On the phase diagram of the Nambu Jona-Lasinio Lagrangian

in collaboration with
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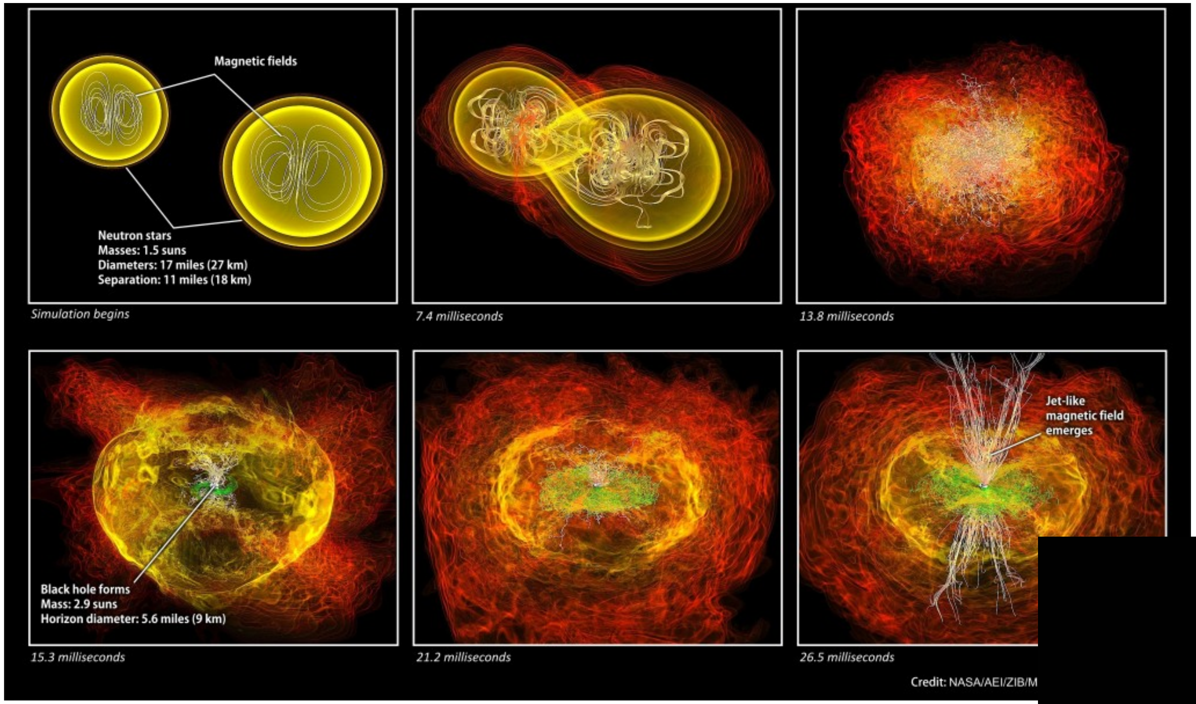
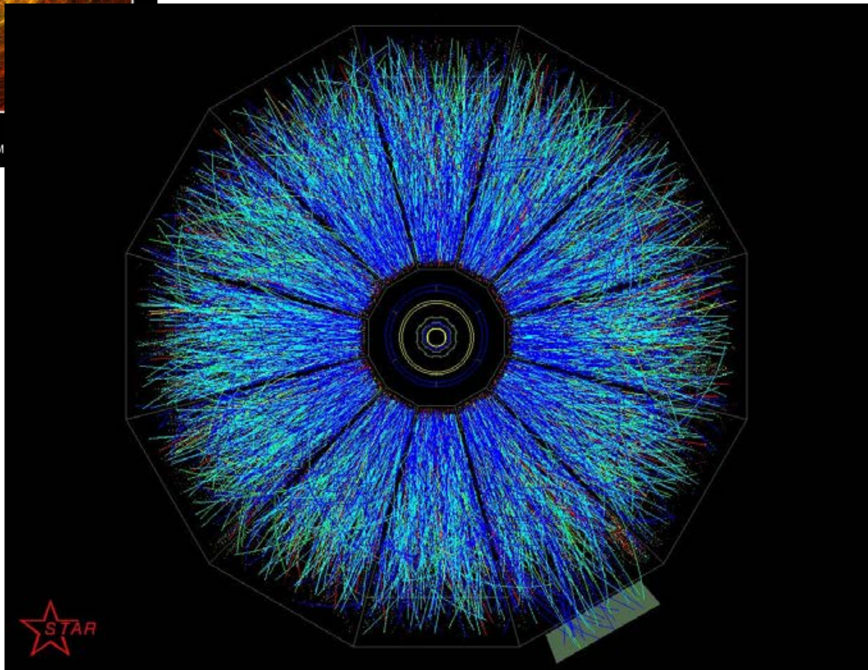
Simulations of Neutron Stars, Neutron Star Collisions and Heavy Ion Collisions

need the same input

PHASE DIAGRAMM OF STRONGLY INTERACING MATTER $s(T,\mu)$, $\varepsilon(T,\mu)$

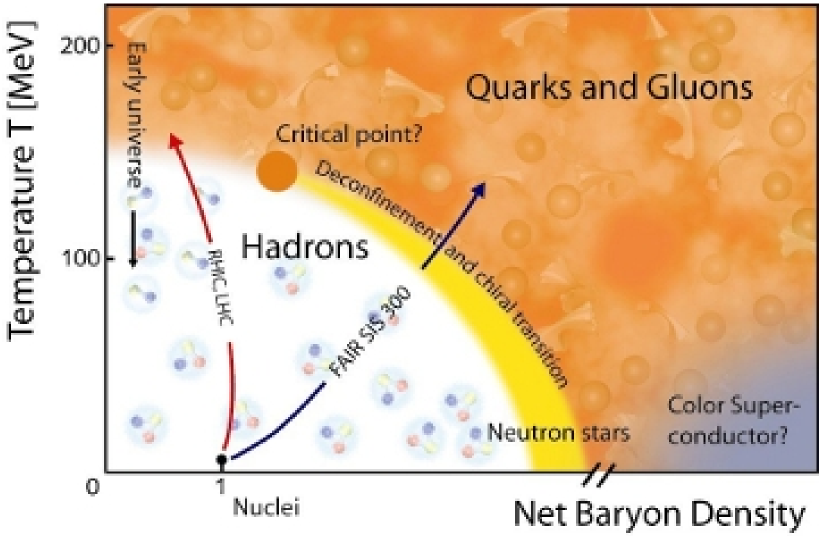


Heavy ion collision:
symmetric nuclear matter
$d = u$
$0 < \rho < 4\rho_0$



Neutron Star collisions
asymmetric matter
$d > u$
$0 < \rho < 8\rho_0$

What are the problems?



Why not calculate simply?
Quantumchromodynamics (QCD) can be calculated on a lattice
but only for μ=0 (same number of quarks and antiquarks)

Taylor expansion allows for calculations for $\mu/T \ll 1$

Neutron Stars as well as Heavy Ion collisions need calculations at
finite chemical potential

- ❑ either assumptions about continuation to finite μ
- ❑ or effective theories which allow for such an extension intrinsically

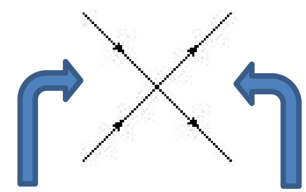
Effective Lagrangian to study phase phase diagram and phase transitions at finite chemical potential (NICA,FAIR, neutron stars)

The Nambu Jona Lasinio Lagrangian is such an effective field theory

Nambu

- allows for predictions for finite T and μ
- needs as input only vacuum values + YM Polyakov loop
- shares the symmetries with the QCD Lagrangian
- can be « derived » from QCD Lagrangian

Jona-Lasinio

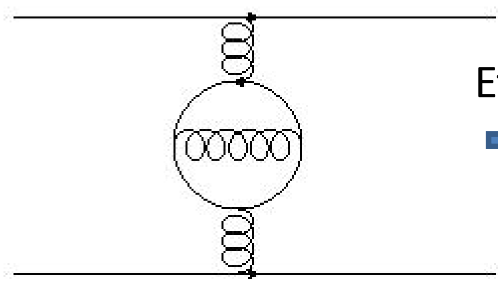


$$\mathscr{L}_{NJL} = \bar{\Psi}_i(i\gamma_\mu\partial^\mu - \hat{M}_0)\Psi_i - G_c^2\ [\bar{\Psi}_i\gamma^\mu T^a\delta_{ij}\Psi_j]\ [\bar{\Psi}_k\gamma_\mu T^a\delta_{kl}\Psi_l]$$

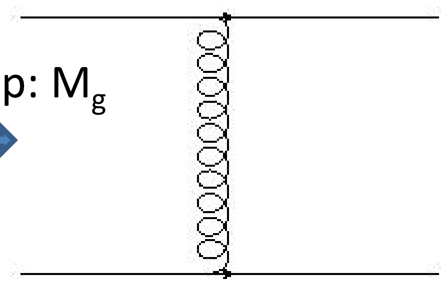
$$+\ H\det_{ij}\ [\bar{\Psi}_i(1-\gamma_5)\Psi_j] - H\det_{ij}\ [\bar{\psi}_i(1+\gamma_5)\psi_j] + \sum_{ij}\ \bar{\psi}_i\mu_{ij}\gamma_0\psi_j$$

NJL Lagrangian

$\Rightarrow$ An *effective Lagrangian* with the same symmetries for the quark degrees of freedom as QCD can be obtained by discarding the gluon dynamics completely.

