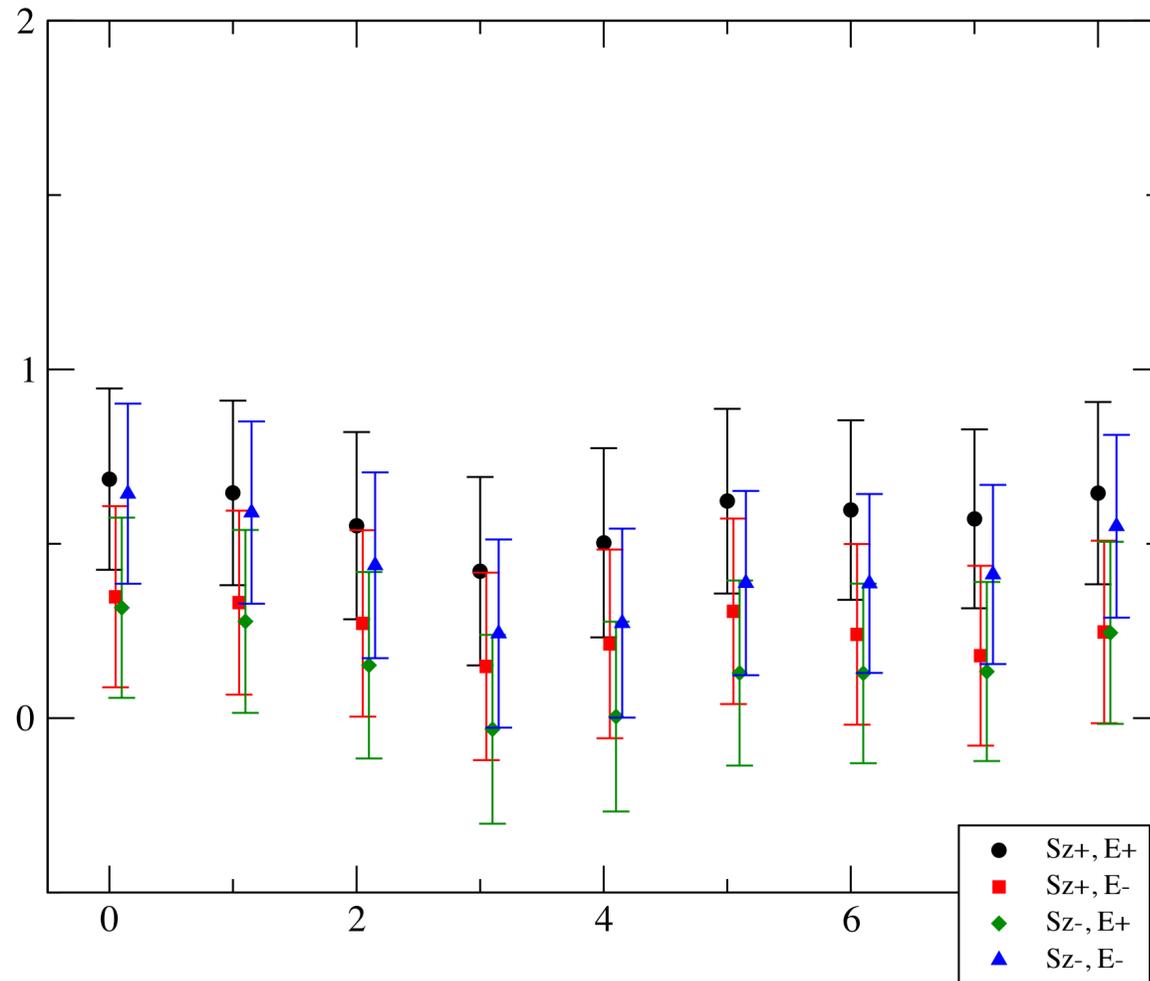
$$\frac{\Delta C_{3pt}(T, \tau)}{C_{2pt}(T)} \rightarrow \delta E \quad (T \rightarrow \infty)$$

