QED$_\infty$ in muon $g-2$, hadron spectroscopy, and beyond

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and
RBC-UKQCD collaborations

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Outline

1. **Introduction**
2. QED_∞ in Muon $g-2$
   Long-distance contribution to the HLbL in position space from the π^0 pole
3. QED_∞ in hadron spectroscopy (X. Feng and L. Jin 2018)
   Pion mass splitting with the infinite volume reconstruction method
4. Conclusion

Muon $g-2$: experiments

$$\vec{\mu} = -g\frac{e}{2m}\vec{s}$$

World Average dominated by BNL

$$a_\mu = (11659208.9 \pm 6.3) \times 10^{-10}$$

Authors	Lab	Muon Anomaly	
Garwin et al. '60	CERN	0.001 13(14)	
Charpak et al. '61	CERN	0.001 145(22)	
Charpak et al. '62	CERN	0.001 162(5)	
Farley et al. '66	CERN	0.001 165(3)	
Bailey et al. '68	CERN	0.001 166 16(31)	
Bailey et al. '79	CERN	0.001 165 923 0(84)	
Brown et al. '00	BNL	0.001 165 919 1(59)	(μ^+)
Brown et al. '01	BNL	0.001 165 920 2(14)(6)	(μ^+)
Bennett et al. '02	BNL	0.001 165 920 4(7)(5)	(μ^+)
Bennett et al. '04	BNL	0.001 165 921 4(8)(3)	(μ^-)

In comparison, for electron

$$a_e = (11596521.8073 \pm 0.0028) \times 10^{-10}$$

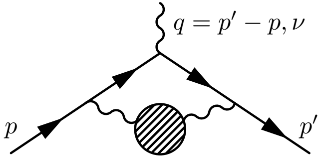
- Fermilab E989 (0.14 ppm) Almost 4 times more accurate then the previous experiment.
- J-PARC E34 also plans to measure muon $g-2$ with similar precision.

Muon $g-2$: theory

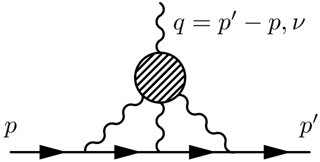
	a_μ	$\times$	10^{10}	
QED incl. 5-loops	11658471.9	$\pm$	0.0	Aoyama, et al, 2012
Weak incl. 2-loops	15.4	$\pm$	0.1	Gnendiger et al, 2013
HVP	692.5	$\pm$	2.7	Talk: C. Lehner (Mon 14:20)
				C. Lehner et al (RBC-UKQCD), 2018
HVP NLO&NNLO	-8.7	$\pm$	0.1	FJ17
HLbL	10.3	$\pm$	2.9	FJ17
				Hadronic Models, "Consensus"
Standard Model	11659181.3	$\pm$	4.0	
Experiment	11659208.9	$\pm$	6.3	E821, The $g-2$ Collab. 2006
Difference (Exp-SM)	27.6	$\pm$	7.5	

- More than 3 standard deviations due to mistake in the highly sophisticated perturbative calculation, inaccuracy of $e^+e^- \to hadrons$ experiments, incorrect hadronic model, or **new physics**?

Hadronic contribution to Muon $g-2$



HVP: Hadronic Vacuum Polarization



HLbL: Hadronic Light by Light

- Dispersive approach with $e^+e^- \to$ *hadrons* experimental data is very successful for HVP.
- HLbL is much more complicated. Anyway, lots of works have been done in this direction.

Pion pole, pion box, rescattering	G. Colangelo, M. Hoferichter, B. Kubis, M. Procura, P. Stoffer.
Light by light scattering sum rule	I. Danilkin, O. Deineka, M. Vanderhaeghen.
Light by light scattering sum rule	L. Dai, M. Pennington.
Schwinger’s sum rule	F. Hagelstein, V. Pascalutsa.

