

Domain wall networks as QCD vacuum

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Mean field approach to QCD vacuum and hadron phenomenology
motivated by the quantum effective action of QCD.

Almost everywhere Abelian (anti-)self-dual homogeneous gauge fields

R. Feynman *et al* PRD 3 1971; H. Pagels, and E. Tomboulis, NPB 143. 1978;
H. Leutwyler, J. Stern, PLB 73, 1978; PLB 77, 1978;
P. Minkowski, NPB 177, 203 (1981); H. Leutwyler, PLB**96**, 154 (1980);
NPB**179**, 129 (1981); A.I. Milstein, Yu.F. Pinelis, PLB 137 (1984);
A. Eichhorn, H. Gies, J. M. Pawłowski, Phys. Rev. D83 (2011).

- **Confinement of both static and dynamical quarks** →

$$W(C) = \langle \text{Tr P } e^{i \int_C dz_\mu \hat{A}_\mu} \rangle$$

$$S(x, y) = \langle \psi(y) \bar{\psi}(x) \rangle$$
- **Dynamical Breaking of chiral $SU_L(N_f) \times SU_R(N_f)$ symmetry** → $\langle \bar{\psi}(x)\psi(x) \rangle$
- **$U_A(1)$ Problem** → η' (χ , Axial Anomaly)
- **Strong CP Problem** → $Z(\theta)$
- **Colorless Hadron Formation:** → Effective action for colorless collective modes:
hadron masses,
form factors, scattering

Light mesons, **Regge spectrum** of excited states of light hadrons,
heavy-light hadrons, **heavy quarkonia**

QCD vacuum as a medium characterized by certain condensates,
quarks and gluons - elementary coloured excitations (confined),
mesons and baryons - collective colorless excitations

Deconfinement, chiral symmetry restoration under "extreme" conditions

Quantum effective action of QCD

- QCD effective action and vacuum gluon configurations
- Gluon condensates and domain wall network as QCD vacuum
- Testing the domain model - static characteristics of QCD vacuum
- Effective meson action
- Meson properties
- Strong electromagnetic field as a trigger of deconfinement
- "Projection" to other approaches: FRG, DSE+BS, 4-dim. oscillator - harmonic confinement, AdS/QCD models
- Confinement-deconfinement: Heterophase fluctuations

QCD effective action and vacuum gluon configurations

In Euclidean functional integral for YM theory one has to define the functional space of integration:

$$Z = N \int_{\mathcal{F}_B} DA \int_{\Psi} D\psi D\bar{\psi} \exp\{-S[A, \psi, \bar{\psi}]\}$$

B.V. Galilo and S.N. ,
Phys. Rev. D84 (2011) 094017

L. D. Faddeev,
[arXiv:0911.1013 [math-ph]]

H. Leutwyler,
Nucl. Phys. B 179 (1981) 129

$$\mathcal{F}_B = \left\{ A : \lim_{V \rightarrow \infty} \frac{1}{V} \int_V d^4x g^2 F_{\mu\nu}^a(x) F_{\mu\nu}^a(x) = B^2 \right\}.$$

$A_\mu^a = B_\mu^a + Q_\mu^a$, background gauge fixing condition $D(B)Q = 0$:

$$1 = \int_{\mathcal{B}} DB \Phi[A, B] \int_{\mathcal{Q}} DQ \int_{\Omega} D\omega \delta[A^\omega - Q^\omega - B^\omega] \delta[D(B^\omega)Q^\omega]$$

Q_μ^a – local (perturbative) fluctuations of gluon field with zero gluon condensate: $Q \in \mathcal{Q}$;
 B_μ^a are long range field configurations with nonzero condensate: $B \in \mathcal{B}$.

$$Z = N' \int_{\mathcal{B}} DB \int_{\mathcal{Q}} DQ \int_{\Psi} D\psi D\bar{\psi} \det[D(B)D(B+Q)] \delta[D(B)Q] \exp\{-S[B+Q, \psi, \bar{\psi}]\}$$

Particular features of background fields B have yet to be identified by the dynamics of fluctuations (L.D. Faddeev, arXiv:0911.1013):

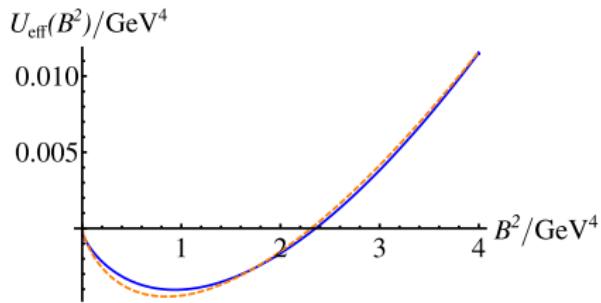
$$\begin{aligned} Z &= N' \int_{\mathcal{B}} DB \int_{\Psi} D\psi D\bar{\psi} \int_{\mathcal{Q}} DQ \det[D(B)D(B+Q)] \delta[D(B)Q] \exp\{-S[B+Q, \psi, \bar{\psi}]\} \\ &= N'' \int_{\mathcal{B}} DB \exp\{-S_{\text{eff}}[B]\} \end{aligned}$$

Global minima of $S_{\text{eff}}[B]$ – field configurations that are dominant in the limit $V \rightarrow \infty$.

Homogeneous Abelian (anti-)self-dual fields are of particular interest.

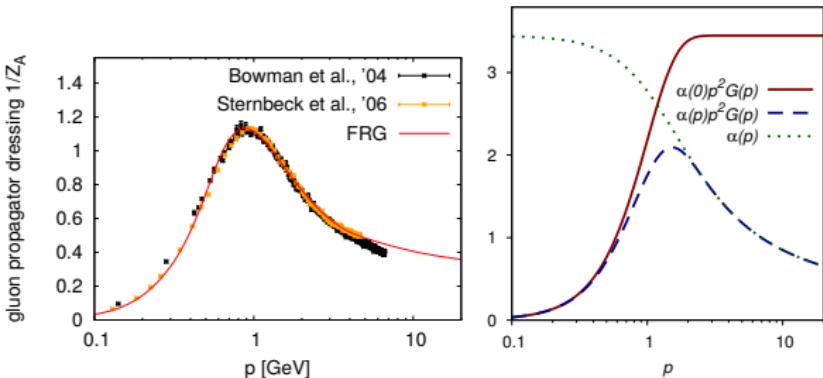
$$B_\mu = -\frac{1}{2}n B_{\mu\nu} x_\nu, \tilde{B}_{\mu\nu} = \pm B_{\mu\nu}$$

$$n = T^3 \cos \xi + T^8 \sin \xi.$$



H. Pagels, and E. Tomboulis,
 Nucl. Phys. B 143 (1978) 485
 P. Minkowski, Nucl. Phys. B 177
 (1981) 203
 H. Leutwyler, Nucl. Phys. B 179
 (1981) 129

A. Eichhorn, H. Gies and
J.M. Pawłowski, Phys. Rev.
D 83, 045014 (2011)



A signature of self-consistency:

Functional RG (used for evaluation of U_{eff}) vs mean field propagators

Analytical properties of the quark and gluon propagators

$$G(z^2) \sim \frac{e^{-Bz^2}}{z^2}, \quad \tilde{G}(p^2) \sim \frac{1}{p^2} \left(1 - e^{-p^2/B}\right)$$

⇒ dynamical color confinement

H. Leutwyler, Phys. Lett. B 96
(1980) 154

Analytical properties of polarization diagram

⇒ confinement

A.I. Milstein, Yu. Pinelis, Phys.
Lett. B 137 (1984)

⇒ Regge mass spectrum of mesons

G.V. Efimov, and S.N.,
Phys. Rev. D 51 (1995)

(Anti-)self-duality ⇒ Chiral symmetry

H. Leutwyler, Phys. Lett. B 96
(1980) 154

A.I. Milshtein, Yu.F. Pinelis, Z.Phys. C27 (1985).

A.G. Grozin, Yu.F. Pinelis, Z.Phys. C33 (1987)

Gluon condensates and domain wall network

Ginzburg-Landau approach:

$$\begin{aligned}\mathcal{L}_{\text{eff}} &= -\frac{1}{4\Lambda^2} \left(D_\nu^{ab} F_{\rho\mu}^b D_\nu^{ac} F_{\rho\mu}^c + D_\mu^{ab} F_{\mu\nu}^b D_\rho^{ac} F_{\rho\nu}^c \right) - U_{\text{eff}} \\ U_{\text{eff}} &= \frac{\Lambda^4}{12} \text{Tr} \left(C_1 F^2 + \frac{4}{3} C_2 F^4 - \frac{16}{9} C_3 F^6 \right),\end{aligned}$$

B.V. Galilo, S.N. , Phys. Part. Nucl. Lett., 8 (2011) 67

D. P. George, A. Ram, J. E. Thompson and R. R. Volkas, Phys. Rev. D 87, 105009 (2013) [arXiv:1203.1048 [hep-th]]

where

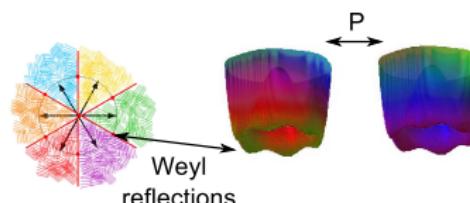
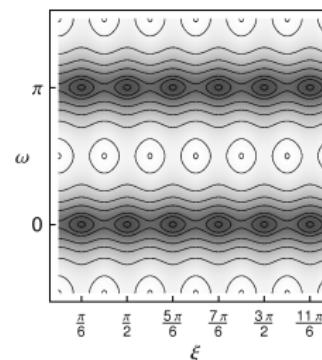
$$\begin{aligned}D_\mu^{ab} &= \delta^{ab} \partial_\mu - i A_\mu^{ab} = \partial_\mu - i A_\mu^c (T^c)^{ab}, \\ F_{\mu\nu}^a &= \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - i f^{abc} A_\mu^b A_\nu^c, \\ F_{\mu\nu} &= F_{\mu\nu}^a T^a, \quad T_{bc}^a = -i f^{abc} \\ C_1 > 0, \quad C_2 > 0, \quad C_3 > 0.\end{aligned}$$