$$|F_{3n}(0)| = \frac{2m_N \delta E}{|\vec{\mathcal{E}}|}$$

$$|\mathcal{E}| = 2$$

$$T = 8$$

$t_{\text{sep}}=8, |\mathcal{E}|=2$



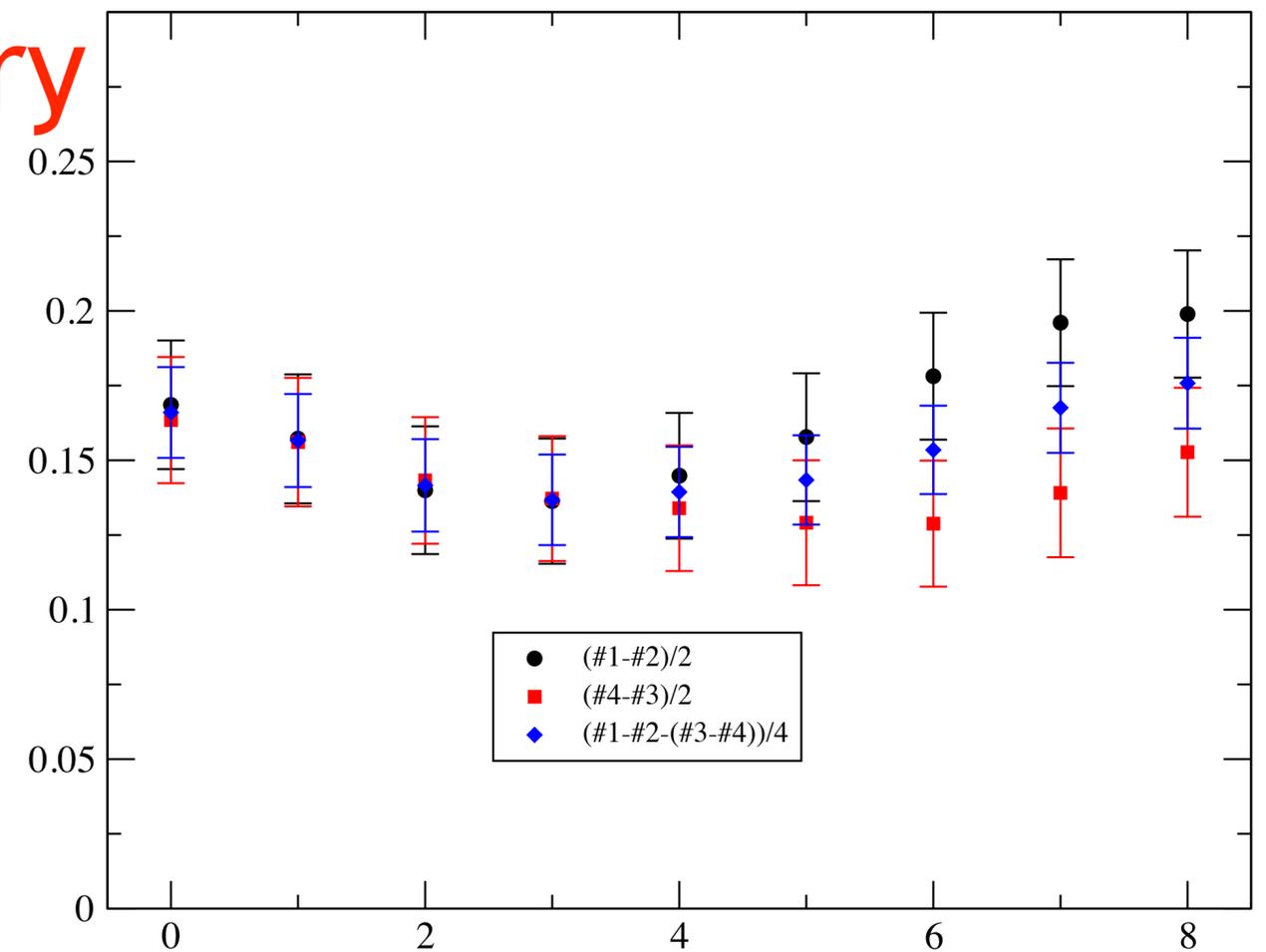
$T$

$m_\pi = 340 \text{ MeV}$

Preliminary

$F_{3n}(0)$

$t_{\text{sep}}=8, |\mathcal{E}|=2$



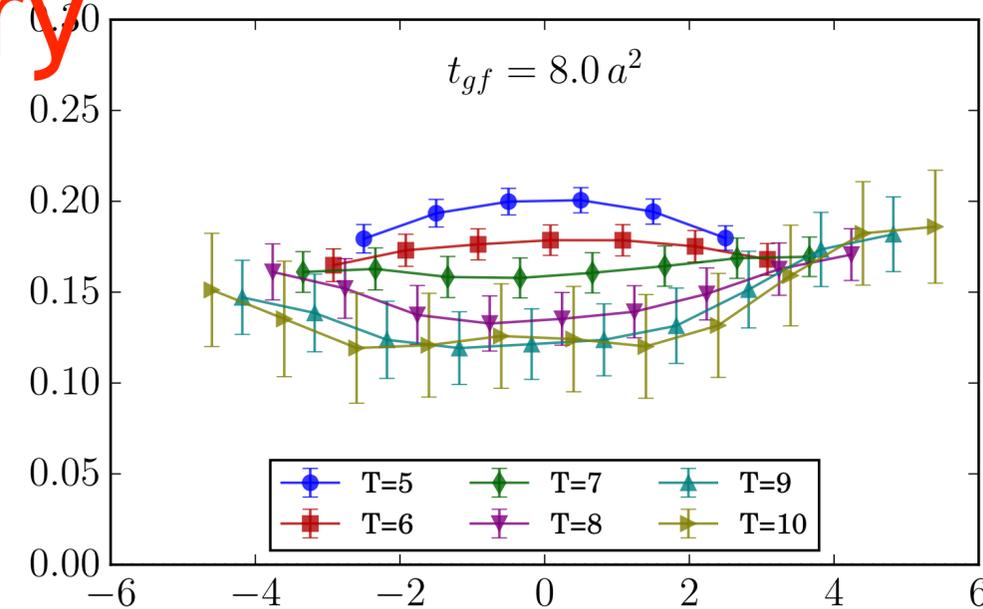
