Meson Screening Masses from 2+1-Flavor Lattice QCD

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Meson Screening Masses

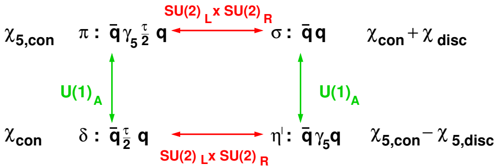
- Screening correlators carry important information about the degrees of freedom of QCD at finite temperature, especially in the important quark-gluon plasma phase.
- The meson screening correlators are defined by

$$G_\Gamma^a(z) = \int_0^\beta d\tau \int dxdy \left\langle \mathcal{M}_\Gamma^a(x,y,z,\tau)\overline{\mathcal{M}_\Gamma^a}(0,0,0,0) \right\rangle,$$

  where $\mathcal{M}_\Gamma^a \equiv \bar{\psi}(\Gamma \otimes t^a)\psi$ is a meson operator and β is the inverse temperature.
- The large-distance fall-off of these correlators is controlled by the respective screening masses viz.

$$G_\Gamma^a(r) \sim \exp(-m_\Gamma^a r), \qquad r \to \infty.$$

Meson Correlators and $U_A(1)$ Restoration



- $2+1$-flavor QCD is well known to undergo a chiral crossover transition at $T_{pc} = 156.5(1.5)$ MeV [A. Bazavov *et al.* (2018)]. In the chiral limit, this becomes a genuine 2$^{\text{nd}}$ order phase transition with $T_c = 132^{+6}_{-3}$ MeV [H.-T. Ding *et al.* (2019)].

- The restoration of various symmetries manifests itself as a degeneracy among various correlation functions.

- In the case of $2+1$-flavor QCD, it suffices to study two-point functions, *i.e.*, meson screening functions.

- Chiral symmetry restoration identifies the vector and axial vector isotriplet correlators while $U_A(1)$ restoration identifies the scalar and pseudoscalar isotriplet correlators.

Setup of the Calculation

- We calculated meson screening masses in 2+1-flavor QCD for temperatures 140 MeV $\lesssim T \lesssim$ 1 GeV.
- Our lattices were generated using the 2+1-flavor Highly Improved Staggered Quark action (HISQ).
- Our strange quark was tuned to its physical value, while the light quark mass was set to one of two values: $m_l = m_s/20$ (nearly physical, high temperatures) and $m_l = m_s/27$ (physical, low temperatures).
- We calculated the screening masses for $N_\tau = 6$, 8, 10 (only for $m_l = m_s/20$), 12 and 16 (only for $m_l = m_s/27$). This allowed us to take the continuum limit.

Staggered Meson Operators

- A meson operator in the staggered formalism is given by

$$\mathcal{M}(x) = \phi(x)\bar{\chi}(x)\chi(x+n),$$

  where $\phi(x)$ is an x-dependent phase factor and n points to a vertex of the unit hypercube based at x.

- If $n = 0$, the operator is said to be a local operator.

- A staggered correlator couples to two mesons of opposite parities:

$$G(n_\sigma) = \sum_{i=0,1,2,\ldots} A_i^{(-)} \cosh\left(am_i^{(-)}\left(n_\sigma - \frac{N_\sigma}{2}\right)\right) - (-1)^{n_\sigma} \sum_{j=0,1,2,\ldots} A_j^{(+)} \cosh\left(am_j^{(+)}\left(n_\sigma - \frac{N_\sigma}{2}\right)\right).$$

- For e.g. the scalar correlator that we study here couples to both the a_0 scalar as well as to one of the tastes of the pion.