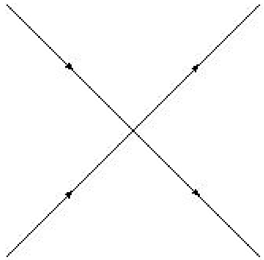


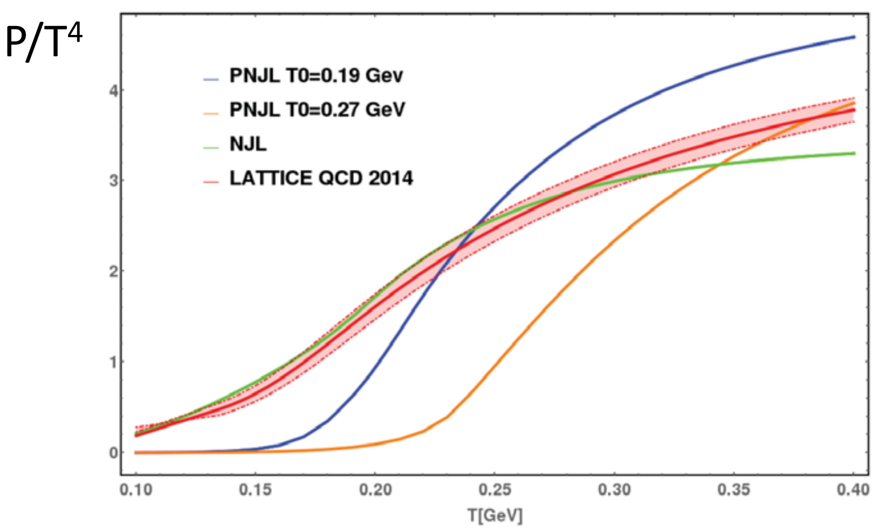
Eff prop: M_g $\Rightarrow$



$$\frac{1}{q^2 - M^2}$$

M>>q $\Rightarrow$





Renewed interest because

Going beyond leading order in N_c + including a gluon mean field potential brings PNJL energy density and entropy density closer to lattice results

Phys.Rev. C96, 045205

Polyakov NJL: gluons on a static level

Eur.Phys.J. C49 (2007) 213-217

It is not possible to introduce gluons as dynamical degrees of freedom without spoiling the simplicity of the NJL Lagrangian which allows for real calculations

but

one can introduce gluons through an effective potential for the Polyakov loop

$$\frac{\mathbf{U}(\mathbf{T},\Phi,\bar{\Phi})}{\mathbf{T}^4} = -\frac{\mathbf{b}_2(\mathbf{T})}{2}\bar{\Phi}\Phi - \frac{\mathbf{b}_3}{6}\left(\Phi^3 + \bar{\Phi}^3\right) + \frac{\mathbf{b}_4}{4}(\bar{\Phi}\Phi)^3$$

$$b_2(T) = a_0 + a_1\frac{T_0}{T} + a_2\left(\frac{T_0}{T}\right)^2 + a_3\left(\frac{T_0}{T}\right)^3$$

a_0	a_1	a_2	a_3	b_3	b_4	T_0	
6.75	-1.95	2.625	-7.44	0.75	7.5	270 MeV ($N_f = 0$)	PRD73,014019
						190 MeV ($N_f = 2+1$)	PRD76,074023

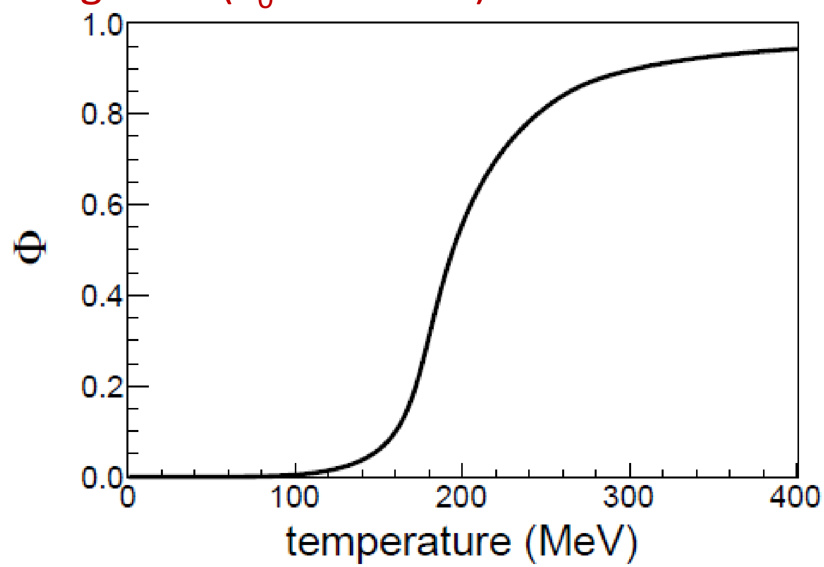
First: Parameter from Yang Mills (T_0=270 MeV)

Now: including (approx.) interaction between quarks and gluons (T_0=190 MeV)

Parameters-> right pressure in the SB limit

Φ is the order parameter of the deconfinement transition

$$\Phi = \frac{1}{N_c}\,\mathrm{Tr}_c\,\langle \mathbf{P}\exp\left(-\int_0^\beta \mathbf{d}\tau\mathbf{A}_0(\mathbf{x},\tau)\right)\rangle$$

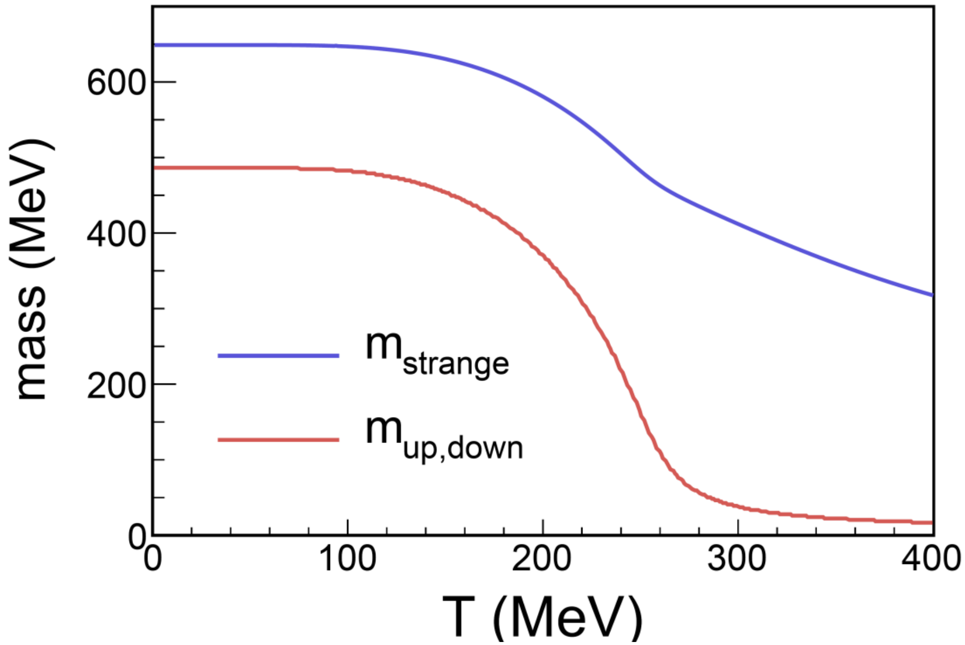


Quark Masses in NJL and PNJL

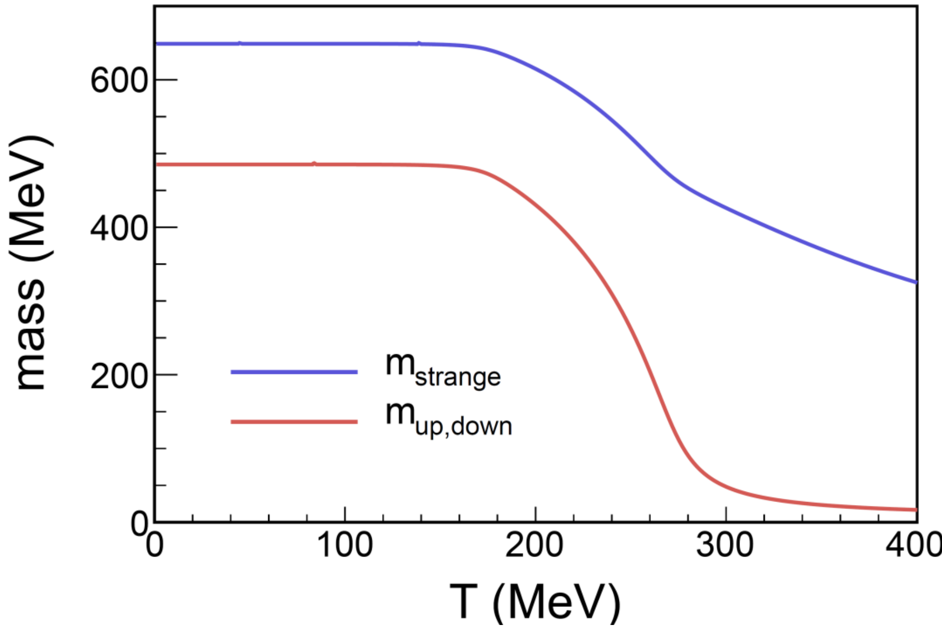
Quark masses are obtained by minimizing the grand canonical potential

$$\mathbf{M} = \hat{\mathbf{M}}_0 - 4\mathbf{G} < \bar{\psi}\psi > +2\mathbf{H} < \bar{\psi}'\psi' >< \bar{\psi}''\psi'' >$$

NJL



PNJL



In PNJL the transition is steeper than in NJL

How can we get mesons?

