

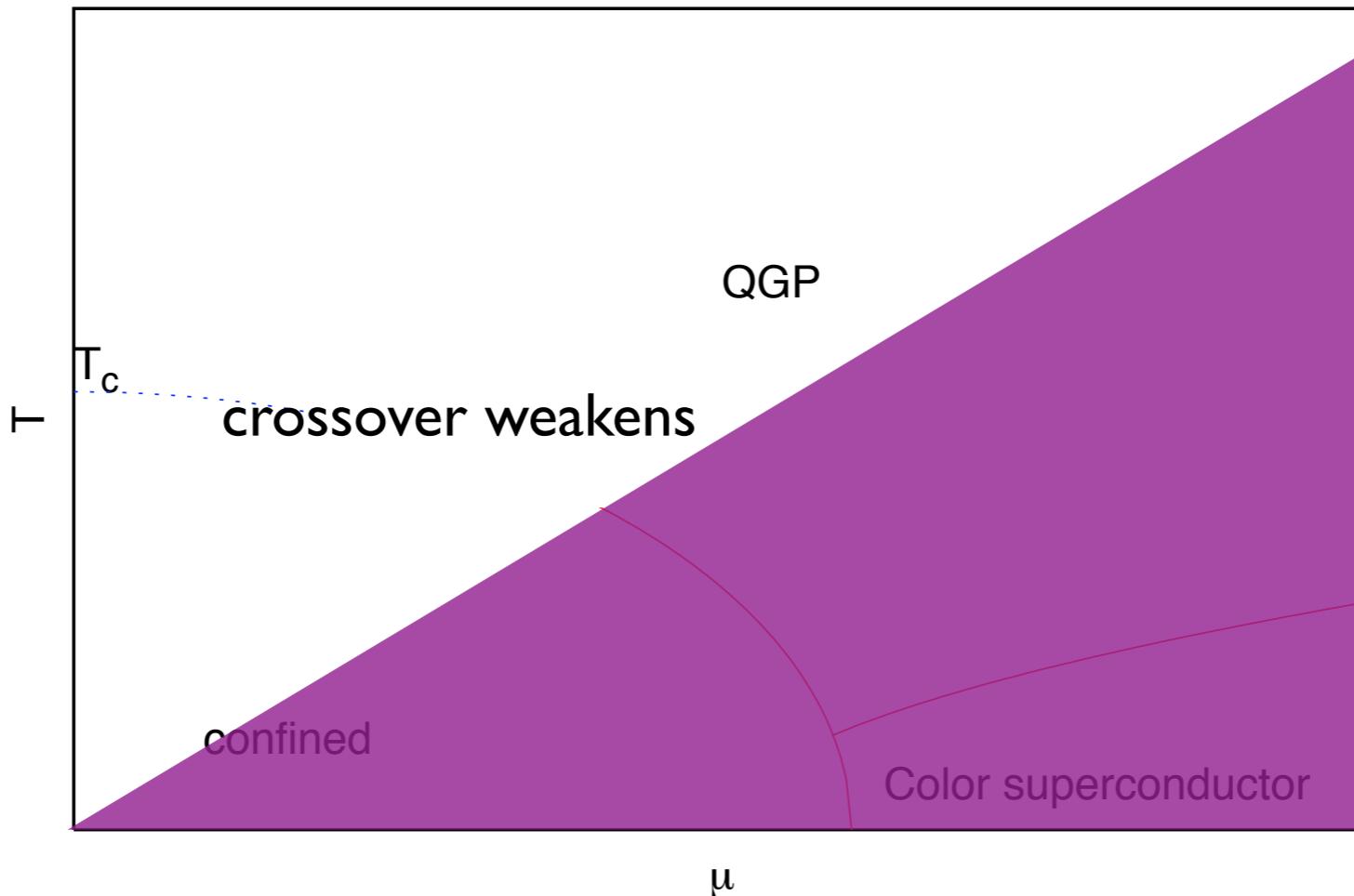
QCD in the heavy dense regime: Large N_c and quarkyonic matter

Owe Philipsen



- Review: effective lattice theory for finite density QCD
- The nuclear liquid gas transition
- What happens at large N_c

The lattice-calculable region of the phase diagram



- Sign problem prohibits direct simulation, circumvented by approximate methods: reweighting, Taylor expansion, imaginary chem. pot., need $\mu/T \lesssim 1$ ($\mu = \mu_B/3$)
- No critical point in the controllable region

Effective lattice theory for heavy dense QCD

O.P. with Fromm, Langelage, Lottini, Neuman, Glesaaen

- Two-step treatment:

- I. Calculate effective theory analytically
- II. Simulate effective theory

- Step I.: split temporal and spatial link integrations:

$$Z = \int DU_0 DU_i \det Q e^{S_g[U]} \equiv \int DU_0 e^{-S_{eff}[U_0]} = \int DL e^{-S_{eff}[L]}$$

Spatial integration after analytic strong coupling and hopping expansion $\sim \frac{1}{g^2}, \frac{1}{m_q}$

- Step II.: mild sign problem of effective theory
- Analytic solution by linked cluster expansion

Effective theory: start from Wilson's lattice action

Pure gauge part: character expansion

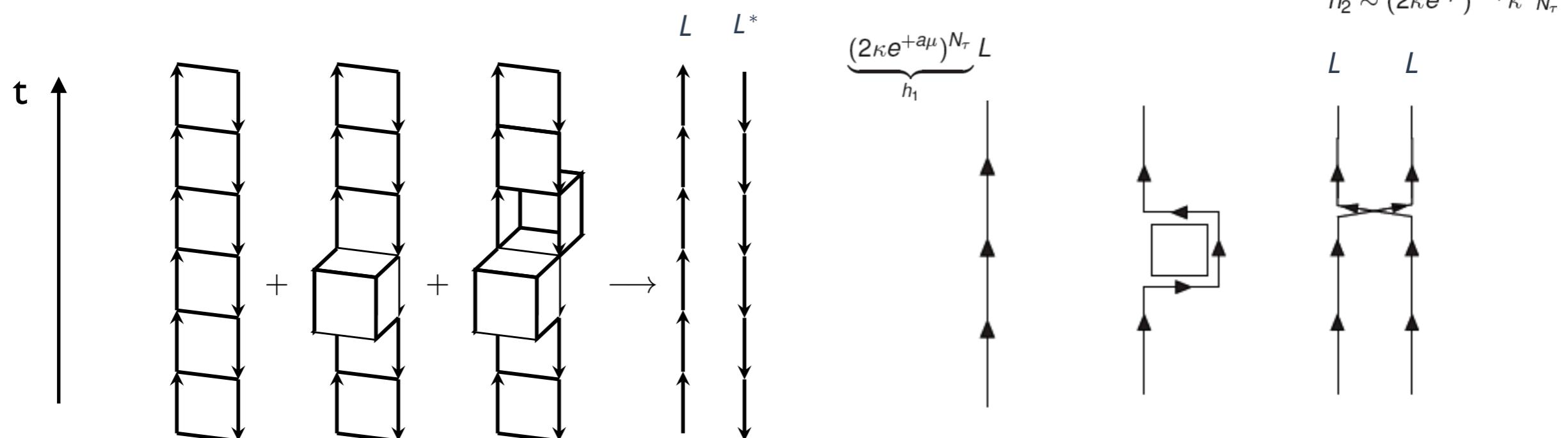
$$u(\beta) = \frac{\beta}{18} + \frac{\beta^2}{216} + \dots < 1$$

$$\beta = \frac{2N_c}{g^2} \quad T = \frac{1}{aN_\tau}$$

Fermion determinant: hopping expansion

$$\kappa = \frac{1}{2am + 8}$$

Generates couplings over all distances, n-pt. couplings, higher reps....:



$$\lambda(u, N_\tau \geq 5) = u^{N_\tau} \exp \left[N_\tau \left(4u^4 + 12u^5 - 14u^6 - 36u^7 + \frac{295}{2}u^8 + \frac{1851}{10}u^9 + \frac{1055797}{5120}u^{10} \right) \right]$$

The effective 3d theory

$$-\mathcal{S}_{\text{eff}} = \sum_i \lambda_i(u, \kappa, N_\tau) S_i^S - 2N_f \sum_i [h_i(u, \kappa, \mu, N_\tau) S_i^A + \bar{h}_i(u, \kappa, \mu, N_\tau) S_i^{\dagger A}]$$

↑
effective couplings →
 $S_i^{A,S} = S_i^{A,S}[L, L^*]$

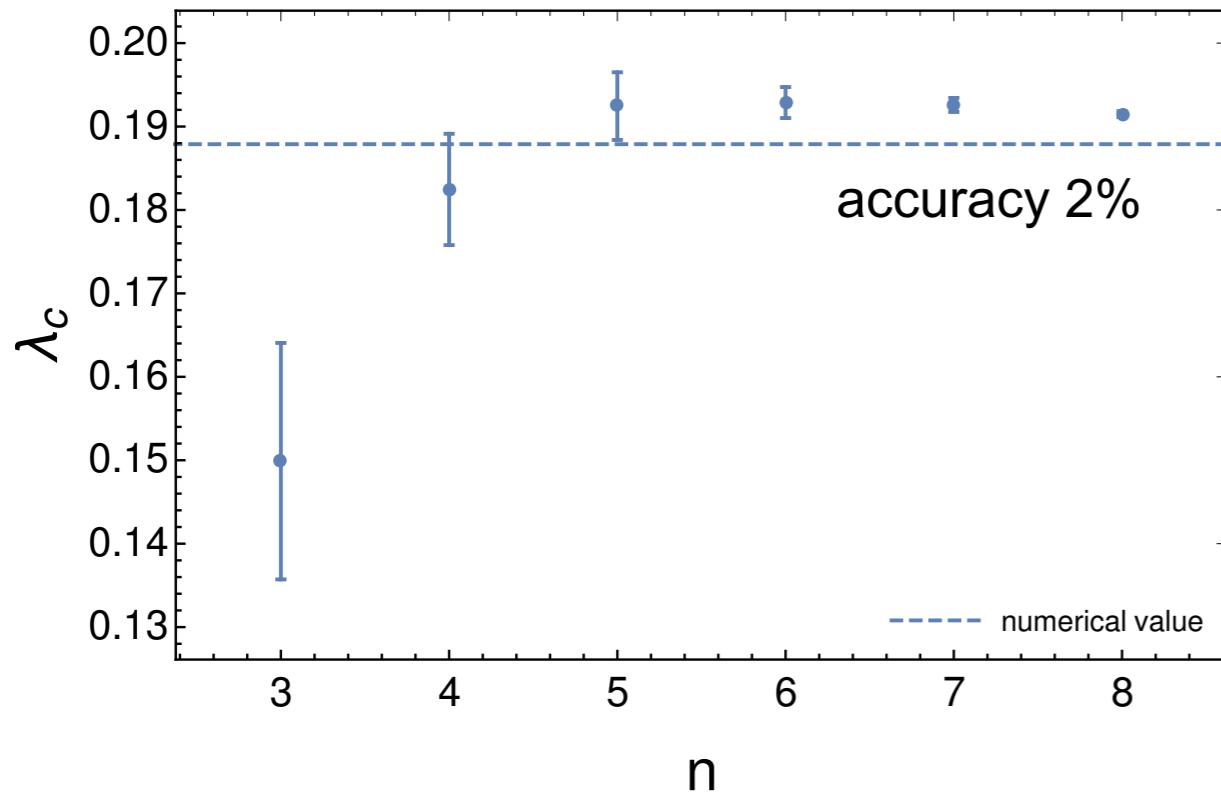
This is a 3d continuous spin model!

cf. Svetitsky-Yaffe conjecture for universality of SU(N) Yang-Mills

$$\begin{aligned} Z &= \int DW \prod_{<\mathbf{x}, \mathbf{y}>} [1 + \lambda(L_{\mathbf{x}}L_{\mathbf{y}}^* + L_{\mathbf{x}}^*L_{\mathbf{y}})] & L &= \text{Tr}W \\ &\times \prod_{\mathbf{x}} [1 + h_1 L_{\mathbf{x}} + h_1^2 L_{\mathbf{x}}^* + h_1^3]^{2N_f} [1 + \bar{h}_1 L_{\mathbf{x}}^* + \bar{h}_1^2 L_{\mathbf{x}} + \bar{h}_1^3]^{2N_f} \\ &\times \prod_{<\mathbf{x}, \mathbf{y}>} \left(1 - h_2 \text{Tr} \frac{h_1 W_{\mathbf{x}}}{1 + h_1 W_{\mathbf{x}}} \text{Tr} \frac{h_1 W_{\mathbf{y}}}{1 + h_1 W_{\mathbf{y}}} \right) \left(1 - h_2 \text{Tr} \frac{\bar{h}_1 W_{\mathbf{x}}^\dagger}{1 + \bar{h}_1 W_{\mathbf{x}}^\dagger} \text{Tr} \frac{\bar{h}_1 W_{\mathbf{y}}^\dagger}{1 + \bar{h}_1 W_{\mathbf{y}}^\dagger} \right) \dots \end{aligned}$$

Yang-Mills transition by series expansion

Solution of eff.th.

