

Partial Wave Analysis as a Tool in Baryon Spectroscopy

Theory of hot matter and relativistic heavy-ion collisions
THOR Annual Meeting
iTÜ , Istanbul, Turkey
September, 2019

Colaboration

Outline
Database
PWA

Laurent +
Pietarinen
(L+P)

- University of Tuzla
 - Jugoslav Stahov
 - Hedim Osmanovic
 - Rifat Omerovic
- Institute Rudjer Boskovic, Zagreb
 - Alfred Svarc
- Institute of Nuclear Physics
Johannes Gutenberg-Universität Mainz
 - Lothar Tiator
 - Viktor Kashevarov
 - Michael Ostrick
- George Washington University
 - Ron Workman

PWA & L+P up to now

Outline

Database

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PWA

- ① Fixed-t analyticity as a constraint in single energy partial wave analyses of meson photoproduction reactions
- ② Single-Energy Partial Wave Analysis for π^0 Photoproduction on Proton with Fixed-t Analyticity Imposed
- ③ Eta and Eta-prime Photoproduction on the Nucleon with the Isobar Model EtaMAID2018
- ④ From Experimental Data to Pole Parameters in a Direct Way (Angle Dependent Continuum Ambiguity and Laurent + Pietarinen Expansion)

L+P

- ① Phys.Rev. C88 (2013) no.3, 035206
- ② Phys.Rev. C89 (2014) no.4, 045205
- ③ Phys.Rev. C89 (2014) no.6, 065208
- ④ Phys.Rev. C91 (2015) no.1, 015207
- ⑤ Phys.Lett. B755 (2016) 452-455
- ⑥ Phys.Rev. C94 (2016) no.6, 065204
- ⑦ Phys.Rev.Lett. 119 (2017) no.6, 062004
- ⑧ Eur.Phys.J. A53 (2017) no.12, 242

PWA & L+P up to now

Outline
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- ① Introducing the Pietarinen expansion method into the single-channel pole extraction problem
- ② Poles of Karlsruhe-Helsinki KH80 and KA84 solutions extracted by using the Laurent-Pietarinen method
- ③ Pole positions and residues from pion photoproduction using the Laurent-Pietarinen expansion method
- ④ Pole structure from energy-dependent and single-energy fits to GWU-SAID πN elastic scattering data
- ⑤ Generalization of the model-independent Laurent-Pietarinen single-channel pole-extraction formalism to multiple channels
- ⑥ Baryon transition form factors at the pole
- ⑦ Strong evidence for nucleon resonances near 1900 MeV
- ⑧ N^* resonances from $K\Lambda$ amplitudes in sliced bins in energy

Experiment

Outline

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Reaction	Experiment	Year	Author (Source)	E _{LAB} range [MeV]	Number of Data	Experiment	Year	Author (Source)	E _{LAB} range [MeV]	Number of Data	
Differential cross section σ_0											
	CB-MAMI	2008	Prakhov PRL111-062004	147 - 795			CB-ELSA	2014	Hartmann PRL113-062001	684 - 917	
	CB-MAMI	2015	Prakhov 2015	218 - 1443	9170		CB-MAMI	2014	Kashevarov	440 - 1430	5126
	CB-MAMI	2015	Prakhov EPT	1428 - 1573			CB-MAMI	2014	Schumann - Otte	144 - 419	
		2017	Dieterle M. PLB-770 523	450 - 1430	340		CB-MAMI	2014	Kashevarov	440-1430	4969
Beam asymmetry Σ											
	TAPS2	2001	Leukel	240 - 440			CB-MAMI	2014	Schumann - Otte	144 - 419	
	GRAAL	2005	Bartelini EPJA26-399	551 - 1475	1356		CB-MAMI	2014	Double - polarisation asymmetry F	144 - 419	
	CB-MAMI	2008	Prakhov PRL111-062004	147 - 317			CB-MAMI	2014	Double - polarisation asymmetry G	144 - 419	
Recoil asymmetry J											
	CB-MAMI	2014	Hartman PRL113-062001	684 - 917			DAPHNE	2005	Ahrens EPJA26-135	340	
	KHARKOV	1983	Belyaev NPB213-201	280 - 450			CB-ELSA	2012	Thiel PRL109-102001	633 - 1300	321
	KHARKOV	1980	Bratashevski NPB166-525	480 - 1275			CB-ELSA	2014	Double - polarisation H	144 - 419	
	DNPL	1972	Prentice NPB41-353	850 - 1250	296		CB-ELSA	2014	Hartmann PRL113-062001	684 - 917	157
	KHARKOV	1976	Debrechinski JETP43-218	585 - 615			CB-ELSA	2017	Double - polarisation asymmetry E	144 - 419	
	KHARKOV	1978	Zybalov SJNP28-218	650, 700			CB-MAMI	2015	Dieterle M. PLB-770 523	450 - 1430	170
	DNPL	1979	Bussey NPB154-492	1300			CB-ELSA	2014	Double - polarisation asymmetry G	144 - 419	
Data in light purple rows are not included in analysis											
Observable sgE											
	DAPHNE	2001	Preobrazenski				DAPHNE	2001	Double - polarisation asymmetry H	144 - 419	

Experiment

Outline
Database
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Reaction	Year	Source - Authors	Energy Range	Number of Data
Differential cross section σ_0				
		A.Ando et al., Physik Daten, Karlsruhe	1203-1517	
	1972	C. Bacciet al., Phys. Lett. C 39, 559	1323-1535	
	1973	Y. Hemmiet al., Nucl. Phys. B 55, 333	1318-1604	106
	1967	Klinesmith, Ph.D Thesis	1611-1869	
	2017	Dieterle M. PLB-770 523	450-1430	
	2018	Dieterle M. PRC-97 065205	430-1450	1290
Beam assymetry Σ				
	2009	R.Di Salvo et al., Eur. Phys. J. A 42, 151	1484-1912	216
Double - polarisation asimmetry E				
	2017	Dieterle M. PLB-770(2017)523	450-1430	170

