

Chiral transport in strong fields from holography

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[1] Y. Bu, T. Demircik and M. Lublinsky, JHEP **05** (2019) 071,
arXiv:1903.00896v2.

Outline:

- Hydrodynamics
- Chiral Transport
- Holography
- **Holographic Model**
- **Gradient Expansion**
- **Gradient Resummation**
- **Conclusion**

Hydrodynamics:

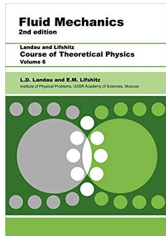
- **Hydrodynamics** is an effective low energy description of many interacting QFTs which is valid for fluctuations about global equilibrium under long wavelength limit.
- **Dynamical content of hydro: Continuity equations**

e.g. : $\partial_t \rho = -\vec{\nabla} \cdot \vec{J}$

(Needs extra input to be able solve!)



Constitutive relations (e.g.: $\vec{J} = -D\vec{\nabla}\rho + \sigma\vec{E} + \dots$)



- **Constitutive relations** are written as gradient expansion of hydro variables.

$$\text{e.g. : } \vec{J} = -\mathcal{D}\vec{\nabla}\rho + \sigma\vec{E} + \dots$$

Any term which respects **symmetries of underlying theory** and **thermodynamical constraints** can be written up to a **coefficients**.



Transport coefficients

(TCs should be determined either from the underlying microscopic theory or experimentally.)

- **Truncation at fixed order** leads **violation of causality** !

(Ideally, one needs to include all orders of gradients \Rightarrow **Ressummation!**)

Chiral Transport:

- For massless Dirac fermions, independent phase rotations of **left handed** and **right handed** spinors are independent symmetries of the classical theory ($U(1)_L$ and $U(1)_R$):

$$i\gamma^\mu \partial_\mu \Psi = 0, \quad \Psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix}, \quad \psi_L \rightarrow e^{i\theta_L} \psi_L \quad \text{and} \quad \psi_R \rightarrow e^{i\theta_R} \psi_R.$$

At the classical level:

$$\partial_\mu J_{L,R}^\mu = 0 \quad \text{where} \quad J_{L,R}^\mu = \bar{\Psi} \gamma^\mu P_\pm \Psi \quad \text{and} \quad P_\pm = (1 \pm \gamma^5)/2.$$

At the quantum level, in the presence EM-field, only one linear combination of these two symmetries is preserved:

$$\partial_\mu J^\mu = 0 \quad \text{and} \quad \partial_\mu J_5^\mu = 12\kappa \vec{E} \cdot \vec{B}$$



Chiral Anomaly!

- **Chiral Anomaly** leads **chiral asymmetry** ($\mu_5 \neq 0$) in a relativistic plasma. \Rightarrow **Modifies the constitutive relation!***

Chiral Magnetic Effect (CME) : $\vec{J} \propto \mu_5 \vec{B}$

Chiral Separation Effect (CSE) : $\vec{J}_5 \propto \mu \vec{B}$

Chiral Electric Separation Effect (CESE) : $\vec{J}_5 \propto \mu_5 \vec{E}$

+

Zoo of anomaly **induced nonlinear transports**

e.g. : **Chiral Hall Effect (CHE)** : $\vec{J} \propto \mu \vec{E} \times \vec{B}$

Anomalous Chiral Hall Effect : $\vec{J} \propto \mu \vec{E} \times \vec{\nabla} \mu_5$

Hall Diffusion : $\vec{J} \propto \vec{B} \times (\mu \vec{\nabla} \mu + \mu_5 \vec{\nabla} \mu_5)$

\vdots

- Note that some of these transports are **Non-dissipative**!

$$\left(\begin{array}{l} \mathcal{T}\text{-even TC} \Rightarrow \text{Nondissipative} \\ \mathcal{T}\text{-odd TC} \Rightarrow \text{Dissipative} \end{array} \right), \quad \text{e.g. CME, CHE, Hall Diffusion, } \dots$$

*CME: [A. Vilenkin, 1980],[K. Fukushima, D.E. Kharzeev, H.J. Warringa, 2008]

- **Chiral Anomaly** leads **new collective modes** which propagate through chiral plasma:

$$\mu \rightarrow \delta\mu e^{-i(\omega t - \vec{q} \cdot \vec{x})}, \quad \mu_5 \rightarrow \delta\mu_5 e^{-i(\omega t - \vec{q} \cdot \vec{x})},$$

$$\omega = \pm v_{\chi B} q \pm v_{\chi E} q + v_{\chi S} q + iDq^2 + \dots$$

CME + CSE \Rightarrow Chiral Magnetic Wave : ($v_{\chi B} \parallel \vec{B}$)

CESE \Rightarrow Chiral Electric Wave : ($v_{\chi B} \parallel \vec{E}$)

CHE \Rightarrow Chiral Hall Density Wave : ($v_{\chi S} \parallel \vec{S}$)