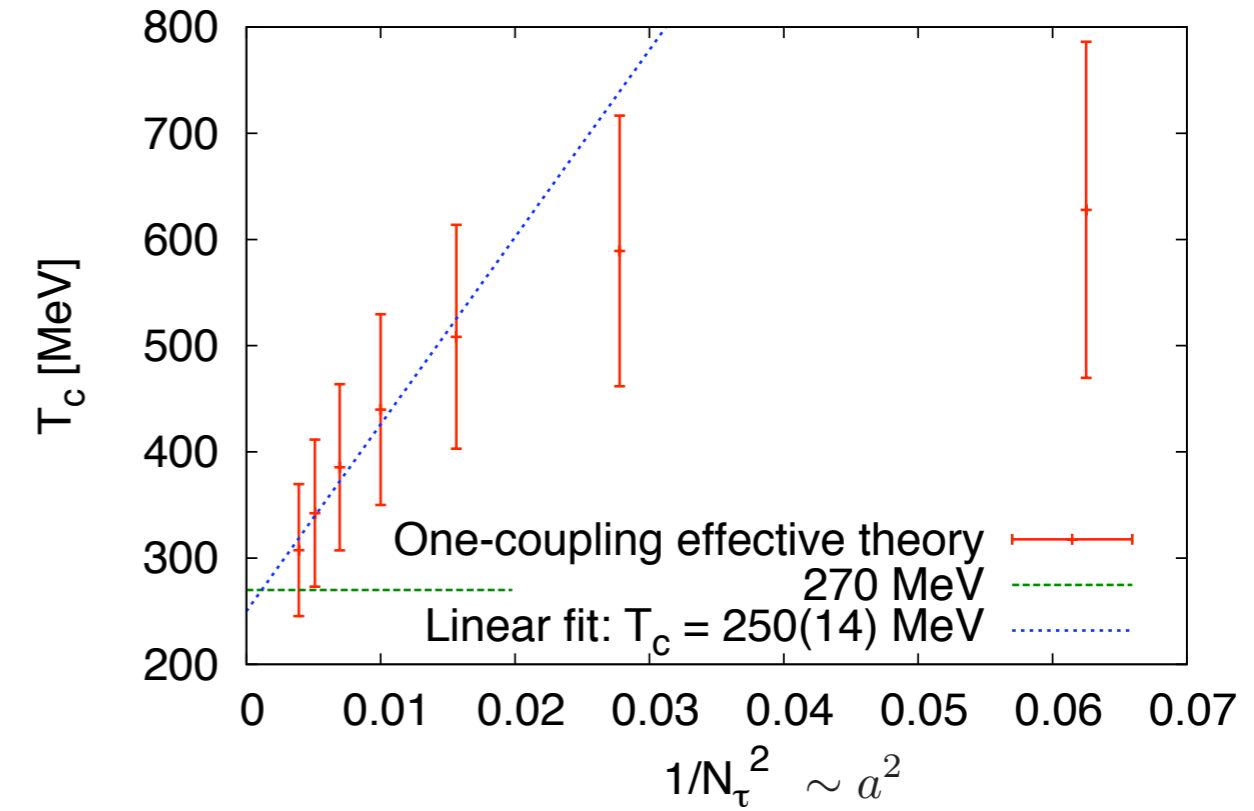
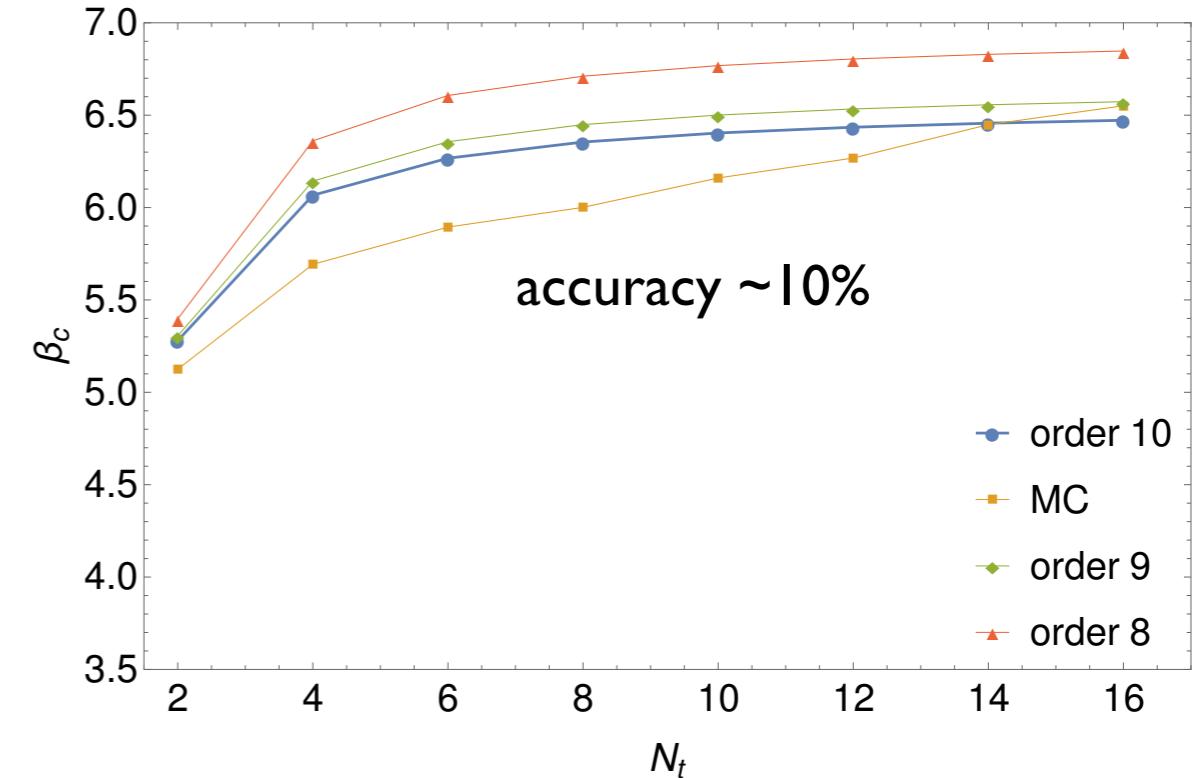


order of expansion

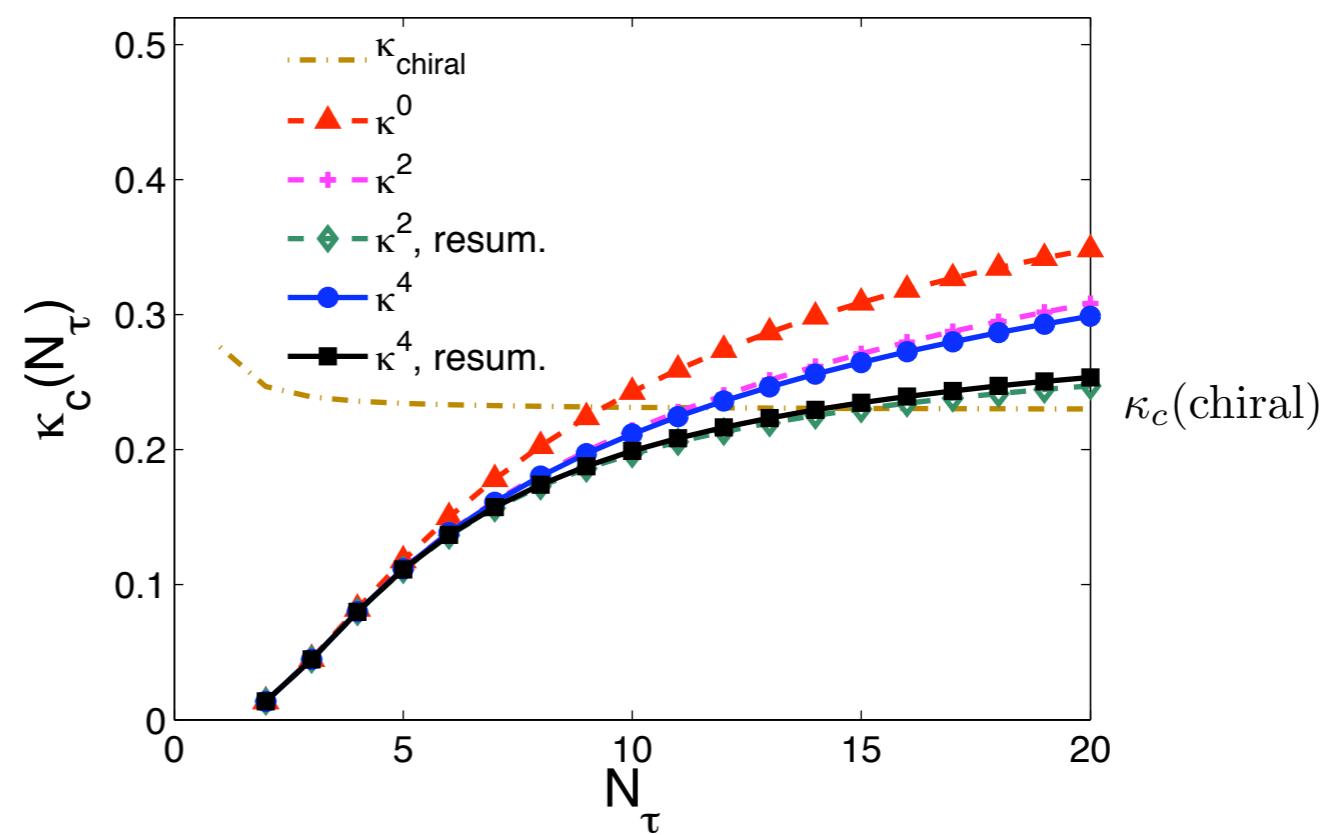
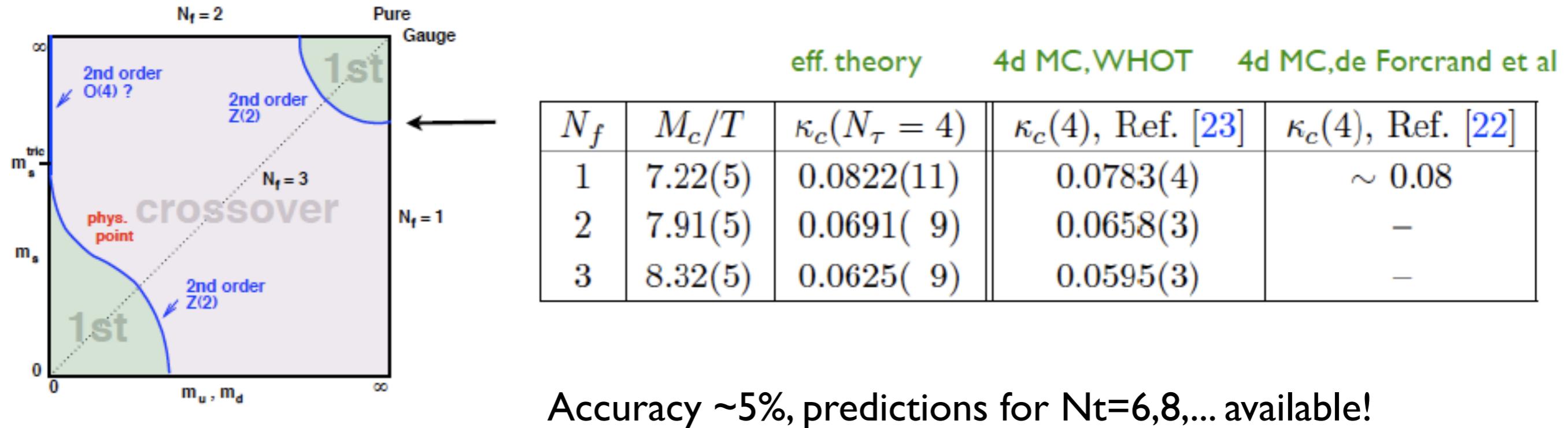
Two calculations:

- 1. by “hand” (Q. Pham, J. Scheunert, GU)
- 2. automatic graph generation (J. Kim, GU)