Quarks are the degrees of freedom of the Lagrangian

To study the phase transition we need mesons

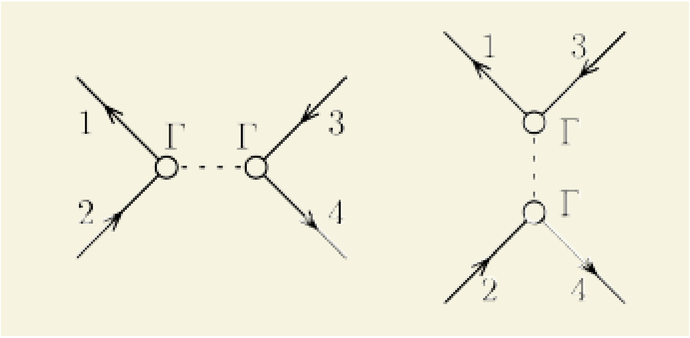
Use a Trick : **Fierz transformation** of the original Lagrangian

Fierz Transformation allows for a reordering of the field operators in 4 point contact interactions. It is simultaneously applied in Dirac, color and flavor space

Example in Dirac space:

$$(\bar{\chi}\gamma^{\mu}\psi)(\bar{\psi}\gamma_{\mu}\chi) = (\bar{\chi}\chi)(\bar{\psi}\psi) - \frac{1}{2}(\bar{\chi}\gamma^{\mu}\chi)(\bar{\psi}\gamma_{\mu}\psi) - \frac{1}{2}(\bar{\chi}\gamma^{\mu}\gamma_5\chi)(\bar{\psi}\gamma_{\mu}\gamma_5\psi) - (\bar{\chi}\gamma_5\chi)(\bar{\psi}\gamma_5\psi)$$

Scalar vector peudovector pseudoscalar



How can we get mesons? II

$$\mathscr{L}_{int} = -\mathbf{G}_c^2 \;\; [\bar{\Psi}_i \gamma^\mu \mathbf{T}^a \delta_{ij} \Psi_j] \;\; [\bar{\Psi}_k \gamma_\mu \mathbf{T}^a \delta_{kl} \Psi_l]$$

Fierz transformation transforms original Lagrangian to one for mesons

$$\mathcal{L}_{\mathrm{Pseudo\ scalar}} = \mathbf{G}\; (\Psi_i \; \tau^a_{il} \; \mathbb{1}_c i\gamma_5 \; \Psi_l)\; (\Psi_k \; \tau^a_{kj} \; \mathbb{1}_c i\gamma_5 \; \Psi_j)\; ; \qquad \mathbf{G} = \frac{\mathbf{N}_c^2 - 1}{\mathbf{N}_c^2} \mathbf{G}_c$$

$\mathscr{K}$

Singulet in color mixing of flavour

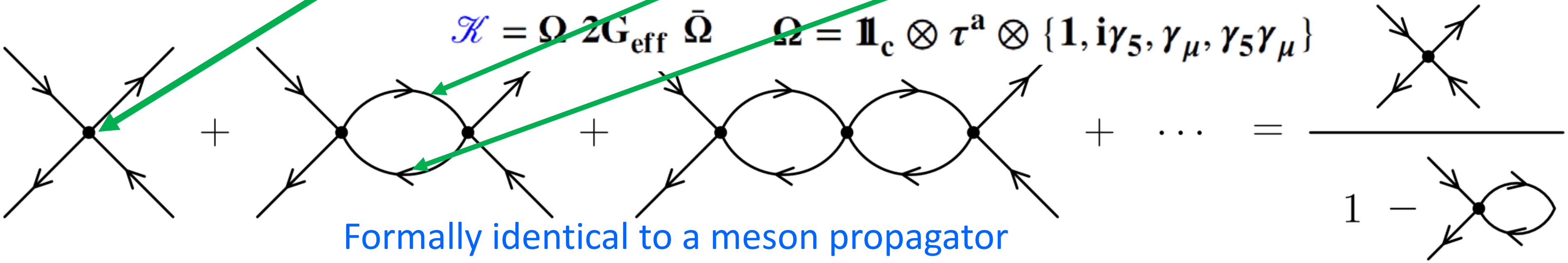
Similar terms can be obtained for
Vector mesons γ_μ
Scalar Mesons 1
Pseudovector mesons $\gamma_\mu \gamma_5$

How can we get mesons? III

We use $\mathscr{K}$ as a kernel for a Bethe-Salpeter equation (relativistic Lippmann-Schwinger eq.)

$$\mathbf{T(p)} = \mathscr{K} + \mathrm{i}\int \frac{\mathrm{d}^4\mathrm{k}}{(2\pi)^4}\,\mathscr{K}\,\mathrm{S}\left(\mathrm{k}+\frac{\mathrm{p}}{2}\right)\mathrm{S}\left(\mathrm{k}-\frac{\mathrm{p}}{2}\right)\mathbf{T(p)}$$

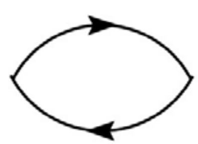
$$\mathscr{K} = \Omega\, 2\mathrm{G}_{\mathrm{eff}}\, \bar{\Omega} \qquad \Omega = \mathbb{1}_c \otimes \tau^a \otimes \{1, i\gamma_5, \gamma_\mu, \gamma_5\gamma_\mu\}$$



Formally identical to a meson propagator

In (P)NJL one can sum up this series analytically:

$$\mathbf{T(p)} = \frac{2\mathrm{G}_{\mathrm{eff}}}{1 - 2\mathrm{G}_{\mathrm{eff}}\,\Pi(\mathrm{p})}\,, \qquad \Pi(\mathrm{p}_0, \mathrm{p}) = -\frac{1}{\beta}\sum_n \int \frac{\mathrm{d}^3\mathrm{k}}{(2\pi)^3}\,\Omega\,\mathrm{S}\left(\mathrm{k}+\frac{\mathrm{p}}{2}\right)\Omega\,\mathrm{S}\left(\mathrm{k}-\frac{\mathrm{p}}{2}\right)$$

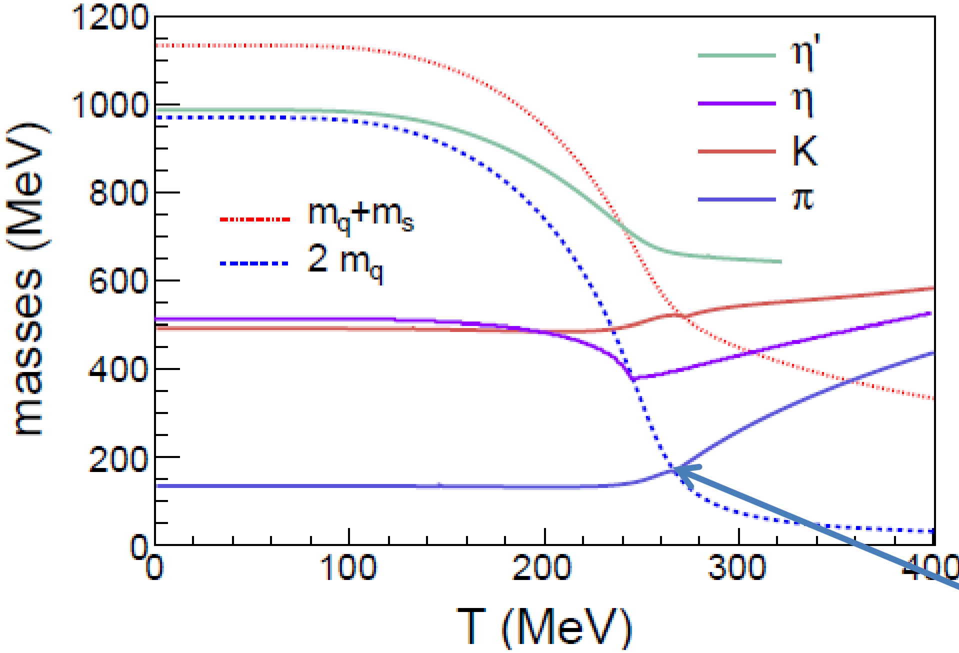


$\equiv \Pi$

How to get mesons? IV

The meson pole mass and the width one obtains by solving:

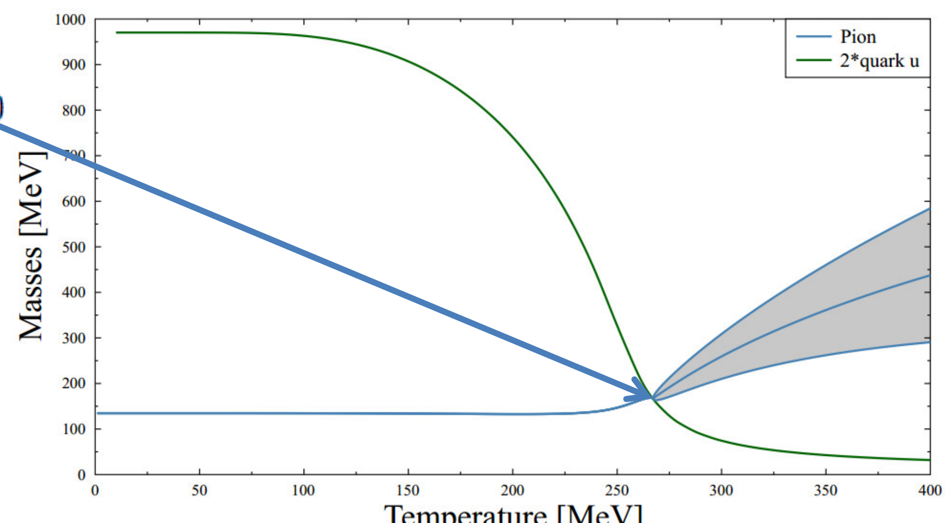
$$1 - 2G_{eff}\ \Pi(p_0 = M_{meson} - i\Gamma_{meson}/2, p = 0) = 0$$



masses of pseudoscalar mesons and of quarks at μ = 0

At T=0 physical and calculated mass agree quite well

When mesons become unstable they develop a width



Looking back

The (P)NJL model describes quite well meson and baryon properties as well as the chiral phase transition with only 5 parameters

Λ = upper cut off of the internal momentum loops
G_c = coupling constant
M_0 = bare mass of u,d and s quarks
H= coupling constant 't Hooft term

These parameters have been adjusted to reproduce in the vacuum m_π, m_K,
the η-η' mass splitting the π decay constant and the chiral condensate ($-241\ \mathrm{MeV})^3$

All masses, cross sections etc. at finite μ and T follow without any new parameters from vacuum observables.