List of Meson Operators

	$\phi(\mathbf{x})$	Γ		J^{PC}	
		NO	O	NO	O
$\mathcal{M}1$	$(-1)^{x+y+\tau}$	$\gamma_3\gamma_5$	1	0^{-+}	0^{++}
$\mathcal{M}2$	1	γ_5	γ_3	0^{-+}	0^{+-}
$\mathcal{M}3$	$(-1)^{y+\tau}$	$\gamma_1\gamma_3$	$\gamma_1\gamma_5$	1^{--}	1^{++}
$\mathcal{M}4$	$(-1)^{x+\tau}$	$\gamma_2\gamma_3$	$\gamma_2\gamma_5$	1^{--}	1^{++}
$\mathcal{M}5$	$(-1)^{x+y}$	$\gamma_4\gamma_3$	$\gamma_4\gamma_5$	1^{--}	1^{++}
$\mathcal{M}6$	$(-1)^{x}$	γ_1	$\gamma_2\gamma_4$	1^{--}	1^{+-}
$\mathcal{M}7$	$(-1)^{y}$	γ_2	$\gamma_1\gamma_4$	1^{--}	1^{+-}
$\mathcal{M}8$	$(-1)^{\tau}$	γ_4	$\gamma_1\gamma_2$	1^{--}	1^{+-}

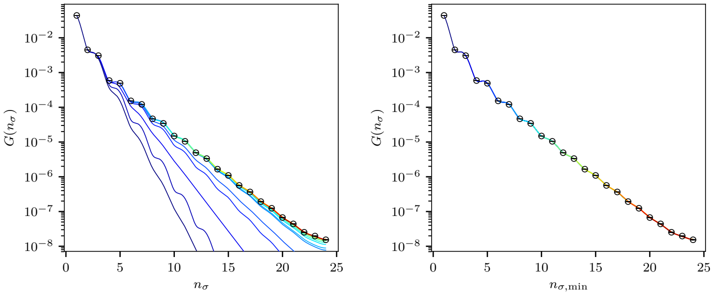
In this study, we only used local operators, and studied the screening masses for spin-0 and spin-1 mesons of both parities.

Fitting the Correlators

Multi-state fits tend to be highly unstable. The number of fit parameters grows and the # degrees of freedom decreases quickly.

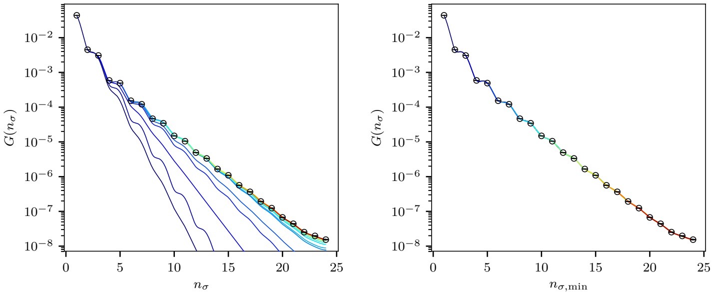
- One-state fits in a narrow fit window $[N_\sigma/2 - \tau, N_\sigma/2 + \tau]$: n.d.f. much reduced. Also, we found that this was not sufficient for all cases.
- Corner wall sources were found to work best for the vector and axial vector correlators below $T \sim 300$ MeV. Comparable results to point wall sources in other cases.
- Effective mass estimators [S. Mukherjee *et al.* (2014)] Split the correlator into oscillating and non-oscillating parts and solve analytically for the effective mass. Only works for one-state fits.
- Bayesian fits [Lepage 2001] Need prior information (screening masses and amplitudes), which we did not have.

Akaike Information Criterion



- Akaike Information Criterion [H. Akaike 1971, 1974] Provides a criterion for measuring the goodness-of-fit of a given model to the data.
  - Akaike Information Criterion (corrected): A correction for small sample sizes. Since AICc tends to AIC as the sample size becomes large, it is always recommended to use AICc over simple AIC.

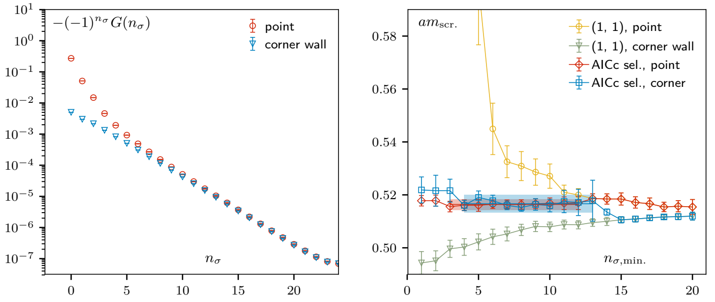
Akaike Information Criterion



(Left) One-state fits, no AICc. (Right) AICc-chosen fits.