Lattice QCD

PHYSICAL REVIEW D VOLUME 10, NUMBER 8 15 OCTOBER 1974

Confinement of quarks*

Kenneth G. Wilson

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(Received 12 June 1974)

A mechanism for total confinement of quarks, similar to that of Schwinger, is defined which requires the existence of Abelian or non-Abelian gauge fields. It is shown how to quantize a gauge field theory on a discrete lattice in Euclidean space-time, preserving exact gauge invariance and treating the gauge fields as angular variables (which makes a gauge-fixing term unnecessary). The lattice gauge theory has a computable strong-coupling limit; in this limit the binding mechanism applies and there are no free quarks. There is unfortunately no Lorentz (or Euclidean) invariance in the strong-coupling limit. The strong-coupling expansion involves sums over all quark paths and sums over all surfaces (on the lattice) joining quark paths. This structure is reminiscent of relativistic string models of hadrons.

- Discrete lattice usually corresponds to hard cut off in momentum space.
- Hard cut off regularization is said to break gauge invariance, while lattice does not.
- Position space formulation can provide new insight.

Muon $g-2$: theory

	a_μ	$\times$	10^{10}	
QED incl. 5-loops	11658471.9	$\pm$	0.0	Aoyama, et al, 2012
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HVP	692.5	$\pm$	2.7	Talk: C. Lehner (Mon 14:20)
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HVP NLO&NNLO	-8.7	$\pm$	0.1	FJ17
HLbL	7.4	$\pm$	6.6	Talk: T. Blum (Tue 16:30)
				Lattice QCD, RBC-UKQCD
Standard Model	11659178.5	$\pm$	7.1	
Experiment	11659208.9	$\pm$	6.3	E821, The $g-2$ Collab. 2006
Difference (Exp-SM)	30.4	$\pm$	9.5	

- More than 3 standard deviations due to mistake in the highly sophisticated perturbative calculation, inaccuracy of $e^+e^- \to hadrons$ experiments, under estimating sys/stat error in lattice calculation, or **new physics**?

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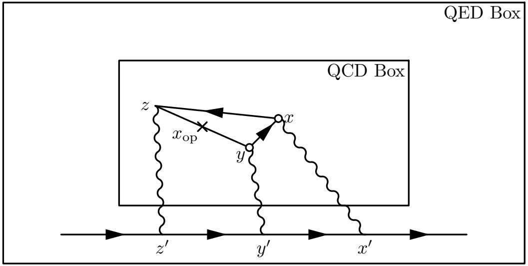
HLbL: QED_∞ to eliminate power-law FV effects

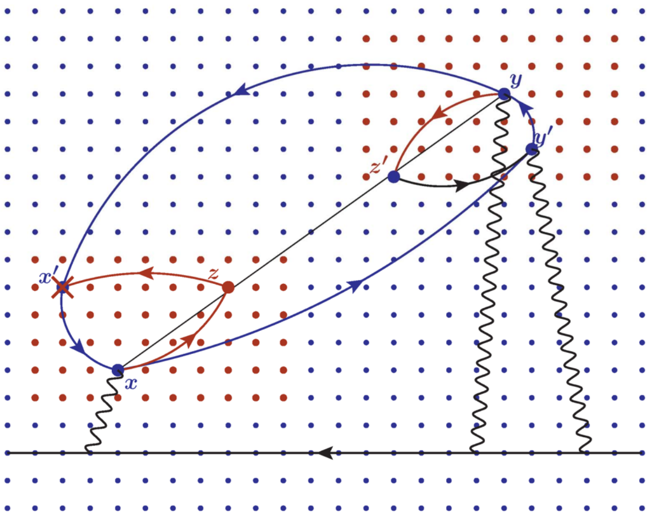
- We start the calculation by simulating both the QCD part and the QED part on the lattice within a finite volume using the *QED_L* formalism. [T. Blum et al, 2014,2016,2017]
- *Mainz*'s group first demonstrated that it is possible to efficiently calculate the QED part semi-analytically in the *infinite volume*. [N. Asmussen et al, LATTICE2016]
- We can improve the infinite volume QED part to reduce the discretization error significantly with a *subtracted QED kernel*. [T. Blum et al, 2017]



At physical point, π^0 is very light, therefore:

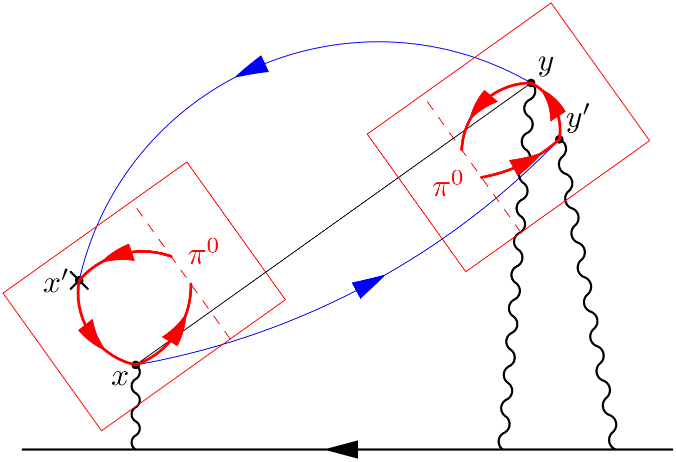
- Large *volume* is needed.
- Large *statistical error* from the long distance region, especially for the disconnected diagrams. [L. Jin et al, 2016] [J. Bijnens and J. Relefors, 2016]

Long-distance part of HLbL from the π^0 pole



- Norman's initial idea for calculating the long-distance part of HLbL from π^0 pole. (Norman's talk at HLbL workshop in UCONN, 2018/03/13)
- The hadronic 4-point function in the long-distance region can be calculated with two three-point functions, which can be directly calculated in a modest size lattice.

Long-distance HLbL: further tweaks

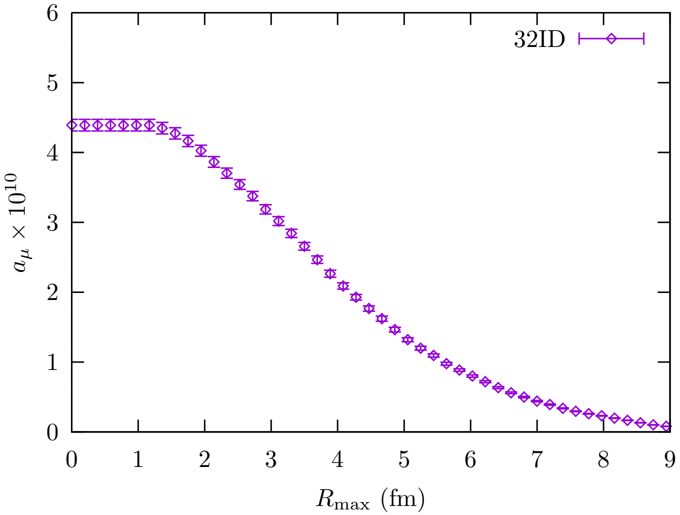


- We can rotate the modest size lattice so its time direction is aligned with the pion propagating direction.
- Only need to measure the $\pi^0 \to \gamma\gamma$ three-point function with zero momentum pion.
- We only need the following three-point function for $\vec{p} = 0$ pion.

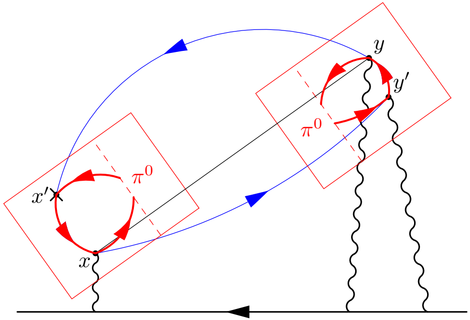
$$H_{\mu,\nu}(x - y) = \langle 0 | J_\mu(x) J_\nu(y) | \pi^0 \rangle$$

Long-distance HLbL: numerical results (prelim)

- UCONN graduate student Cheng Tu did the calculation.



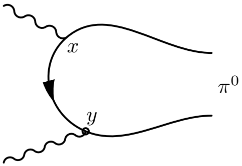
- 32ID: $32^3 \times 64$, $a^{-1} = 1.015$ GeV, $M_\pi = 142$ MeV.
- $R_{\max} = \max(|x-y|, |x-y'|, |y-y'|)$. Reverse partial sum plotted.

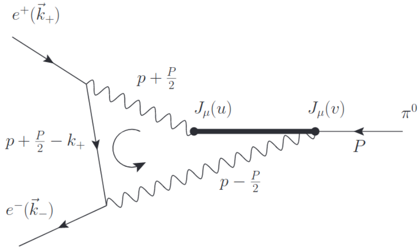


The three-point function: $\pi^0 \to \gamma\gamma$

$$H_{\mu,\nu}(x-y) = \langle 0|J_\mu(x)J_\nu(y)|\pi^0\rangle$$

- We calculated it using one point source propagator from y and wall source propagators seperated by enough distance from both x and y to create the pion.
- We averaged it within a configuration, saved it to disk, then performed furthur contractions.