U_{eff} possesses degenerate discrete minima:

$$B_\mu = -\frac{1}{2}n_k B_{\mu\nu}x_\nu, \tilde{B}_{\mu\nu} = \pm B_{\mu\nu}, B_{\mu\nu}B_{\mu\nu} = 4B^2$$

matrix n_k belongs to the Cartan subalgebra of $su(3)$

$$n_k = T^3 \cos(\xi_k) + T^8 \sin(\xi_k), \quad \xi_k = \frac{2k+1}{6}\pi, \quad k = 0, 1, \dots, 5,$$
$$\vec{E}\vec{H} = B^2 \cos(\omega)$$

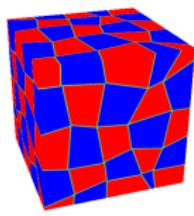


The general kink configuration can be parametrized as

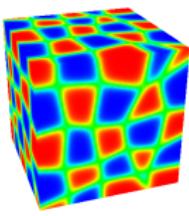
$$\zeta(\mu_i, \eta_\nu^i x_\nu - q^i) = \frac{2}{\pi} \arctan \exp(\mu_i(\eta_\nu^i x_\nu - q^i)).$$

The general domain wall network in R^4

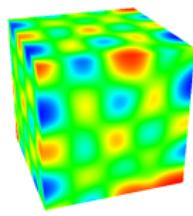
$$\omega = \pi \sum_{j=1}^{\infty} \prod_{i=1}^8 \zeta(\mu_{ij}, \eta_\nu^{ij} x_\nu - q^{ij})$$



$$\langle F^2 \rangle = B^2$$
$$\langle |F\tilde{F}| \rangle = B^2$$



$$\langle F^2 \rangle = B^2$$
$$\langle |F\tilde{F}| \rangle \ll B^2$$



WHAT COULD STABILIZE A FINITE MEAN SIZE OF DOMAINS?

TOPOLOGY?

Lower dimensional defects?

Wall junctions - nonabelian and inhomogeneous field

ENERGY?

Gluon and Quark (quasi-)zero modes?

Minimum of the free energy density for finite size of domains

In general near the boundaries

$$\operatorname{div} \vec{H} \neq 0, \quad \operatorname{div} \vec{E} \neq 0$$

The description of the domain walls as well as separation of the Abelian part in the general network in terms of the vector potential requires application of the gauge field parametrization suggested by L.D. Faddeev, A. J. Niemi (2007); K.-I. Kondo, T. Shinohara, T. Murakami(2008); Y.M. Cho (1980, 1981); L.Prokhorov, S.V. Shabanov (1989,1999)

The Abelian part $\hat{V}_\mu(x)$ of the gauge field $\hat{A}_\mu(x)$ is separated manifestly,

$$\begin{aligned}\hat{A}_\mu(x) &= \hat{V}_\mu(x) + \hat{X}_\mu(x), \quad \hat{V}_\mu(x) = \hat{B}_\mu(x) + \hat{C}_\mu(x), \\ \hat{B}_\mu(x) &= [n^a A_\mu^a(x)] \hat{n}(x) = B_\mu(x) \hat{n}(x), \\ \hat{C}_\mu(x) &= g^{-1} \partial_\mu \hat{n}(x) \times \hat{n}(x), \\ \hat{X}_\mu(x) &= g^{-1} \hat{n}(x) \times \left(\partial_\mu \hat{n}(x) + g \hat{A}_\mu(x) \times \hat{n}(x) \right),\end{aligned}$$

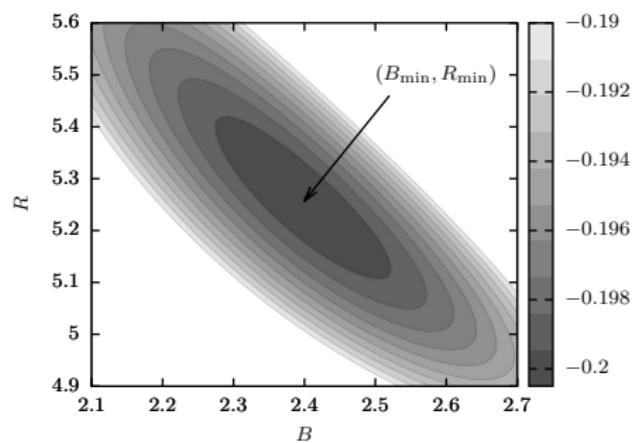
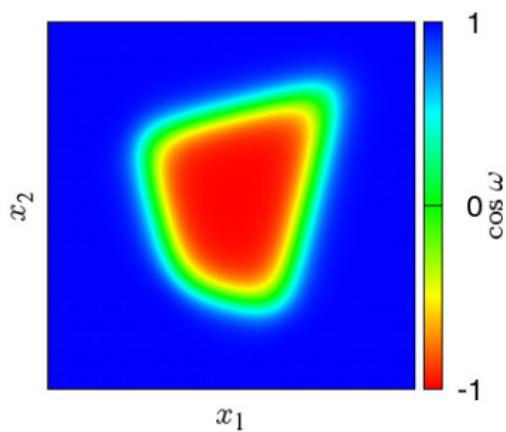
where $\hat{A}_\mu(x) = A_\mu^a(x)t^a$, $\hat{n}(x) = n_a(x)t^a$, $n^a n^a = 1$, and

$$\partial_\mu \hat{n} \times \hat{n} = i f^{abc} \partial_\mu n^a n^b t^c, \quad [t^a, t^b] = i f^{abc} t^c.$$

$$[\hat{V}_\mu(x), \hat{V}_\nu(x)] = 0$$

Both the color and space orientation of the field can become frustrated at the junction location and, thus, develop the singularities in the vector potential. Potentially singularities cover the whole range of defects – vortex-like, monopole/dyon-like and instanton-like defects.

Finite size effects in the free energy density for Abelian (anti-)self-dual gluon field in $SU(3)$ gluodynamics.



Quasi-zero gluon and quark modes - **minimum** of free energy density at finite values of the **field strength and size of the domain**.

V.Voronin& SN arXiv:1906.00432 (2019)

The role of (quasi-)zero modes

$$\begin{aligned} U &= V_R F(B, R) = U^{\text{cl}} + \delta U \\ &= \frac{\pi^2 B^2 R^4}{2g^2} - \text{Tr} \ln \left[-\check{D}^2 \right] + \frac{1}{2} \text{Tr} \ln \left[-\check{D}^2 \delta_{\mu\nu} + 2i\check{B}_{\mu\nu} \right]' - \text{Tr} \ln \left[\hat{P} \right]' + \delta U_0 \end{aligned}$$

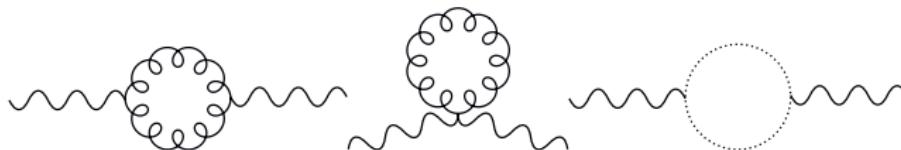
Normalization $U|_{B=0} = 0$.

Evaluation of zeta-function regularized determinants (M. Bordag *J. Math. Phys.* 37 (1996))

$$\text{Tr} \log \Delta = - \frac{d}{ds} \sum_j \lambda_j^{-s} \Big|_{s=0} = - \frac{d}{ds} \zeta(s) \Big|_{s=0}$$

An appropriate Gaussian measure for integration over quasi-zero modes is generated due to interaction of quasi-zero and normal modes, which has to be taken into account (Leutwyler NPB'81)

In pure gluodynamics



Domain bulk - harmonic confinement

Elementary color charged excitations - fluctuations, eigenmodes decay in all four directions.

Eigenvalue problem for scalar field in \mathbb{R}^4 :

H. Leutwyler, Nucl. Phys. B 179 (1981);

$$B_\mu = B_{\mu\nu}x_\nu, \tilde{B}_{\mu\nu} = \pm B_{\mu\nu}, B_{\mu\alpha}B_{\nu\alpha} = B^2\delta_{\mu\nu}.$$

$$-(\partial_\mu - iB_\mu)^2 G = \delta \quad \longrightarrow \quad G(x-y) \sim \frac{e^{-B(x-y)^2/4}}{(x-y)^2}$$

$$-\left(\partial_\mu - i\check{B}_\mu\right)^2 \Phi = \lambda \Phi \quad \longrightarrow \quad \left[\beta_\pm^+ \beta_\pm + \gamma_+^+ \gamma_+ + 1\right] \Phi = \frac{\lambda}{4B} \Phi,$$

$$\beta_\pm = \frac{1}{2}(\alpha_1 \mp i\alpha_2), \quad \gamma_\pm = \frac{1}{2}(\alpha_3 \mp i\alpha_4), \quad \alpha_\mu = \frac{1}{\sqrt{B}}x_\mu + \partial_\mu,$$

$$\beta_\pm^+ = \frac{1}{2}(\alpha_1^+ \pm i\alpha_2^+), \quad \gamma_\pm^+ = \frac{1}{2}(\alpha_3^+ \pm i\alpha_4^+), \quad \alpha_\mu^+ = \frac{1}{\sqrt{B}}x_\mu - \partial_\mu.$$