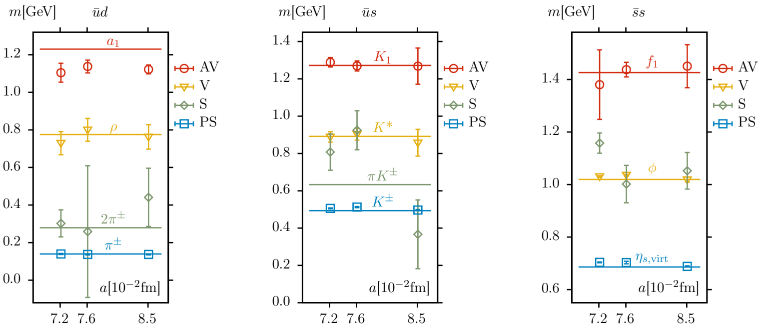
Multi-state fits for multiple fit windows; allow AICc to pick the best fit for each window.

Point versus Corner Wall sources



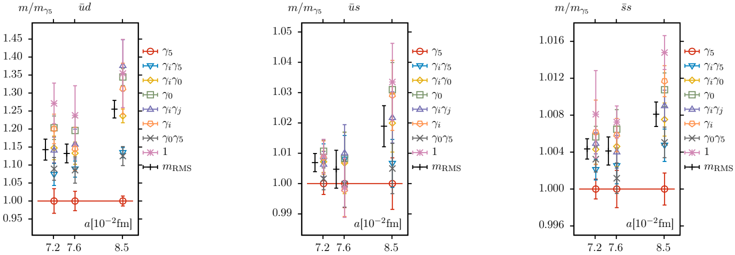
- Select effective mass plateaus by hand.
- We found that point and corner wall fits performed comparably.
- We used corner wall sources for vector and axial vector correlators below $T \sim 300$ MeV, and point sources in all other cases.

Spectrum at $T = 0$



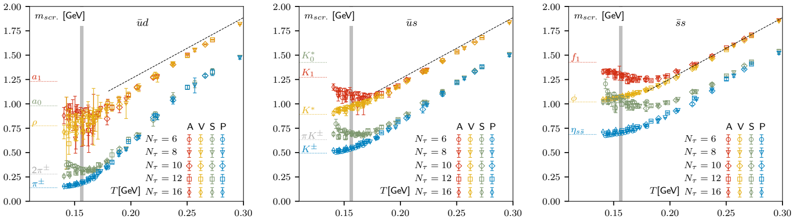
- No determination of the flavored scalar meson ($a_0(980)$).
- This is because the staggered scalar decays to two pions [Prelovsek *et al.* 2004; Prelovsek 2005].
- Unphysical contribution from the various taste sectors cancels out in the continuum; more on this later.

Taste-Splitting in the Pion Sector



Our results may be compared to earlier results on taste-splittings for the HISQ action [A. Bazavov and P. Petreczky [HotQCD]], PoS LATTICE2010, 169.

Screening Masses: 140 MeV $\lesssim T \lesssim$ 300 MeV



- The screening masses tend to the mass of the respective $T = 0$ mesons as the temperature is decreased.
- However, this is not true for the case of the scalar screening mass.