Conversion to 4d YM

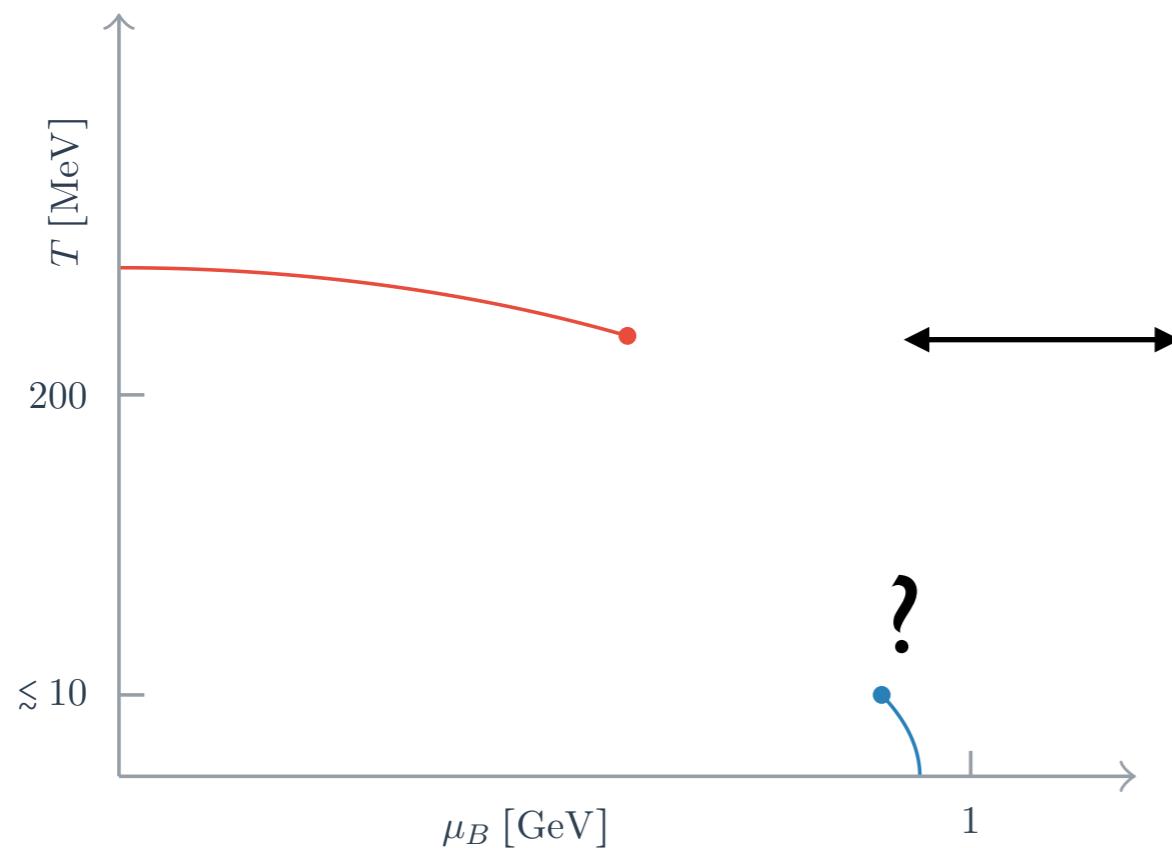


The deconfinement transition for heavy quarks

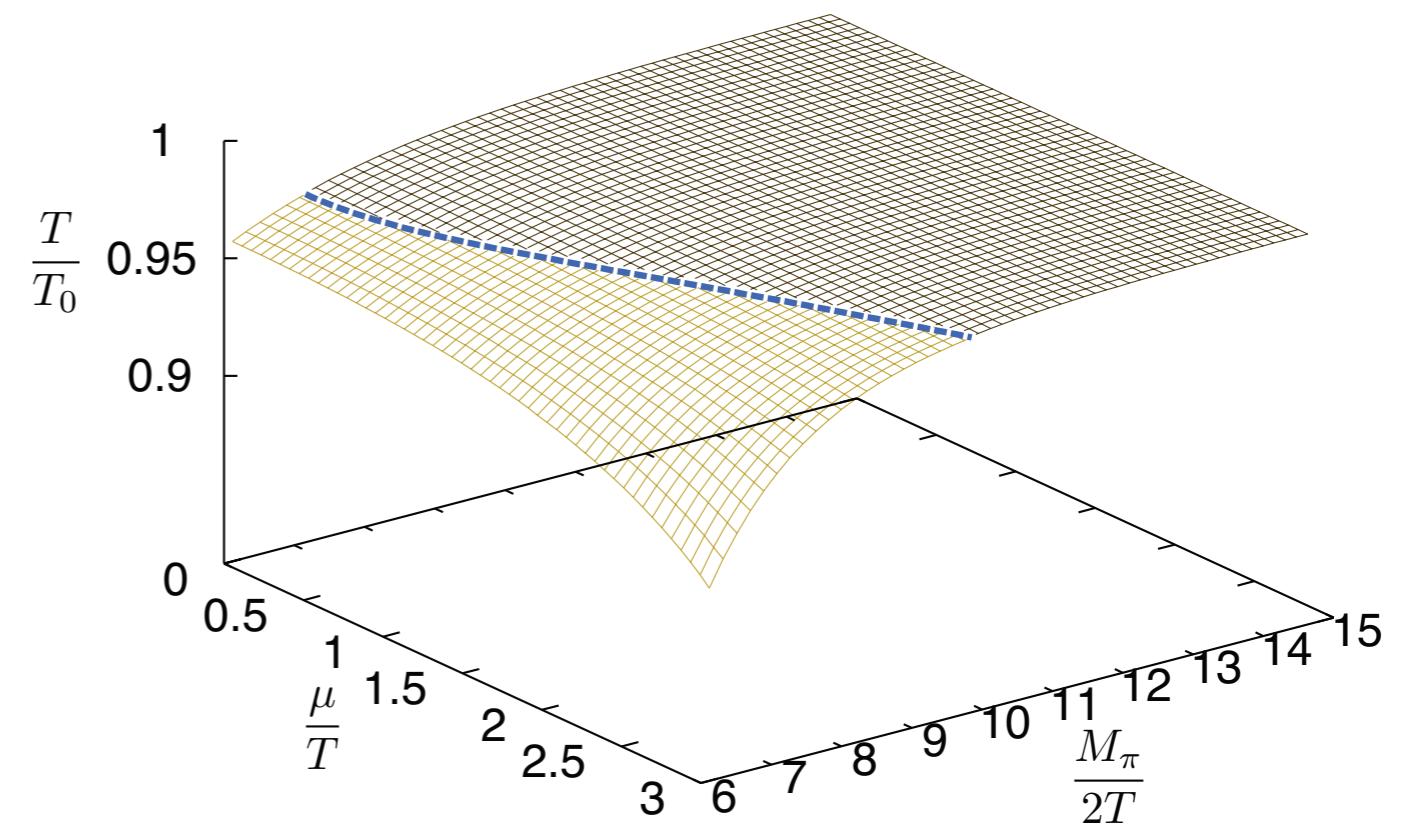


The fully calculated deconfinement transition

"Heavy QCD" phase diagram



Same phase structure:
continuum, functional methods:
Fischer, Lücker, Pawłowski 15



Fromm, Langelage, Lottini, O.P. 11

Cold and dense: static strong coupling limit

Fromm, Langelage, Lottini, Neuman, O.P., PRL 13

T=0: anti-fermions decouple:

$$h_1 = (2\kappa e^{a\mu})^{N_\tau} = e^{\frac{\mu-m}{T}}$$
$$\bar{h}_1 = (2\kappa e^{-a\mu})^{N_\tau} = e^{\frac{-\mu-m}{T}}$$

$$Z(\beta = 0) \xrightarrow{T \rightarrow 0} \left[\prod_f \int dW (1 + h_1 L + h_1^2 L^* + h_1^3)^2 \right]^V = z_0^V$$

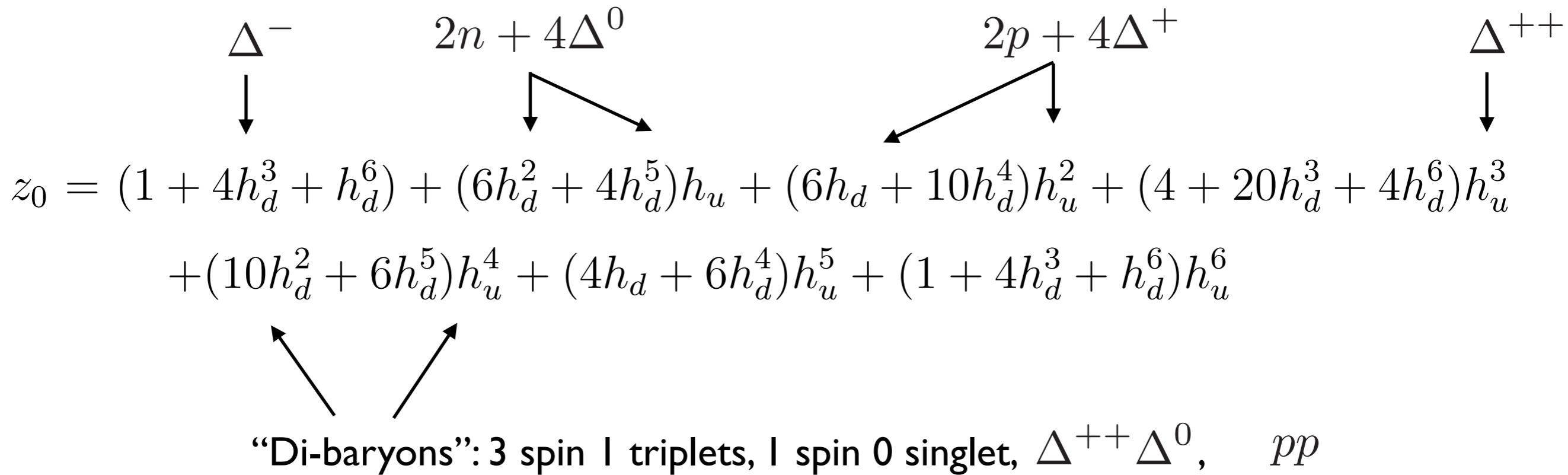
$$N_f = 1 : \quad z_0 = 1 + 4h_1^3 + h_1^6 \quad \text{free baryon gas}$$

↑ ↑
spin 3/2, 0

Silver blaze phenomenon + Pauli principle: $\lim_{T \rightarrow 0} a^3 n = \begin{cases} 0, & \mu < m \\ 2N_c, & \mu > m \end{cases}$

1st order phase transition from vacuum to saturated baryon crystal

$N_f = 2$:

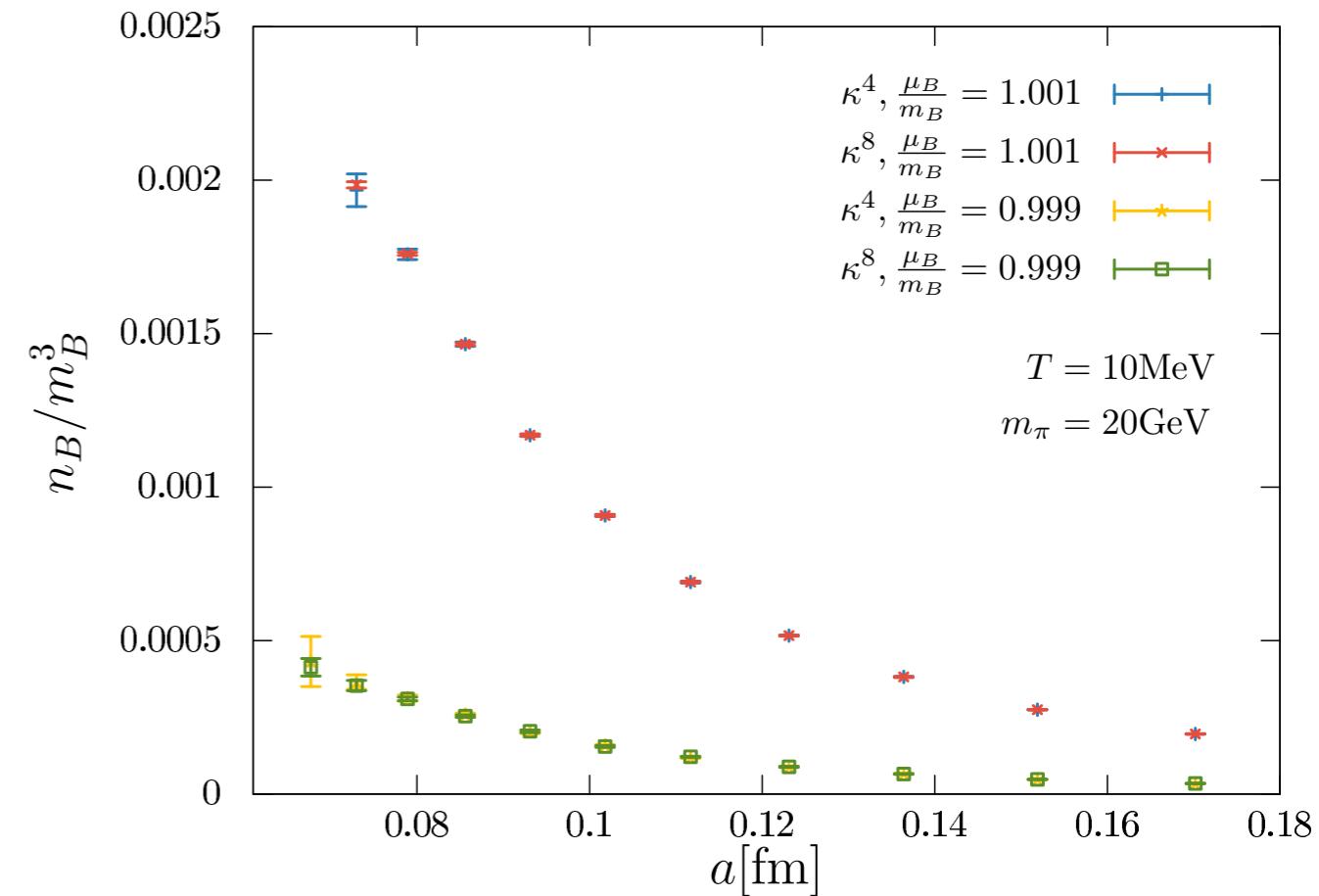
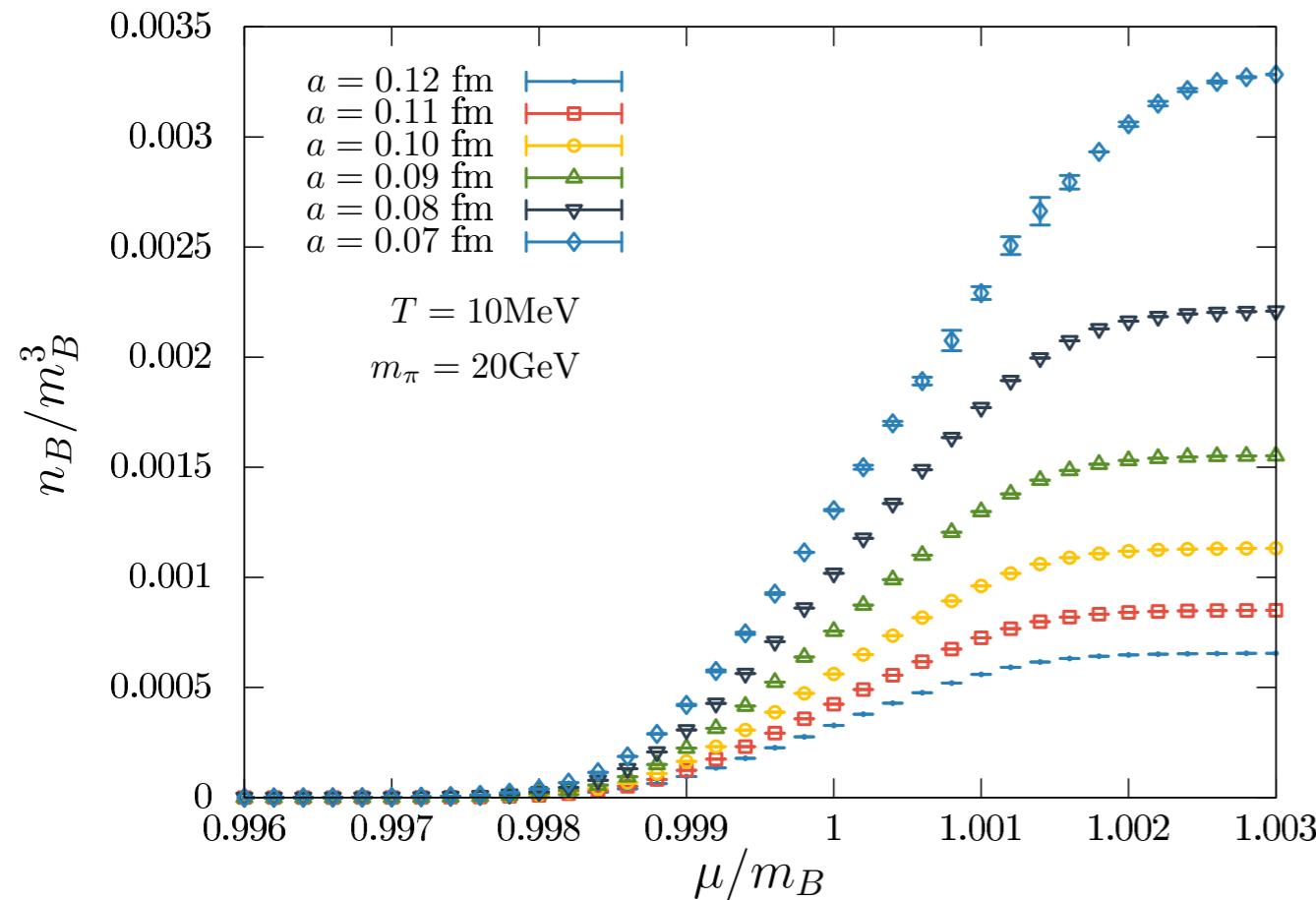


Complete spin-flavour structure of baryons (mesons for isospin chemical potential)

Gauge and Lorentz symmetries!