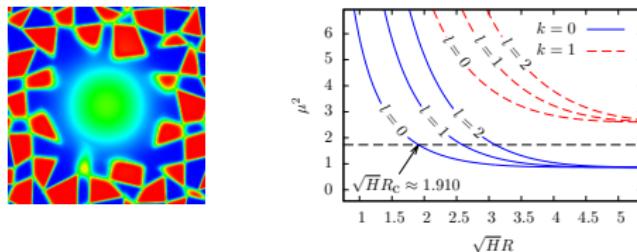
The eigenfunctions and eigenvalues - 4-dim. harmonic oscillator

$$\Phi_{nmkl}(x) = \frac{1}{\pi^2 \sqrt{n!m!k!l!}} \left(\beta_+^+\right)^k \left(\beta_-^+\right)^l \left(\gamma_+^+\right)^n \left(\gamma_-^+\right)^m \Phi_{0000}, \quad \Phi_{0000} = e^{-\frac{1}{2}Bx^2}$$

$$\lambda_r = 4B(r+1), \quad r = k+n \text{ self-dual field, } r = l+n \text{ anti-self-dual field}$$

Domain wall junctions - deconfinement

S.N. , V.E. Voronin, Eur.Phys.J. A51 (2015) 4



The color charged scalar field inside junction:

$$-\left(\partial_\mu - i\check{B}_\mu\right)^2 \Phi = 0, \quad \Phi(x) = 0, \quad x \in \partial\mathcal{T}, \quad \mathcal{T} = \{x_1^2 + x_2^2 < R^2, (x_3, x_4) \in \mathbf{R}^2\}$$

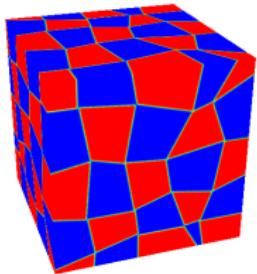
The solutions are quasi-particle excitations

$$\phi^a(x) = \sum_{lk} \int_{-\infty}^{+\infty} \frac{dp_3}{2\pi} \frac{1}{\sqrt{2\omega_{alk}}} \left[a_{akl}^+(p_3) e^{ix_0\omega_{akl}-ip_3x_3} + b_{akl}(p_3) e^{-ix_0\omega_{akl}+ip_3x_3} \right] e^{il\vartheta} \phi_{alk}(r),$$

$$\phi^{a\dagger}(x) = \sum_{lk} \int_{-\infty}^{+\infty} \frac{dp_3}{2\pi} \frac{1}{\sqrt{2\omega_{alk}}} \left[b_{akl}^+(p_3) e^{-ix_0\omega_{akl} + ip_3 x_3} + a_{akl}(p_3) e^{ix_0\omega_{akl} - ip_3 x_3} \right] e^{-il\vartheta} \phi_{alk}(r),$$

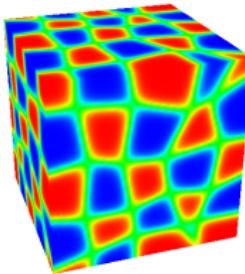
$$p_0^2 = p_3^2 + \mu_{akl}^2, \quad p_0 = \pm \omega_{akl}(p_3), \quad \omega_{akl} = \sqrt{p_3^2 + \mu_{akl}^2},$$

$$k = 0, 1, \dots, \infty, \quad l \in \mathbb{Z},$$



$$\langle F^2 \rangle = B^2$$
$$\langle |F\tilde{F}| \rangle = B^2$$

Confinement - colorless
hadrons



$$\langle F^2 \rangle = B^2$$
$$\langle |F\tilde{F}| \rangle \ll B^2$$

Deconfinement - color
charged quasi-particles

"Phase transitions and heterophase fluctuations",
V. I. Yukalov, Phys. Rep. 208, 396 (1991)

Testing the model: characteristics of the domain wall network ensemble

Spherical domains

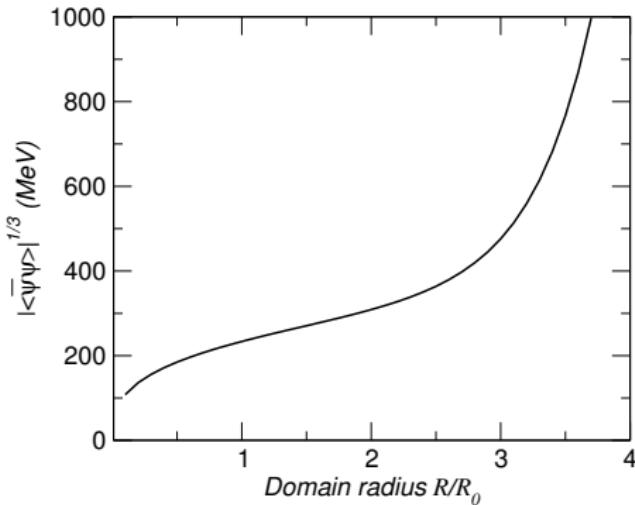
A.C. Kalloniatis and S.N. , Phys. Rev. D 64 (2001); Phys. Rev. D 69 (2004); Phys. Rev. D 71 (2005); Phys. Rev. D 73 (2006), Eur.Phys.J. A51 (2015), arXiv:1603.01447 [hep-ph] (2016)

Area law

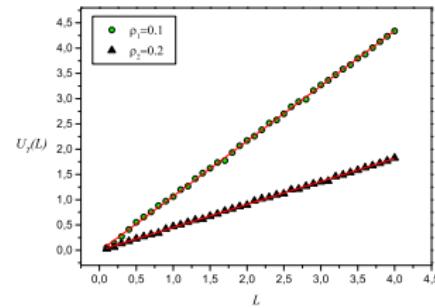
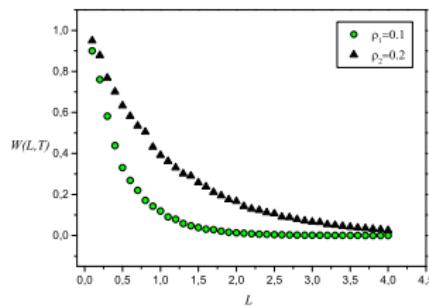
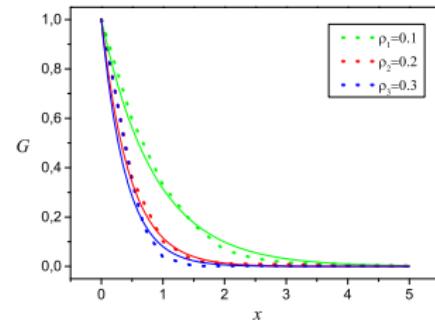
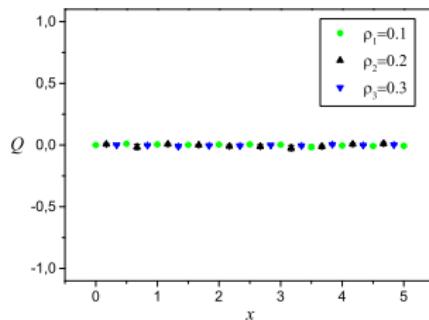
Spontaneous chiral symmetry breaking

$U_A(1)$ is broken by anomaly

There is no strong CP violation



Noninteracting walls: pure glue, 4d domains (mean topological charge, two-point correlator of top. charge density, Wilson loop and static potential)



P. Olesen, "Confinement and random fluxes", Nucl. Phys. B, Volume 200 (1982) 381-390.

Hadronization

G.V. Efimov and S.N. , Phys. Rev. D 51 (1995); Phys. Rev. D 54 (1996)

A.C. Kalloniatis and S.N. , Phys. Rev. D 64 (2001); Phys. Rev. D 69 (2004); Phys. Rev. D 71 (2005);
Phys. Rev. D 73 (2006)

$$\mathcal{Z} = \int dB \int_{\Psi} \mathcal{D}\psi \mathcal{D}\bar{\psi} \int_{\mathcal{Q}} \mathcal{D}Q \delta[D(B)Q] \Delta_{\text{FP}}[B, Q] e^{-S^{\text{QCD}}[Q+B, \psi, \bar{\psi}]} = \\ \int dB \int_{\Psi} \mathcal{D}\psi \mathcal{D}\bar{\psi} \exp \left\{ \int dx \bar{\psi} (i\partial + g\beta - m) \psi \right\} W[j]$$

$$W[j] = \exp \left\{ \sum_n \frac{g^n}{n!} \int dx_1 \cdots \int dx_n j_{\mu_1}^{a_1}(x_1) \cdots j_{\mu_n}^{a_n}(x_n) G_{\mu_1 \cdots \mu_n}^{a_1 \cdots a_n}(x_1, \dots, x_n | B) \right\} \\ j_{\mu}^a = \bar{\psi} \gamma_{\mu} t^a \psi,$$

Next step: $W[j]$ is truncated up to the term including two-point gluon correlation function.

$$\mathcal{Z} = \int dB \int_{\Psi} \mathcal{D}\psi \mathcal{D}\bar{\psi} \exp \left\{ \int dx \bar{\psi} (i \not{\partial} + g \not{\beta} - m) \psi + \frac{g^2}{2} \int dx_1 dx_2 G_{\mu_1 \mu_2}^{a_1 a_2} (x_1, x_2 | B) j_{\mu_1}^{a_1} (x_1) j_{\mu_2}^{a_2} (x_2) \right\}$$