- **Remark: Realistic plasmas are exposed strong E/M fields!**
e.g. **Quark-Gluon-Plasma** [X.G. Huang, 2015], **Primordial plasma** in early universe [D. Grasso, H.R. Rubinstein, 2000]

Holography (AdS/CFT):

- **Holography (AdS/CFT)** is a duality between **strongly-coupled gauge theory in 4D** and **weakly-coupled gravity theory in 5D**. (It is a powerful tool to analyze strongly-coupled gauge theories by using classical gravitational theories!) [$N_c^2 \propto (L^3/G_5)$, $\lambda \propto (L/l_s)^4$]
- **Gubser-Klebanov-Polyakov-Witten**-relation:

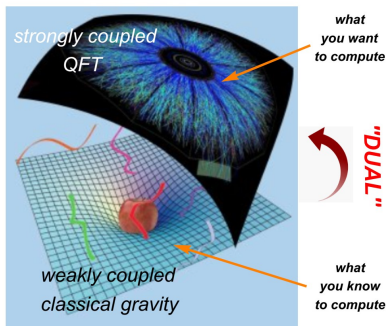
$$\mathcal{Z}_{gauge} = \mathcal{Z}_{AdS}$$

$$\left\langle \exp \left(i \int \phi^{(0)} O \right) \right\rangle = \exp \left(i \bar{S}[\phi|_{r=r_B} = \phi^{(0)}] \right)$$

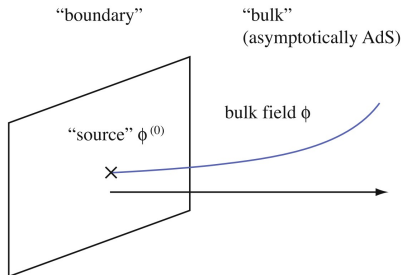


(i.e. Bulk fields act as external sources of boundary operator!)

$$\left(\begin{array}{ccc} \text{e.g. Boundary operator} & \text{External sources} & \text{Bulk fields} \\ T^{\mu\nu} & \leftrightarrow g_{\mu\nu}^{(0)} & \rightarrow g_{MN} \\ J^\mu & \leftrightarrow A_\mu^{(0)} & \rightarrow A_M \end{array} \right)$$



(a) Image: [M. Baggioli, 2019]



(b) Image: [M. Natsuume, 2014]

1. The Large N limit of superconformal field theories and supergravity

(14822) Juan Martin Maldacena (Harvard U.). Nov 1997. 21 pp.

Published in *Int.J.Theor.Phys.* **38** (1999) 1113-1133, *Adv.Theor.Math.Phys.* **2** (1998) 231-252

HUTP-97-A097, HUTP-98-A097

DOI: [10.1023/A:1026654312961](https://doi.org/10.1023/A:1026654312961), [10.4310/ATMP.1998.v2.n2.a1](https://doi.org/10.4310/ATMP.1998.v2.n2.a1)

e-Print: [hep-th/9711200](https://arxiv.org/abs/hep-th/9711200) | [PDF](#)

[References](#) | [BibTeX](#) | [LaTeX\(US\)](#) | [LaTeX\(EU\)](#) | [Harvmac](#) | [EndNote](#)

[ADS Abstract Service](#); [AMS MathSciNet](#); [OSTI.gov Server](#)

Detailed record - Cited by 14822 records 1000+

(c) The original AdS/CFT paper has been cited in all physics arXivs!

Holographic setup: $U(1)_V \times U(1)_A$

The Holographic model is **Maxwell-Chern-Simons theory** in the **Schwarzschild-AdS₅**. The bulk action

$$S = \int d^5x \sqrt{-g} \mathcal{L} + S_{\text{c.t.}},$$

where

$$\begin{aligned} \mathcal{L} = & -\frac{1}{4}(F^V)_{MN}(F^V)^{MN} - \frac{1}{4}(F^a)_{MN}(F^a)^{MN} + \frac{\kappa \epsilon^{MNPQR}}{2\sqrt{-g}} \\ & \times [3A_M(F^V)_{NP}(F^V)_{QR} + A_M(F^a)_{NP}(F^a)_{QR}], \\ S_{\text{c.t.}} = & \frac{1}{4} \log r \int d^4x \sqrt{-\gamma} [(F^V)_{\mu\nu}(F^V)^{\mu\nu} + (F^a)_{\mu\nu}(F^a)^{\mu\nu}], \end{aligned}$$

and the background metric is

$$ds^2 = g_{MN} dx^M dx^N = 2dt dr - r^2 f(r) dt^2 + r^2 \delta_{ij} dx^i dx^j,$$

where $f(r) = 1 - 1/r^4$, $r \in [1, \infty)$. Temperature is $\pi T = 1$.