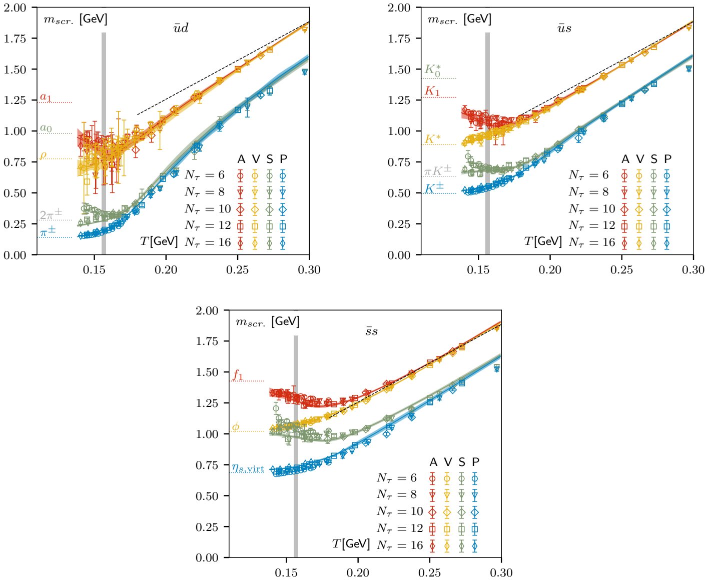
The staggered scalar correlator

- The scalar mass tends to $2m_\pi$, rather than m_{a_0}, at low temperatures.
- As already noted, this is because the staggered a_0 can undergo the unphysical decay $a_0 \to \pi\pi$.
- The decay arises from contributions of various tastes beyond tree level to the staggered correlator [S. Prelovsek (2006), S. Prelovsek *et al.* (2004)].
- These contributions cancel out in the continuum limit. In our case however, we calculate the screening mass first and then take the continuum limit.
- Beyond the question of screening masses, this also poses questions regarding $U_A(1)$ restoration.

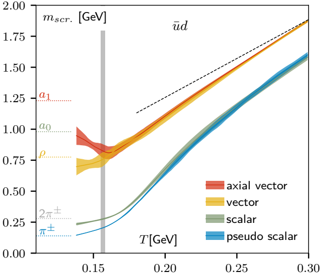
Taking the Continuum Limit

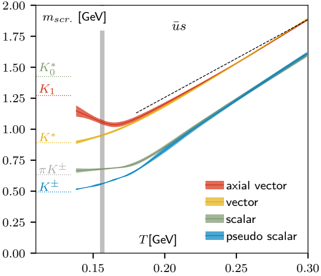
- We have screening mass results for $N_\tau = 6$, 8, 10 (only for $m_l = m_s/20$), 12 and 16 (only for $m_l = m_s/27$).
- This allowed us to make a continuum extrapolation. Since we did not have different N_τ for the same temperature, we fitted the data to piecewise smooth splines with N_τ-dependent coefficients.
- The spline knots are placed in such a way that one has the same number of points between successive knots. This means more knots at lower temperatures and less knots at higher ones.
- The fits are stabilized by constraining the spline derivative to be zero at $T = 25$ and 50 MeV, and the spline value to be $2\pi T$ at $T = 1.5$ GeV. Our spline extrapolations were performed for 140 MeV $\lesssim T \lesssim 1$ GeV, so these constraints lie well outside the fit region.
- The errors were estimated by repeating the fits for several bootstrap samples. The effect of fixed knots was removed by slightly randomizing the knot positions.

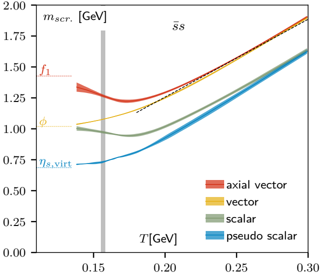
Continuum-Extrapolated Results



Continuum-Extrapolated Results







The question of $U_A(1)$ Symmetry Restoration

- It is an intriguing and open question regarding whether $U_A(1)$ symmetry is also restored at the chiral phase transition [E. Shuryak (1994), M. Birse, T. Cohen and J. McGovern (1996), S. Lee and T. Hatsuda (1996), N. Evans, S. Hsu and M. Schwetz (1996), S. Aoki *et al.* (2012)].