Fierz transform, center of mass coordinates $\rightarrow \int dz dx G(z|B) J^{aJ}(x, z) J^{aJ}(x, z)$

$$\alpha_s \text{~~~} \curvearrowleft \text{~~~} \curvearrowright = \alpha_s(0) \text{~~~} \curvearrowleft \text{~~~} \curvearrowright \left[1 + \Pi^R(p^2) \right]; \quad \Pi^R(0) = 0$$

$$0 \text{~~~} \curvearrowleft \text{~~~} z \rightarrow \frac{e^{-\frac{1}{4}Bz^2}}{4\pi^2 z^2} \quad \int dx_1 dx_2 \begin{array}{c} x_1 \\ \curvearrowleft \\ x_2 \end{array} = \int dx \sum_{aJln} \quad \begin{array}{c} x \\ aJln \end{array} \bullet \begin{array}{c} x \\ aJln \end{array}$$

$$\rightarrow \alpha_s(p) \frac{1 - \exp(-p^2/B)}{p^2}$$

$$J^{aJ}(x, z) = \sum_{nl} (z^2)^{l/2} f_{\mu_1 \dots \mu_l}^{nl}(z) J_{\mu_1 \dots \mu_l}^{aJln}(x), \quad J_{\mu_1 \dots \mu_l}^{aJln}(x) = \bar{q}(x) V_{\mu_1 \dots \mu_l}^{aJln} \left(\frac{\overset{\leftrightarrow}{D}(x)}{B} \right) q(x),$$

$$f_{\mu_1 \dots \mu_l}^{nl} = L_{nl} (z^2) T_{\mu_1 \dots \mu_l}^{(l)}(n_z), \quad n_z = \frac{z}{\sqrt{z}}.$$

$T_{\mu_1 \dots \mu_l}^{(l)}$ are irreducible tensors of four-dimensional rotational group

$$\int_0^\infty du \rho_l(u) L_{nl}(u) L_{n'l}(u) = \delta_{nn'}, \quad \rho_l(u) = u^l e^{-u} \leftrightarrow \frac{e^{-Bz^2}}{z^2} \quad \text{gluon propagator}$$

Effective meson action for composite colorless fields:

$$Z = \mathcal{N} \lim_{V \rightarrow \infty} \int D\Phi_{\mathcal{Q}} \exp \left\{ -\frac{B}{2} \frac{h_{\mathcal{Q}}^2}{g^2 C_{\mathcal{Q}}} \int dx \Phi_{\mathcal{Q}}^2(x) - \sum_k \frac{1}{k} W_k[\Phi] \right\}, \quad \mathcal{Q} = (aJln)$$

$$1 = \frac{g^2 C_{\mathcal{Q}}}{B} \tilde{\Gamma}_{\mathcal{Q}\mathcal{Q}}^{(2)}(-M_{\mathcal{Q}}^2 | B), \quad h_{\mathcal{Q}}^{-2} = \frac{d}{dp^2} \tilde{\Gamma}_{\mathcal{Q}\mathcal{Q}}^{(2)}(p^2)|_{p^2=-M_{\mathcal{Q}}^2}$$

$$W_k[\Phi] = \sum_{\mathcal{Q}_1 \dots \mathcal{Q}_k} h_{\mathcal{Q}_1} \dots h_{\mathcal{Q}_k} \int dx_1 \dots \int dx_k \Phi_{\mathcal{Q}_1}(x_1) \dots \Phi_{\mathcal{Q}_k}(x_k) \Gamma_{\mathcal{Q}_1 \dots \mathcal{Q}_k}^{(k)}(x_1, \dots, x_k | B)$$

$$\Gamma_{\mathcal{Q}_1 \mathcal{Q}_2}^{(2)} = \overline{G_{\mathcal{Q}_1 \mathcal{Q}_2}^{(2)}(x_1, x_2)} - \Xi_2(x_1 - x_2) \overline{G_{\mathcal{Q}_1}^{(1)} G_{\mathcal{Q}_2}^{(1)}},$$

$$\begin{aligned} \Gamma_{\mathcal{Q}_1 \mathcal{Q}_2 \mathcal{Q}_3}^{(3)} &= \overline{G_{\mathcal{Q}_1 \mathcal{Q}_2 \mathcal{Q}_3}^{(3)}(x_1, x_2, x_3)} - \frac{3}{2} \Xi_2(x_1 - x_3) \overline{G_{\mathcal{Q}_1 \mathcal{Q}_2}^{(2)}(x_1, x_2) G_{\mathcal{Q}_3}^{(1)}(x_3)} \\ &\quad + \frac{1}{2} \Xi_3(x_1, x_2, x_3) \overline{G_{\mathcal{Q}_1}^{(1)}(x_1) G_{\mathcal{Q}_2}^{(1)}(x_2) G_{\mathcal{Q}_3}^{(1)}(x_3)}, \end{aligned}$$

$$\begin{aligned} \Gamma_{\mathcal{Q}_1 \mathcal{Q}_2 \mathcal{Q}_3 \mathcal{Q}_4}^{(4)} &= \overline{G_{\mathcal{Q}_1 \mathcal{Q}_2 \mathcal{Q}_3 \mathcal{Q}_4}^{(4)}(x_1, x_2, x_3, x_4)} - \frac{4}{3} \Xi_2(x_1 - x_2) \overline{G_{\mathcal{Q}_1}^{(1)}(x_1) G_{\mathcal{Q}_2 \mathcal{Q}_3 \mathcal{Q}_4}^{(3)}(x_2, x_3, x_4)} \\ &\quad - \frac{1}{2} \Xi_2(x_1 - x_3) \overline{G_{\mathcal{Q}_1 \mathcal{Q}_2}^{(2)}(x_1, x_2) G_{\mathcal{Q}_3 \mathcal{Q}_4}^{(2)}(x_3, x_4)} \\ &\quad + \Xi_3(x_1, x_2, x_3) \overline{G_{\mathcal{Q}_1}^{(1)}(x_1) G_{\mathcal{Q}_2}^{(1)}(x_2) G_{\mathcal{Q}_3 \mathcal{Q}_4}^{(2)}(x_3, x_4)} \\ &\quad - \frac{1}{6} \Xi_4(x_1, x_2, x_3, x_4) \overline{G_{\mathcal{Q}_1}^{(1)}(x_1) G_{\mathcal{Q}_2}^{(1)}(x_2) G_{\mathcal{Q}_3}^{(1)}(x_3) G_{\mathcal{Q}_4}^{(1)}(x_4)}. \end{aligned}$$

$$\overline{G_{\mathcal{Q}_1 \dots \mathcal{Q}_k}^{(k)}(x_1, \dots, x_k)} = \int dB_j \text{Tr} V_{\mathcal{Q}_1} \left(x_1 | B^{(j)} \right) S \left(x_1, x_2 | B^{(j)} \right) \dots \\ \dots V_{\mathcal{Q}_k} \left(x_k | B^{(j)} \right) S \left(x_k, x_1 | B^{(j)} \right)$$

$$\overline{G_{\mathcal{Q}_1 \dots \mathcal{Q}_l}^{(l)}(x_1, \dots, x_l) G_{\mathcal{Q}_{l+1} \dots \mathcal{Q}_k}^{(k)}(x_{l+1}, \dots, x_k)} = \\ \int dB_j \text{Tr} \left\{ V_{\mathcal{Q}_1} \left(x_1 | B^{(j)} \right) S \left(x_1, x_2 | B^{(j)} \right) \dots V_{\mathcal{Q}_k} \left(x_l | B^{(j)} \right) S \left(x_l, x_1 | B^{(j)} \right) \right\} \\ \times \text{Tr} \left\{ V_{\mathcal{Q}_{l+1}} \left(x_{l+1} | B^{(j)} \right) S \left(x_{l+1}, x_{l+2} | B^{(j)} \right) \dots V_{\mathcal{Q}_k} \left(x_k | B^{(j)} \right) S \left(x_k, x_{l+1} | B^{(j)} \right) \right\},$$

Bar denotes integration over all configurations of the background field with measure dB_j .

$$\langle \exp(iB_{\mu\nu}J_{\mu\nu}) \rangle = \frac{\sin W}{W}$$

$$W = \sqrt{2B^2 \left(J_{\mu\nu}J_{\mu\nu} \pm J_{\mu\nu}\tilde{J}_{\mu\nu} \right)}$$

$J_{\mu\nu}$ is a tensor, in general composed of the momenta $p_{1\mu_1} \dots p_{n\mu_n}$ - arguments of the meson interaction vertex

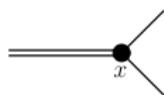
$$\tilde{\Gamma}^{(n)}(p_{1\mu_1} \dots p_{n\mu_n})$$

η'
↓

$$\Gamma_{\mathcal{Q}_1 \mathcal{Q}_2}^{(2)} = \text{---} \xrightarrow{p} \bullet \text{---} \xleftarrow{p} + \text{---} \xrightarrow{p} \bullet \text{---} \xleftarrow{p}$$

$$\Gamma_{\mathcal{Q}_1 \mathcal{Q}_2 \dots \mathcal{Q}_n}^{(n)} = \text{---} \xrightarrow{\quad} \bullet \text{---} \xleftarrow{\quad} + \dots + \text{---} \xrightarrow{\quad} \bullet \text{---} \xleftarrow{\quad} + \dots + \dots$$

Meson-quark vertex operators $\Leftarrow J_{\mu_1 \dots \mu_l}^{aJln} = \bar{q}(x) V_{\mu_1 \dots \mu_l}^{aJln} q(x)$



$$V_{\mu_1 \dots \mu_l}^{aJln}(x) = M^a \Gamma^J \left\{ \left\{ F_{nl} \left(\frac{\overset{\leftrightarrow}{D}^2(x)}{B^2} \right) T_{\mu_1 \dots \mu_l}^{(l)} \left(\frac{1}{i} \frac{\overset{\leftrightarrow}{D}(x)}{B} \right) \right\} \right\},$$

$$F_{nl}(s) = s^n \int_0^1 dt t^{n+l} \exp(st) = \int_0^1 dt t^{n+l} \frac{\partial^n}{\partial t^n} \exp(st),$$

$$\overset{\leftrightarrow}{D} = \overset{\leftarrow}{D} \xi_{f'} - \vec{D} \xi_f, \xi_f = \frac{m_f}{m_f + m_{f'}}$$