Bulk equations can be split into **dynamical** ($EV^\mu = EA^\mu = 0$) and **constraint** components ($EV^r = EA^r = 0$), where

$$EV^M \equiv \nabla_N (F^V)^{NM} + \frac{3\kappa\epsilon^{MNPQR}}{\sqrt{-g}} (F^a)_{NP} (F^V)_{QR},$$

$$EA^M \equiv \nabla_N (F^a)^{NM} + \frac{3\kappa\epsilon^{MNPQR}}{2\sqrt{-g}} [(F^V)_{NP} (F^V)_{QR} + (F^a)_{NP} (F^a)_{QR}].$$

The boundary currents in terms **the bulk fiels** are

$$J^\mu = \lim_{r \rightarrow \infty} \sqrt{-\gamma} \left\{ (F^V)^{\mu M} n_M + \frac{6\kappa\epsilon^{M\mu NQR}}{\sqrt{-g}} n_M A_N (F^V)_{QR} - \tilde{\nabla}_\nu (F^V)^{\nu\mu} \log r \right\},$$

$$J_5^\mu = \lim_{r \rightarrow \infty} \sqrt{-\gamma} \left\{ (F^a)^{\mu M} n_M + \frac{2\kappa\epsilon^{M\mu NQR}}{\sqrt{-g}} n_M A_N (F^a)_{QR} - \tilde{\nabla}_\nu (F^a)^{\nu\mu} \log r \right\},$$

Near boundary asymptotic expansion of the bulk gauge fields:

$$V_\mu = \mathcal{V}_\mu + \frac{V_\mu^{(1)}}{r} + \frac{V_\mu^{(2)}}{r^2} - \frac{2V_\mu^L}{r^2} \log r + \mathcal{O}\left(\frac{\log r}{r^3}\right), \quad A_\mu = \frac{A_\mu^{(2)}}{r^2} + \mathcal{O}\left(\frac{\log r}{r^3}\right),$$

where

$$V_{\mu}^{(1)} = \mathcal{F}_{t\mu}^V, \quad 4V_{\mu}^L = \partial^{\nu} \mathcal{F}_{\mu\nu}^V.$$

The currents in terms of coefficients of near boundary expansion:

$$J^\mu = \eta^{\mu\nu}(2V_{\nu}^{(2)} + 2V_{\nu}^L + \eta^{\sigma\tau}\partial_{\sigma}\mathcal{F}_{t\nu}^V), \quad J_5^\mu = \eta^{\mu\nu}2A_{\nu}^{(2)}.$$

(Near boundary coefficients at $\mathcal{O}(r^{-2})$ determine the boundary currents!)

Fluid/Gravity correspondence*:

The most general static and homogeneous profiles for the bulk gauge fields:

$$V_\mu = \mathcal{V}_\mu - \frac{\rho}{2r^2} \delta_{\mu t}, \quad A_\mu = -\frac{\rho_5}{2r^2} \delta_{\mu t},$$

with the boundary currents:

$$J^t = \rho, \quad J^i = 0; \quad J_5^t = \rho_5, \quad J_5^i = 0.$$

Promoting \mathcal{V}_μ , ρ , ρ_5 into **arbitrary functions of the boundary coordinates**

$$\mathcal{V}_\mu \rightarrow \mathcal{V}_\mu(x_\alpha), \quad \rho \rightarrow \rho(x_\alpha), \quad \rho_5 \rightarrow \rho_5(x_\alpha),$$

causes them to cease to be solution. **Corrections** should be introduced

$$V_\mu(r, x_\alpha) = \mathcal{V}_\mu(x_\alpha) - \frac{\rho(x_\alpha)}{2r^2} \delta_{\mu t} + \mathbb{V}_\mu(r, x_\alpha),$$
$$A_\mu(r, x_\alpha) = -\frac{\rho_5(x_\alpha)}{2r^2} \delta_{\mu t} + \mathbb{A}_\mu(r, x_\alpha).$$

*:[S.Bhattacharyya, V.E. Hubeny, S. Minwalla, M. Rangamani, 2008]

BCs:

- (i) regularity over $r \in [1, \infty)$
- (ii) $\mathbb{V}_\mu \rightarrow 0, \quad \mathbb{A}_\mu \rightarrow 0 \quad \text{as } r \rightarrow \infty$
- (iii) the Landau frame convention ($J^t = \rho(x_\alpha), \quad J_5^t = \rho_5(x_\alpha)$)

Equations for the corrections:

$$\begin{aligned} 0 &= r^3 \partial_r^2 \mathbb{V}_t + 3r^2 \partial_r \mathbb{V}_t + r \partial_r \partial_k \mathbb{V}_k + 12\kappa \epsilon^{ijk} [\partial_r \mathbb{A}_i (\partial_j \mathcal{V}_k + \partial_j \mathbb{V}_k) + \partial_r \mathbb{V}_i \partial_j \mathbb{A}_k], \\ 0 &= (r^5 - r) \partial_r^2 \mathbb{V}_i + (3r^4 + 1) \partial_r \mathbb{V}_i + 2r^3 \partial_r \partial_t \mathbb{V}_i - r^3 \partial_r \partial_i \mathbb{V}_t + r^2 (\partial_t \mathbb{V}_i - \partial_i \mathbb{V}_t) \\ &\quad + r (\partial^2 \mathbb{V}_i - \partial_i \partial_k \mathbb{V}_k) - \frac{1}{2} \partial_i \rho + r^2 (\partial_t \mathcal{V}_i - \partial_i \mathcal{V}_t) + r (\partial^2 \mathcal{V}_i - \partial_i \partial_k \mathcal{V}_k) \\ &\quad + 12\kappa r^2 \epsilon^{ijk} \left(\frac{1}{r^3} \rho_5 \partial_j \mathcal{V}_k + \frac{1}{r^3} \rho_5 \partial_j \mathbb{V}_k + \partial_r \mathbb{A}_t \partial_j \mathcal{V}_k + \partial_r \mathbb{A}_t \partial_j \mathbb{V}_k \right) \\ &\quad - 12\kappa r^2 \epsilon^{ijk} \partial_r \mathbb{A}_j \left[(\partial_t \mathcal{V}_k - \partial_k \mathcal{V}_t) + (\partial_t \mathbb{V}_k - \partial_k \mathbb{V}_t) + \frac{1}{2r^2} \partial_k \rho \right] \\ &\quad - 12\kappa r^2 \epsilon^{ijk} \left\{ \partial_r \mathbb{V}_j \left[(\partial_t \mathbb{A}_k - \partial_k \mathbb{A}_t) + \frac{1}{2r^2} \partial_k \rho_5 \right] - \partial_j \mathbb{A}_k \left(\partial_r \mathbb{V}_t + \frac{1}{r^3} \rho \right) \right\}, \end{aligned}$$

$$\begin{aligned}
0 &= r^3 \partial_r^2 \mathbb{A}_t + 3r^2 \partial_r \mathbb{A}_t + r \partial_r \partial_k \mathbb{A}_k + 12\kappa \epsilon^{ijk} [\partial_r \mathbb{V}_i (\partial_j \mathcal{V}_k + \partial_j \mathbb{V}_k) + \partial_r \mathbb{A}_i \partial_j \mathbb{A}_k], \\
0 &= (r^5 - r) \partial_r^2 \mathbb{A}_i + (3r^4 + 1) \partial_r \mathbb{A}_i + 2r^3 \partial_r \partial_t \mathbb{A}_i - r^3 \partial_r \partial_i \mathbb{A}_t + r^2 (\partial_t \mathbb{A}_i - \partial_i \mathbb{A}_t) \\
&\quad + r (\partial^2 \mathbb{A}_i - \partial_i \partial_k \mathbb{A}_k) - \frac{1}{2} \partial_i \rho_5 + 12\kappa r^2 \epsilon^{ijk} (\partial_j \mathcal{V}_k + \partial_j \mathbb{V}_k) \left(\partial_r \mathbb{V}_t + \frac{1}{r^3} \rho \right) \\
&\quad - 12\kappa r^2 \epsilon^{ijk} \partial_r \mathbb{V}_j \left[(\partial_t \mathcal{V}_k - \partial_k \mathcal{V}_t) + (\partial_t \mathbb{V}_k - \partial_k \mathbb{V}_t) + \frac{1}{2r^2} \partial_k \rho \right] \\
&\quad - 12\kappa r^2 \epsilon^{ijk} \left\{ \partial_r \mathbb{A}_j \left[(\partial_t \mathbb{A}_k - \partial_k \mathbb{A}_t) + \frac{1}{2r^2} \partial_k \rho_5 \right] - \partial_j \mathbb{A}_k \left(\partial_r \mathbb{A}_t + \frac{1}{r^3} \rho_5 \right) \right\}.
\end{aligned}$$

I) Gradient expansion:

Introducing λ as a **gradient expansion parameter** ($\partial_\mu \rightarrow \lambda \partial_\mu$), the corrections of bulk fields are expandable in powers of λ :

$$\mathbb{V}_\mu = \sum_{n=0}^{\infty} \lambda^n \mathbb{V}_\mu^{[n]}, \quad \mathbb{A}_\mu = \sum_{n=0}^{\infty} \lambda^n \mathbb{A}_\mu^{[n]}.$$

Accordingly:

$$\vec{J} = \sum_{n=0}^{\infty} \lambda^n \vec{J}^{[n]}, \quad \vec{J}_5 = \sum_{n=0}^{\infty} \lambda^n \vec{J}_5^{[n]}.$$

The background fields \vec{E} and \vec{B} are treated as *zeroth order* in the gradient expansion, as opposed to our previous studies!

$$\Downarrow \\ \vec{E}, \vec{B} \sim \mathcal{O}(\partial^0) \Rightarrow \text{TCs} \rightarrow \text{TCs}[\mathbf{E}^2, \mathbf{B}^2, (\vec{E} \cdot \vec{B})^2]$$