Cold and dense regime: onset of baryon matter

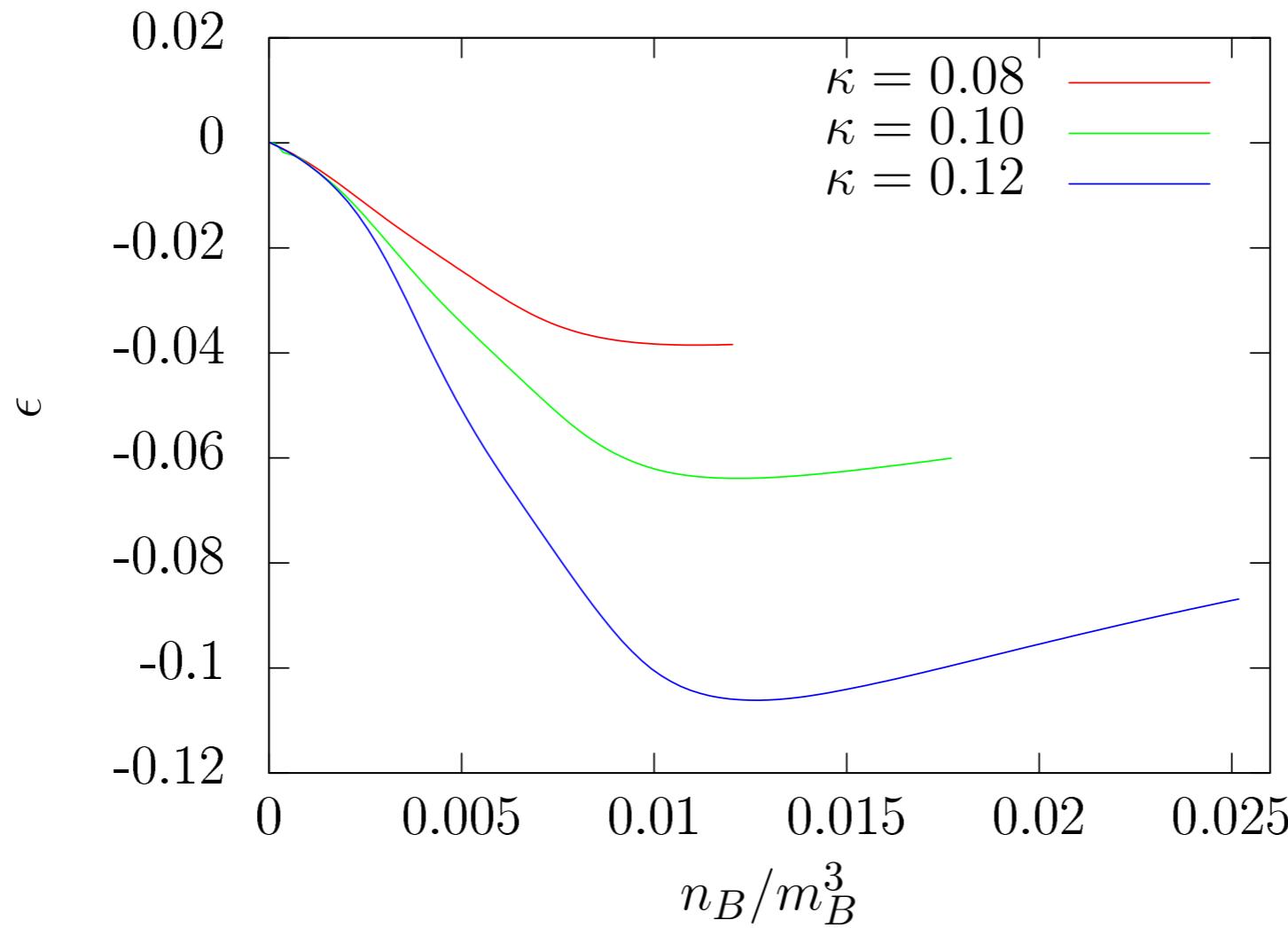
Glesaaen, Neuman, O.P., JHEP 15



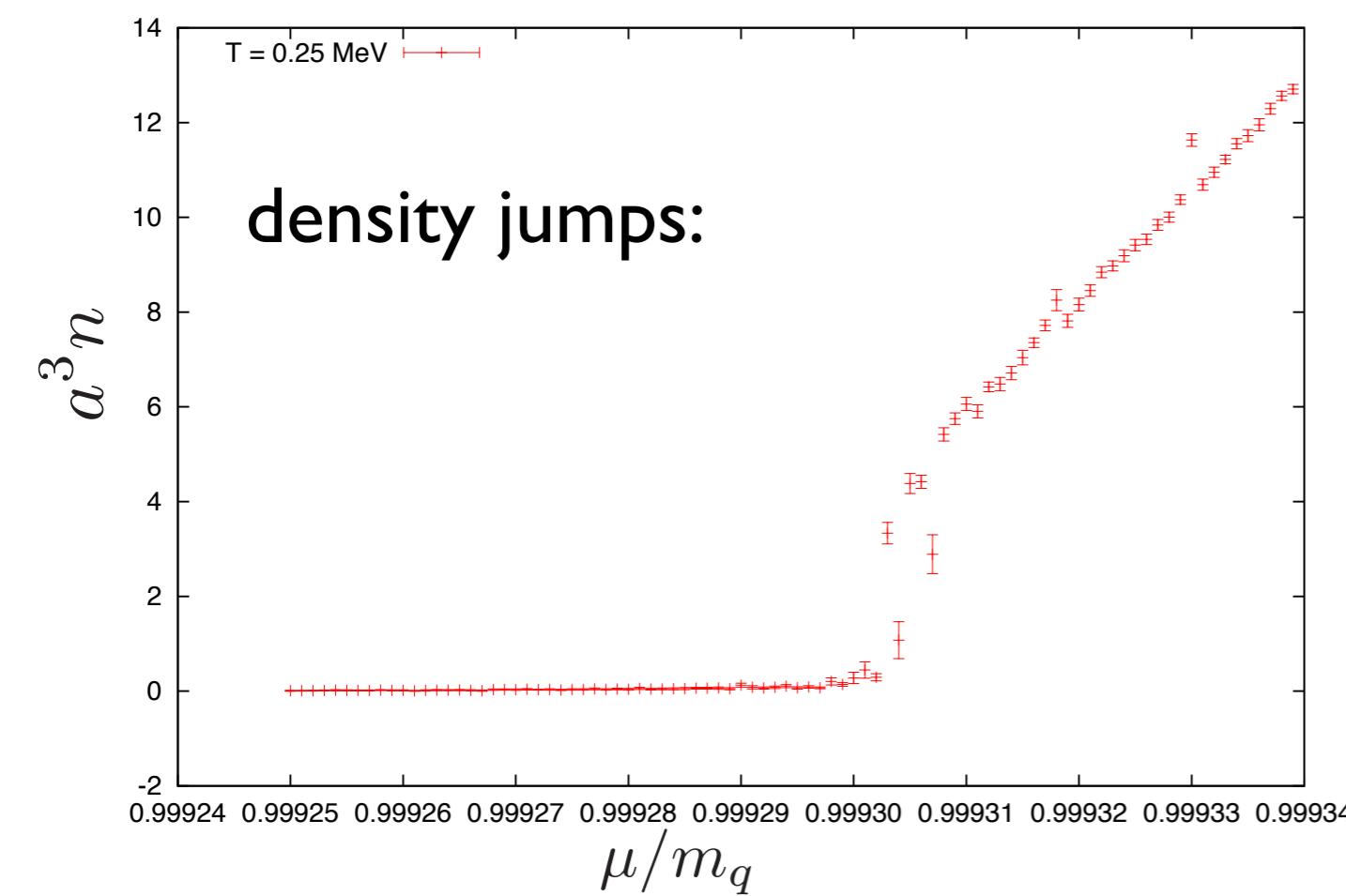
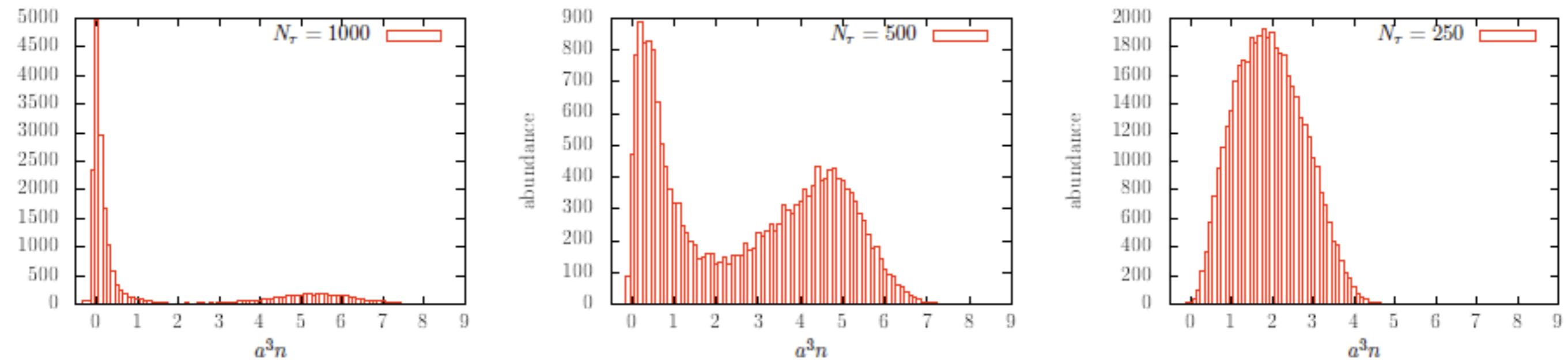
- Continuum approach $\sim a$ as expected for Wilson fermions
- Cut-off effects grow rapidly beyond onset transition: **lattice saturation!**
- Finer lattice necessary for larger density!

Binding energy per nucleon

$$\epsilon \equiv \frac{e - n_B m_B}{n_B m_B} \stackrel{LO}{=} -\frac{4}{3} \frac{1}{a^3 n_B} \left(\frac{z_3}{z_0} \right)^2 \kappa^2 = -\frac{1}{3} \frac{1}{a^3 n_B} \left(\frac{z_3}{z_0} \right)^2 e^{-am_M}$$

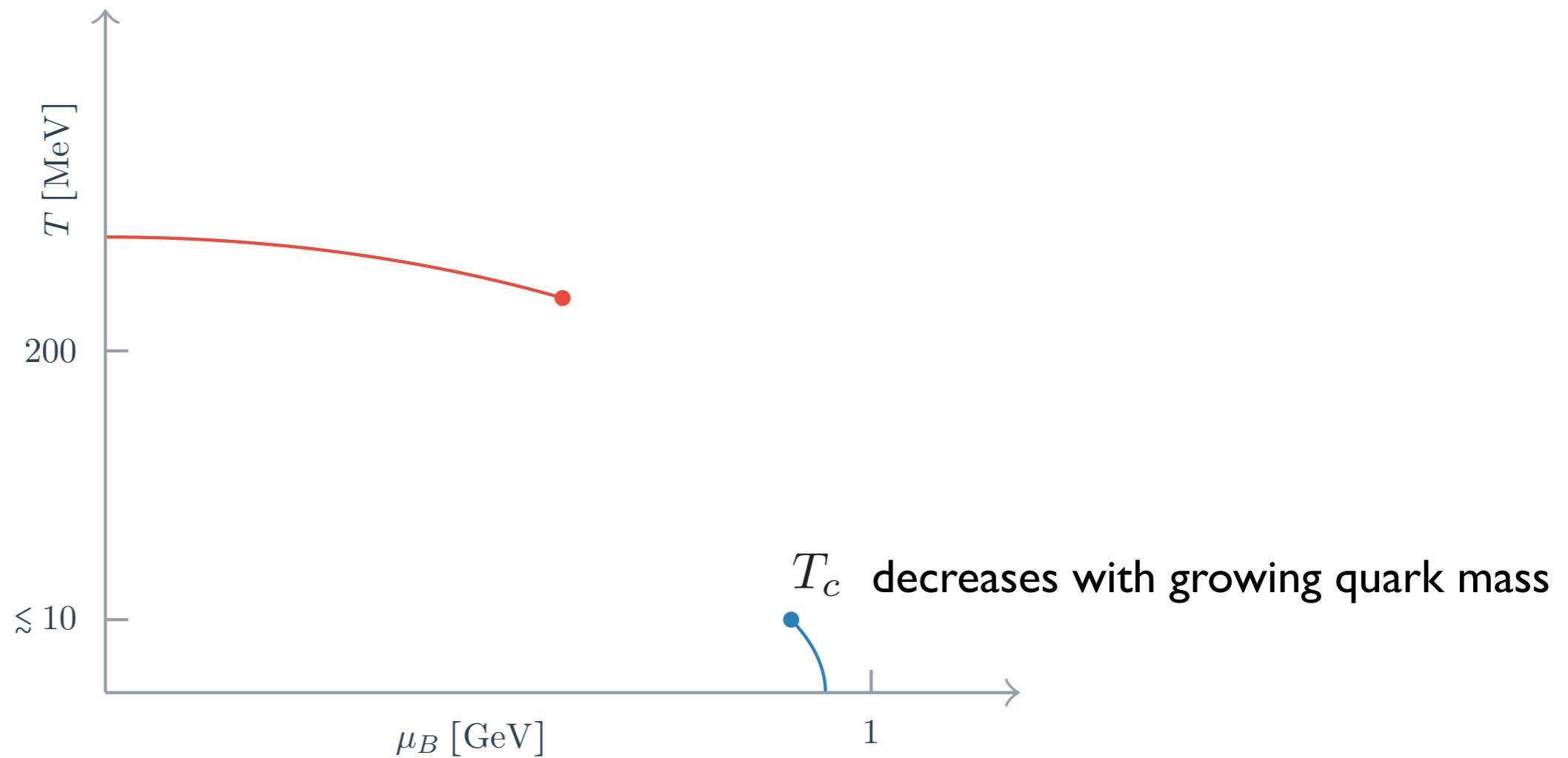


Light quarks: first order transition + endpoint



- phase coexistence: first order
- for higher $T = \frac{1}{aN_\tau}$ crossover
- nuclear liquid gas transition!