BUT:

It does not reproduce the lattice results of
the thermal properties at μ=0.

Can one improve ?

- Modify the (little known) change of U due to the presence of quarks
- Add higher order terms in N_C in the partition function

$$\frac{U(T,\Phi,\bar{\Phi})}{T^4} = -\frac{b_2(T)}{2}\bar{\Phi}\Phi - \frac{b_3}{6}\left(\Phi^3+\bar{\Phi}^3\right) + \frac{b_4}{4}(\bar{\Phi}\Phi)^3$$

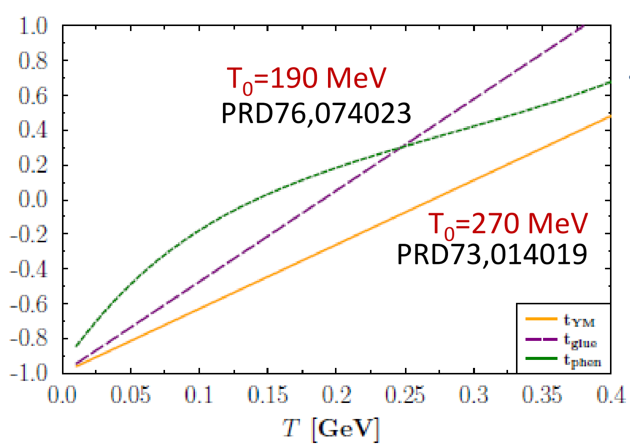
$$b_2(T) = a_0 + \left(\frac{a_1}{1+\tau}\right) + \frac{a_2}{(1+\tau)^2} + \frac{a_3}{1+\tau)^3} \qquad \tau = f\frac{T-T_{glue}}{T_{glue}} \qquad T_{glue} = a + bT + cT^2 + dT^3 + e\frac{1}{T}$$

a_0	a_1	a_2	a_3	b_3	b_4	a	b	c	d	e	f
6.75	-1.95	2.625	-7.44	0.75	7.5	0.086	0.36	0.57	-1.15	-0.0005	0.57

$$t_{YM} = \frac{T-T_{YM}^{cr}}{T_{YM}^{cr}} = 0.57\,\frac{T-T_{glue}^{cr}}{T_{glue}^{cr}} = 0.57 t_{glue}\,.$$

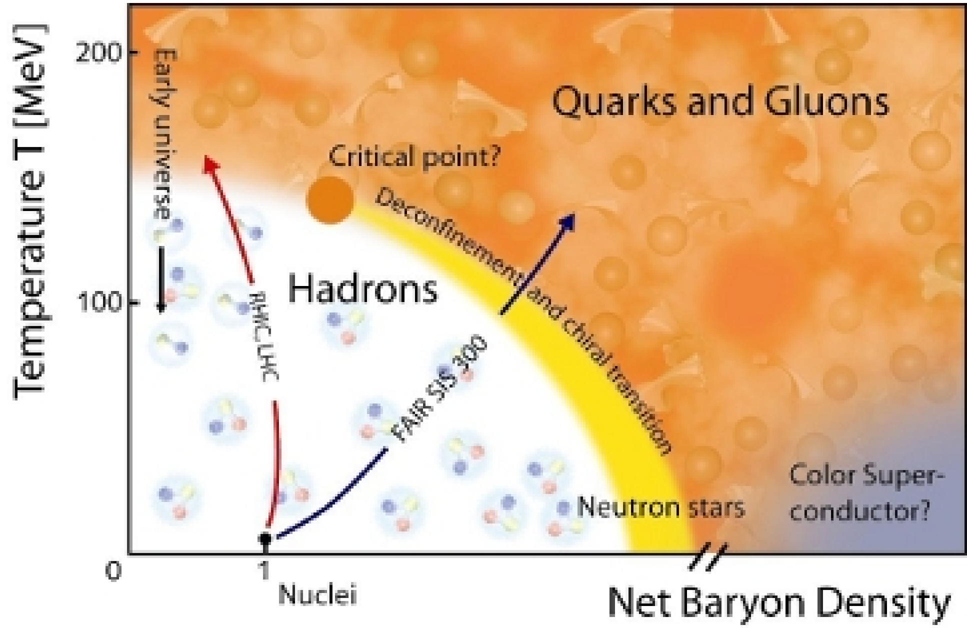
$$\tau_{phen} = 0.57\frac{T-T_{phen}(T)}{T_{phen}(T)}.$$

$$T_{phen}(T) = a + bT + cT^2 + dT^3 + e\frac{1}{T}.$$



our parametrization
right asymptotic limit

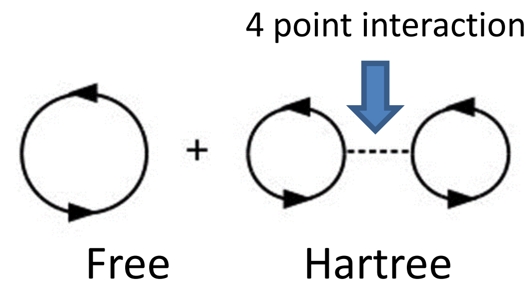
The Phase diagram of PNJL in T and μ



To obtain the phase diagram
one starts from the partition function

$$Z = Tr[\exp -\beta(H - \mu N)] = \exp(-\beta\Omega)$$

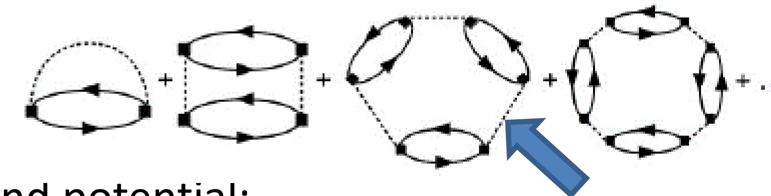
and obtains in order N_c , $(1/N_C)^{-1}$, ,the number of colors:



$$\begin{aligned}
&\Omega_q^{(-1)}(T, \mu_i; \langle\bar{\psi}_i\psi_i\rangle, \Phi, \bar{\Phi}) \\
= \quad & \ln(Tr[\exp(-\beta \int dx^3(-\bar{\psi}(i\partial\!\!\!/ - m)\psi - \mu\bar{\psi}\psi))]) \\
+ \quad & 2G\sum_k < \bar{\psi}_k\psi_k >^2 - 4K\prod_i < \bar{\psi}_k\psi_k > + U_{PNJL}
\end{aligned}$$

In this order the lattice data cannot be reproduced

Go to the order $O(N_c = 0)$ for the partition sum



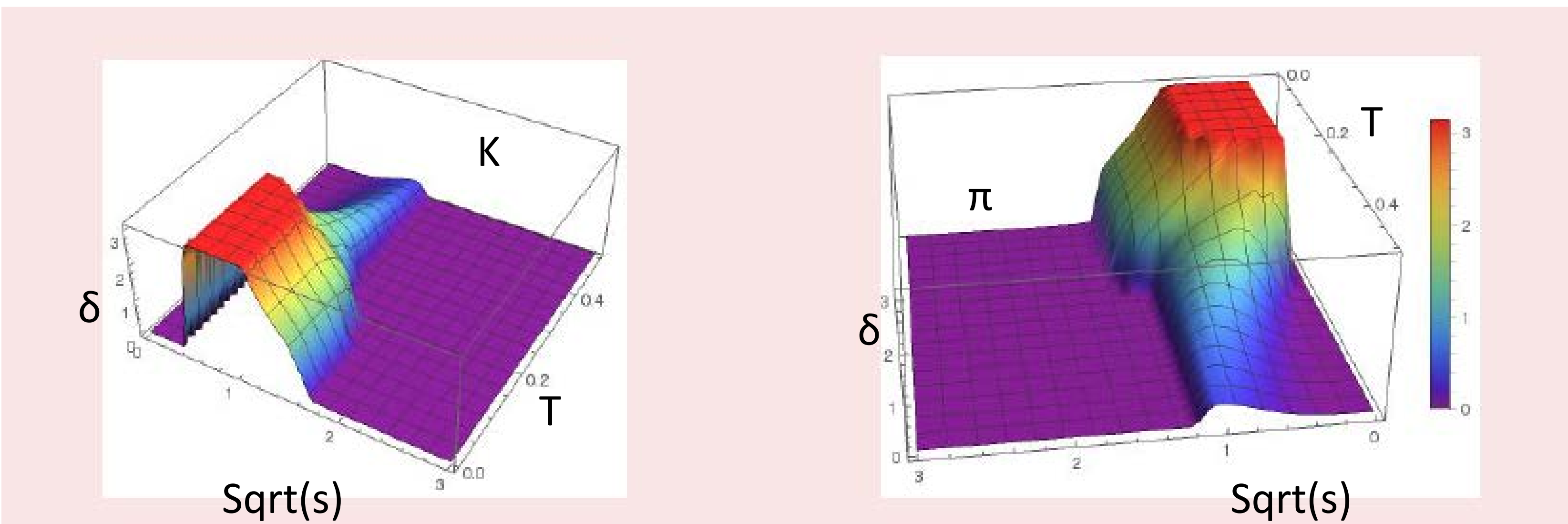
4 point interaction
PRC96,045205

In this order meson loops contribute to the grand potential:

$$\Omega_q^{(0)}(T,\mu_i) = \sum_{M\in J^\pi=\{0^+,0^-\}} \Omega_M^{(0)}(T,\mu_M(\mu_i))$$