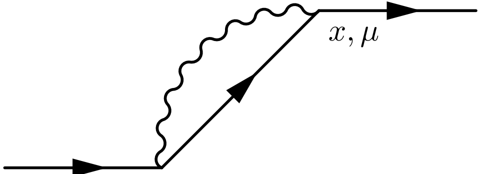


- $\pi^0 \to \gamma\gamma$ decay width from a coordinate-space method.
  Poster: X. Feng (Tue 17:50).
- Yidi Zhao and Norman Christ's very noval calculation $\pi^0 \to e^+e^-$, which serves as the starting point for a more adventurous calculation $K_L \to \mu^+\mu^-$.
  Talk: N. Christ (Tue 17:10).
  Talk: Y. Zhao (Tue 17:30).
- Likely more applications.

Outline

QED correction to hadron masses

$$
\begin{aligned}
\Delta M = \mathcal{I} &= \frac{1}{2}\int d^4x\, \mathcal{H}_{\mu,\nu}(x) S^{\gamma}_{\mu,\nu}(x) \\
\mathcal{H}_{\mu,\nu}(x) &= \frac{1}{2M}\langle N | T J_\mu(x) J_\nu(0) | N\rangle \\
S^{\gamma}_{\mu,\nu}(x) &= \frac{\delta_{\mu,\nu}}{4\pi^2 x^2}
\end{aligned}
$$



- We can evaluate the QED part, the photon propagator, in infinite volume.

- The hadronic function do not always fall exponentially in the long distance region.

$$
\Delta M = \mathcal{I} = \mathcal{I}^{(s)} + \mathcal{I}^{(l)}
$$

$$
\begin{aligned}
\mathcal{I}^{(s)} &= \frac{1}{2}\int_{-t_s}^{t_s} dt \int d^3x\, \mathcal{H}_{\mu,\nu}(x) S^{\gamma}_{\mu,\nu}(x) \\
\mathcal{I}^{(l)} &= \int_{t_s}^{\infty} dt \int d^3x\, \mathcal{H}_{\mu,\nu}(x) S^{\gamma}_{\mu,\nu}(x)
\end{aligned}
$$

- To eliminate all power-law suppressed finite volume effects, a different treatment for the long distance part is *required*. ($t_s \lesssim L$)

The infinite volume reconstruction method

- For the short distance part, $\mathcal{I}^{(s)}$ can be directly calculated on a finite volume lattice:

$$\mathcal{I}^{(s)} \approx \mathcal{I}^{(s,L)} = \frac{1}{2}\int_{-t_s}^{t_s} dt \int_{L/2}^{L/2} d^3x\, \mathcal{H}_{\mu,\nu}(x) S^{\gamma}_{\mu,\nu}(x)$$

- For the long distance part, we need to evaluate $\mathcal{H}_{\mu,\nu}(x)$ indirectly.
  Note that when t is large, the intermediate states between the two currents are mostly ground states (possibly with small momentum). Therefore:

$$\mathcal{H}_{\mu,\nu}(x) \approx \int \frac{d^3p}{(2\pi)^3}\left[\frac{1}{2E_{\vec{p}}}\frac{1}{2M}\langle N|J_\mu(0)|N(\vec{p})\rangle\langle N(\vec{p})|J_\nu(0)|N\rangle\right] e^{i\vec{p}\cdot\vec{x}-(E_{\vec{p}}-M)t}$$

  - We only need to calculate the form factors: $\langle N(\vec{p})|J_\nu(0)|N\rangle$!
  - Values for all $\vec{p}$ are needed. Simply inverse Fouier transform the above relation!

$$\int d^3x \mathcal{H}_{\mu,\nu}(t_s,\vec{x}) e^{-i\vec{p}\cdot\vec{x}+(E_{\vec{p}}-M)t_s} \approx \frac{1}{2E_{\vec{p}}}\frac{1}{2M}\langle N|J_\mu(0)|N(\vec{p})\rangle\langle N(\vec{p})|J_\nu(0)|N\rangle$$