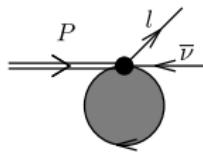
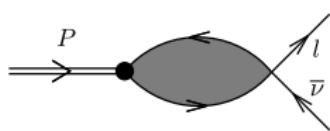
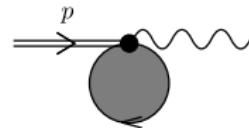
Quark propagator in homogeneous Abelian (anti-)self-dual field

$$\overrightarrow{\text{---}} = \overrightarrow{\text{---}}_{m(0)} \left[1 + \Sigma^R(p^2) \right]; \quad \Sigma^R(0) = 0 \quad S(x, y) = \exp \left(-\frac{i}{2} x_\mu B_{\mu\nu} y^\nu \right) H(x - y),$$

$$\begin{aligned} \tilde{H}_f(p|B) = & \frac{1}{vB^2} \int_0^1 ds e^{(-p^2/vB^2)s} \left(\frac{1-s}{1+s} \right)^{m_f^2/2vB^2} \left[p_\alpha \gamma_\alpha \pm i s \gamma_5 \gamma_\alpha \frac{B_{\alpha\beta}}{vB^2} p_\beta + \right. \\ & \left. + m_f \left(P_\pm + P_\mp \frac{1+s^2}{1-s^2} - \frac{i}{2} \gamma_\alpha \frac{B_{\alpha\beta}}{vB^2} \gamma_\beta \frac{s}{1-s^2} \right) \right] \end{aligned}$$

$$\begin{aligned} \tilde{H}_f(p|B) = & \frac{m}{2v\Lambda^2} \mathcal{H}_S(p^2) \mp \gamma_5 \frac{m}{2v\Lambda^2} \mathcal{H}_P(p^2) + \gamma_\alpha \frac{p_\alpha}{2v\Lambda^2} \mathcal{H}_V(p^2) \pm i \gamma_5 \gamma_\alpha \frac{f_{\alpha\beta} p_\beta}{2v\Lambda^2} \mathcal{H}_A(p^2) \quad (1) \\ & + \sigma_{\alpha\beta} \frac{m f_{\alpha\beta}}{4v\Lambda^2} \mathcal{H}_T(p^2). \end{aligned}$$

Weak and electromagnetic interactions

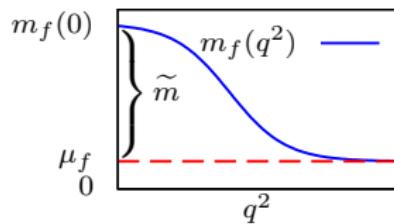


Masses of radially excited mesons

The parameters of the model are

$$\alpha_s(0) \quad m_{u/d}(0) \quad m_s(0) \quad m_c(0) \quad m_b(0) \quad B \quad R$$
$$\langle \alpha_s F^2 \rangle = \frac{B^2}{\pi} \quad \chi_{\text{YM}} = \frac{B^4 R^4}{128\pi^2}$$

Dynamical chiral symmetry breaking:



$$\tilde{m} = 136 \text{ MeV}$$
$$\mu_{u/d} = m_{u/d} - \tilde{m}$$
$$\mu_s = m_s - \tilde{m}$$
$$\frac{\mu_s}{\mu_{u/d}} = 26.7$$

$$\Lambda^2 \Phi_{Q_1}^{(0)} = \sum_{k=1}^{\infty} \frac{g^k}{k} \sum_{Q_1 \dots Q_k} \Phi_{Q_2}^{(0)} \dots \Phi_{Q_k}^{(0)} \Gamma_{Q_1 \dots Q_k}^{(k)},$$

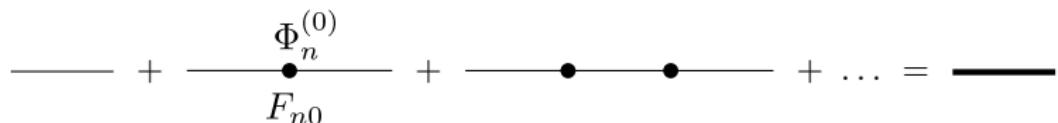


Figure: Mass corrections to the quark propagator due to the constant scalar condensates $\Phi_n^{(0)}$ coupled to nonlocal form factor F_{n0} . Summation over the radial number n is assumed.

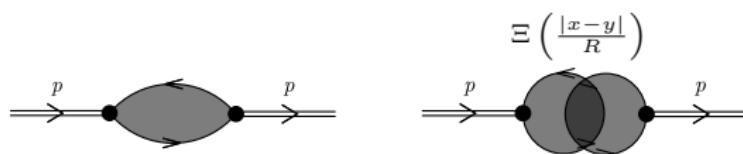
Asymptotic Regge spectrum :

$$M_n^2 \sim Bn, \quad n \gg 1$$

G.V. Efimov and S.N. , Phys. Rev. D 51 (1995)

$$M_l^2 \sim Bl, \ l \gg 1$$

η and η' !



Polarization operator

Polarization operation for $l = 0$:

$$\begin{aligned} \Pi_J^{nn'}(-M^2; m_f, m_{f'}; B) = \\ \frac{B}{4\pi^2} \text{Tr}_v \int_0^1 dt_1 \int_0^1 dt_2 \int_0^1 ds_1 \int_0^1 ds_2 \left(\frac{1-s_1}{1+s_1} \right)^{m_f^2/4vB} \left(\frac{1-s_2}{1+s_2} \right)^{m_{f'}^2/4vB} \times \\ \times t_1^n t_2^{n'} \frac{\partial^n}{\partial t_1^n} \frac{\partial^{n'}}{\partial t_2^{n'}} \frac{1}{\Phi_2^2} \left[\frac{M^2}{B} \frac{F_1^{(J)}}{\Phi_2^2} + \frac{m_f m_{f'}}{B} \frac{F_2^{(J)}}{(1-s_1^2)(1-s_2^2)} + \frac{F_3^{(J)}}{\Phi_2} \right] \exp \left\{ \frac{M^2}{2vB} \frac{\Phi_1}{\Phi_2} \right\}. \end{aligned}$$

$$\Phi_1 = s_1 s_2 + 2(\xi_1^2 s_1 + \xi_2^2 s_2)(t_1 + t_2)v,$$

$$\Phi_2 = s_1 + s_2 + 2(1 + s_1 s_2)(t_1 + t_2)v + 16(\xi_1^2 s_1 + \xi_2^2 s_2)t_1 t_2 v^2,$$

$$\begin{aligned} F_1^{(P)} = (1 + s_1 s_2) [2(\xi_1 s_1 + \xi_2 s_2)(t_1 + t_2)v + \\ 4\xi_1 \xi_2 (1 + s_1 s_2)(t_1 + t_2)^2 v^2 + s_1 s_2 (1 - 16\xi_1 \xi_2 t_1 t_2 v^2)], \end{aligned}$$

$$\begin{aligned} F_1^{(V)} = \left(1 - \frac{1}{3} s_1 s_2 \right) [s_1 s_2 + 16\xi_1 \xi_2 t_1 t_2 v^2 + 2(\xi_1 s_1 + \xi_2 s_2)(t_1 + t_2)v] + \\ 4\xi_1 \xi_2 (1 - s_1^2 s_2^2)(t_1 - t_2)^2 v^2, \end{aligned}$$

$$F_2^{(P)} = (1 + s_1 s_2)^2, \quad F_2^{(V)} = (1 - s_1^2 s_2^2),$$

$$F_3^{(P)} = 4v(1 + s_1 s_2)(1 - 16\xi_1 \xi_2 t_1 t_2 v^2), \quad F_3^{(V)} = 2v(1 - s_1 s_2)(1 - 16\xi_1 \xi_2 t_1 t_2 v^2).$$

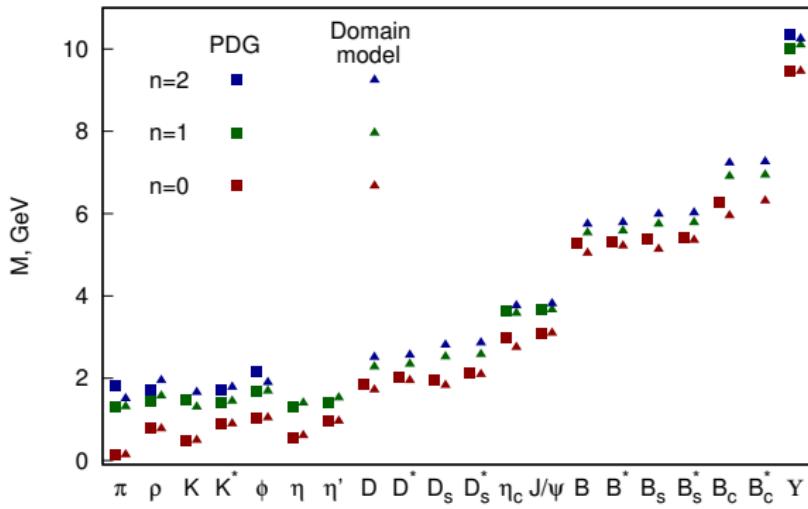


Table: Model parameters fitted to the masses of π , ρ , K , K^* , η' , J/ψ , Υ and used in calculation of all other quantities.

$m_{u/d}$, MeV	m_s , MeV	m_c , MeV	m_b , MeV	Λ , MeV	α_s	R , fm
145	376	1566	4879	416	3.45	1.12

Table: Masses of light mesons. \tilde{M} denotes the value in the chiral limit.

Meson	n	M_{exp} (MeV)	M (MeV)	\tilde{M} (MeV)	Meson	n	M_{exp} (MeV)	M (MeV)	\tilde{M} (MeV)
π	0	140	140	0	ρ	0	775	775	769
$\pi(1300)$	1	1300	1310	1301	$\rho(1450)$	1	1450	1571	1576
$\pi(1800)$	1	1812	1503	1466	ρ	2	1720	1946	2098
K	0	494	494	0	K^*	0	892	892	769
$K(1460)$	1	1460	1302	1301	$K^*(1410)$	1	1410	1443	1576
K	2		1655	1466	$K^*(1717)$	1	1717	1781	2098
η	0	548	621	0	ω	0	775	775	769
η'	0	958	958	872	ϕ	0	1019	1039	769
$\eta(1295)$	1	1294	1138	1361	$\phi(1680)$	1	1680	1686	1576
$\eta(1475)$	1	1476	1297	1516	ϕ	2	2175	1897	2098

Table: Masses of heavy-light mesons and their lowest radial excitations .