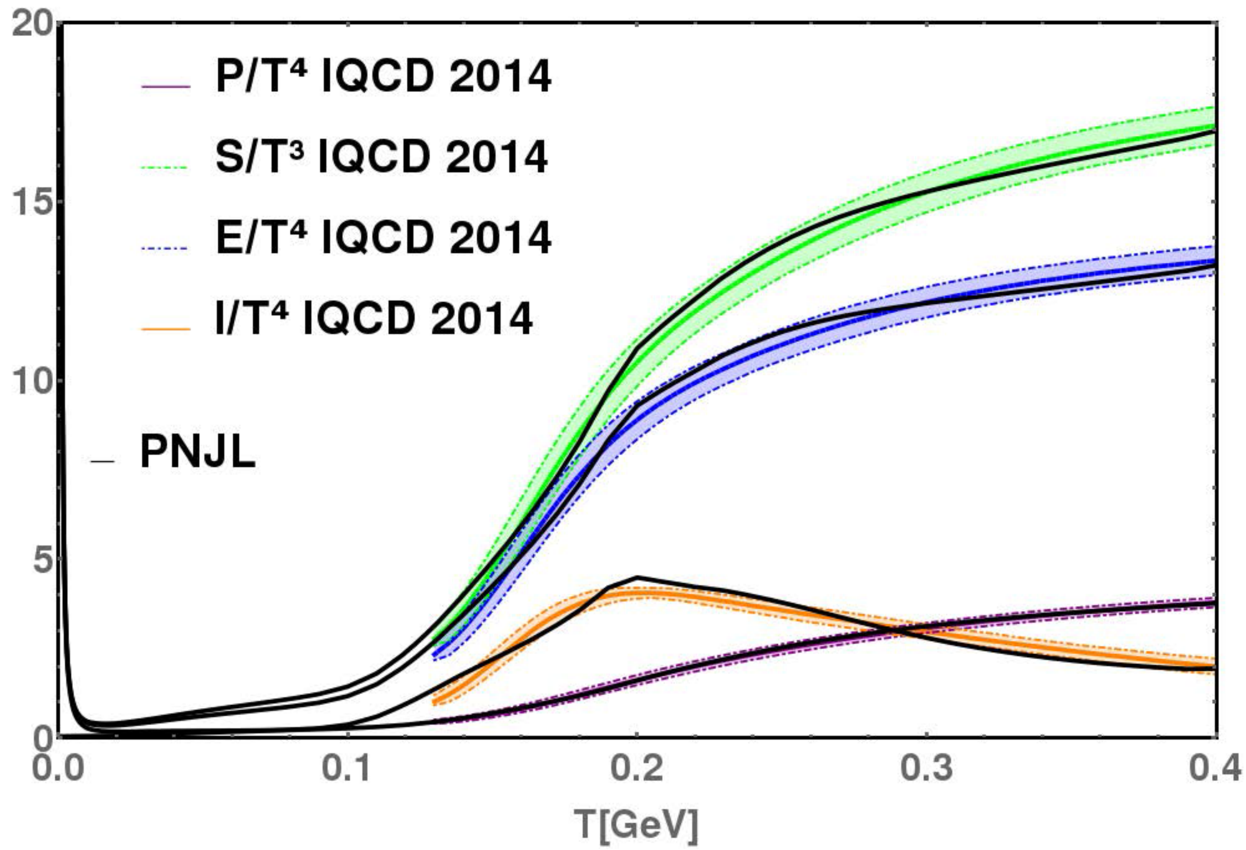
$$\Omega_M^{(0)}(T,\mu_M) = -\frac{g_M}{2\pi}\int \frac{d^3p}{(2\pi)^3}\int_0^{+\infty} d\omega \left[\frac{1}{e^{\beta(\omega-\mu_M)}-1} + \frac{1}{e^{\beta(\omega+\mu_M)}-1}\right] \delta(\omega,\mathbf{p};T,\mu_M)$$

with the phase shifts δ



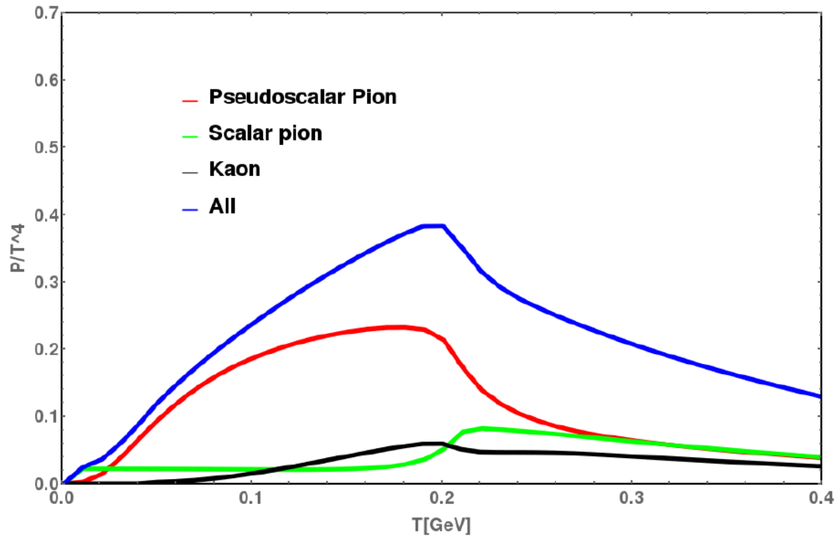
Comparison with lattice results for μ =0

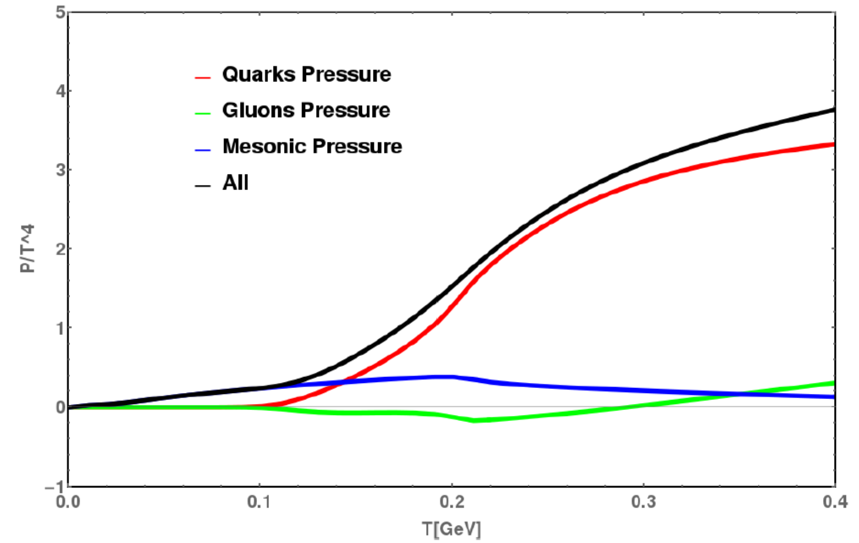
In the order O(N_c =0) and including the modfied g-q interaction
we can reproduce Pressure P, entropy density s, energy density E and interaction measure I of the lattice calculations at μ=0



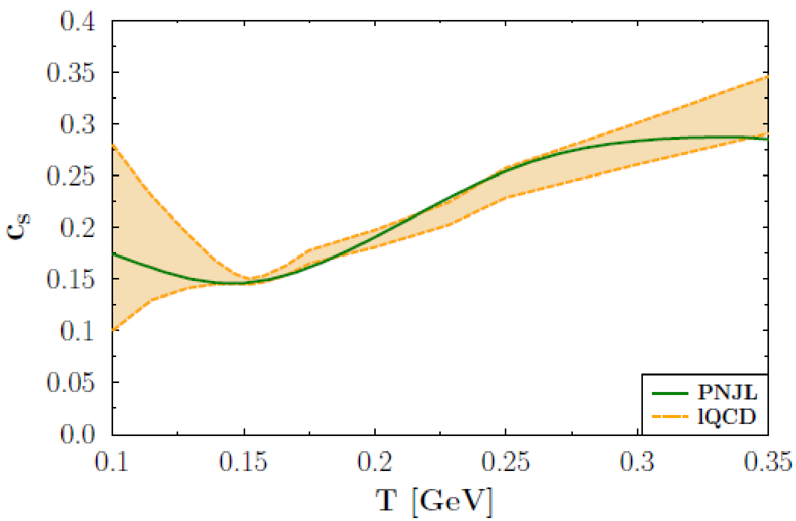
This allows to explore the phase diagram in the whole T,μ plane

Where does the pressure come from?





Speed of sound

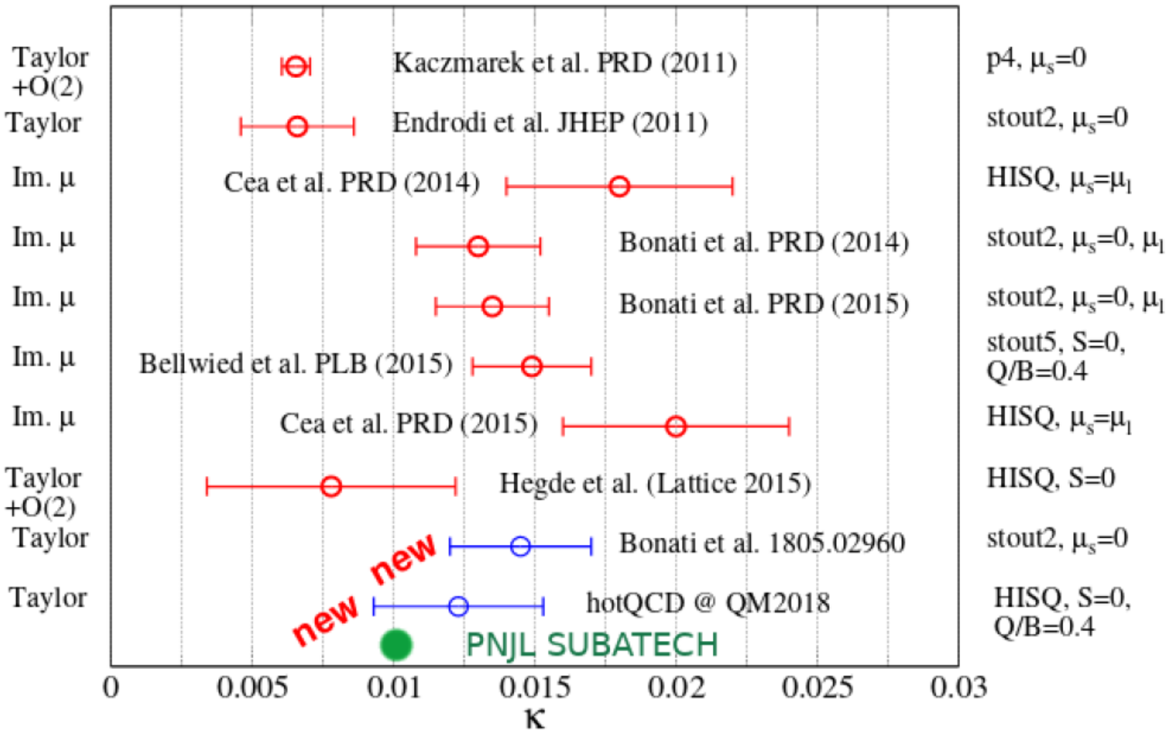


Also the speed of sound comes close to the lattice results.