Master formula for QED self-energy

- The final expression for QED correction to hadron mass is split into two parts:

$$\Delta M = \mathcal{I} \approx \mathcal{I}^{(s,L)} + \mathcal{I}^{(l,L)}$$

- For the short distance part: $\mathcal{I}^{(s)} \approx \mathcal{I}^{(s,L)} = \frac{1}{2}\int_{-t_s}^{t_s} dt \int_{L/2}^{L/2} d^3x\, \mathcal{H}_{\mu,\nu}(x) S^{\gamma}_{\mu,\nu}(x)$

- For the long distance part: $\mathcal{I}^{(l)} \approx \mathcal{I}^{(l,L)} = \int_{-L/2}^{L/2} d^3x\, \mathcal{H}^{L}_{\mu,\nu}(t_s,\vec{x}) L_{\mu,\nu}(t_s,\vec{x})$

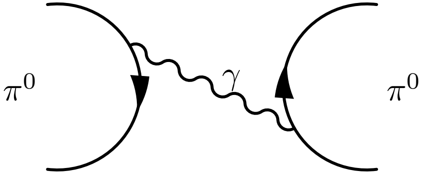
- For Feynman gauge:

$$S^{\gamma}_{\mu,\nu}(x) = \frac{\delta_{\mu,\nu}}{4\pi^2 x^2} \qquad L_{\mu,\nu}(t_s,\vec{x}) = \frac{\delta_{\mu,\nu}}{2\pi^2}\int_0^{\infty} dp \frac{\sin(p|\vec{x}|)}{2(p+E_p-M)|\vec{x}|} e^{-pt_s}$$

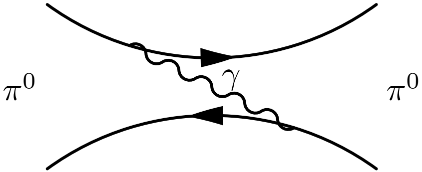
- Only exponentially suppressed finite volume errors.

  Power-law suppressed finite volume errors are removed to all orders.

Pion mass splitting: $M_{\pi^+} - M_{\pi^0}$



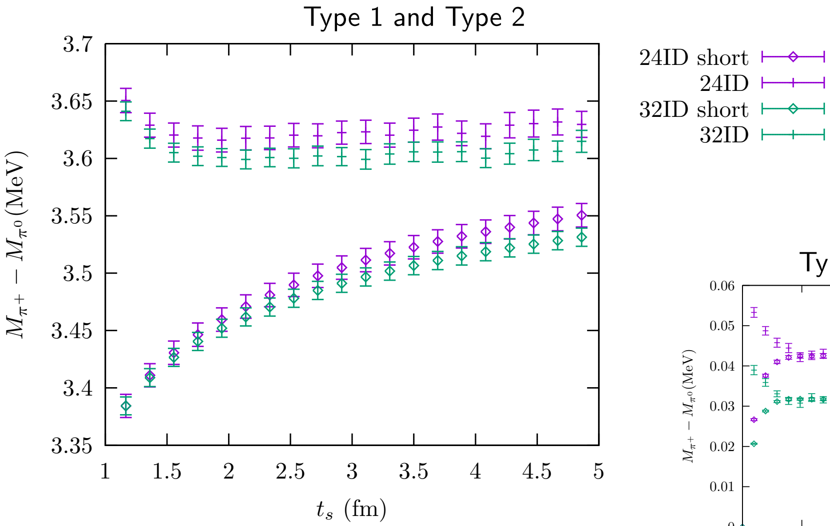
Type 1



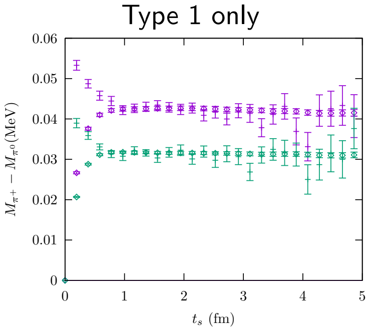
Type 2

- Diagrams are very similar to the $\pi^- \to \pi^+ ee$ $0\nu 2\beta$ decay calculation. (D. Murphy and W. Detmold, 2018.) Poster: X. Tuo (Tue 17:50).
- All UV divergence and other disconnected diagrams are cancelled. (RM123 2013)
- RM123 2013 (type 2 only): $M^2_{\pi^+} - M^2_{\pi^0} = 1.44(13)_{\text{stat}}(16)_{\text{chiral}} \times 10^3$ MeV^2
- A. Risch and H. Wittig, 2018.
- Talk: J. Richings (Wed 11:50).