$\mathcal{O}(\lambda^0)$:

$$\vec{J}^{[0]} = \sigma_e^0 \vec{\mathbf{E}} + \sigma_\chi^0 \kappa \rho_5 \vec{\mathbf{B}} + \delta \sigma_\chi^0 \kappa^2 (\vec{\mathbf{E}} \cdot \vec{\mathbf{B}}) \vec{\mathbf{B}} + \sigma_{\chi H}^0 \kappa^2 \rho \vec{\mathbf{B}} \times \vec{\mathbf{E}} + \sigma_{\chi e}^0 \kappa^3 \rho_5 (\vec{\mathbf{B}} \cdot \vec{\mathbf{E}}) \vec{\mathbf{E}}$$

$$\vec{J}_5^{[0]} = \sigma_\chi^0 \kappa \rho \vec{\mathbf{B}} + \sigma_{\chi H}^0 \kappa^2 \rho_5 \vec{\mathbf{B}} \times \vec{\mathbf{E}} + \sigma_{\chi e}^0 \kappa^3 \rho (\vec{\mathbf{B}} \cdot \vec{\mathbf{E}}) \vec{\mathbf{E}} + \sigma_s^0 \kappa^3 (\vec{\mathbf{E}} \cdot \vec{\mathbf{B}}) \vec{\mathbf{B}} \times \vec{\mathbf{E}}$$

Except σ_s^0 , all terms have already appeared in the literature. **The main novelty is:**

$$\sigma_e^0 = \sigma_e^0 [\mathbf{E}^2, \mathbf{B}^2, (\vec{\mathbf{B}} \cdot \vec{\mathbf{E}})^2]; \quad \sigma_\chi^0 = \sigma_\chi^0 [\mathbf{E}^2, \mathbf{B}^2, (\vec{\mathbf{B}} \cdot \vec{\mathbf{E}})^2]; \quad \text{etc...}$$

Alternatively:

$$J_i^{[0]} = \sigma_e^0 \left(\delta_{ij} - \frac{\mathbf{B}_i \mathbf{B}_j}{\mathbf{B}^2} \right) \mathbf{E}_j + \sigma_e^{0L} \frac{\mathbf{B}_i \mathbf{B}_j}{\mathbf{B}^2} \mathbf{E}_j + \sigma_\chi^0 \kappa \rho_5 \left(\delta_{ij} - \frac{\mathbf{E}_i \mathbf{E}_j}{\mathbf{E}^2} \right) \mathbf{B}_j \\ + \sigma_\chi^{0L} \kappa \rho_5 \frac{\mathbf{E}_i \mathbf{E}_j}{\mathbf{E}^2} \mathbf{B}_j + \sigma_{\chi H}^0 \kappa^2 \rho (\vec{\mathbf{B}} \times \vec{\mathbf{E}})_i,$$

$$\text{where: } \sigma_e^{0L} = \sigma_e^0 + \kappa^2 \mathbf{B}^2 \delta \sigma_\chi^0,$$

$$\sigma_\chi^{0L} = \sigma_\chi^0 + \kappa^2 \mathbf{E}^2 \sigma_{\chi e}^0.$$

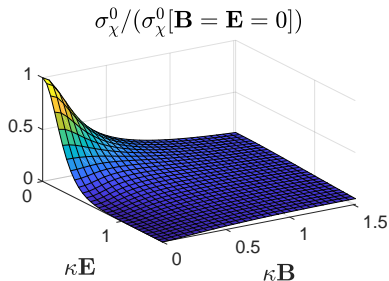
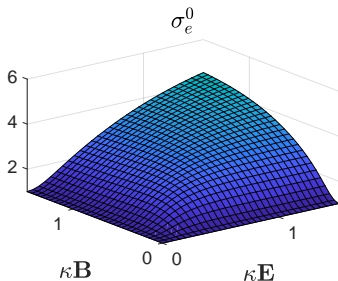


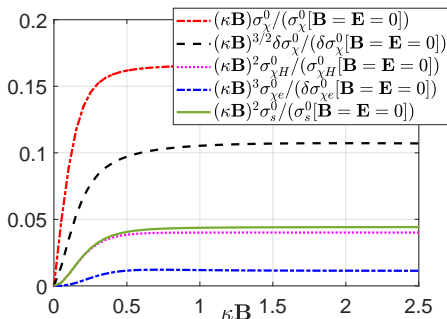
Figure: Zeroth order TCs as functions of e/m fields when $\vec{\mathbf{E}} \parallel \vec{\mathbf{B}}$

- Ohmic conductivity σ_e^0 gets **enhancement!**



(Without Anomaly: [G.T. Horowitz, N. Iqbal, J.E. Santos, 2013],[H.B. Zeng, Y.Tian, Z.Y. Fan, C.M. Fan, 2016, 2017], **Anomaly induced:New!**)

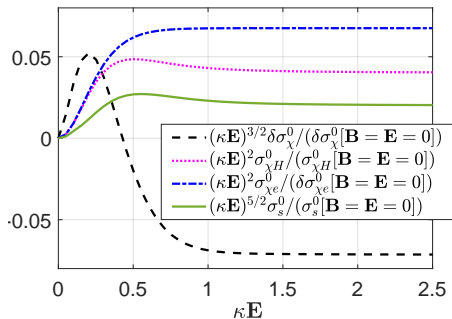
- Other TCs behave similarly as σ_χ^0 , i.e. **decrease dramatically and vanish asymptotically!**