Extension to small but finite chemical potential

Like in lattice calculation: Taylor expansion around μ = 0

$$\frac{T_c(\mu_B)}{T_c(0)} = 1 - \kappa \left(\frac{\mu_B}{T_c(\mu_B)}\right)^2 + \dots \qquad \kappa = \left.\frac{\partial^2 \frac{T_c(\mu_B)}{T_c(0)}}{\partial \mu_B^2}\right|_{\mu_B=0}$$



We find

T_c = 138 MeV
κ = 0.01

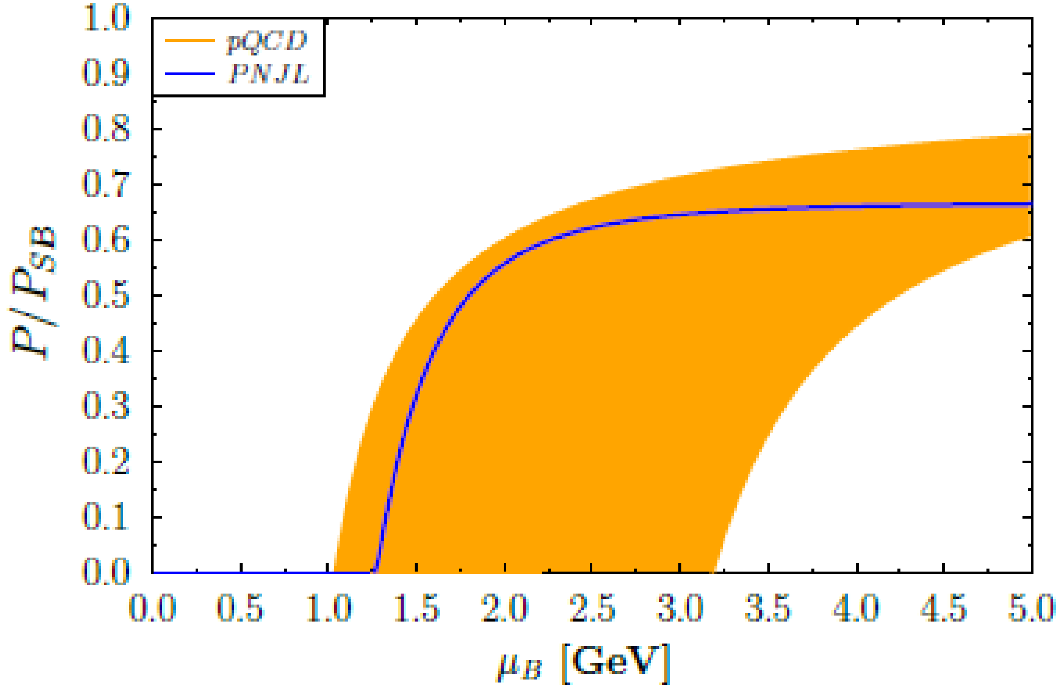
Very similar to lattice calculations

Also for small but finite μ we reproduce the lattice results: confidence for larger μ

Limit of (very) large chemical potential

For (very) large μ contact with perturbative QCD calculations (PRL 117 042501)

PNJL in the error bars of pQCD



Having verified that at small μ and at very large μ the PNJL thermodyn quantities agree with QCD based approaches

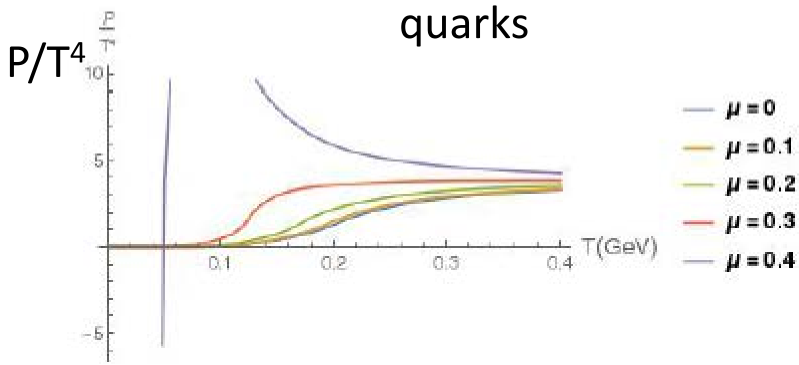
we can explore the finite μ region

The PNJL equation of state for finite μ

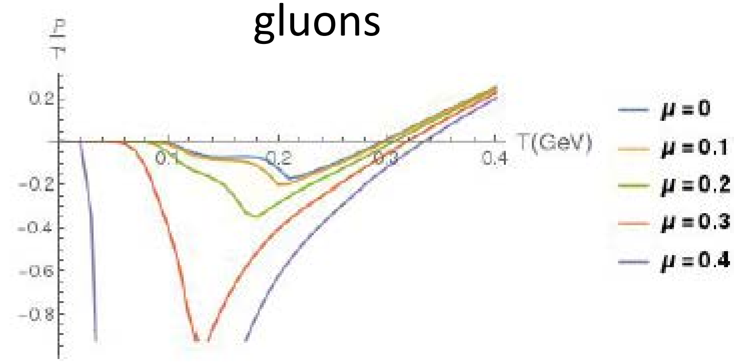
Calculation of thermal quantities at finite μ is straight forward in PNJL

Contribution to the pressure of the different particles

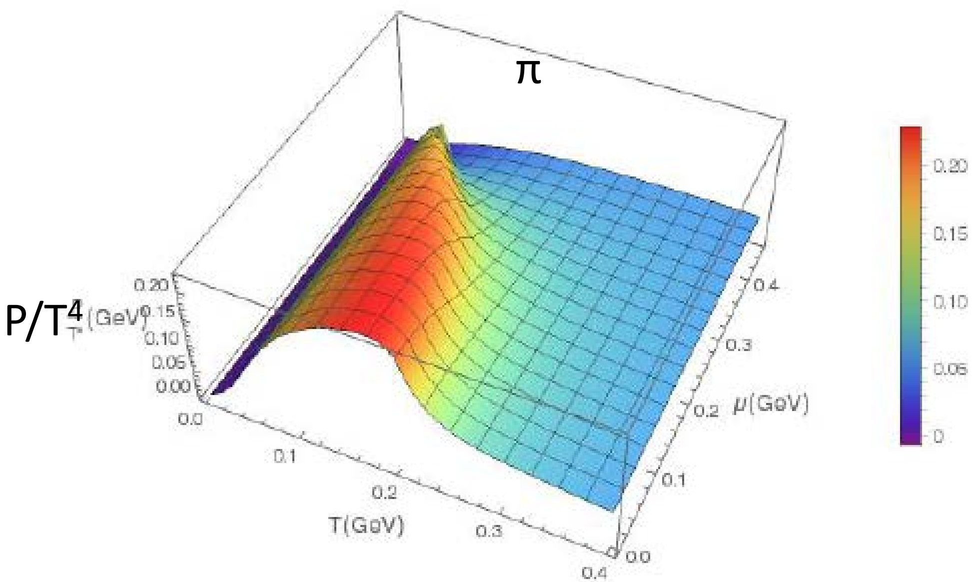
quarks



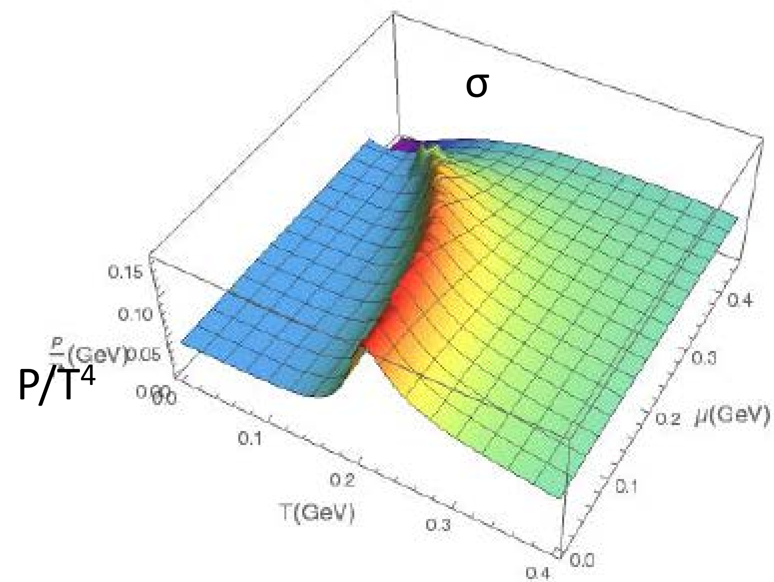
gluons



π

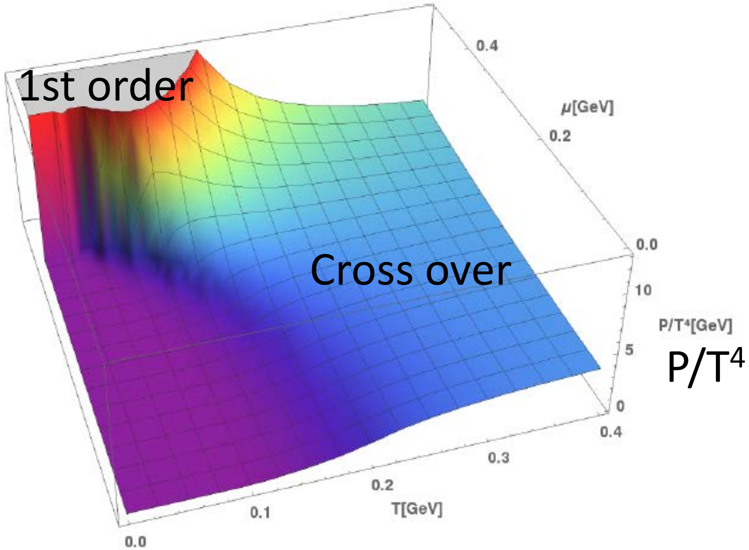


σ



Phase diagram at finite μ

Total P(T,μ)