"Heavy QCD" phase diagram



QCD at large N_c

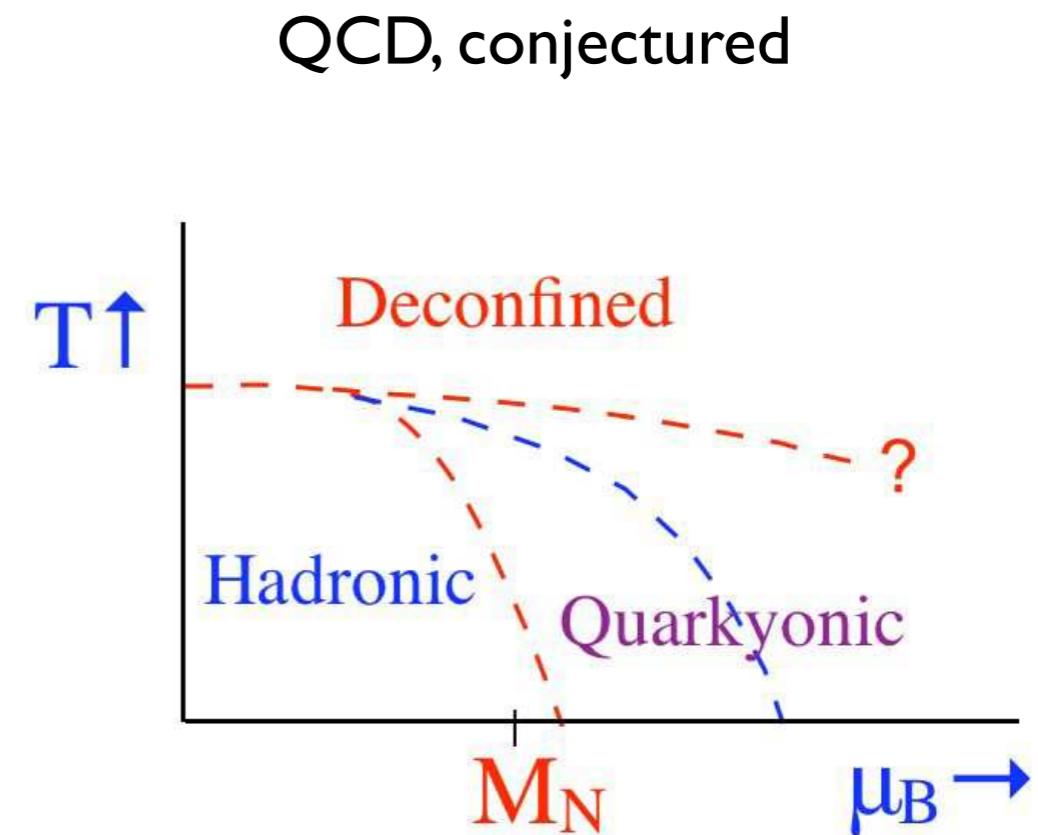
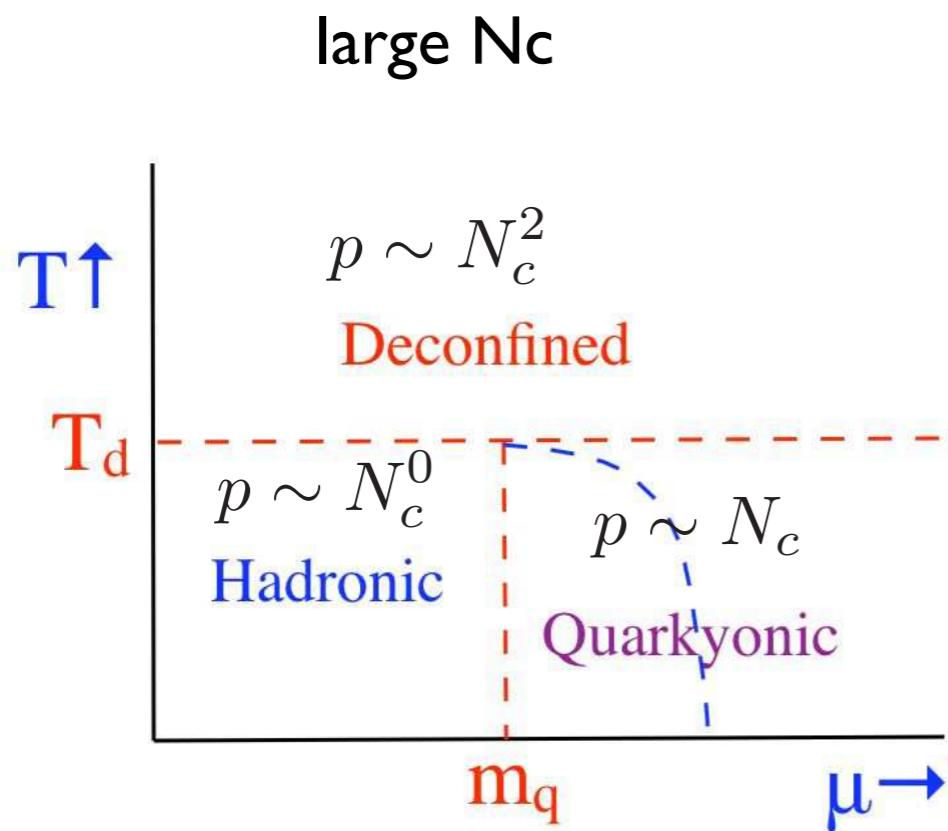
Definition, 't Hooft 1974 : $N_c \rightarrow \infty, g^2 N_c = \text{const.}$

- suppresses quark loops in Feynman diagrams
- mesons are free;
corrections: cubic interactions $\sim 1/\sqrt{N_c}$, quartic int. $\sim 1/N_c$
- meson masses $\sim \Lambda_{QCD}$
- baryons: N_c quarks, baryon masses $\sim N_c \Lambda_{QCD}$
- baryon interactions: $\sim N_c$

Witten 1979

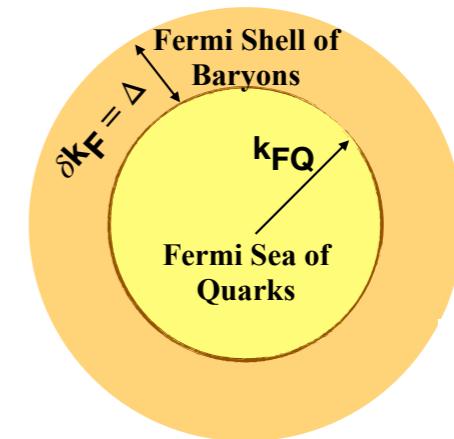
Implications on the phase diagram

McLerran, Pisarski 07:



Quarkyonic matter:

can smoothly vary from baryons to quark matter



The effective theory for large N_c

O.P., Jonas Scheunert 19

Recalculate for general N_c , start with strong coupling limit, need new SU(N) integrals!

Static determinant:

$$\int_{SU(N)} dU \det(1 + h_1 U)^{2N_f} = \sum_{p=0}^{N_f} \left(\prod_{i=1}^p \frac{(i-1+2N_f-p+N)^{2N_f-p}}{(i-1+2N_f-p)^{2N_f-p}} \right) \left(h_1^{pN} + h_1^{(2N_f-p)N} \right) \left(1 - \frac{\delta_{p,N_f}}{2} \right)$$

And corrections:

$$\begin{aligned} & \int_{SU(N)} dU \det(1 + h_1 U)^{2N_f} \operatorname{tr} \left(\frac{(h_1 U)^n}{(1 + h_1 U)^m} \right) \\ &= h_1^{N(2N_f+1)} \sum_{r=\max(0, N-m)}^{2N_f+N-m} (-1)^{r+N+1} \binom{N+r-1}{r} (r+m-1)^{N-1} \frac{(2N_f)^{2N_f+1-r-m}}{(N+2N_f-r-m)} \\ &+ \sum_{p=0}^{2N_f} h_1^{Np} \det_{1 \leq i,j \leq N} \left[\binom{2N_f}{i-j+p} \right] \sum_{\mu=1}^N \sum_{r=\max(0, \mu-m)}^{\mu+p-m} (-1)^r \binom{r+n-1}{r} \\ &\times \frac{(-1)^{\mu+1}}{r+m} \frac{(r+m+N-\mu)^{r+m}}{(r+m-\mu)!(\mu-1)!} \frac{(\mu+p-1)^{r+m}}{(N+2N_f-p+r+m-\mu)^{r+m}}. \end{aligned}$$

Results for $N_f = 2$:

Static determinant:

$$z_0 = 1 + \frac{1}{6}(h_1^N + h_1^{3N})(N+3)(N+2)(N+1) + \frac{1}{12}h_1^{2N}(N+3)(N+2)^2(N+1) + h_1^{4N}$$

Curious: spin degeneracy of a baryon determined by N!

Correction:

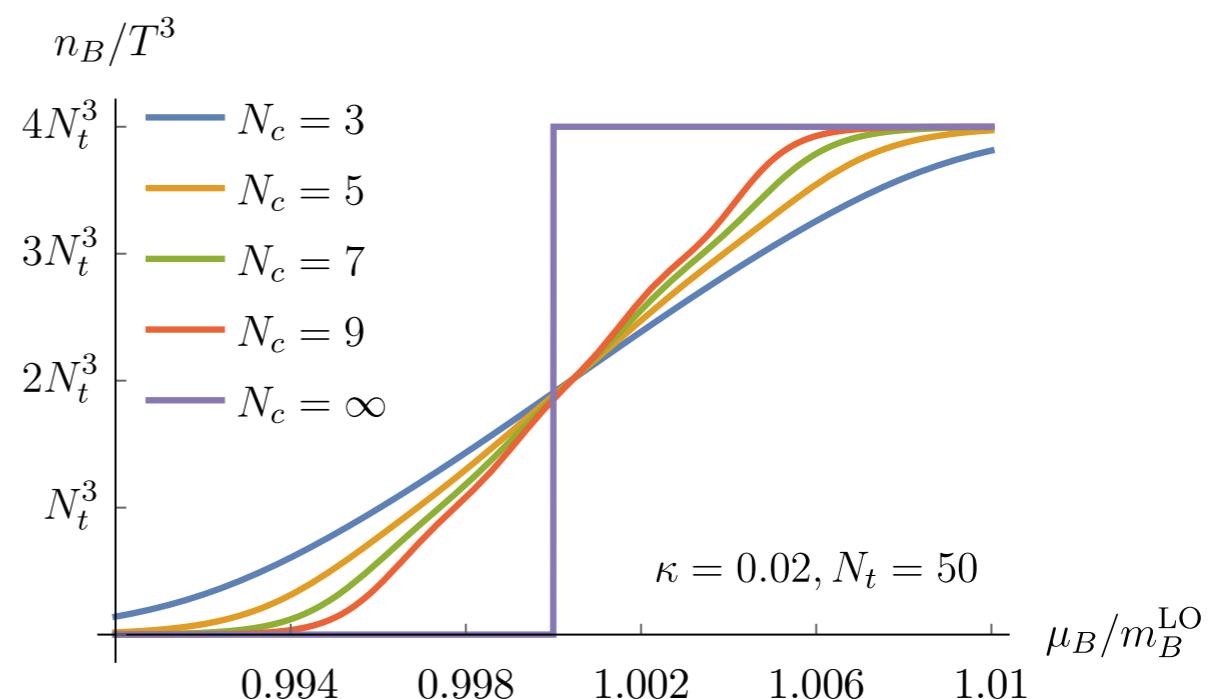
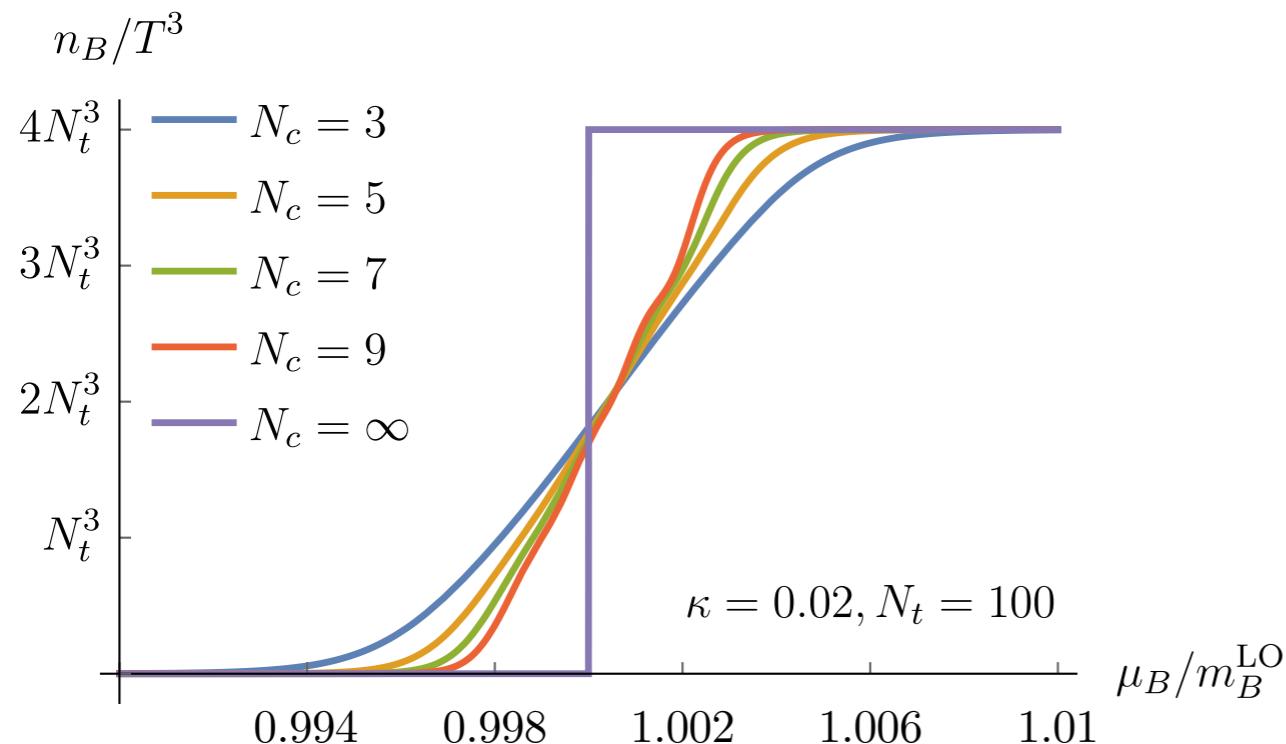
$$\begin{aligned} z_{11} = & \frac{1}{24}h_1^N(N+3)(N+2)(N+1)N + \frac{1}{24}h_1^{2N}(N+3)(N+2)^2(N+1)N \\ & + \frac{1}{8}h_1^{3N}(N+3)(N+2)(N+1)N + h_1^{4N}N \end{aligned}$$