- For $\kappa \mathbf{E} = 0$, asymptotic behaviors for large- $\kappa \mathbf{B}$ can be extracted numerically:

$$\begin{aligned} \sigma_{\chi}^0[\kappa \mathbf{B} \rightarrow \infty, \kappa \mathbf{E} = 0] &\rightarrow \frac{1}{\kappa \mathbf{B}}, & \delta \sigma_{\chi}^0[\kappa \mathbf{B} \rightarrow \infty, \kappa \mathbf{E} = 0] &\rightarrow \frac{3.349}{(\kappa \mathbf{B})^{3/2}}, \\ \sigma_{\chi H}^0[\kappa \mathbf{B} \rightarrow \infty, \kappa \mathbf{E} = 0] &\rightarrow -\frac{1}{(\kappa \mathbf{B})^2}, & \sigma_{\chi e}^0[\kappa \mathbf{B} \rightarrow \infty, \kappa \mathbf{E} = 0] &\rightarrow \frac{0.977}{(\kappa \mathbf{B})^3}, \\ \sigma_s^0[\kappa \mathbf{B} \rightarrow \infty, \kappa \mathbf{E} = 0] &\rightarrow -\frac{6.751}{(\kappa \mathbf{B})^2}. \end{aligned}$$

(Note: σ_{χ}^0 is consistent with analytic results of [K.Landsteiner, Y.Liu, Y.W.Sun, 2015], [M.Ammon, S.Grieninger, A.J. Alba, 2016], [Y.Bu, M.Lublinsky, A.Sharon, 2017].)



- For $\kappa \mathbf{B} = 0$, asymptotic behaviors for large- $\kappa \mathbf{E}$ can be extracted numerically:

$$\begin{aligned} \delta\sigma_{\chi}^0[\kappa\mathbf{B}=0, \kappa\mathbf{E}\rightarrow\infty] &\rightarrow -\frac{2.243}{(\kappa\mathbf{E})^{3/2}}, & \sigma_{\chi H}^0[\kappa\mathbf{B}=0, \kappa\mathbf{E}\rightarrow\infty] &\rightarrow -\frac{1}{(\kappa\mathbf{E})^2}, \\ \sigma_{\chi e}^0[\kappa\mathbf{B}=0, \kappa\mathbf{E}\rightarrow\infty] &\rightarrow \frac{6.04}{(\kappa\mathbf{E})^2}, & \sigma_s^0[\kappa\mathbf{B}=0, \kappa\mathbf{E}\rightarrow\infty] &\rightarrow -\frac{3.069}{(\kappa\mathbf{E})^{5/2}}. \end{aligned}$$

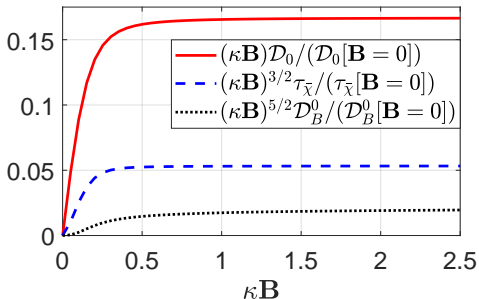
Except for $\sigma_{\chi}^0[\kappa \mathbf{B} = 0]$ which decays much faster than any other TCs, and asymptotically does not scale as a power function of $\kappa \mathbf{E}$.

$\mathcal{O}(\lambda^1)$:

We consider the cases of either $\mathbf{E} = 0$ or $\mathbf{B} = 0$ separately:

a) $\mathbf{E} = 0$:

$$\vec{J}^{[1]} = -\mathcal{D}_0 \vec{\nabla} \rho + \tau_{\bar{\chi}} \kappa \partial_t \rho_5 \vec{\mathbf{B}} + \mathcal{D}_B^0 \kappa^2 (\vec{\mathbf{B}} \cdot \vec{\nabla} \rho) \vec{\mathbf{B}}, \quad \vec{J}_5^{[1]} : \rho \leftrightarrow \rho_5.$$

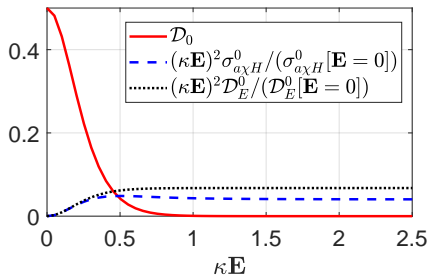


$$\mathcal{D}_0[\kappa \mathbf{B} \rightarrow \infty, \kappa \mathbf{E} = 0] \rightarrow \frac{0.083}{(\kappa \mathbf{B})}, \quad \tau_{\bar{\chi}}[\kappa \mathbf{B} \rightarrow \infty, \kappa \mathbf{E} = 0] \rightarrow -\frac{0.36}{(\kappa \mathbf{B})^{3/2}},$$