Meson	n	M_{exp} (MeV)	M (MeV)	Meson	n	M_{exp} (MeV)	M (MeV)
D	0	1864	1715	D^*	0	2010	1944
D	1		2274	D^*	1		2341
D	2		2508	D^*	2		2564
D_s	0	1968	1827	D_s^*	0	2112	2092
D_s	1		2521	D_s^*	1		2578
D_s	2		2808	D_s^*	2		2859
B	0	5279	5041	B^*	0	5325	5215
B	1		5535	B^*	1		5578
B	2		5746	B^*	2		5781
B_s	0	5366	5135	B_s^*	0	5415	5355
B_s	1		5746	B_s^*	1		5783
B_s	2		5988	B_s^*	2		6021
B_c	0	6277	5952	B_c^*	0		6310
B_c	1		6904	B_c^*	1		6938
B_c	2		7233	B_c^*	2		7260

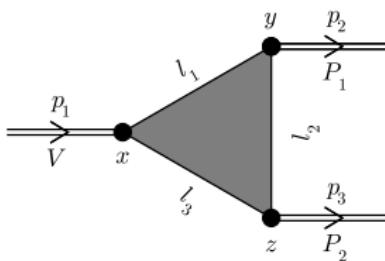
Table: Masses of heavy quarkonia.

Meson	n	M_{exp} (MeV)	M (MeV)
$\eta_c(1S)$	0	2981	2751
$\eta_c(2S)$	1	3639	3620
η_c	2		3882
$J/\psi(1S)$	0	3097	3097
$\psi(2S)$	1	3686	3665
$\psi(3770)$	2	3773	3810
$\Upsilon(1S)$	0	9460	9460
$\Upsilon(2S)$	1	10023	10102
$\Upsilon(3S)$	2	10355	10249

Table: Decay and transition constants of various mesons

Meson	n	f_P^{exp} (MeV)	f_P (MeV)	Meson	n	$g_{V\gamma}^{\text{exp}}$	$g_{V\gamma}$
π	0	130	140	ρ	0	0.2	0.2
$\pi(1300)$	1	—	29	ρ	1		0.034
K	0	156	175	ω	0	0.059	0.067
$K(1460)$	1	—	27	ω	1		0.011
D	0	205	212	ϕ	0	0.074	0.069
D	1	—	51	ϕ	1		0.025
D_s	0	258	274	J/ψ	0	0.09	0.057
D_s	1	—	57	J/ψ	1		0.024
B	0	191	187	Υ	0	0.025	0.011
B	1	—	55	Υ	1		0.0039
B_s	0	253	248				
B_s	1	—	68				
B_c	0	489	434				
B_c	1		135				

Strong decays: $gVPP$



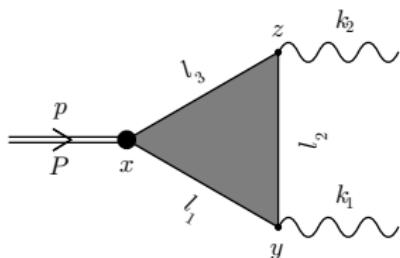
Decay	g_{VPP} [*]	g_{VPP}
$\rho^0 \rightarrow \pi^+ \pi^-$	5.95	7.58
$\omega \rightarrow \pi^+ \pi^-$	0.17	0
$K^{*\pm} \rightarrow K^\pm \pi^0$	3.23	3.54
$K^{*\pm} \rightarrow K^0 \pi^\pm$	4.57	5.01
$\varphi \rightarrow K^+ K^-$	4.47	5.02
$D^{*\pm} \rightarrow D^0 \pi^\pm$	8.41	7.9
$D^{*\pm} \rightarrow D^\pm \pi^0$	5.66	5.59

local color
gauge
invariance

[*] K.A. Olive et al. (Particle Data Group) Chinese Phys. C 38,090001, 2014

Pion transition form factor

$$T_a^{\mu\nu}(x, y, z) = h_P \sum_n u_n^a \int d\sigma_B \text{Tr} t_a e_f^2 V^n(x) \gamma_5 S(x, y|B) \gamma_\mu S(y, z|B) \gamma_\nu S(z, x|B),$$



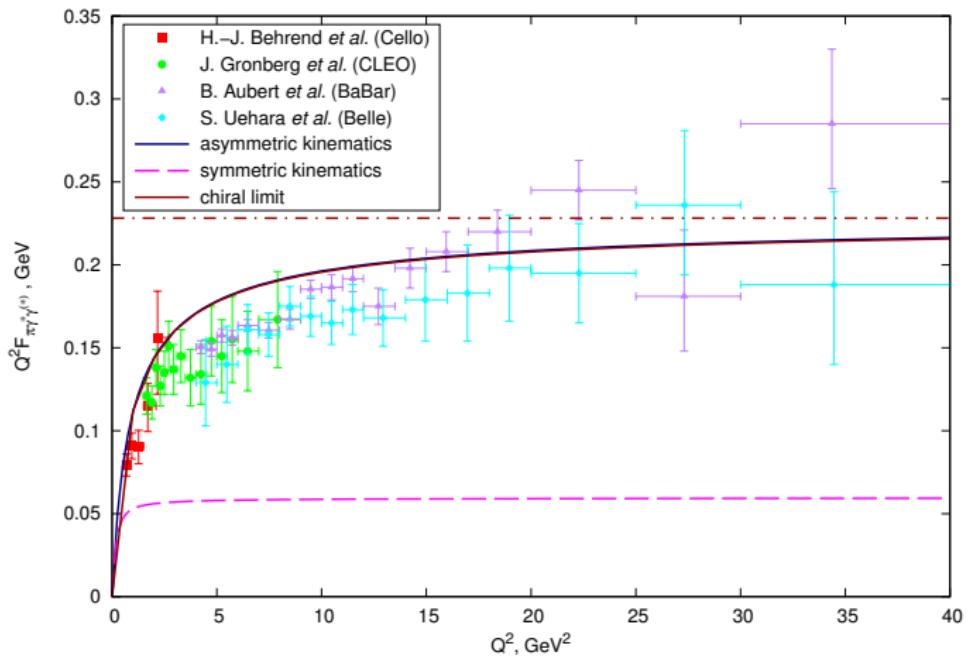
In momentum representation, the diagram has the following structure:

$$T_a^{\mu\nu}(p^2, k_1^2, k_2^2) = ie^2 \delta^{(4)}(p - k_1 - k_2) \epsilon_{\mu\nu\alpha\beta} k_{1\alpha} k_{2\beta} T_a(p^2, k_1^2, k_2^2).$$

$$F_{P\gamma}(Q^2) = T(-M_P^2, Q^2, 0).$$

$$\Gamma(P \rightarrow \gamma\gamma) = \frac{\pi}{4} \alpha^2 M_P^3 g_{P\gamma\gamma}^2$$

$$g_{P\gamma\gamma} = T(-M_P^2, 0, 0) = F_{P\gamma}(0).$$



$$g_{\pi\gamma\gamma} = 0.272 \text{GeV}^{-1} \quad (g_{\pi\gamma\gamma}^{\text{exp}} = 0.274 \text{GeV}^{-1}).$$

$$F_{\pi\gamma^*\gamma^*}(Q^2) = T(-M_P^2, Q^2, Q^2).$$

$$F_{\pi\gamma^*\gamma} \sim \kappa_{\gamma^*\gamma} \frac{\sqrt{2}f_\pi}{Q^2}, \quad \kappa_{\gamma^*\gamma} = 1.23, \quad (2)$$

$Q^2 F_{\pi\gamma^*\gamma}$ approaches a constant value at large Q^2 in qualitative agreement with factorization prediction, but the value of constant $\kappa_{\gamma^*\gamma}$ substantially differs from unity.

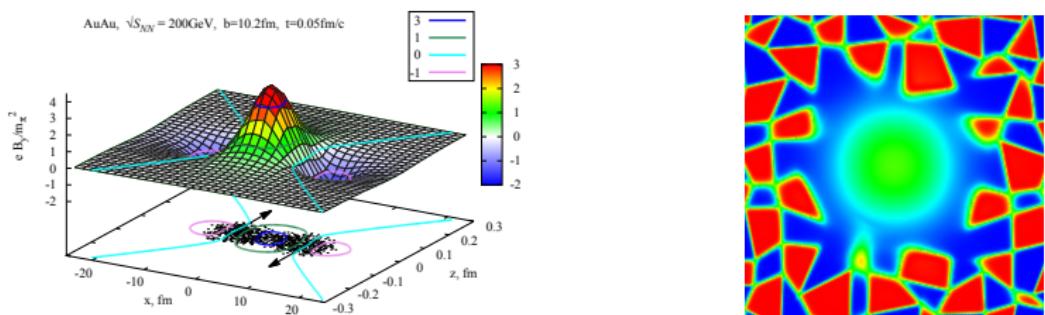
At the same time, asymptotic behavior of form factor in symmetric kinematics with two photons with equal virtuality Q^2 ,

$$F_{\pi\gamma^*\gamma^*} \sim \kappa_{\gamma^*\gamma^*} \frac{\sqrt{2}f_\pi}{3Q^2}, \quad \kappa_{\gamma^*\gamma^*} = 1. \quad (3)$$

”Polarization” of QCD vacuum by the strong electromagnetic fields

- Relativistic heavy ion collisions - strong electromagnetic fields

V. Skokov, A. Y. Illarionov and V. Toneev, *Int. J. Mod. Phys. A* **24** (2009) 5925
V. Voronyuk, V. D. Toneev, W. Cassing, E. L. Bratkovskaya,
V. P. Konchakovski and S. A. Voloshin, *Phys. Rev C* **84** (2011)



Strong electro-magnetic field plays catalyzing role for deconfinement and anisotropies!