Thermodynamic functions for large N_c

Order hopping expansion		κ^0	κ^2	κ^4
$h_1 < 1$	$a^4 p$	$\sim \frac{1}{6N_\tau} N_c^3 h_1^{N_c}$	$\sim -\frac{1}{48} N_c^7 h_1^{2N_c}$	$\sim \frac{3N_\tau \kappa^4}{800} N_c^8 h_1^{2N_c}$
	$a^3 n_B$	$\sim \frac{1}{6} N_c^3 h_1^{N_c}$	$\sim -\frac{N_\tau}{24} N_c^7 h_1^{2N_c}$	$\sim \frac{(9N_\tau+1)N_\tau}{1200} N_c^8 h_1^{2N_c}$
	$a^4 e$	$\sim -\frac{\ln(2\kappa)}{6} N_c^4 h_1^{N_c}$	$\sim \frac{N_\tau \ln(2\kappa)}{48} N_c^8 h_1^{2N_c}$	
	ϵ	0	$\sim -\frac{1}{4} N_c^3 h_1^{N_c}$	
$h_1 > 1$	$a^4 p$	$\sim \frac{4 \ln(h_1)}{N_\tau} N_c$	$\sim -12N_c$	$\sim 198N_c$
	$a^3 n_B$	~ 4	$\sim -N_\tau \frac{N_c^4}{h_1^{N_c}}$	$\sim -\frac{(59N_\tau-19)N_\tau}{20} \frac{N_c^5}{h_1^{N_c}}$
	$a^4 e$	$\sim -4 \ln(2\kappa) N_c$	$\sim 24 \ln(2\kappa) N_c$	
	ϵ	0	~ -6	

Beyond the onset transition: $p \sim N_c$ **definition of quarkyonic matter!**

The baryon onset transition for growing N_c



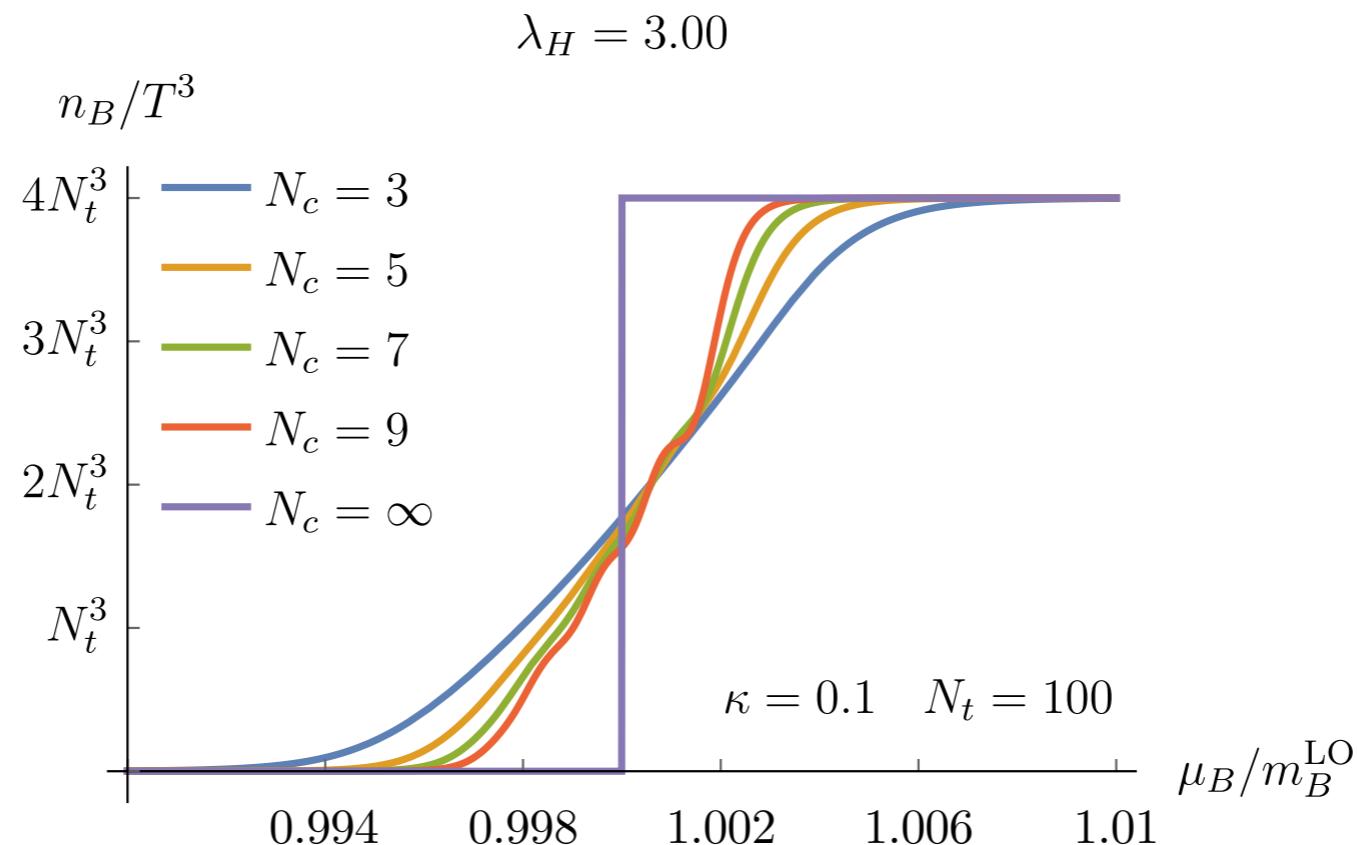
Transition becomes more strongly first-order!

Gauge corrections

So far strong coupling limit, not consistent with 't Hooft scaling

$$u(\beta) = \frac{1}{\lambda_H} = \frac{1}{g^2 N_c} < 1$$

Gross,Witten 80

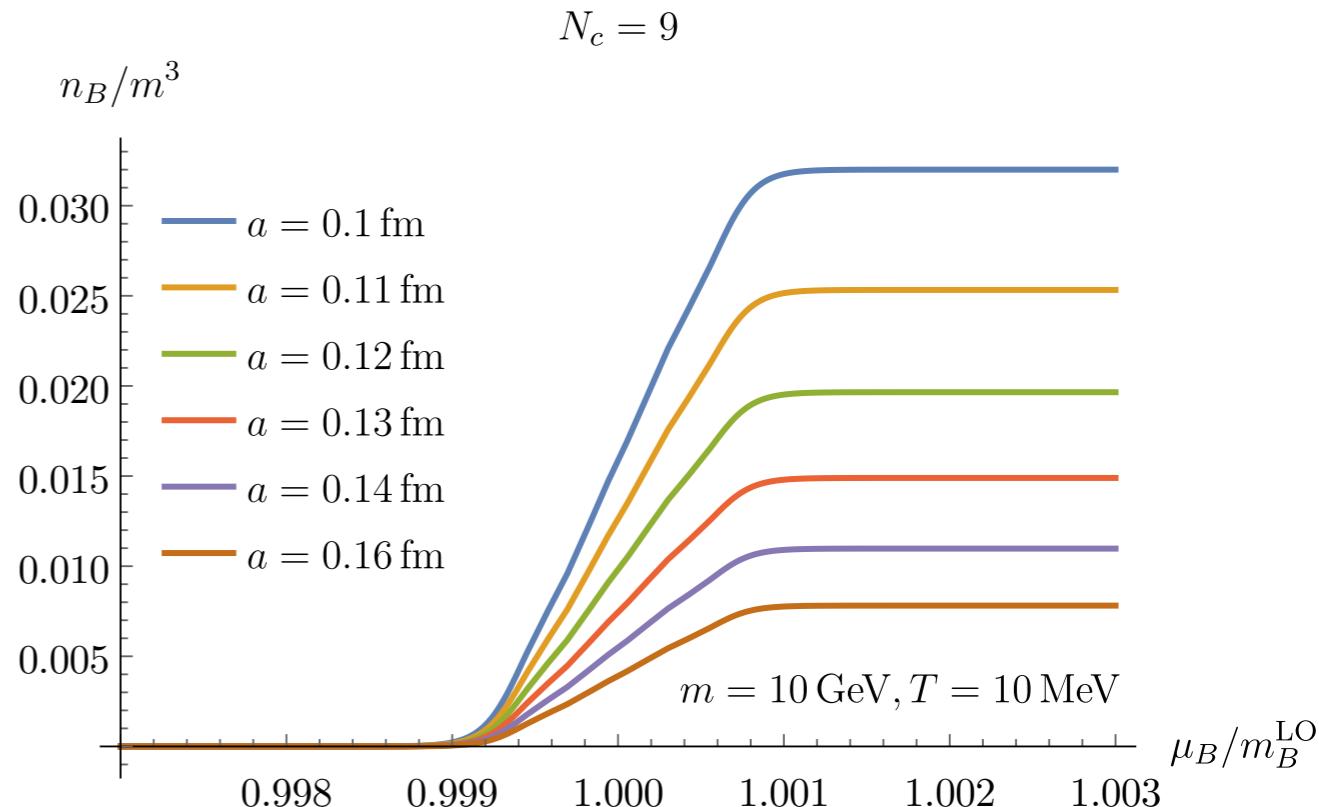
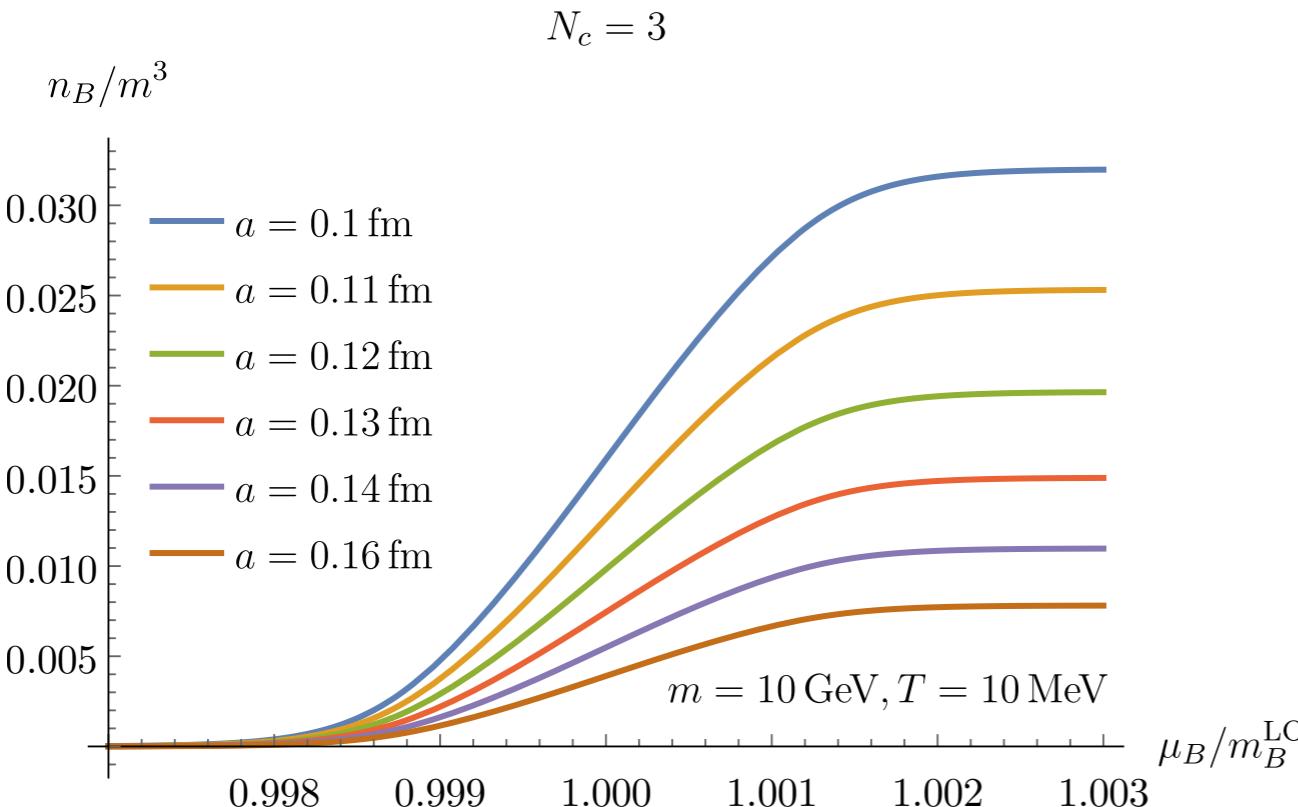


Transition still steepens, N_c -scaling in condensed phase unaffected

Continuum approach

Gross,Witten 80: interchange of strong coupling and large N_c -limit “highly suspicious” in $I+I\bar{d}$

Same here: system immediately jumps to lattice saturation, unphysical
take continuum limit first!