Pion mass splitting: $M_{\pi^+} - M_{\pi^0}$ (prelim)

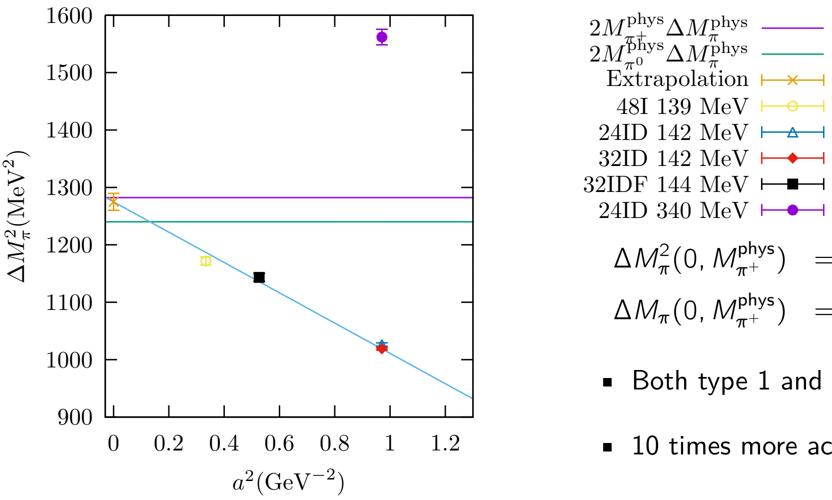


- $M_\pi = 142$ MeV.
- $a^{-1} = 1.015$ GeV,
- 24ID: $24^3 \times 64$,
- 32ID: $32^3 \times 64$,



Pion mass splitting: $M_{\pi^+} - M_{\pi^0}$ (prelim)

$$\Delta M_\pi^2(a, M_\pi) = 2M_\pi \Delta M_\pi = \Delta M_\pi^2(0, M_{\pi^+}^{\text{phys}}) + c_1 a^2 + c_2(M_\pi^2 - (M_{\pi^+}^{\text{phys}})^2)$$



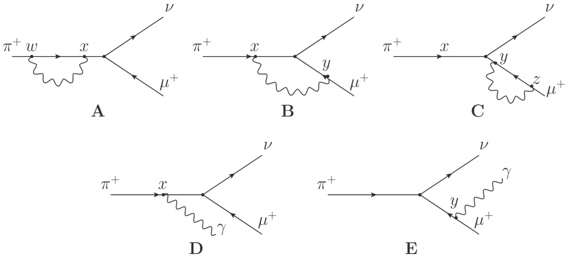
$$\Delta M_\pi^2(0, M_{\pi^+}^{\text{phys}}) = 1.275(15) \times 10^3 \text{ MeV}$$

$$\Delta M_\pi(0, M_{\pi^+}^{\text{phys}}) = 4.57(6) \text{ MeV}$$

- Both type 1 and type 2 diagrams included.
- 10 times more accurate than previous.

QED correction for meson leptonic decay

> The calculation of QED correction to QCD observables, and the appropriate treatment of the problems arising due to the finite size of the simulated lattice, is a *hot topic in the field*; nonetheless the main aspects debated at the moment concern actually *the infrared singularities* appearing in the matrix elements due to the presence of a finite box, which is not the subject of the present paper.
>
> – Anonymous



Talk: X. Feng (Wed 11:10).

- Infrared divergence shall *cancel analytically* as it always does in infinite volume perturbative calculations.

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Conclusion

- In many cases, it is very beneficial to view the problem from a position space perspective.
- For a class of problems, we can split the contribution into two different regions and adopt different treatment.
  - Muon $g-2$:
    - * Window method and GEVP for HVP C. Lehner et al, 2018.
    - * Long-distance π^0 pole contribution for HLbL.
  - Hadron Spectroscopy:
    - * The infinite volume reconstruction method. X. Feng and L. Jin 2018. Applied in calculating $M_{\pi^+} - M_{\pi^0}$. Work in progress for leptonic decay.
- More applications?

Thank You!