Data in light purple rows are not included in analysis



Experiment

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Reaction	Year	Source - Authors	Energy Range W [MeV]	Number of Data	Year	Source - Authors	Energy Range W [MeV]	Number of Data
Differential cross section σ_0								
S.D.Ecklund, R.L.Walker, Phys Rev. 159, 1195 (1967)								
		C.Betourne et al., Phys Rev. 172, 1343 (1968)				P.J.Bussey et al., Nucl.Phys. B154, 205 (1979)		
		B.Bouquet et al., Phys.Rev.Lett. 27, 1244 (1971)			1979-1981	V.A.Getman et al., Nucl.Phys. B188, 397 (1981)	1201-2259	252
		T.Fujii et al., Phys Rev Lett. 26, 1672 (1971)				K.Egawa et al., Nucl.Phys. B188, 11 (1981)		
		K.Ekstrand et al., Phys.Rev. D6, 1 (1972)						
	1967-2001	T.Fujii et al., Nucl.Phys. B120, 395 (1977)	1481-2201	2534				
		I.Arai et al., J.Phys.Soc.Jap. 43, 363 (1977)						
		E.J.Durwen, Ph.D.Thesis (1980); BONN-IR-80-7						
		K.H.Althoff et al., Z.Phys. C18, 199 (1983)						
		W.Hesse, Ph.D.Thesis (1988); BONN-IR-88-06						
		K.Buechler et al., Nucl.Phys. A570, 580 (1994)						
		H.W.Dernhausen et al., Eur.Phys.J. A11, 441 (2001)						
	2009	M.Dugger et al., Phys.Rev. C79, 065206	1497-2505					
	2004	J.Ahrens et al., Eur.Phys.J. A21, 323	1178-1313					
	2006	J.Ahrens et al., Phys.Rev. C74, 045204	1323-1533					
Beam asymmetry Σ								
G.Bianqued et al., Phys.Rev. C64, 025203 (2001)								
		J.Bouquet et al., AIP Conf.Proc. 503, 499 (2001)						
		R.E.Taylor, R.F.Mozley, Phys.Rev. 117, 835 (1960)						
		R.C.Smith, R.F.Mozley, Phys.Rev. 138, 2429 (1963)						
		J.Akspector et al., Phys.Rev.Lett. 28, 1403 (1972)						
		G.Knies et al., Phys.Rev. D10, 2778 (1974)						
	1960-2001	V.B.Gorenko et al., Yad.Fiz. 23, 100 (1976)	1201-2259	128B				
		P.J.Bussey et al., Nucl.Phys. B154, 205 (1979)						
		V.A.Getman et al., Nucl.Phys. B188, 397 (1981)						
		P.Hampe, Ph.D.Thesis, 1980						
		R.Beck et al., Phys.Rev. C61, 035204 (2000)						
		J.Ajaka et al., Phys.Lett. B475, 372 (2000)						
	2014	M.Dugger et al., PRC 88, 065203 (2013); PRC 89, 029901	1724-2093					
Recoil asymmetry P								
P.J.Bussey et al., Nucl.Phys. B154, 205 (1979)								
		V.A.Getman et al., Nucl.Phys. B188, 397 (1981)						
		K.Egawa et al., Nucl.Phys. B188, 11 (1981)						
Target asymmetry T								
P.J.Bussey et al., Nucl.Phys. B154, 205 (1979)								
		V.A.Getman et al., Nucl.Phys. B188, 397 (1981)						
		K.H.Althoff et al., Nucl.Phys. B53, 9 (1973)						
		S.Arai et al., Nucl.Phys. B48, 397 (1972)						
		P.Feller et al., Nucl.Phys. B102, 207 (1976)						
		K.H.Althoff et al., Phys.Lett. B59, 93 (1975)						
	1972-1996	H.Genzel et al., Nucl.Phys. B92, 196 (1975)	1201-2360	912				
		K.H.Althoff et al., Phys.Lett. B63, 107 (1978)						
		K.H.Althoff et al., Nucl.Phys. B131, 1 (1977)						
		M.Fukushima et al., Nucl.Phys. B130, 486 (1977)						
		V.A.Getman et al., Yad.Fiz. 32, 1008 (1980)						
		K.Fujii et al., Nucl.Phys. B197, 365 (1982)						
		H.Dutz et al., Nucl.Phys. A601, 319 (1996)						
	2013	V.Kashevarev, PWAT Camogli	1300-1650					
Double polarisation asymmetry G								
J.Ahrens et al., Eur.Phys.J. A26, 135 (2005)								
		P.J.Bussey et al., Nucl.Phys. B169, 403 (1980)	1217-2097	86				
		A.A.Belyaev et al., Yad.Fiz. 40, 133 (1984)						
Double polarisation H								
P.J.Bussey et al., Nucl.Phys. B169, 403 (1980)								
		A.A.Belyaev et al., Yad.Fiz. 43, 1469 (1986)	1217-2052	128				
		A.A.Belyaev et al., Yad.Fiz. 40, 133 (1984)						
Double-polarisation asymmetry F								
V.Kashevarev, PWAT Camogli								
		1300-1650	251					