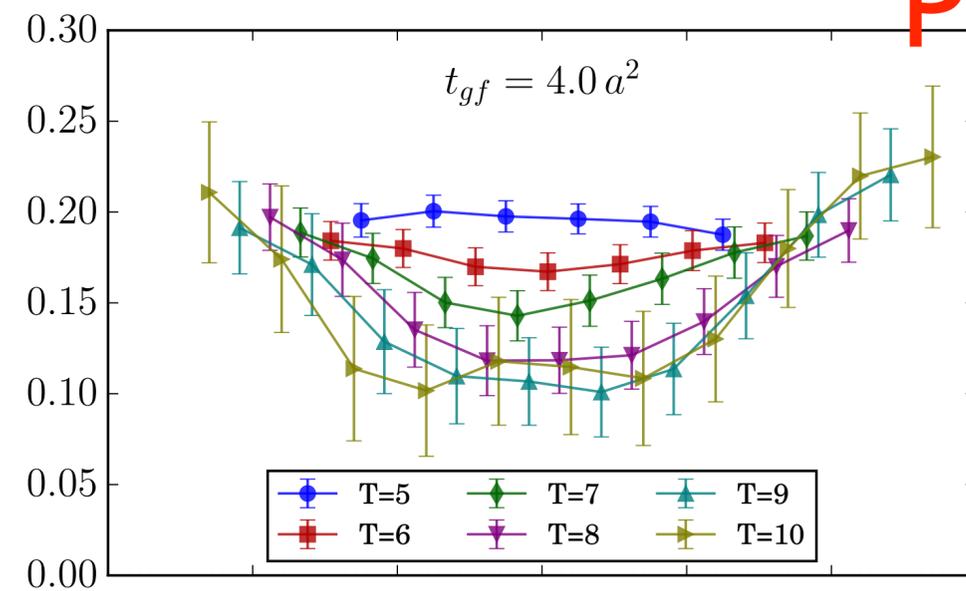
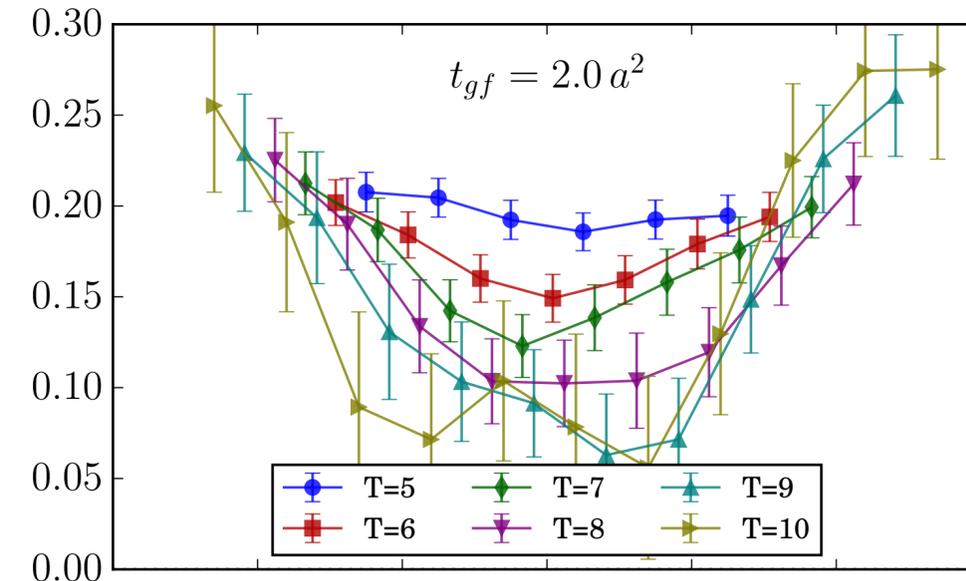
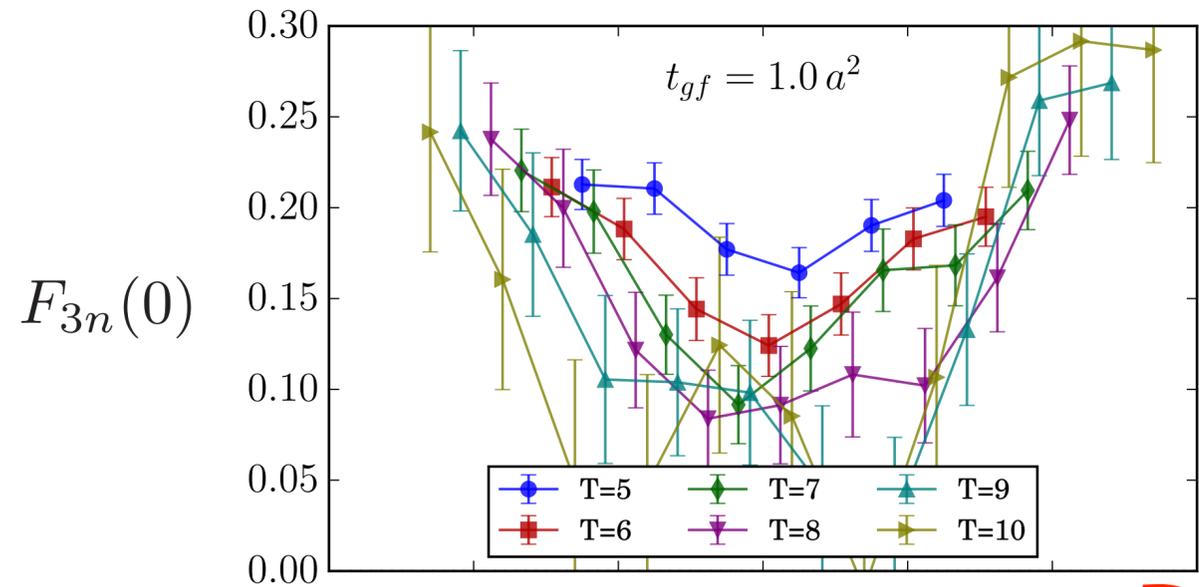
$T$