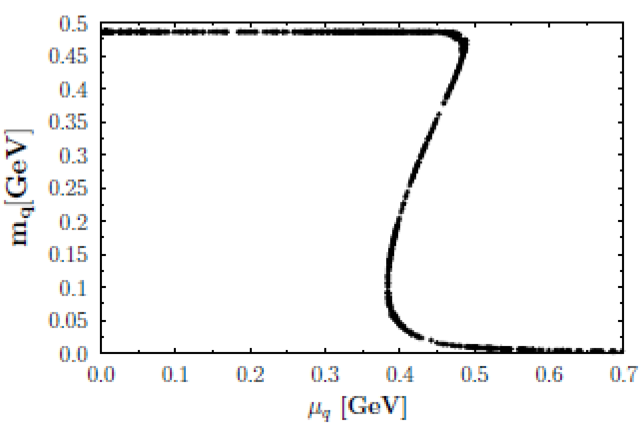
Cross over at $\mu = 0$

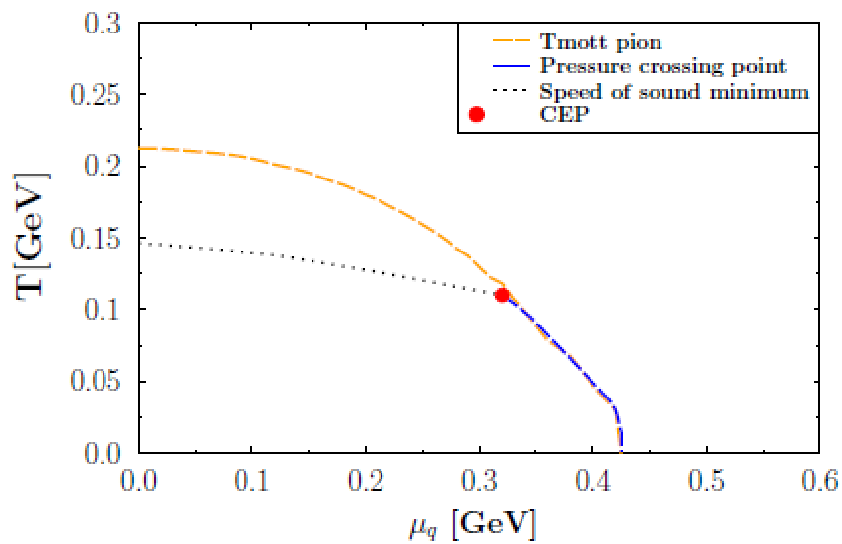
1st order phase transition for $\mu >> 0$ with a CEP

$T^{CEP} = 110$ MeV

$\mu_q^{CEP} = 320$ MeV

For small temperatures the equation of state shows a first order phase transition with the quark mass (chiral condensate) as order parameter





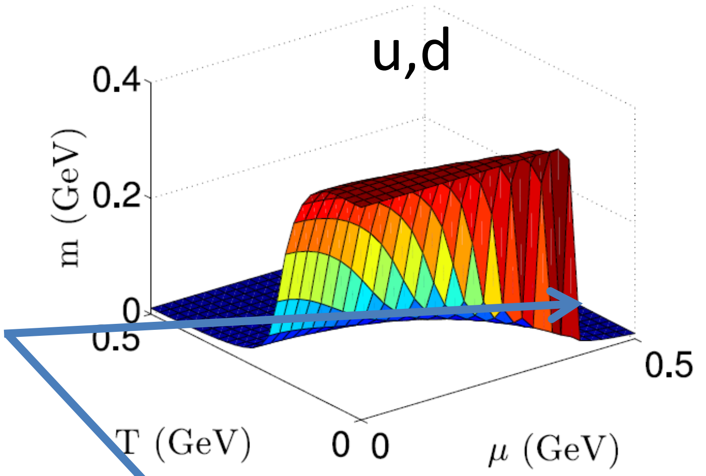
Masses close to the tricritical point

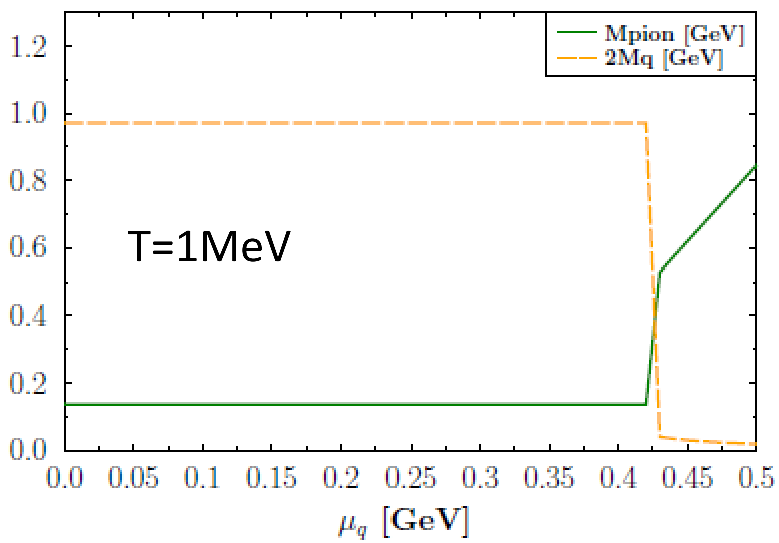
PNJL Lagrangian:
transition between quarks and hadrons

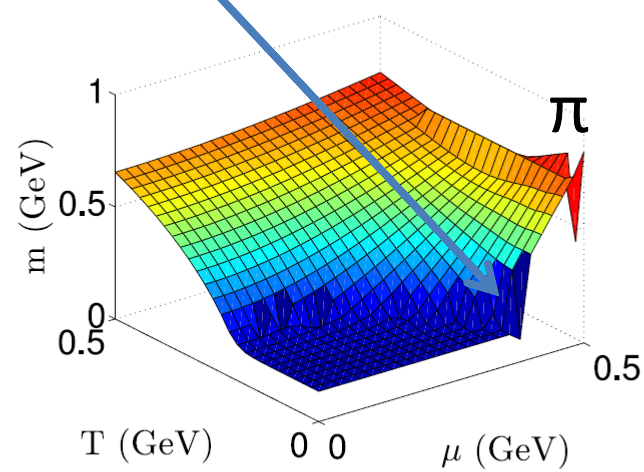
Cross over at μ = 0

1st order transition μ >>0

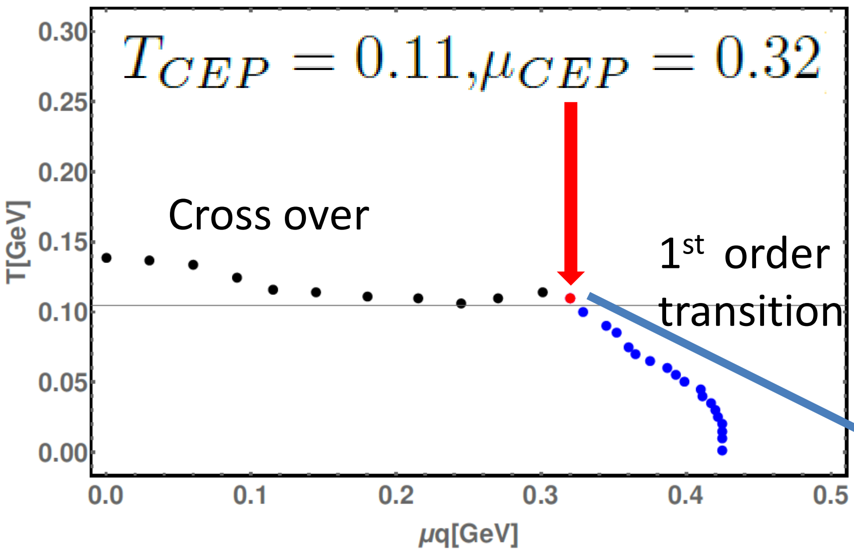
sudden change of q and meson mass





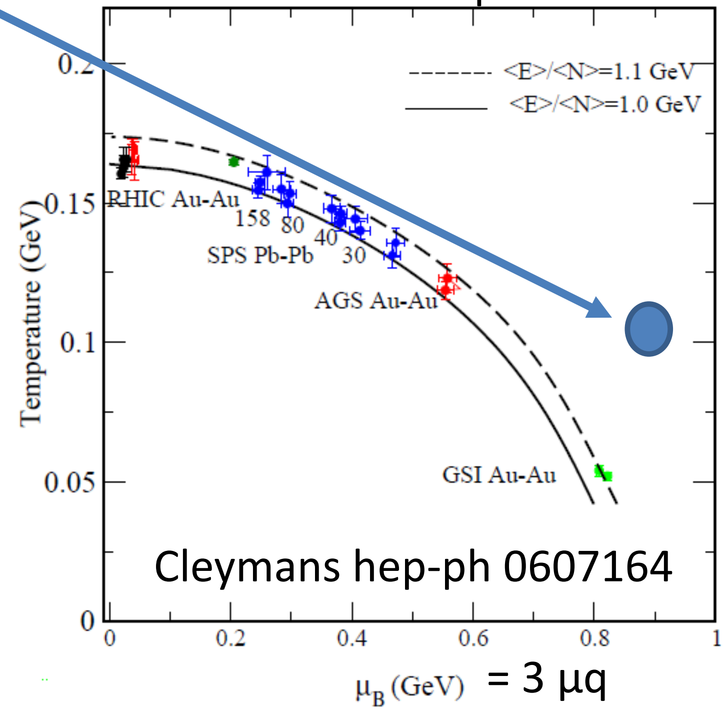


Borderline between quark gluon plasma and hadrons



Freeze out condition

Statistical model prediction



Phase transition point may be reachable in experiments at NICA and FAIR

Summary of our long way

Starting point: (P)NJL Lagrangian which shares the symmetries with QCD

Fierz transformation -> color less meson channels and qq channels -> baryons

Bethe Salpeter equation in $q\bar{q}$ → mesons as pole masses

All masses described (10% precision) by 5 parameters fitted to ground state properties