Experiment

Outline

Database

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Reaction	Year	Source - Authors	Energy Range W [MeV]	Number of Data	Year	Source - Authors	Energy Range W [MeV]	Number of Data
Differential cross section σ_0								
	1978	P E Argan et al., Nucl. Phys. A 296, 373	1188-1312			F V Adamian et al., J. Phys. G 15, 1797	1575-2061	
	1974	M Beneventano et al., Nuovo Cim. A 19, 529	1336-1587			J Alspector et al., Phys. Rev. Lett. 28, 1403	1483-2154	
	1977	T Fujii et al., Nucl. Phys. B 120, 395				G Kries et al., Phys. Rev. D 10, 2778	1438-1539	
	1981	K Fujii et al., Nucl. Phys. B 187, 53 (1981)	1174-1761			K Kondo et al., Phys. Rev. D 9, 529	1203-1388	
	1974	G von Holtey et al., Nucl. Phys. B 70, 379	1187-1279	1494		G Mandaglio et al., Phys. Rev. C 82, 045209	1516-1894	304
	1960	G Neugebauer et al., Phys. Rev. 119, 1726	1350-1662			L O Abramian, Sov. J. Nucl. Phys. 32, 69	1604-1996	
	1974	P E Scheffler, P L Walden, Nucl. Phys. B 75, 125	1418-1798			V B Ganenko, Sov. J. Nucl. Phys. 23, 511	1203-1350	
	2012	W Chen et al., Phys. Rev. C 86, 015206	1690-2620			F F Liu et al., Phys. Rev. B 136, 1183	1226-1315	
Target asymmetry T								
	1989	V L Agranovich et al., VANT 8, 5	1187-1279			A M Sandorfi, Proc. Conf., 05/30/95.	1188-1220	
	1975	K H Althoff et al., Nucl. Phys. B 96, 497	1315-2154			Recoil asymmetry P		
	1976	K H Althoff et al., Nucl. Phys. B 116, 253	1315-2154	105		H Takeda et al., Nucl. Phys. B 168, 17	1494-1764	
	1977	T Fujii et al., Nucl. Phys. B 120, 395	1393-1604			J P Kenemuth, P C Stein, Phys. Rev. 129, 2259	1492	27
	1981	K Fujii et al., Nucl. Phys. B 187, 53	1393-1604			M Beneventano et al., Nuovo Cim. A 19, 529	1360-1492	

$\gamma n \rightarrow \pi^- p$

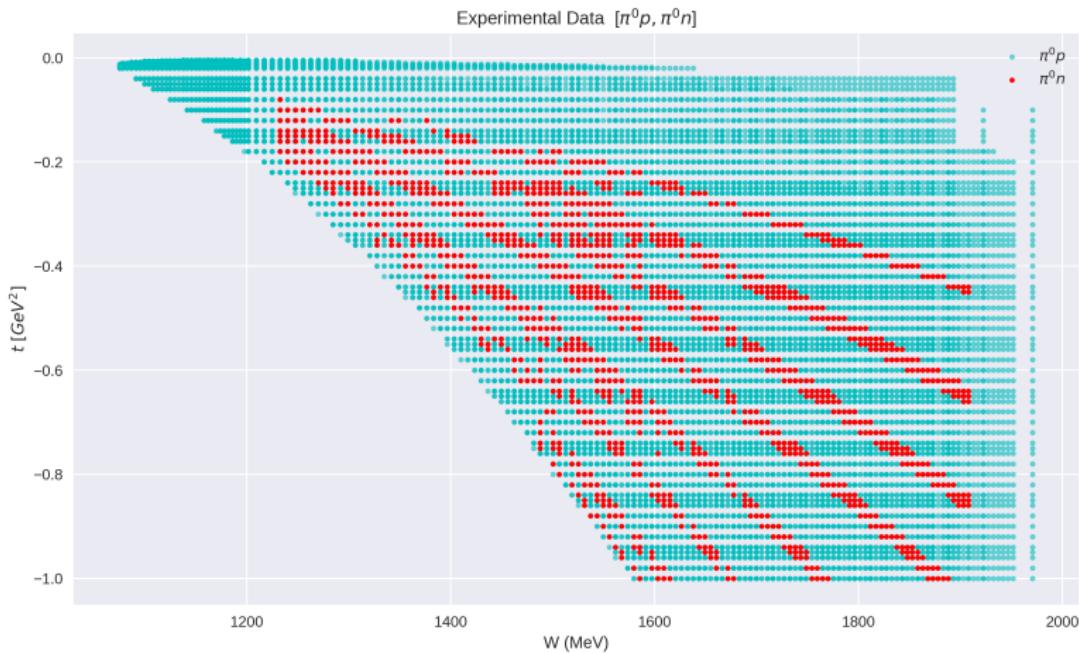
Experimental data

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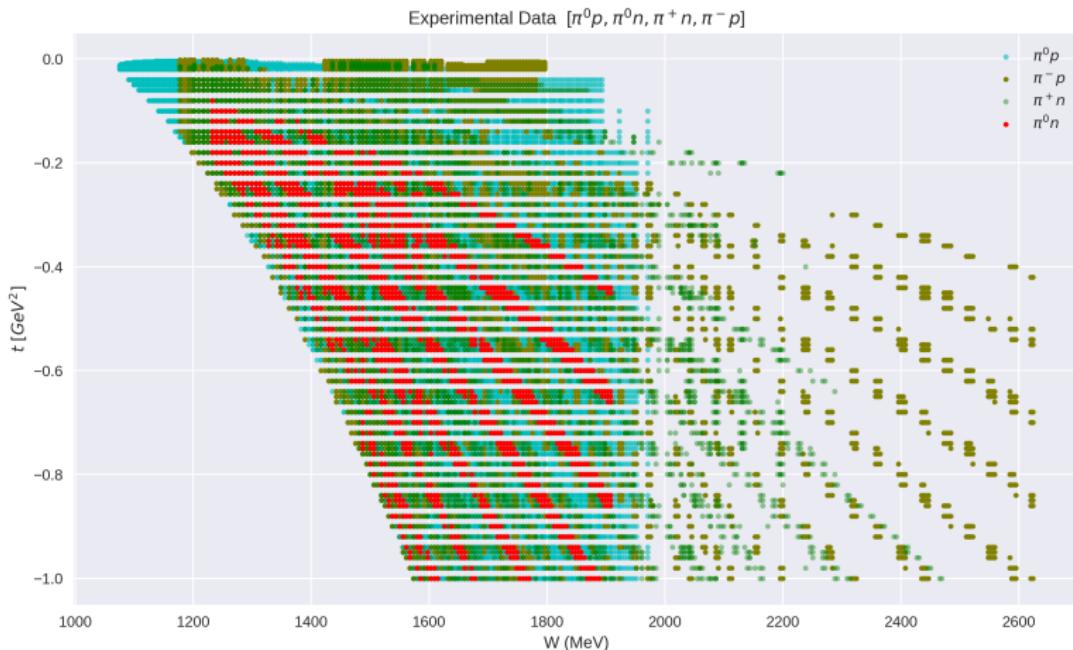
Experimental data