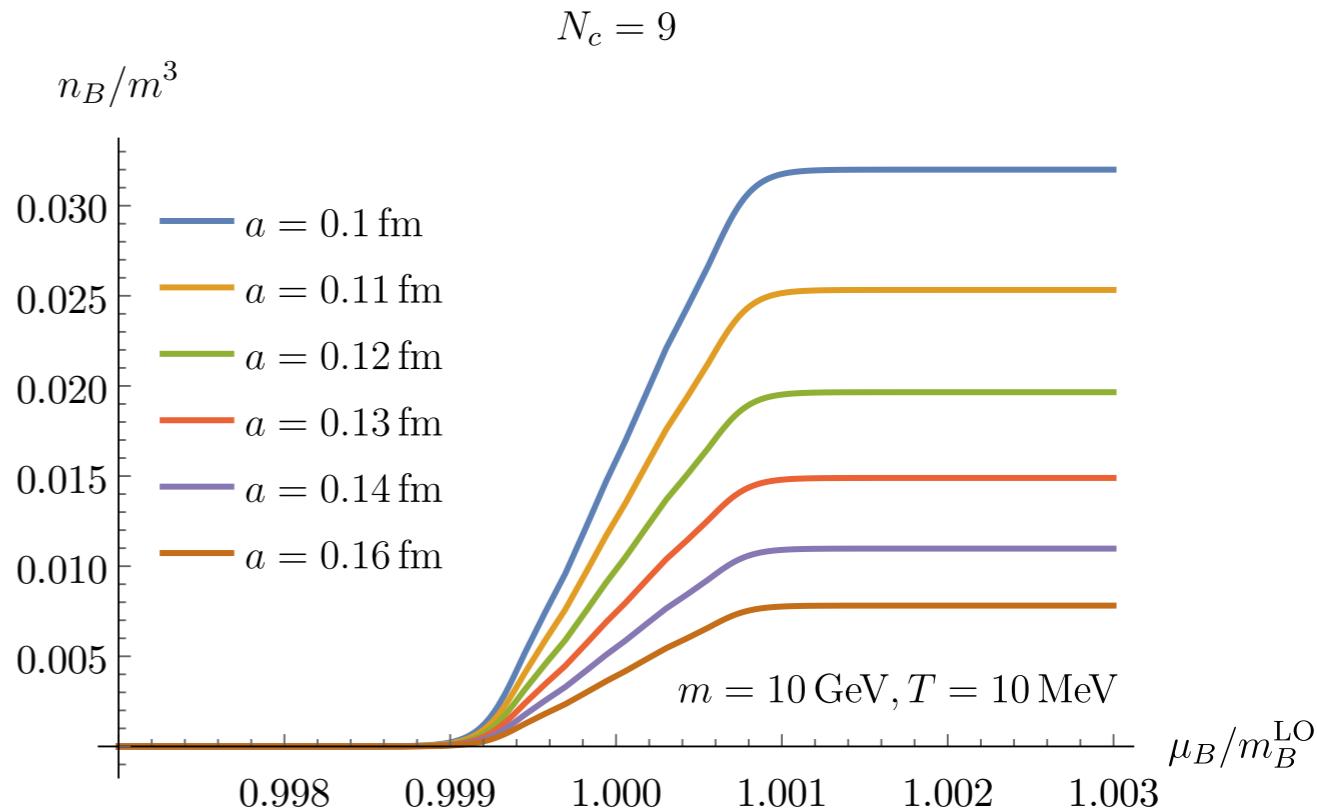
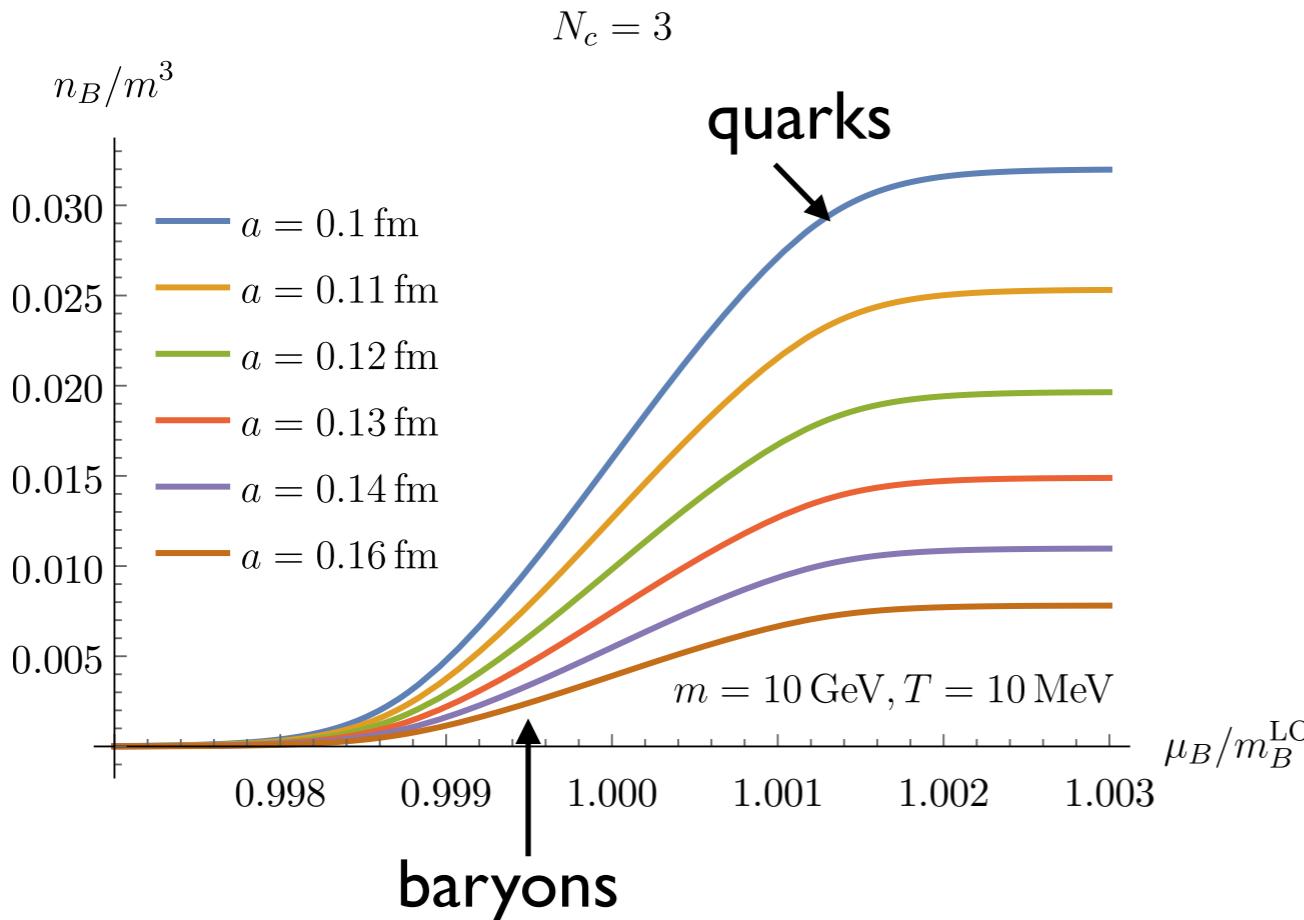
Not enough orders to take limits, but steepening of transition clearly observed!

Quarkyonic matter on the lattice?

Continuum approach

Gross,Witten 80: interchange of strong coupling and large N_c -limit “highly suspicious” in I+Id

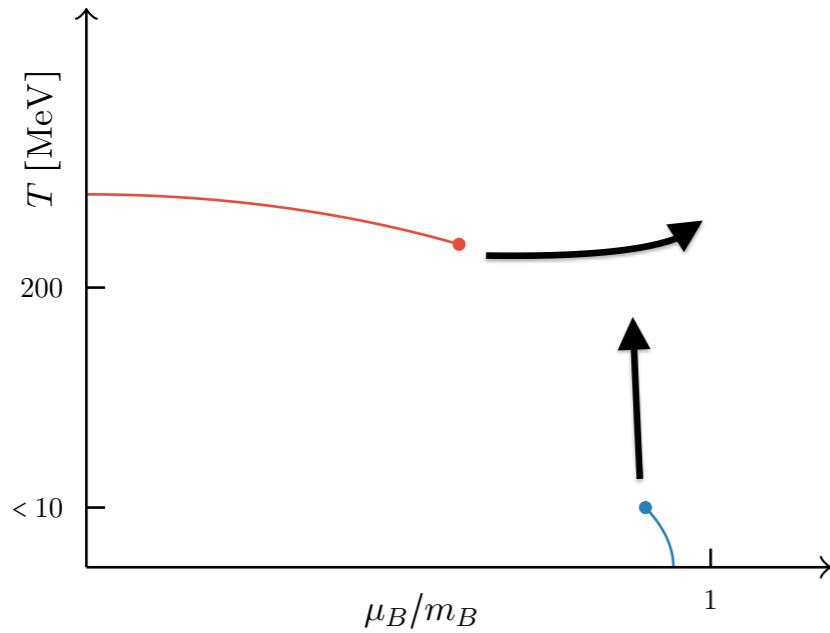
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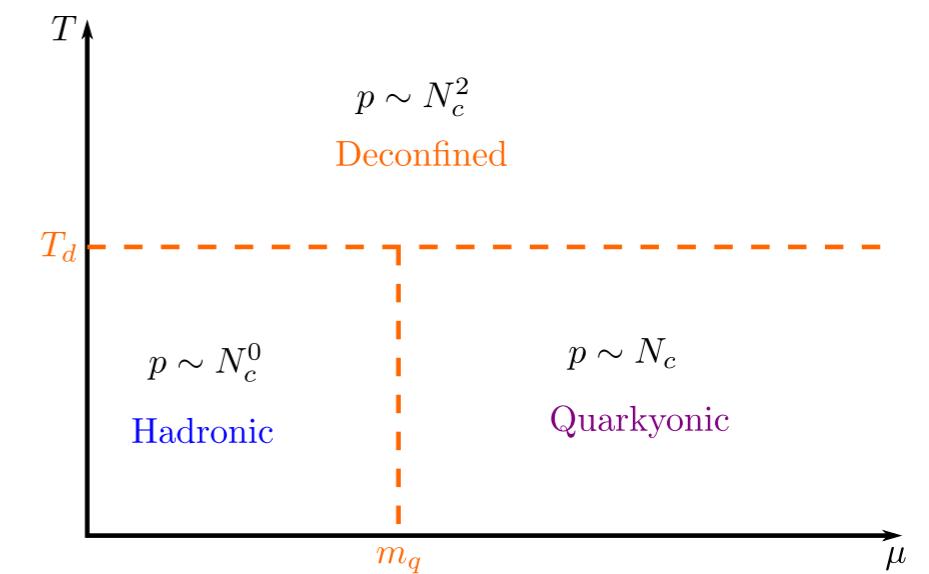
Not enough orders to take limits, but steepening of transition clearly observed!

Quarkyonic matter on the lattice?

Altogether:



large N_c



Smooth transition of phase diagram to conjectured limit, scaling beyond baryon onset!