# Test of flow time and T dependence

$$\Delta C_{3pt}(T, \tau) = \langle N(T) \bar{Q}(\tau) \bar{N}(0) \rangle_{\mathcal{E}}$$

$$m_\pi = 340 \text{ MeV}$$

Preliminary



$$|\mathcal{E}| = 2$$

$$\tau - T/2$$

$$\tau - T/2$$

Gradient flow reduces the topological charge fluctuations.

Plateau is obtained at T=9 and 10 for  $t_{GF}=8$ .  $\rightarrow F_{3n}(0) = 0.12(3)$

# Summary

**Lattice computation of lattice  $\theta$ -EDM is very challenging.**

## **Form factor methods**

- **Noise reduction techniques for Q-samplings have been developed in recent years.**
- **Very clear signal at heavier pion mass region**
- **The error becomes larger at the physical point.**
- **Need to understand  $\pi$  mass (and  $Q^2$ ) dependence of  $F_3(Q^2)$  form factor**

## **A new method -matrix element approach-**

- **Potentially better control of the systematic uncertainties  
(no need  $Q^2$  extrapolation, no need to extend outside sink-source position)**
- **linearity of  $|E|$  should be checked**
- **Need to study at the physical point**

**Thank you**

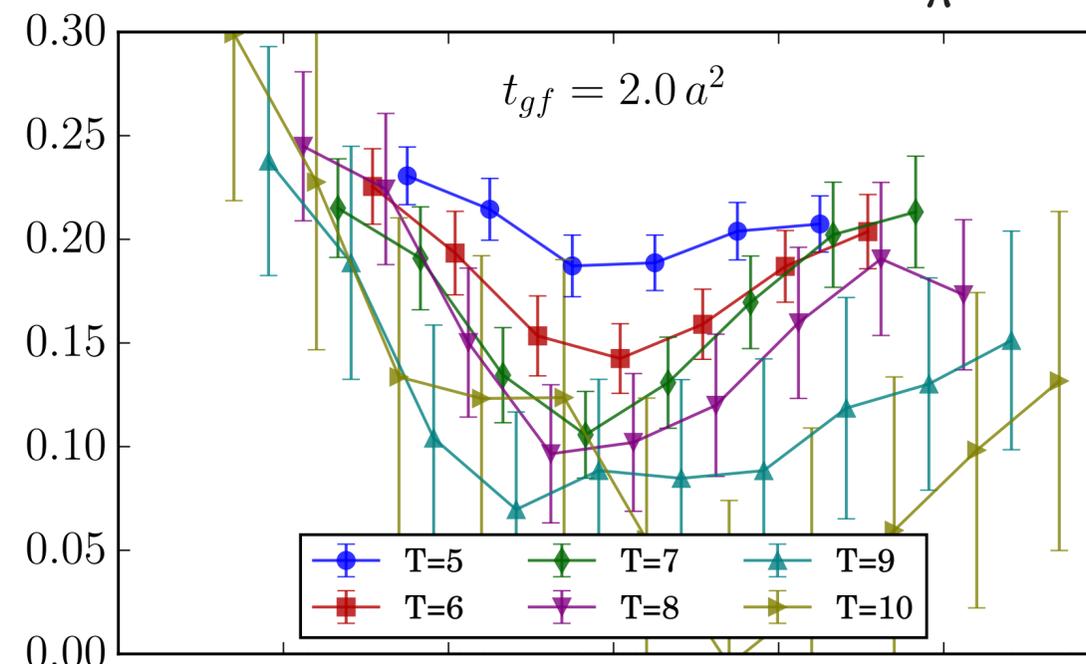
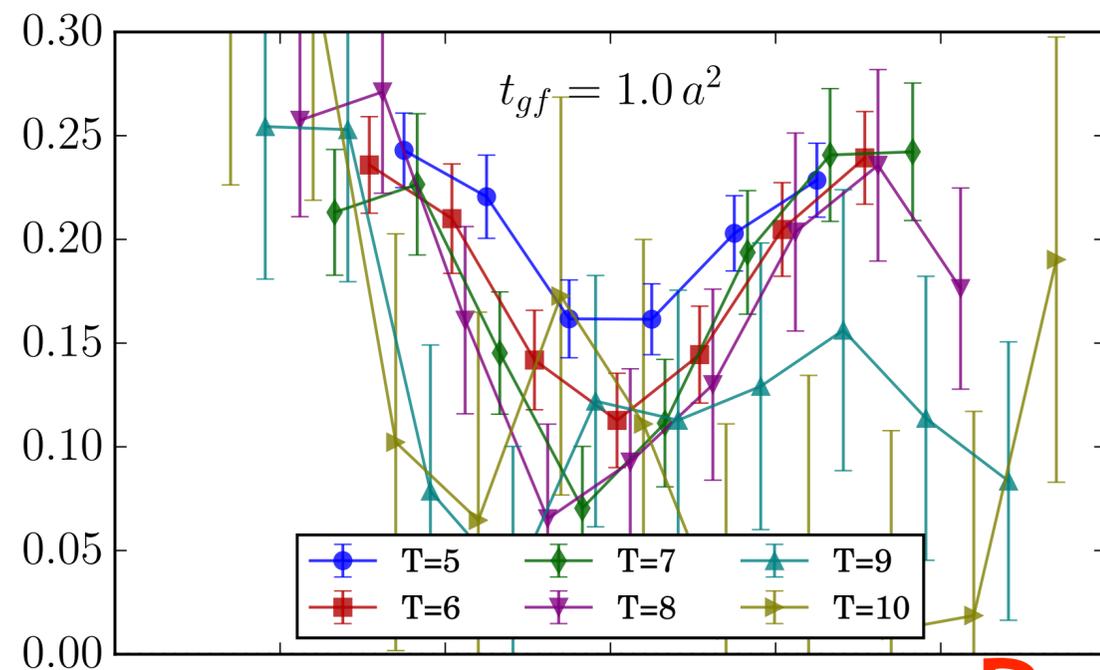
**Back up**