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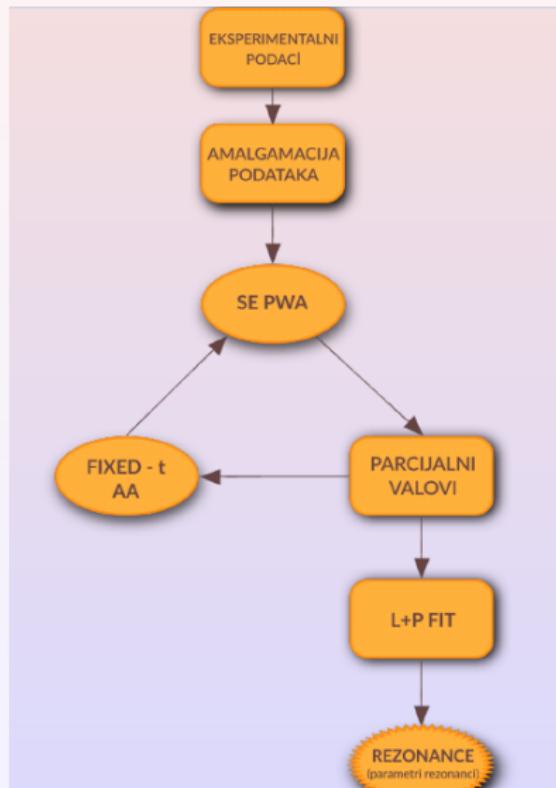
Experimental data

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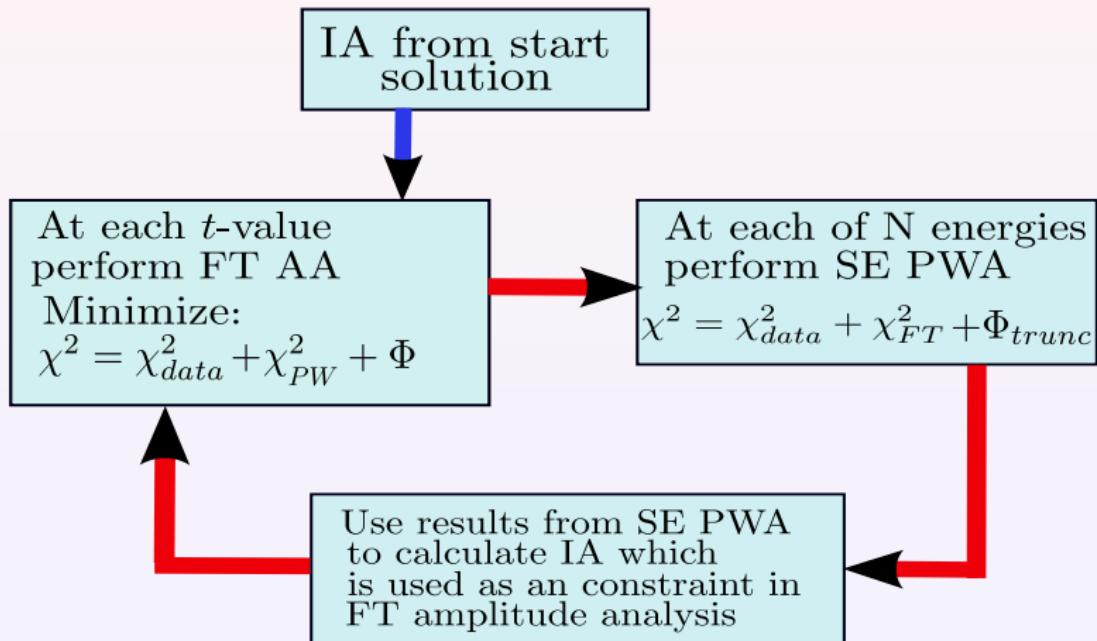
Experiment -> Poles

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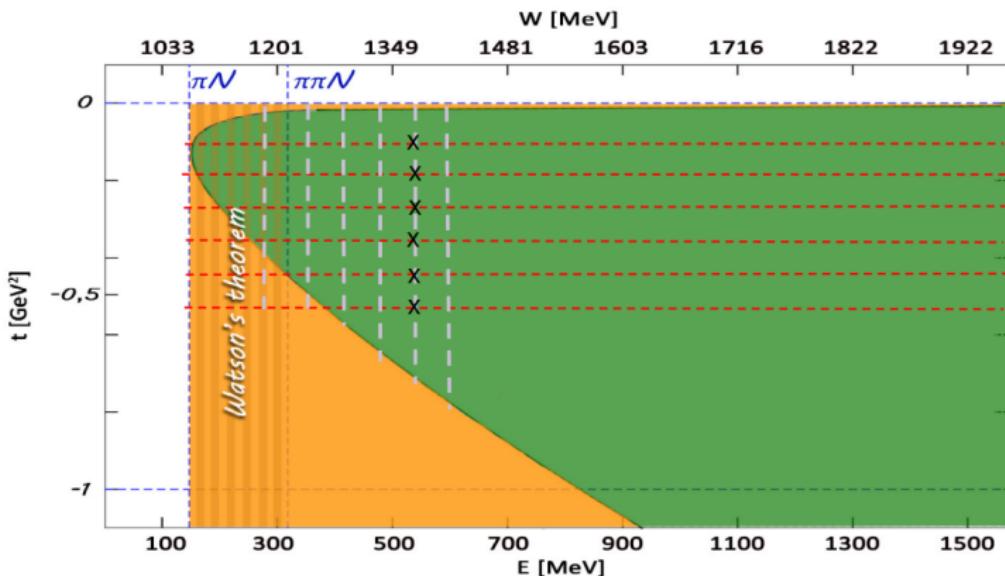
Fixed-t <> Single Energy