Large N_c limit independent of current quark masses, the same starting from physical QCD

No statement on chiral transition possible yet

Conclusions

- Sign problem beaten by effective lattice theory for heavy quarks
- Nuclear liquid gas transition and equation of state calculable in the heavy mass region
- Varying N_c : dense QCD is consistent with quarkyonic matter

Backup slides

Subleading couplings

Subleading contributions for next-to-nearest neighbours:

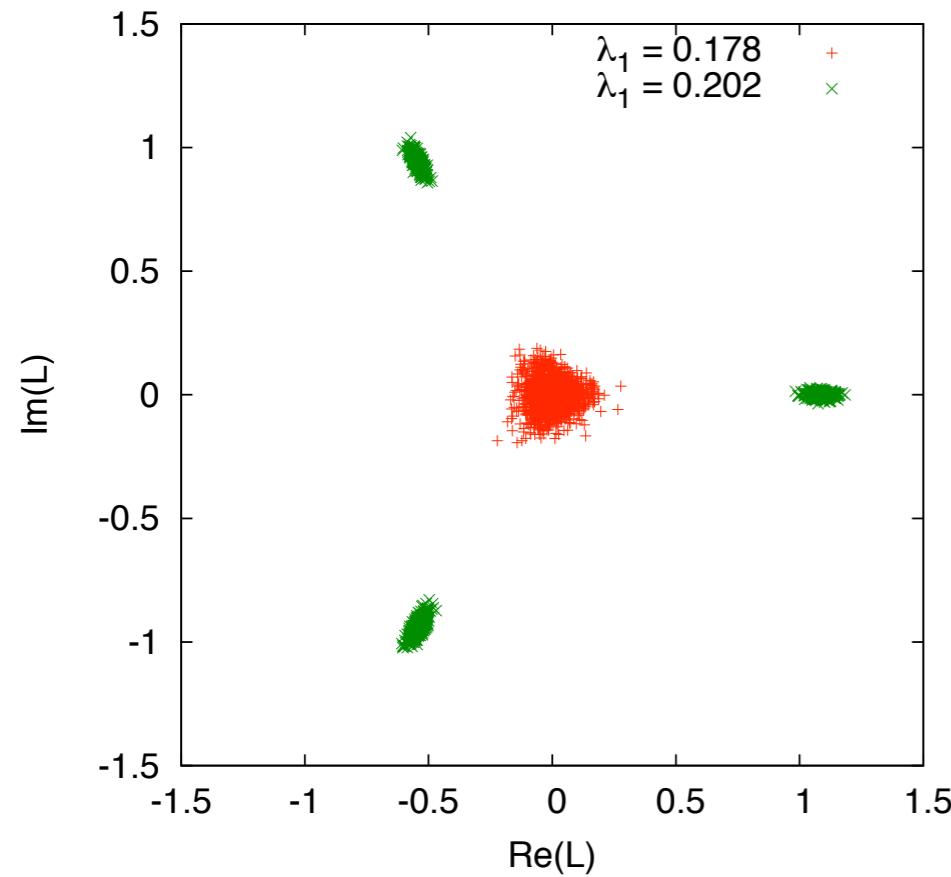
$$\lambda_2 S_2 \propto u^{2N_\tau+2} \sum'_{[kl]} 2\text{Re}(L_k L_l^*) \quad \text{distance} = \sqrt{2}$$

$$\lambda_3 S_3 \propto u^{2N_\tau+6} \sum''_{\{mn\}} 2\text{Re}(L_m L_n^*) \quad \text{distance} = 2$$

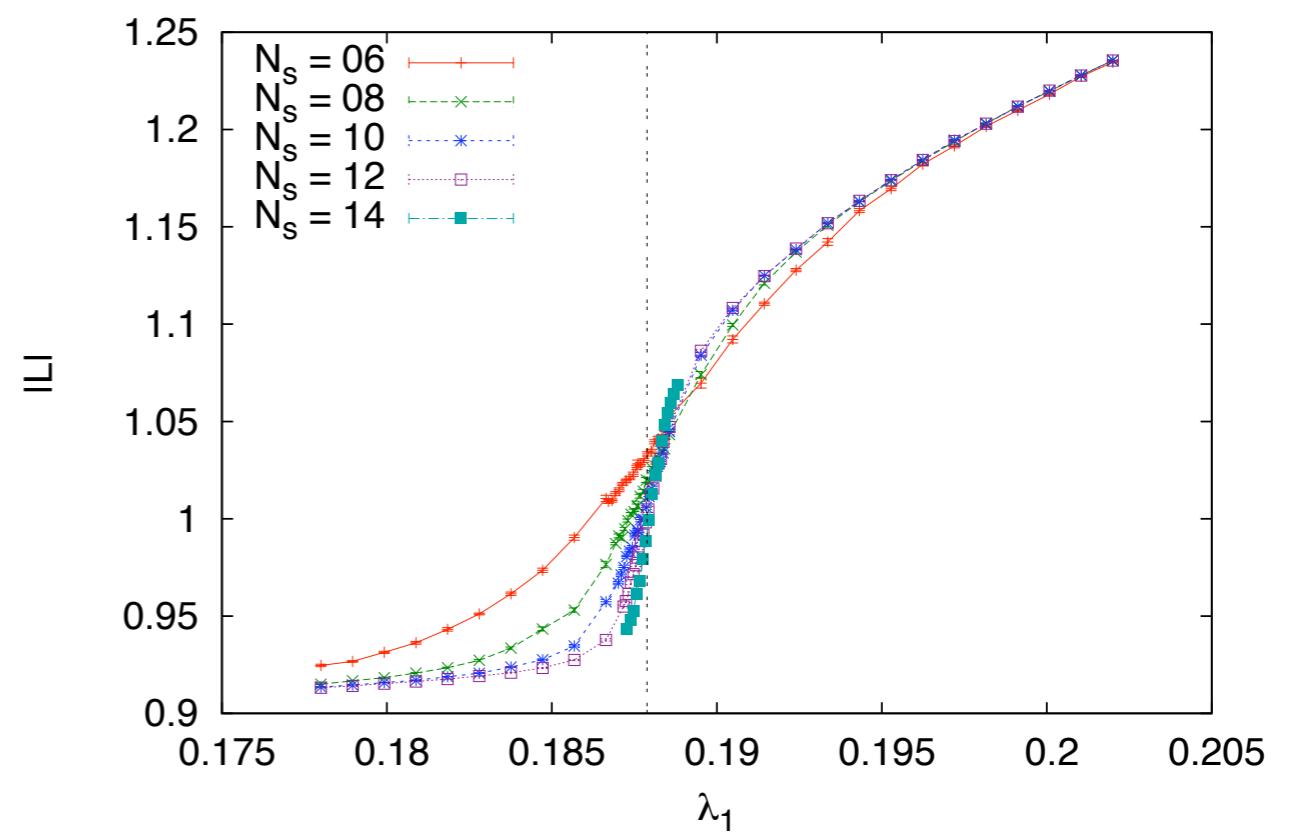
as well as terms from loops in the *adjoint* representation:

$$\lambda_a S_a \propto u^{2N_\tau} \sum_{\langle ij \rangle} \text{Tr}^{(a)} W_i \text{Tr}^{(a)} W_j \quad ; \quad \text{Tr}^{(a)} W = |L|^2 - 1$$

Numerical results for SU(3), one coupling



Order-disorder transition
=Z(3) breaking



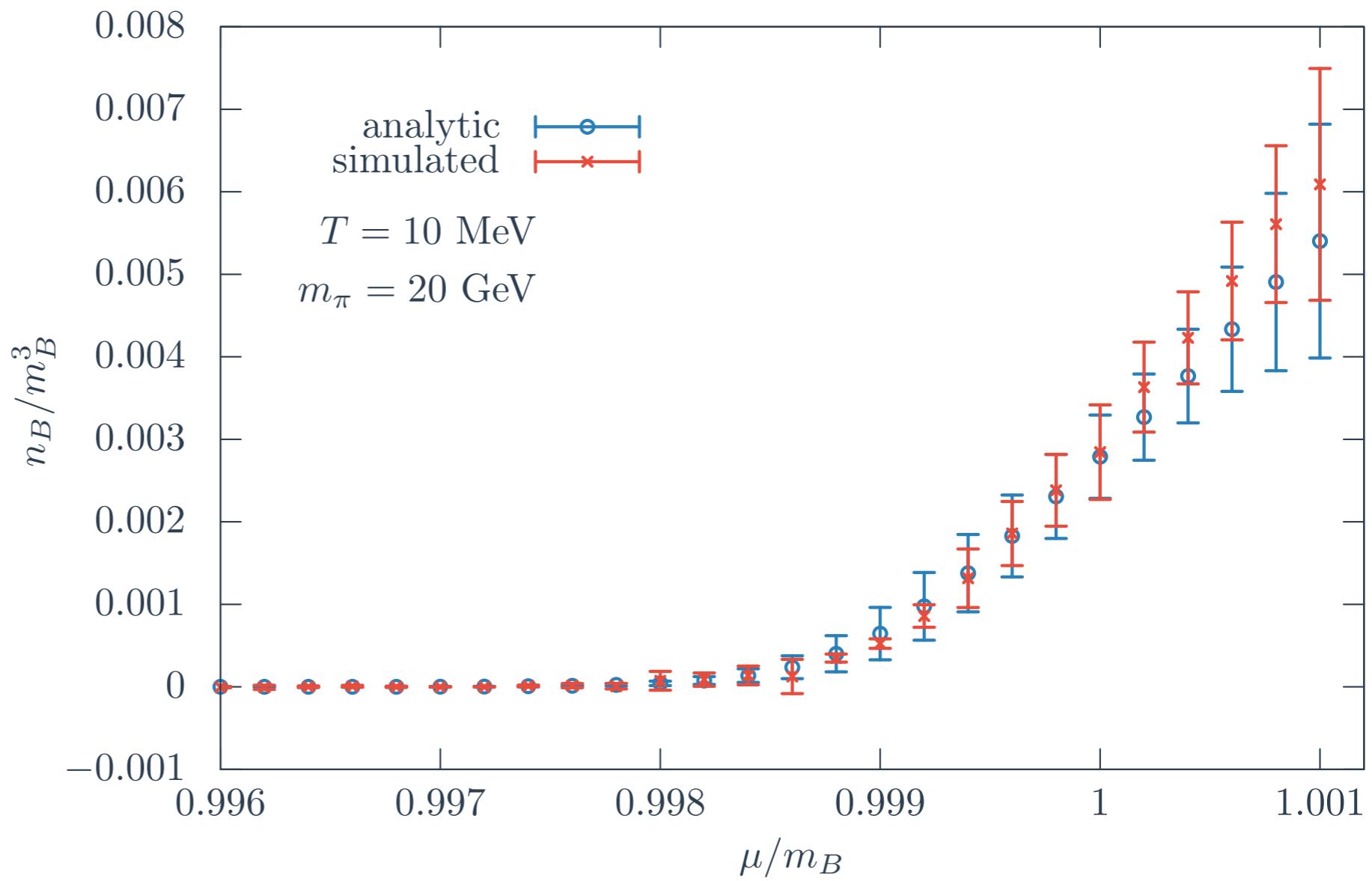
Linked cluster expansion of effective theory

$$\mathcal{Z} = \int \mathcal{D}\phi e^{-S_0[\phi] + \frac{1}{2} \sum v_{ij}(x,y)\phi_i(x)\phi_j(y) + \frac{1}{3!} \sum u_{ijk}(x,y,z)\phi_i(x)\phi_j(y)\phi_k(z) + \dots}$$

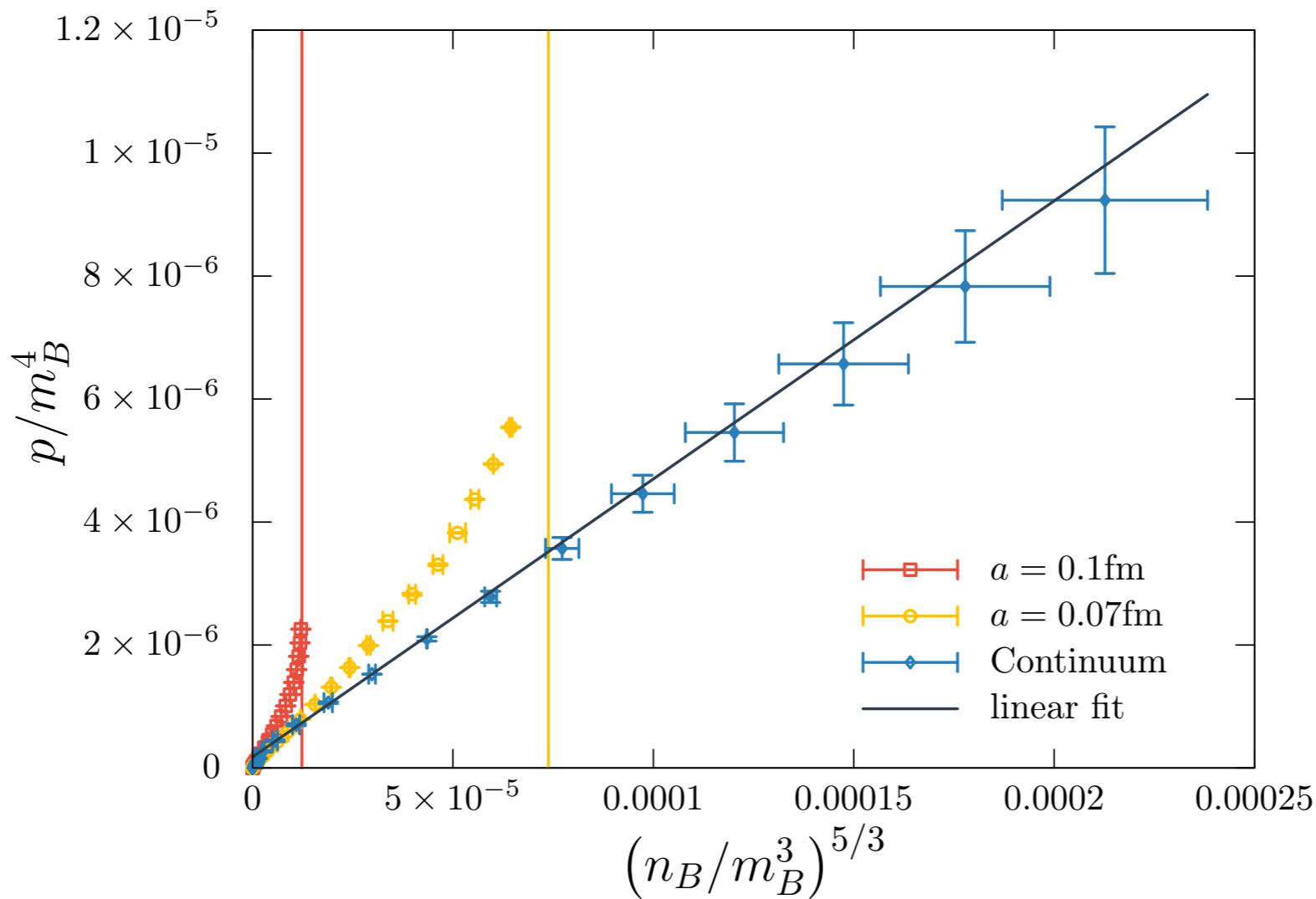
“perturbation theory” in effective couplings

Glesaaen, Neuman, O.P. 15

through $u^5 \kappa^8$



Equation of state of heavy nuclear matter, continuum



- EoS fitted by polytrope, non-relativistic fermions!
- Can we understand the pre-factor? Interactions, mass-dependence...