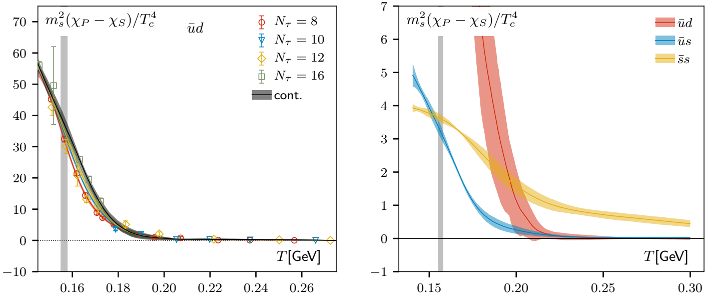
- One way of studying $U_A(1)$ restoration on the lattice is by looking for a degeneracy between the π and a_0 (δ) correlators [HotQCD Collaboration (2012), M. Buchoff *et al.* (2013), G.Cossu *et al.* (2012, 2013, 2017), R. Gavai, S. Gupta and R. Lacaze (2001), T.-W. Chiu *et al.* (2013)].

- Easier to determine the degeneracy between the corresponding susceptibilities viz.

$$\chi_\pi = \sum_{n_\sigma=0}^{N_\sigma-1} \mathcal{M}2(n_\sigma), \qquad \chi_\delta = -\sum_{n_\sigma=0}^{N_\sigma-1} (-1)^{n_\sigma} \mathcal{M}1(n_\sigma).$$

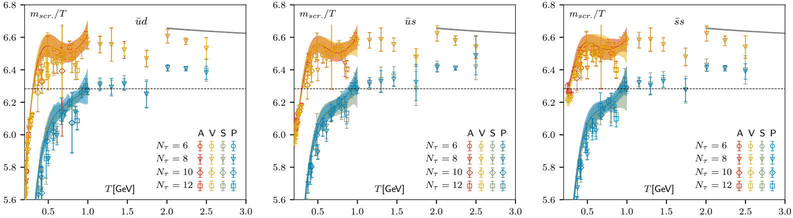
(The oscillating phase factor is only needed in the staggered case).

$U_A(1)$ Symmetry Restoration on the Lattice



- Taking the continuum limit of the susceptibilities is equivalent to taking the continuum limit of the correlators.
- We find that $m_s^2(\chi_\pi - \chi_\delta)$ goes to zero very slowly and not at the chiral crossover temperature itself.
- Note however that the question of $U_A(1)$ restoration only makes sense in the chiral limit. A systematic chiral extrapolation needs to be carried out before the question can really be addressed.

Screening Masses: 300 MeV $\lesssim T \lesssim$ 1 GeV



- It is an interesting question whether the quark-gluon plasma is perturbative for $T \sim$ 2-3 T_{pc} [C. DeTar and J. Kogut (1987), MTc collaboration (1991), R. Gavai *et al.* (2001, 2011), E. Laermann and F. Pucci (2012), S. Gupta and N. Karthik (2013)].
- We compare our results to the predictions of dimensionally reduced QCD [M. Laine and M. Vepsalainen (2003), M. Laine and Y. Schroeder (2005)].
- We find a difference between our results and EQCD predictions out to $T \sim$ 1 GeV. In any case, the spin-0 and spin-1 masses are very different, whereas all masses receive the same corrections in perturbation theory.

Conclusions

- We calculated meson screening masses in $2+1$-flavor QCD for temperatures 140 MeV $\lesssim T \lesssim 1$ GeV.
- We were able to take the continuum limit owing to having results for multiple lattice spacings.
- We compared these results to predictions from resummed perturbation theory at high temperatures. We found that the system remained non-perturbative up to temperatures $T \sim 1$ GeV.
- The low-temperature limit of the vector, axial vector and pseudoscalar screening masses was as expected. The scalar mass had the wrong $T \to 0$ limit due to staggered artifacts. These artifacts disappear when the continuum limit of the correlator is taken first. We calculated the continuum limit of $\chi_\pi - \chi_\delta$ and found that the difference goes to zero well above the chiral crossover temperature.