# Test of flow time and T dependence

$$\Delta C_{3pt}(T, \tau) = \langle N(T) \bar{Q}(\tau) \bar{N}(0) \rangle_{\vec{\mathcal{E}}}$$

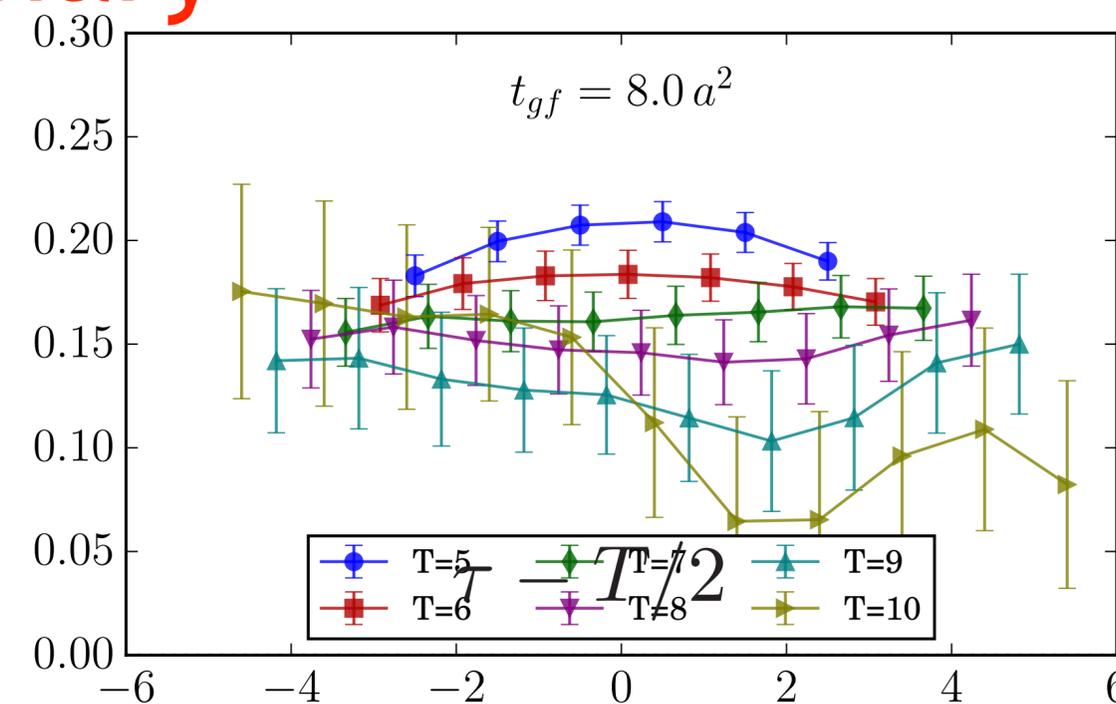
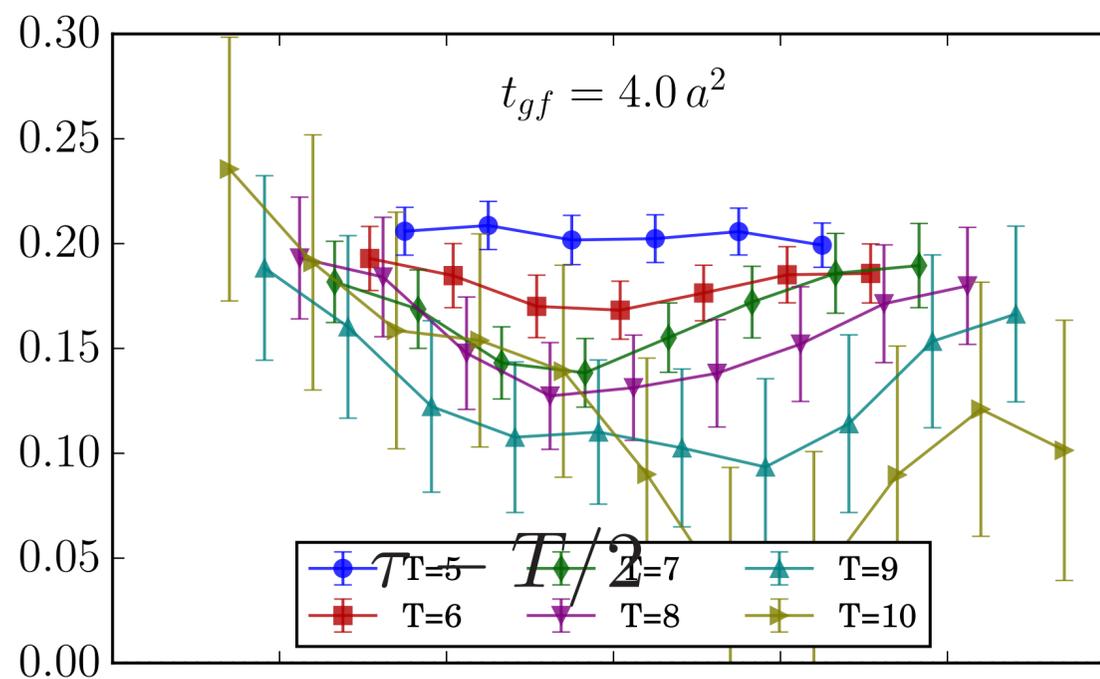
$m_\pi = 340 \text{ MeV}$