(PNJL needs additional parameters to fix the Polyakov loop)

Going to next to leading order in the partition sum and introducing an effective quark gluon interaction (guided by more fundamental approaches) we can reproduce lattice equation of state at μ=0 ,

lattice expansion coeff for finite μ

pQCD calculations at very large μ

makes extension to finite T and μ meaningful (without any new parameter)

We obtain the equation of state and the phase diagram in the (T, μ) plane necessary for neutron star, neutron star collisions and heavy ion physics

We find a first order phase transition for finite μ.

and can explore now the consequences by employing

transport approaches for heavy ion and neutron star collisions

Baryons

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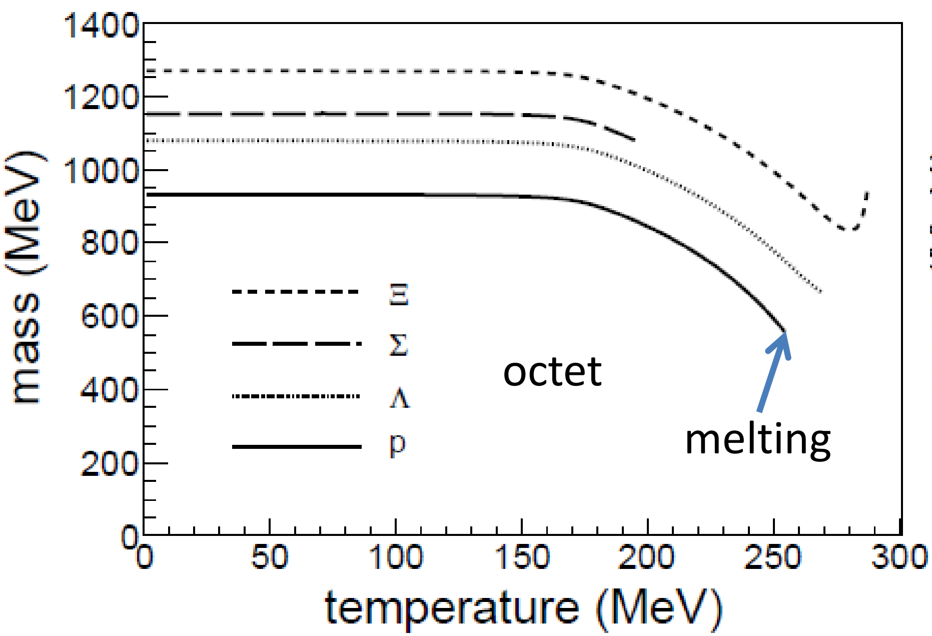
Omitting Dirac and flavor structure :

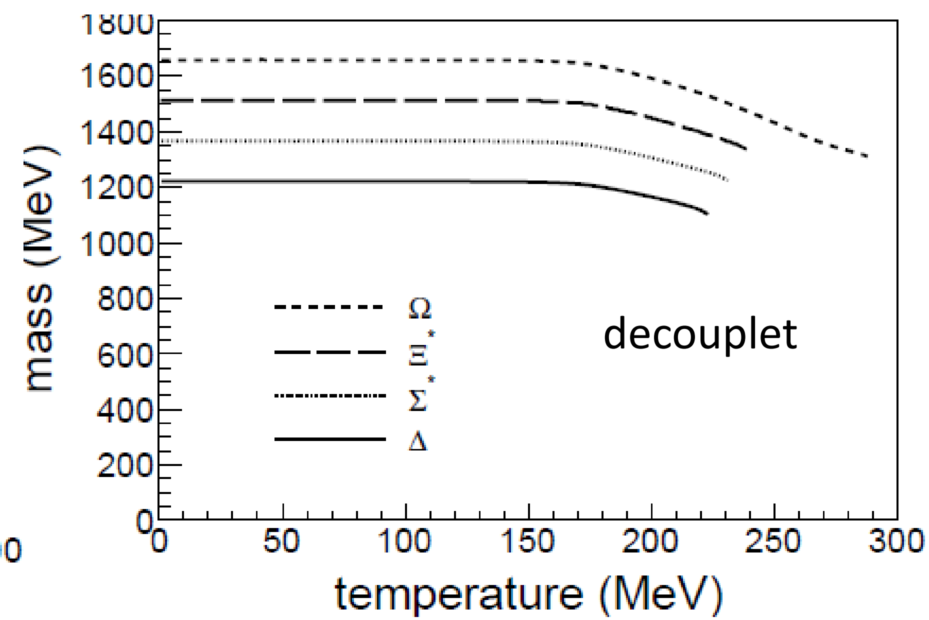
$$\left[1 - \frac{2}{m_{\mathrm{quark}}} \frac{1}{\beta}\sum_{n} \int \frac{d^3q}{(2\pi)^3} S_q(i\omega_n, q)\; t_D(i\nu_l - i\omega_n, -q)\right]\Bigg|_{i\nu_l \to P_0 + i\epsilon = M_{\mathrm{Baryon}}} = 0$$

where we approximated the quark propagator for the exchanged quark by:

$$S_q(q) = \frac{1}{\not{q} - m_{\mathrm{quark}}} \to -\frac{\mathbb{1}_{\mathrm{Dirac}}}{m_{\mathrm{quark}}}$$

5% error (Buck et al. (92))

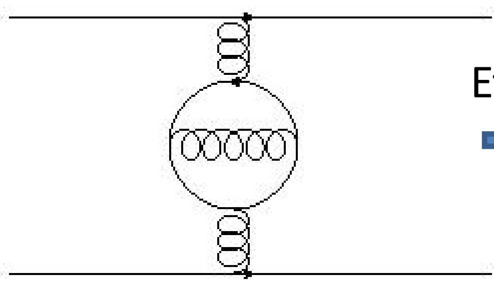




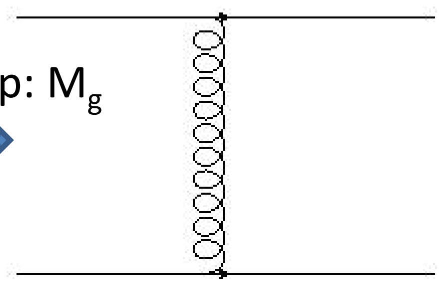
The more strange quarks the higher the melting temperature

NJL Lagrangian

$\Rightarrow$ An *effective Lagrangian* with the same symmetries for the quark degrees of freedom as QCD can be obtained by discarding the gluon dynamics completely.

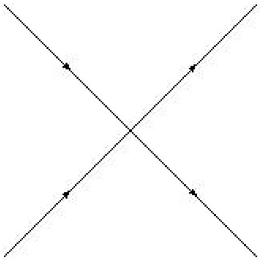


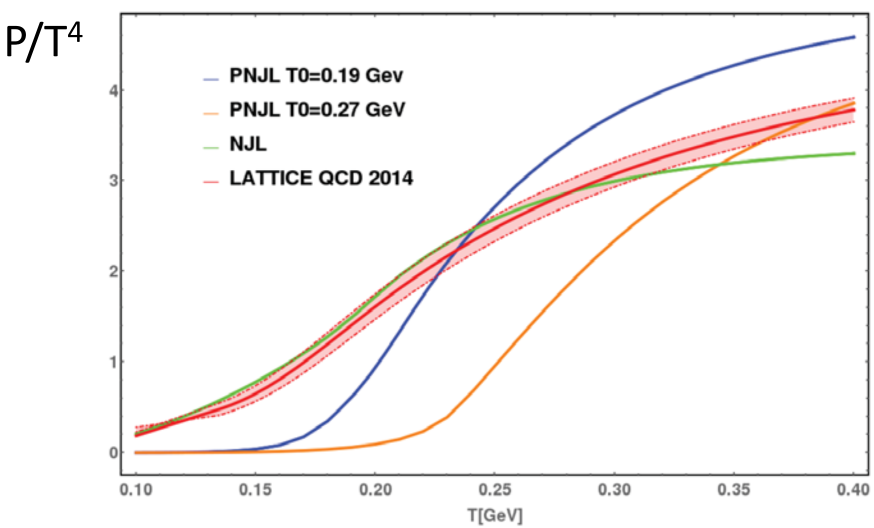
Eff prop: M_g $\Rightarrow$



$$\frac{1}{q^2 - M^2}$$

M>>q $\Rightarrow$





Renewed interest because

Going beyond leading order in N_c + including a gluon mean field potential brings PNJL energy density and entropy density closer to lattice results

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