One-loop quark contribution to the effective potential in the presence of arbitrary homogenous Abelian fields

$$U_{\text{eff}}(G) = -\frac{1}{V} \ln \frac{\det(iD - m)}{\det(i\partial - m)} = \frac{1}{V} \int_V d^4x \text{Tr} \int_m^\infty dm' [S(x, x|m') - S_0(x, x|m')] |$$

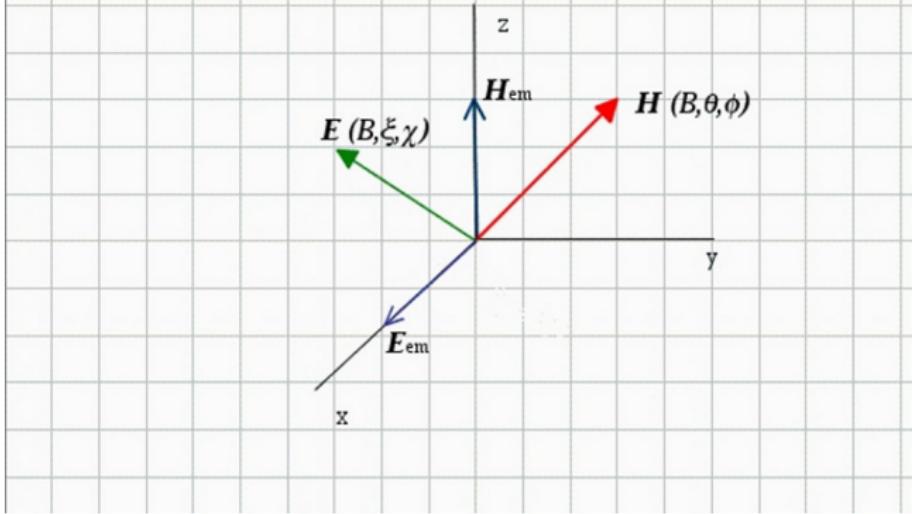
$$U_{\text{eff}}^{\text{ren}}(G) = \frac{B^2}{8\pi^2} \int_0^\infty \frac{ds}{s^3} \text{Tr}_n \left[s\varkappa_+ \coth(s\varkappa_+) s\varkappa_- \coth(s\varkappa_-) - \mathbf{1} - \frac{s^2}{3} (\varkappa_+^2 + \varkappa_-^2) \right] e^{-\frac{m^2}{B}s},$$

$$\varkappa_\pm = \frac{1}{2B} \sqrt{\mathcal{Q}\sigma_\pm} = \frac{1}{2B} \left(\sqrt{2(\mathcal{R} + \mathcal{Q})} \pm \sqrt{2(\mathcal{R} - \mathcal{Q})} \right),$$

$$\mathcal{R} = (H^2 - E^2)/2 + \hat{n}^2 B^2 + \hat{n}B(H \cos(\theta) + iE \cos(\chi) \sin(\xi))$$

$$\mathcal{Q} = \hat{n}BH \cos(\xi) + i\hat{n}BE \sin(\theta) \cos(\phi) + \hat{n}^2 B^2 (\sin(\theta) \sin(\xi) \cos(\phi - \chi) + \cos(\theta) \cos(\xi))$$

Y. M. Cho and D. G. Pak, Phys.Rev. Lett., 6 (2001) 1047

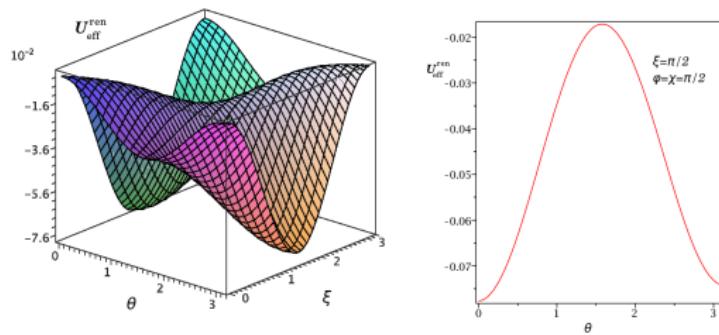


$$H_i = H\delta_{i3}, \quad E_j = E\delta_{j1}, \quad H^c = \{B, \theta, \phi\}, \quad E^c = \{B, \xi, \chi\}$$

$H \neq 0$, $E \neq 0$ and arbitrary gluon field

$$\Im(U_{\text{eff}}) = 0 \implies \cos(\chi)\sin(\xi) = 0, \sin(\theta)\cos(\phi) = 0$$

Effective potential (in units of $B^2/8\pi^2$) for the electric $E = .5B$ and the magnetic $H = .9B$ fields as functions of angles θ and ξ ($\phi = \chi = \pi/2$)



Minimum is at $\theta = \pi$ and $\xi = \pi/2$:

orthogonal to each other chromomagnetic and chromoelectric fields: $\mathcal{Q} = 0$.

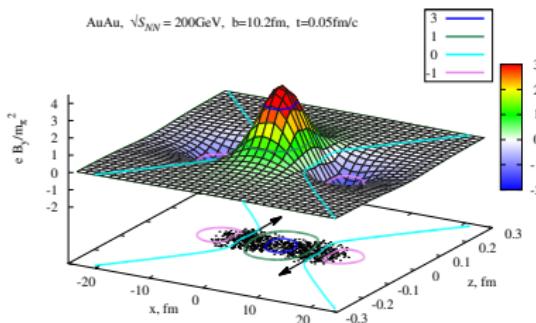
Strong electro-magnetic field plays catalyzing role for deconfinement and anisotropies?!

B.V. Galilo and S.N., Phys. Rev. D84 (2011) 094017.

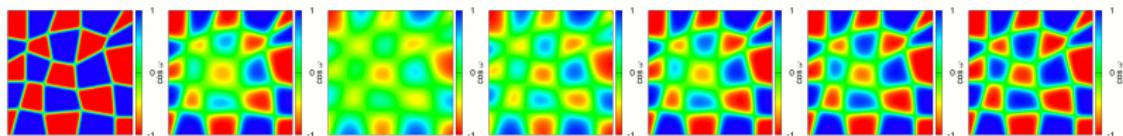
M. D'Elia, M. Mariti and F. Negro, Phys. Rev. Lett. **110**, 082002 (2013)

G. S. Bali, F. Bruckmann, G. Endrodi, F. Gruber and A. Schaefer, JHEP **1304**, 130 (2013)

V. Voronyuk, V. D. Toneev, W. Cassing, E. L. Bratkovskaya,
V. P. Konchakovski and S. A. Voloshin, Phys. Rev C 84 (2011)

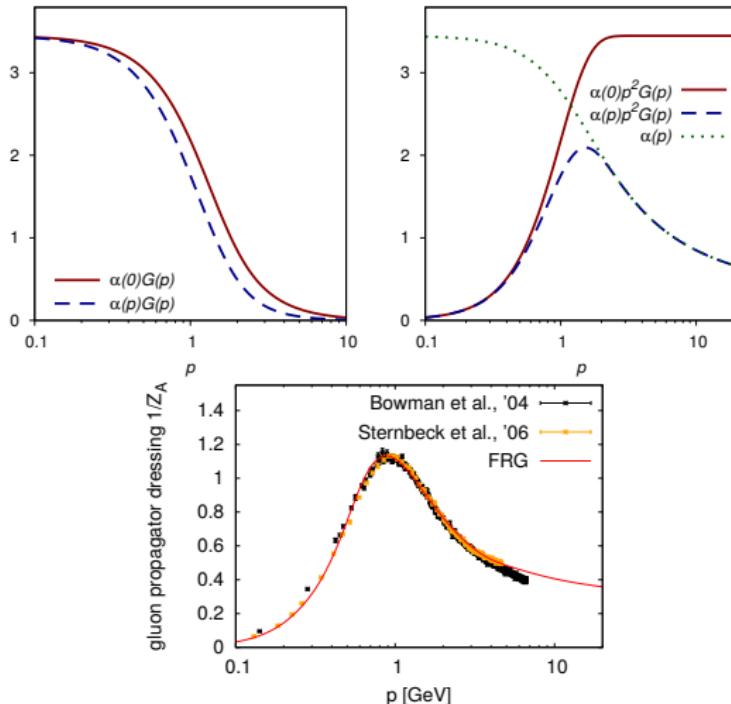


Magnetic field $eB \gtrsim m_\pi^2$ in the region $5fm \times 5fm \times .2fm \times .2fm/c$

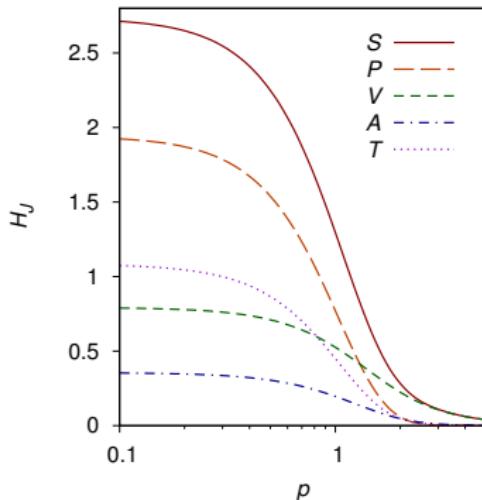


Green region ("Spaghetti vacuum") and the color charged quasi-particles

"Projection" to other methods/models



Functional RG, DSE, Lattice QCD



$$\tilde{H}(p) = \frac{m}{2v\Lambda^2} \mathcal{H}_S(p^2) \mp \gamma_5 \frac{m}{2v\Lambda^2} \mathcal{H}_P(p^2) + \gamma_\alpha \frac{p_\alpha}{2v\Lambda^2} \mathcal{H}_V(p^2) \pm i\gamma_5 \gamma_\alpha \frac{f_{\alpha\beta} p_\beta}{2v\Lambda^2} \mathcal{H}_A(p^2) + \sigma_{\alpha\beta} \frac{mf_{\alpha\beta}}{4v\Lambda^2} \mathcal{H}_T(p^2). \quad (4)$$