$$\mathcal{D}_B^0[\kappa \mathbf{B} \rightarrow \infty, \kappa \mathbf{E} = 0] \rightarrow -\frac{0.269}{(\kappa \mathbf{B})^{5/2}}.$$

a) $\mathbf{B} = \mathbf{0}$:

$$\vec{J}^{[1]} = -\mathcal{D}_0 \vec{\nabla} \rho + \sigma_{axH}^0 \kappa \vec{\mathbf{E}} \times \vec{\nabla} \rho_5 + \mathcal{D}_E^0 \kappa^2 (\vec{\mathbf{E}} \cdot \vec{\nabla} \rho) \vec{\mathbf{E}}, \quad \vec{J}_5^{[1]} : \rho \leftrightarrow \rho_5.$$



$$\sigma_{a\chi H}^0[\kappa\mathbf{B}=0, \kappa\mathbf{E}\rightarrow\infty]\rightarrow -\frac{0.141}{(\kappa\mathbf{E})^2}, \quad \mathcal{D}_E^0[\kappa\mathbf{B}=0, \kappa\mathbf{E}\rightarrow\infty]\rightarrow -\frac{0.298}{(\kappa\mathbf{E})^2}.$$

- \mathcal{D}_0 was shown to receive **negative \mathbf{E}^2 - and \mathbf{B}^2 -corrections** induced by the chiral anomaly ([Y.Bu, M.Lublinsky, A.Sharon, 2016],[Y.Bu, T.Demircik, M.Lublinsky, 2018]). **\Rightarrow Now it vanishes asymptotically!**

II) Gradient Resummation :

- The framework of **all order resummation** was invented and improved through: [M.Lublinsky, E.Shuryak, 2009],
[Y.Bu, M.Lublinsky, 2014, 2015, 2015, 2016]
- The essence is: **Transport Coefficient Functions**

$$\mathbf{TCs} \rightarrow \mathbf{TCFs}[\partial_t, \vec{\nabla}^2]$$

i) contain information about **infinitely many gradients**.

ii) valid **beyond hydro limit** ($\omega, q^2 \ll 1$).

iii) introduce **memory functions**.

- In this study, we **relax weak field assumption** ($\mathbf{E} = \mathbf{0}$) for and employ **linearization in ρ, ρ_5** :

$$\rho \rightarrow \epsilon\rho, \quad \rho_5 \rightarrow \epsilon\rho_5, \quad \mathbf{B} \sim \mathcal{O}(\epsilon^0).$$

\Downarrow

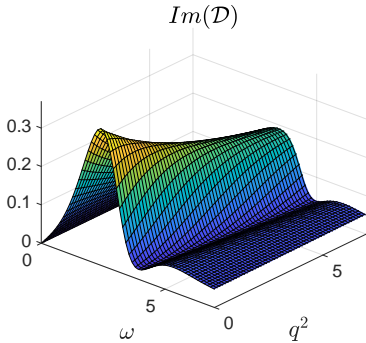
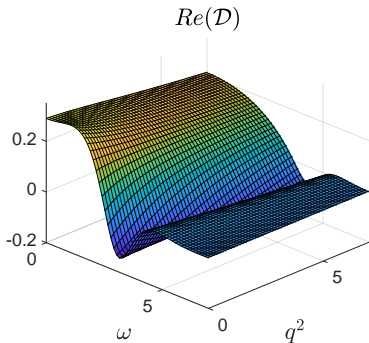
$$\mathbf{TCFs}[\partial_t, \vec{\nabla}^2] \rightarrow \mathbf{TCFs}[\partial_t, \vec{\nabla}^2, (\kappa\mathbf{B})^2, (\kappa\vec{\mathbf{B}} \cdot \vec{q})^2]$$

(The main **novelty!**)

$\mathcal{O}(\epsilon)$:

$$\vec{J} = -\mathcal{D}\vec{\nabla}\rho + \mathcal{D}_B\kappa^2\vec{\mathbf{B}}(\vec{\mathbf{B}} \cdot \vec{\nabla}\rho) + \bar{\sigma}_{\bar{\chi}}\kappa\vec{\mathbf{B}}\rho_5 + \mathcal{D}_{\chi}\kappa(\vec{\mathbf{B}} \cdot \vec{\nabla})\vec{\nabla}\rho_5, \quad \vec{J}_5 : \rho \leftrightarrow \rho_5.$$

where: $\mathcal{D}[\partial_t, \vec{\nabla}^2, (\kappa\vec{\mathbf{B}})^2, (\kappa\vec{\mathbf{B}} \cdot \vec{q})^2]$, $\mathcal{D}_B[\partial_t, \vec{\nabla}^2, (\kappa\vec{\mathbf{B}})^2, (\kappa\vec{\mathbf{B}} \cdot \vec{q})^2]$, etc..



- All the **TCFs** exhibit similar behavior: **relatively weak dependence** on q^2 ; **damped oscillations** in ω .