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Fixed-t <> Single Energy

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Single channel formalism

Laurent series - L

- Laurent expansion of a complex analytic function

$$f(z) = \sum_{n=0}^{\infty} a_n (z - z_0)^n + \sum_{n=1}^{\infty} a_{-n} (z - z_0)^{-n}$$

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Single channel formalism

Laurent series - L

- Laurent expansion of a complex analytic function

$$f(z) = \sum_{n=0}^{\infty} a_n (z - z_0)^n + \sum_{n=1}^{\infty} a_{-n} (z - z_0)^{-n}$$

- Applied to a single channel scattering matrix

$$T(W) = \frac{a_{-1}}{W_0 - W} + \sum_{n=0}^{\infty} a_n (W - W_0)^n$$

Single channel formalism

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- Applied to a single channel scattering matrix

$$T(W) = \frac{a_{-1}}{W_0 - W} + \sum_{n=0}^{\infty} a_n (W - W_0)^n$$

- Generalized Laurent expansion for the function with k poles

$$T(W) = \sum_{i=1}^k \frac{a_{-1}^{(i)}}{W_i - W} + B^L(W)$$

Single channel formalism

Laurent series - L

- Laurent expansion of a complex analytic function

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- Generalized Laurent expansion for the function with k poles

$$T(W) = \sum_{i=1}^k \frac{a_{-1}^{(i)}}{W_i - W} + B^L(W)$$

- k - number of poles, $a_{-1}^{(i)}$ and W_i are residues and pole positions for i -th pole, $B^L(W)$ regular function in all $W \neq W_i$

Single channel formalism

Laurent series - L

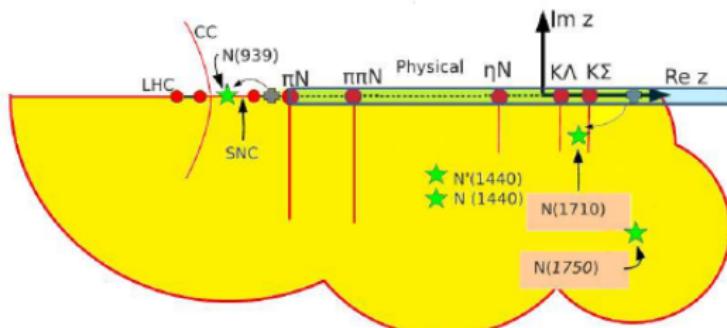
Outline

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Laurent expansion is valid only locally



Single channel formalism

Pietarinen series - P

- Using different approach than standard power series for the regular part of Laurent expansion

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Single channel formalism

Pietarinen series - P

- Using different approach than standard power series for the regular part of Laurent expansion
- S. Ciulli and J. Fischer in Nucl. Phys. 24, 465 (1961).
I. Ciulli, S. Ciulli, and J. Fisher, Nuovo Cimento 23, 1129
E. Pietarinen, Nuovo Cimento Soc. Ital. Fis. 12A, 522 (1972).

Single channel formalism

Pietarinen series - P

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- S. Ciulli and J. Fischer in Nucl. Phys. 24, 465 (1961).
I. Ciulli, S. Ciulli, and J. Fisher, Nuovo Cimento 23, 1129
E. Pietarinen, Nuovo Cimento Soc. Ital. Fis. 12A, 522 (1972).
- It has been used, with great success in the KH PWA

Single channel formalism

Pietarinen series - P

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E. Pietarinen, Nuovo Cimento Soc. Ital. Fis. 12A, 522 (1972).
- It has been used, with great success in the KH PWA
- To avoid discussing the arbitrariness of all possible choices for the background function $B^L(W)$ by replacing it with **rapidly converging** Pietarinen power series defined by a complete set of functions with well known analytic properties.

Single channel formalism

Pietarinen series - P

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- S. Ciulli and J. Fischer in Nucl. Phys. 24, 465 (1961).
I. Ciulli, S. Ciulli, and J. Fisher, Nuovo Cimento 23, 1129
E. Pietarinen, Nuovo Cimento Soc. Ital. Fis. 12A, 522 (1972).
- It has been used, with great success in the KH PWA
- To avoid discussing the arbitrariness of all possible choices for the background function $B^L(W)$ by replacing it with **rapidly converging** Pietarinen power series defined by a complete set of functions with well known analytic properties.
- If $F(W)$ is analytic function having a cut starting at $W = x_P$ then

$$F(W) = \sum_{n=0}^N c_n Z^n(W) \quad \text{where} \quad Z(W) = \frac{\alpha - \sqrt{x_p - W}}{\alpha + \sqrt{x_p - W}}$$

Single channel formalism

Pietarinen series - P

- LHC, RHC, Poles

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Single channel formalism

Pietarinen series - P

- LHC, RHC, Poles
- One Pietarinen series to represent each cut

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Single channel formalism

Pietarinen series - P

- LHC, RHC, Poles
- One Pietarinen series to represent each cut
- As we have too many cuts in PW we will group them into two categories

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Single channel formalism

Pietarinen series - P

- LHC, RHC, Poles
- One Pietarinen series to represent each cut
- As we have too many cuts in PW we will group them into two categories
 - all negative energy cuts are approximated with only one, effective negative energy cut represented with one Pietarinen series

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Single channel formalism

Pietarinen series - P

- LHC, RHC, Poles
- One Pietarinen series to represent each cut
- As we have too many cuts in PW we will group them into two categories
 - all negative energy cuts are approximated with only one, effective negative energy cut represented with one Pietarinen series
 - each physical cut is represented with its own Pietarinen series with branch points determined by the physics of the process.