$F_{3n}(0)$



Preliminary

$|\mathcal{E}| = 1$



# Nucleon 2 point function with a constant Ez-field

$$C_{2pt}^{CP}(\vec{p} = 0, t, \mathcal{E}) = |Z|^2 e^{i\alpha\gamma_5} \left( \frac{1 + \gamma_4}{2} \right) \left[ \frac{1 + \Sigma_z}{2} e^{-(m+\delta E)t} + \frac{1 - \Sigma_z}{2} e^{-(m-\delta E)t} \right] e^{i\alpha\gamma_5} + \mathcal{O}((\kappa, \mathcal{E})^2)$$

$$\sim |Z|^2 e^{-mt} \left[ \underbrace{\frac{1 + \gamma_4}{2}}_{\text{(CP-even)}} + \underbrace{i\alpha\gamma_5 - \Sigma_z \delta E t}_{\text{(CP-odd)}} \right]$$

(t >> 1)

spin dependent interaction energy

Energy shift :  $\delta E = -\frac{\zeta}{2m}(i\mathcal{E})$

“Effective” energy shift (extraction of the term proportion to linear-time)

$$\zeta^{eff} = 2m F_3^{eff}(0) = -\frac{2m}{\mathcal{E}_z} [R_z(t+1) - R_z(t)],$$

$$R_z(t) = \frac{\text{Tr}[T_{S_z}^+ C_{2pt}^{CP-odd}(t, \mathcal{E})]}{\text{Tr}[T^+ C_{2pt}(t, \mathcal{E})]}$$

$$C_{2pt}^{CP-odd}(t, \mathcal{E}) = \langle N(t)N(0) \sum_x [\mathcal{O}_{cEDM}(x)] \rangle_{\mathcal{E} \neq 0}$$