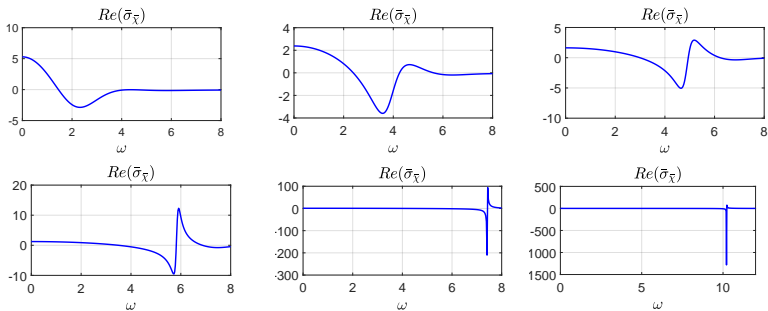


Figure: $Re(\bar{\sigma}_{\chi})$ as a function of ω when $q = 0$ and $\kappa\mathbf{B} = 0.1 \rightarrow \kappa\mathbf{B} = 0.4 \rightarrow \kappa\mathbf{B} = 0.6 \rightarrow \kappa\mathbf{B} = 0.8 \rightarrow \kappa\mathbf{B} = 1.2 \rightarrow \kappa\mathbf{B} = 2.2$.

- TCFs become **singular** when **magnetic field is large enough** ($\kappa\mathbf{B} \gtrsim 0.5$) !
- At larger values of $\kappa\mathbf{B}$, **additional singularities** emerge at larger ω .
- Locations of these additional singularities are **symmetric** about the origin.

- These singularities are the **Quasi Normal Modes!**

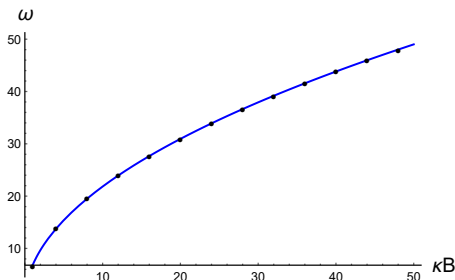


Figure: Location of the singularity as a function of κB : numerical result (black dots) is best fitted by $\omega = -0.155157 + 6.95306\sqrt{\kappa B}$ (blue curve).

- The lowest **QNM** exhibit **Landau level** behavior:

$$\text{Re}[\omega] \sim \sqrt{\kappa B}, \quad \text{Im}[\omega] \rightarrow 0$$

[M.Ammon, S.Grieninger, A.J.Alba, R.P. Macedo, L.Melgar, 2016]

- Similar phenomenon was found recently and named as "**anomalous resonance**" in [M.Haack, D. Sarkar, A.Yarom, 2019].

- Modifications on **CMW**:

$$\omega = \pm (\bar{\sigma}_{\bar{\chi}} - q^2 \mathcal{D}_{\chi}) \kappa \vec{\mathbf{B}} \cdot \vec{\mathbf{q}} - i \left(q^2 \mathcal{D} - \mathcal{D}_B (\kappa \vec{\mathbf{B}} \cdot \vec{\mathbf{q}})^2 \right).$$

- Can **CMW** be **fully non-dissipative** \Rightarrow **Yes!**

[Y.Bu, M.Lublinsky, T.Demircik, 2018]

- Can **CMW** be **fully non-dissipative** at **large magnetic field**? \Rightarrow **Yes!**

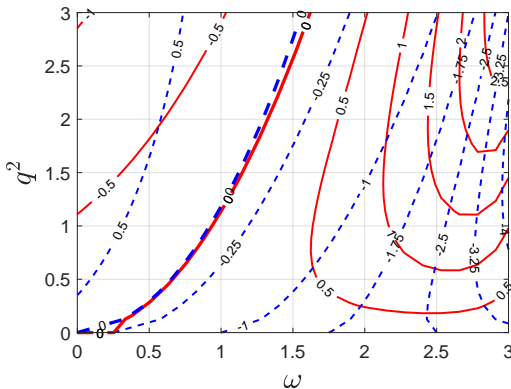


Figure: Contour plots for the functions ϕ_R (blue dashed) and ϕ_I (red solid) at $\kappa \mathbf{B} = 0.33$.

Conclusion:

- In this work we focused on **influence of strong background e/m fields on chiral anomaly-induced transport phenomena** for a holographically defined thermal plasma.
- Constitutive relations for \vec{J} , \vec{J}_5 were evaluated within two complimentary approximation schemes: a **fixed order gradient expansion** (up to first order) and an **all-order gradient resummation** (linear in ρ, ρ_5).
- **TCs** are found to be suppressed by the external fields and vanish at asymptotically strong fields, **except the Ohmic conductivity** which gets enhanced in parallel electrical and magnetic fields.
- When $\vec{E} = 0$, **the all-order resummed constitutive relations** are parameterised by **four independent TCFs, which are functions of ω, \vec{q} and \vec{B}** . The TCFs are found to show a common **singularity at B** is strong enough. This singularity is identified as **QNM** and obey **Landau level behavior**.
- Previously, discovered **non-dissipative and thus long-lived CMW mode** is examined CMW exactly without the weak field approximation.

Thanks!