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Single channel formalism

Pietarinen series - P

- LHC, RHC, Poles
- One Pietarinen series to represent each cut
- As we have too many cuts in PW we will group them into two categories
 - all negative energy cuts are approximated with only one, effective negative energy cut represented with one Pietarinen series
 - each physical cut is represented with its own Pietarinen series with branch points determined by the physics of the process.
- Equations which define Laurent expansion + Pietarinen series method (L+P method):

$$T(W) = \sum_{i=1}^k \frac{x_i + \imath y_i}{W_i - W} + \sum_{k=1}^K c_k X(W)^k + \sum_{l=1}^L d_l Y(W)^l + \sum_{m=1}^M e_m Z(W)^m$$

$$X(W) = \frac{\alpha - \sqrt{x_P - W}}{\alpha + \sqrt{x_P - W}}; \quad Y(W) = \frac{\beta - \sqrt{x_Q - W}}{\beta + \sqrt{x_Q - W}}; \quad Z(W) = \frac{\gamma - \sqrt{x_R - W}}{\gamma + \sqrt{x_R - W}}$$

$$D_{dp} = \frac{1}{2N_E} \sum_{i=1}^{N_E} \left[\left(\frac{\Re T_i^{fit} - \Re T_i}{\text{Err}_i^{\Re}} \right)^2 + \left(\frac{\Im T_i^{fit} - \Im T_i}{\text{Err}_i^{\Im}} \right)^2 \right]$$

Multi/Coupled - channel/multipole... formalism

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- Correlated multipoles in π and η photoproduction, and partial wave amplitudes in coupled-channel models can only be treated in a sequence of independent single-channel procedures, missing the constraint that poles in all such situations must be the same.
- Also, in some cases, all existing poles may not be recognized in each individual process, and that in particular happens if a resonance coupling to a particular channel is weak.

Multi/Coupled - channel/multipole... formalism

The generalization of L+P method to MC L+P is performed in the following way:

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$$T^{(a)}(W) = \sum_{i=1}^k \frac{x_i^{(a)} + \imath y_i^{(a)}}{W_i - W} + \sum_{k=1}^K c_k^{(a)} X^{(a)}(W)^k + \sum_{l=1}^L d_l^{(a)} Y^{(a)}(W)^l + \sum_{m=1}^M e_m^{(a)} Z^{(a)}(W)^m$$

$$X^{(a)}(W) = \frac{\alpha^{(a)} - \sqrt{x_P^{(a)} - W}}{\alpha^{(a)} + \sqrt{x_P^{(a)} - W}}; \quad Y^{(a)}(W) = \frac{\beta^{(a)} - \sqrt{x_Q^{(a)} - W}}{\beta^{(a)} + \sqrt{x_Q^{(a)} - W}}; \quad Z^{(a)}(W) = \frac{\gamma^{(a)} - \sqrt{x_R^{(a)} - W}}{\gamma^{(a)} + \sqrt{x_R^{(a)} - W}}$$

$$D_{dp} = \sum_{(a)} D_{dp}^{(a)} = \frac{1}{2N_{data}} \sum_{i=1}^{N_{data}} \left[\left(\frac{\Re T_{(a)}^{fit}(W_i) - \Re T_{(a)}(W_i)}{\text{Err}_{i,(a)}^{\Re}} \right)^2 + \left(\frac{\Im T_{(a)}^{fit}(W_i) - \Im T_{(a)}(W_i)}{\text{Err}_{i,(a)}^{\Im}} \right)^2 \right]$$

Multi/Coupled - channel/multipole... formalism

The generalization of L+P method to MC L+P is performed in the following way:

- Separate Laurent expansions and Pietarinen series for each channel/multipole;

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$$T^{\mathbf{a}}(W) = \sum_{i=1}^k \frac{x_i^{(\mathbf{a})} + \imath y_i^{(\mathbf{a})}}{W_i - W} + \sum_{k=1}^K c_k^{(\mathbf{a})} X^{(\mathbf{a})}(W)^k + \sum_{l=1}^L d_l^{(\mathbf{a})} Y^{(\mathbf{a})}(W)^l + \sum_{m=1}^M e_m^{(\mathbf{a})} Z^{(\mathbf{a})}(W)^m$$

$$X^{(\mathbf{a})}(W) = \frac{\alpha^{(\mathbf{a})} - \sqrt{x_P^{(\mathbf{a})} - W}}{\alpha^{(\mathbf{a})} + \sqrt{x_P^{(\mathbf{a})} - W}}; \quad Y^{(\mathbf{a})}(W) = \frac{\beta^{(\mathbf{a})} - \sqrt{x_Q^{(\mathbf{a})} - W}}{\beta^{(\mathbf{a})} + \sqrt{x_Q^{(\mathbf{a})} - W}}; \quad Z^{(\mathbf{a})}(W) = \frac{\gamma^{(\mathbf{a})} - \sqrt{x_R^{(\mathbf{a})} - W}}{\gamma^{(\mathbf{a})} + \sqrt{x_R^{(\mathbf{a})} - W}}$$

$$D_{dp} = \sum_{(\mathbf{a})} D_{dp}^{(\mathbf{a})} = \frac{1}{2N_{data}} \sum_{i=1}^{N_{data}} \left[\left(\frac{\Re T_{(\mathbf{a})}^{fit}(W_i) - \Re T_{(\mathbf{a})}(W_i)}{\text{Err}_{i,(\mathbf{a})}^{\Re}} \right)^2 + \left(\frac{\Im T_{(\mathbf{a})}^{fit}(W_i) - \Im T_{(\mathbf{a})}(W_i)}{\text{Err}_{i,(\mathbf{a})}^{\Im}} \right)^2 \right]$$

Multi/Coupled - channel/multipole... formalism

The generalization of L+P method to MC L+P is performed in the following way:

- Separate Laurent expansions and Pietarinen series for each channel/multipole;
- Pole positions are the same for all channels/multipoles,