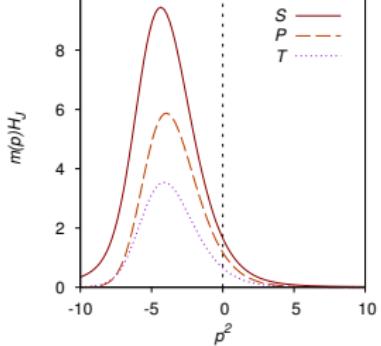
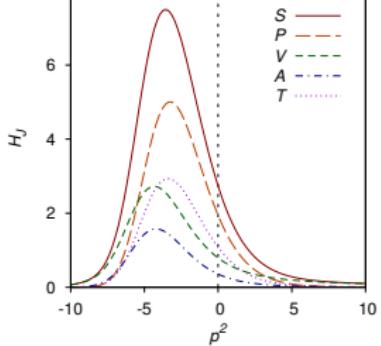
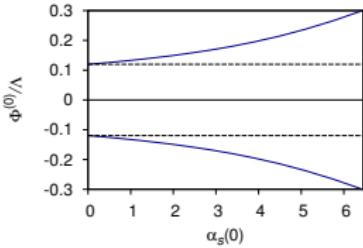


Figure: Scalar quark condensate (LHS). Momentum dependence of the scalar (solid line), pseudoscalar (long dash), vector (dash), axial (dash dot) and tensor (dot) form factors (central plot) in the quark propagator (6), and scalar, pseudoscalar and tensor form factors (RHS plot) multiplied by the quark mass.

$$\Lambda^2 \Phi_{\mathcal{Q}_1}^{(0)} = \sum_{k=1}^{\infty} \frac{g^k}{k} \sum_{\mathcal{Q}_1 \dots \mathcal{Q}_k} \Phi_{\mathcal{Q}_2}^{(0)} \dots \Phi_{\mathcal{Q}_k}^{(0)} \Gamma_{\mathcal{Q}_1 \dots \mathcal{Q}_k}^{(k)},$$

$$m(p) = \bar{m}(0) F_{00}(p^2), \quad F_{00}(p) = \left[1 - \exp \left(-\frac{p^2}{\Lambda^2} \right) \right] \frac{\Lambda^2}{p^2}, \quad \bar{m}(0) = \frac{1}{3} g \Phi^{(0)}, \quad (5)$$

$$\begin{aligned} \tilde{H}(p) = & \frac{m}{2v\Lambda^2} \mathcal{H}_S(p^2) \mp \gamma_5 \frac{m}{2v\Lambda^2} \mathcal{H}_P(p^2) + \gamma_\alpha \frac{p_\alpha}{2v\Lambda^2} \mathcal{H}_V(p^2) \pm i\gamma_5 \gamma_\alpha \frac{f_{\alpha\beta} p_\beta}{2v\Lambda^2} \mathcal{H}_A(p^2) \\ & + \sigma_{\alpha\beta} \frac{mf_{\alpha\beta}}{4v\Lambda^2} \mathcal{H}_T(p^2). \end{aligned} \quad (6)$$

DSE in combination with Bethe-Salpeter approach

S. Kubrak, C. S. Fischer and R. Williams, arXiv:1412.5395 [hep-ph]
C. S. Fischer, S. Kubrak and R. Williams, Eur. Phys. J. A **51**, no. 1, 10 (2015) [arXiv:1409.5076 [hep-ph]]
C. S. Fischer, S. Kubrak and R. Williams, Eur. Phys. J. A **50**, 126 (2014) [arXiv:1406.4370 [hep-ph]].

S. M. Dorkin, L. P. Kaptari and B. Kampfer, arXiv:1412.3345 [hep-ph]
S. M. Dorkin, L. P. Kaptari, T. Hilger and B. Kampfer, Phys. Rev. C **89**, no. 3, 034005 (2014)
[arXiv:1312.2721 [hep-ph]]

$$S^{-1}(p) = Z_2 S_0^{-1}(p) + 4\pi Z_2^2 C_F \int \frac{d^4 k}{(2\pi)^4} \gamma^\mu S(k+p) \gamma^\nu (\delta_{\mu\nu} - k_\mu k_\nu / k^2) \frac{\alpha_{\text{eff}}(k^2)}{k^2},$$
$$\alpha_{\text{eff}}(q^2) = \pi \eta^7 x^2 e^{-\eta^2 x} + \frac{2\pi \gamma_m (1 - e^{-y})}{\ln [e^2 - 1 + (1+z)^2]}, \quad x = q^2/\Lambda^2, \quad y = q^2/\Lambda_t^2, \quad z = q^2/\Lambda_{\text{QCD}}^2$$

Harmonic confinement - 4-dim. oscillator

R. P. Feynman, M, Kislinger, and F. Ravndal, Phys. Rev. D **3** (1971) 2706.

H. Leutwyler and J. Stern, "Harmonic Confinement: A Fully Relativistic Approximation to the Meson Spectrum," Phys. Lett. B **73** (1978) 75;

H. Leutwyler and J. Stern, "Relativistic Dynamics on a Null Plane," Annals Phys. **112** (1978) 94.

Laguerre polynomials

$$\begin{aligned} \mathcal{S}_2 = & -\frac{1}{2} \int d^4x \int d^4z D(z) \Phi_{Jc}^2(x, z) \\ & -2g^2 \int d^4x d^4x' d^4z d^4z' D(z) D(z') \Phi_{Jc}(x, z) \Pi_{Jc, J'c'}(x, x'; z, z') \Phi_{J'c'}(x', z'), \\ \Phi^{aJ}(x, z) = & \sum_{nl} (z^2)^{l/2} \varphi^{nl}(z) \Phi^{aJln}(x). \end{aligned}$$

Soft-wall AdS/QCD models

A. Karch, E. Katz, D. T. Son and M. A. Stephanov, Phys. Rev. D **74**, 015005 (2006) [hep-ph/0602229]
T. Branz, T. Gutsche, V. E. Lyubovitskij, I. Schmidt and A. Vega, Phys. Rev. D **82**, 074022 (2010)
[arXiv:1008.0268 [hep-ph]].

G. F. de Teramond and S. J. Brodsky, Phys. Rev. Lett. **102**, 081601 (2009)

$$S_\Phi = \frac{(-1)^J}{2} \int d^d x \ dz \left(\frac{R}{z} \right)^{d+1} e^{-\kappa^2 z^2} \left(\partial_N \Phi_J \partial^N \Phi_J - \mu_J^2(z) \Phi_J \Phi^J \right)$$

$$M_{nJ} = 4\kappa^2 \left(n + \frac{l+J}{2} \right) \quad \Phi_J(x, z) = \sum_n \phi_{nJ} \Phi_{nJ}(x), \\ \phi_{nJ} = R^{J-(d-1)/2} \kappa^{1+l} z^{l-J+2} L_n^l(\kappa^2 z^2)$$

G.V. Efimov and S.N. Nedelko, Phys. Rev. D **51**, 176 (1995)

$$J^{aJ}(x, z) = \sum_{nl} (z^2)^{l/2} f_J^{nl}(z) J_J^{aJln}(x), \quad J_J^{aJln}(x) = \bar{q}(x) V_J^{aJln}(x) q(x)$$

$$M_n^2 \propto Bn \quad f_{\mu_1 \dots \mu_l}^{nl} = L_n^l(z^2) T_{\mu_1 \dots \mu_l}^{(l)}(nz), \quad nz = \frac{z}{\sqrt{z}}, \\ M_l^2 \propto Bl \quad \int_0^\infty du \rho_l(u) L_n^l(u) L_{n'}^l(u) = \delta_{nn'}, \quad \rho_l(u) = u^l e^{-u}.$$

Strong electromagnetic fields, finite temperature and density

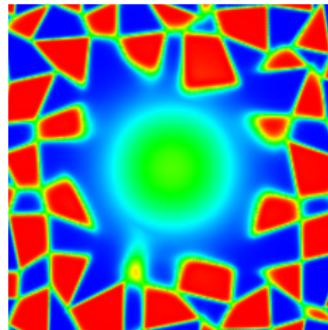
Two-stage phase transformation:

I. Colorless hadrons: $\langle |F\tilde{F}| \rangle = \langle F^2 \rangle^2 \neq 0$

→ II. Charged quasiparticles: $\langle |F\tilde{F}| \rangle = 0, \langle F^2 \rangle \neq 0$

→ III Weakly interacting QGP: $\langle |F\tilde{F}| \rangle = 0, \langle F^2 \rangle = 0$

Free energy density $F(B, R; H_{\text{em}}, T, \mu)$, EOS, ...: finite size effects, anisotropy, instabilities, ...



Summary

$\langle g^2 F^2 \rangle \neq 0 \longrightarrow$ domain wall network, almost everywhere abelian (anti-)self-dual gluon fields.

An ensemble of almost everywhere Abelian homogeneous (anti-)self-dual gluon fields represented by the domain wall networks looks like a suitable framework for studying mechanisms of confinement (both static and dynamical quarks), chiral symmetry realisation and hadronization.

Background of domain wall networks - harmonic confinement.

Meson effective action - quantitatively correct phenomenology both with respect to confinement and chiral symmetry.

Polarization effects in QCD vacuum due to the strong electromagnetic fields, deconfinement, chiral symmetry restoration.

Electromagnetic fields as trigger of deconfinement. Confirmed by lQCD calculations.

Quark and gluon propagators - qualitative agreement with FRG and DSE.

As a matter of fact the picture of confinement of dynamical quarks based on harmonic oscillator enters many approaches.