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$$T^{(\alpha)}(W) = \sum_{i=1}^k \frac{x_i^{(\alpha)} + \gamma y_i^{(\alpha)}}{W_i - W} + \sum_{k=1}^K c_k^{(\alpha)} X^{(\alpha)}(W)^k + \sum_{l=1}^L d_l^{(\alpha)} Y^{(\alpha)}(W)^l + \sum_{m=1}^M e_m^{(\alpha)} Z^{(\alpha)}(W)^m$$

$$X^{(\alpha)}(W) = \frac{\alpha^{(\alpha)} - \sqrt{x_P^{(\alpha)} - W}}{\alpha^{(\alpha)} + \sqrt{x_P^{(\alpha)} - W}}; \quad Y^{(\alpha)}(W) = \frac{\beta^{(\alpha)} - \sqrt{x_Q^{(\alpha)} - W}}{\beta^{(\alpha)} + \sqrt{x_Q^{(\alpha)} - W}}; \quad Z^{(\alpha)}(W) = \frac{\gamma^{(\alpha)} - \sqrt{x_R^{(\alpha)} - W}}{\gamma^{(\alpha)} + \sqrt{x_R^{(\alpha)} - W}}$$

$$D_{dp} = \sum_{(\alpha)} D_{dp}^{(\alpha)} = \frac{1}{2N_{data}} \sum_{i=1}^{N_{data}} \left[\left(\frac{\Re T_{(\alpha)}^{fit}(W_i) - \Re T_{(\alpha)}(W_i)}{\text{Err}_{i,(\alpha)}^{\Re}} \right)^2 + \left(\frac{\Im T_{(\alpha)}^{fit}(W_i) - \Im T_{(\alpha)}(W_i)}{\text{Err}_{i,(\alpha)}^{\Im}} \right)^2 \right]$$

Multi/Coupled - channel/multipole... formalism

The generalization of L+P method to MC L+P is performed in the following way:

- Separate Laurent expansions and Pietarinen series for each channel/multipole;
- Pole positions are the same for all channels/multipoles,
- Residua and all Pietarinen coefficients free;

$$T^{(a)}(W) = \sum_{i=1}^k \frac{x_i^{(a)} + \imath y_i^{(a)}}{w_i - W} + \sum_{k=1}^K c_k^{(a)} X^{(a)}(W)^k + \sum_{l=1}^L d_l^{(a)} Y^{(a)}(W)^l + \sum_{m=1}^M e_m^{(a)} Z^{(a)}(W)^m$$

$$X^{(a)}(W) = \frac{\alpha^{(a)} - \sqrt{x_P^{(a)} - W}}{\alpha^{(a)} + \sqrt{x_P^{(a)} - W}}; \quad Y^{(a)}(W) = \frac{\beta^{(a)} - \sqrt{x_Q^{(a)} - W}}{\beta^{(a)} + \sqrt{x_Q^{(a)} - W}}; \quad Z^{(a)}(W) = \frac{\gamma^{(a)} - \sqrt{x_R^{(a)} - W}}{\gamma^{(a)} + \sqrt{x_R^{(a)} - W}}$$

$$D_{dp} = \sum_{(a)} D_{dp}^{(a)} = \frac{1}{2N_{data}} \sum_{i=1}^{N_{data}} \left[\left(\frac{\Re T_{(a)}^{fit}(W_i) - \Re T_{(a)}(W_i)}{\text{Err}_{i,(a)}^{\Re}} \right)^2 + \left(\frac{\Im T_{(a)}^{fit}(W_i) - \Im T_{(a)}(W_i)}{\text{Err}_{i,(a)}^{\Im}} \right)^2 \right]$$

Multi/Coupled - channel/multipole... formalism

The generalization of L+P method to MC L+P is performed in the following way:

- Separate Laurent expansions and Pietarinen series for each channel/multipole;
- Pole positions are the same for all channels/multipoles,
- Residua and all Pietarinen coefficients free;
- Branch-points exactly as for the single-channel model;

$$T^{(a)}(W) = \sum_{i=1}^k \frac{x_i^{(a)} + \gamma_i^{(a)}}{W_i - W} + \sum_{k=1}^K c_k^{(a)} X^{(a)}(W)^k + \sum_{l=1}^L d_l^{(a)} Y^{(a)}(W)^l + \sum_{m=1}^M e_m^{(a)} Z^{(a)}(W)^m$$

$$X^{(a)}(W) = \frac{\alpha^{(a)} - \sqrt{x_P^{(a)} - W}}{\alpha^{(a)} + \sqrt{x_P^{(a)} - W}}; \quad Y^{(a)}(W) = \frac{\beta^{(a)} - \sqrt{x_Q^{(a)} - W}}{\beta^{(a)} + \sqrt{x_Q^{(a)} - W}}; \quad Z^{(a)}(W) = \frac{\gamma^{(a)} - \sqrt{x_R^{(a)} - W}}{\gamma^{(a)} + \sqrt{x_R^{(a)} - W}}$$

$$D_{dp} = \sum_{(a)} D_{dp}^{(a)} = \frac{1}{2N_{data}} \sum_{i=1}^{N_{data}} \left[\left(\frac{\Re T_{(a)}^{fit}(W_i) - \Re T_{(a)}(W_i)}{\text{Err}_{i,(a)}^{\Re}} \right)^2 + \left(\frac{\Im T_{(a)}^{fit}(W_i) - \Im T_{(a)}(W_i)}{\text{Err}_{i,(a)}^{\Im}} \right)^2 \right]$$

Multi/Coupled - channel/multipole... formalism

The generalization of L+P method to MC L+P is performed in the following way:

- Separate Laurent expansions and Pietarinen series for each channel/multipole;
- Pole positions are the same for all channels/multipoles,
- Residua and all Pietarinen coefficients free;
- Branch-points exactly as for the single-channel model;
- Generalize the single-channel discrepancy function $D_{dp}^{(a)}$

$$T^{(a)}(W) = \sum_{i=1}^k \frac{x_i^{(a)} + \imath y_i^{(a)}}{W_i - W} + \sum_{k=1}^K c_k^{(a)} X^{(a)}(W)^k + \sum_{l=1}^L d_l^{(a)} Y^{(a)}(W)^l + \sum_{m=1}^M e_m^{(a)} Z^{(a)}(W)^m$$

$$X^{(a)}(W) = \frac{\alpha^{(a)} - \sqrt{x_P^{(a)} - W}}{\alpha^{(a)} + \sqrt{x_P^{(a)} - W}}; \quad Y^{(a)}(W) = \frac{\beta^{(a)} - \sqrt{x_Q^{(a)} - W}}{\beta^{(a)} + \sqrt{x_Q^{(a)} - W}}; \quad Z^{(a)}(W) = \frac{\gamma^{(a)} - \sqrt{x_R^{(a)} - W}}{\gamma^{(a)} + \sqrt{x_R^{(a)} - W}}$$

$$D_{dp}^{(a)} = \sum_{(a)} D_{dp}^{(a)} = \frac{1}{2N_{data}} \sum_{i=1}^{N_{data}} \left[\left(\frac{\Re T_{(a)}^{fit}(W_i) - \Re T_{(a)}(W_i)}{\text{Err}_{i,(a)}^{\Re}} \right)^2 + \left(\frac{\Im T_{(a)}^{fit}(W_i) - \Im T_{(a)}(W_i)}{\text{Err}_{i,(a)}^{\Im}} \right)^2 \right]$$

L+P fits

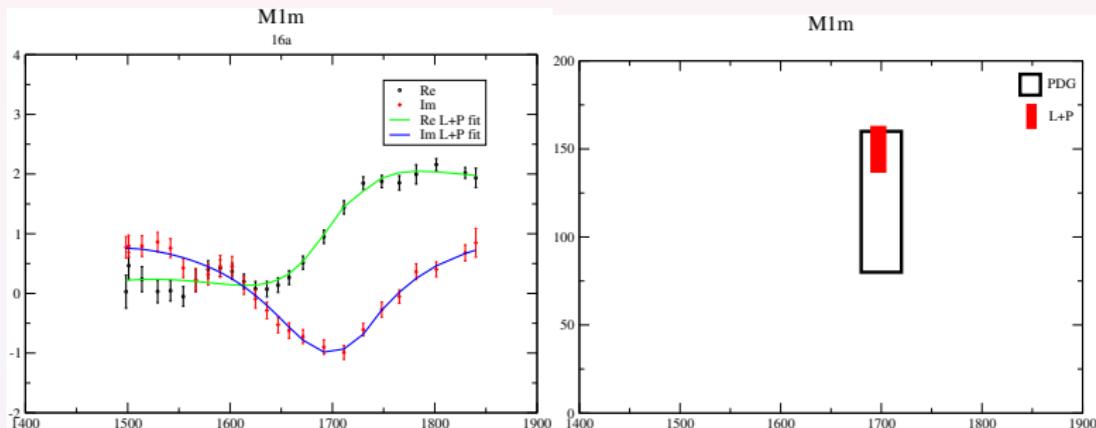
Solution 16a

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Resonance	$\text{Re } W_p$	$-2\text{Im } W_p$	$ \text{residue} $	θ
$N(1710) \frac{1}{2}^+$	1698^{+5}_{-7}	149^{+13}_{-12}	143	-178

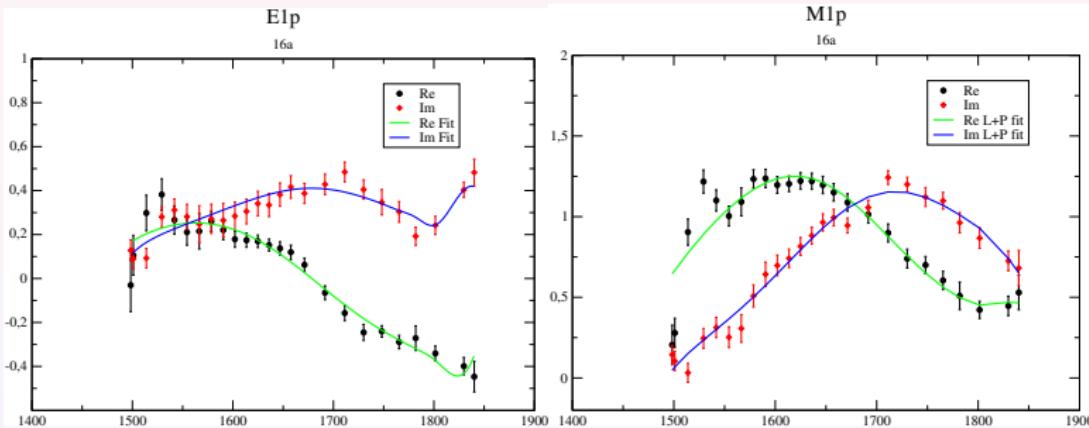
L+P fits
Solution 16a

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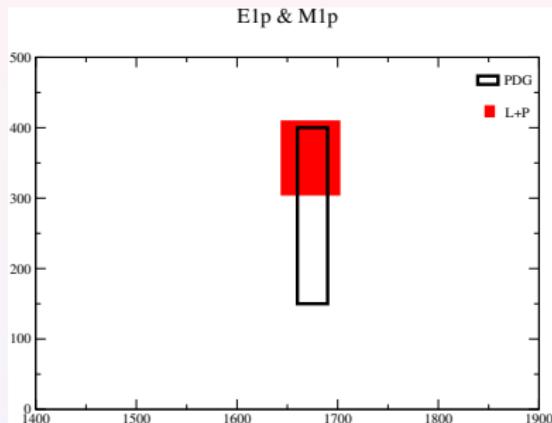
Laurent +
Pietarinen
(L+P)



Resonance	$\text{Re } W_p$	$-2\text{Im } W_p$	$ \text{residue} $	θ
$N(1720) \, 3/2^+$	1671^{+30}_{-26}	356^{+52}_{-50}	112 291	-11 -40

L+P fits

Solution 16a



Resonance	$\text{Re } W_p$	$-2\text{Im } W_p$	$ \text{residue} $	θ
$N(1720) \frac{3}{2}^+$	1671^{+30}_{-26}	356^{+52}_{-50}	$\begin{array}{r} 112 \\ 291 \end{array}$	$\begin{array}{r} -11 \\ -40 \end{array}$

Conclusion

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