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Dense nuclear and quark matter from holography

- motivation: theoretical challenges in dense QCD and relevance for compact stars
- nuclear matter in the "decompactified" limit of the Sakai-Sugimoto model
- holographic quark-hadron continuity?

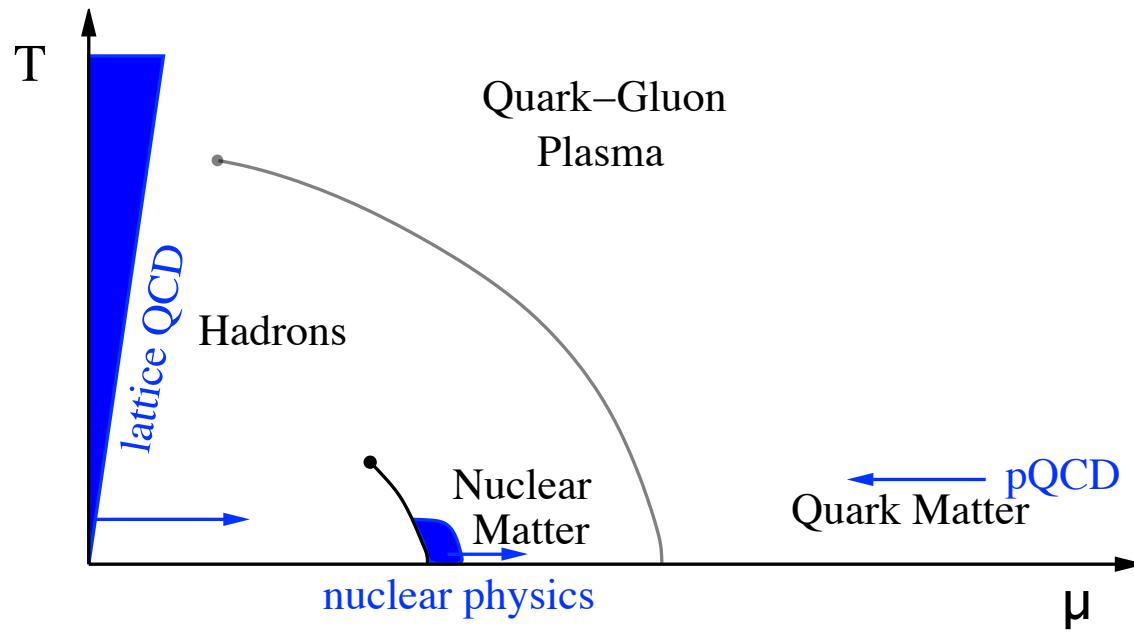
S.-w. Li, A. Schmitt, Q. Wang, PRD 92, 026006 (2015)

F. Preis, A. Schmitt, JHEP 1607, 001 (2016); EPJ Web Conf. 137, 09009 (2017)

K. Bitaghsir Fadafan, F. Kazemian, A. Schmitt, JHEP 1903, 183 (2019)

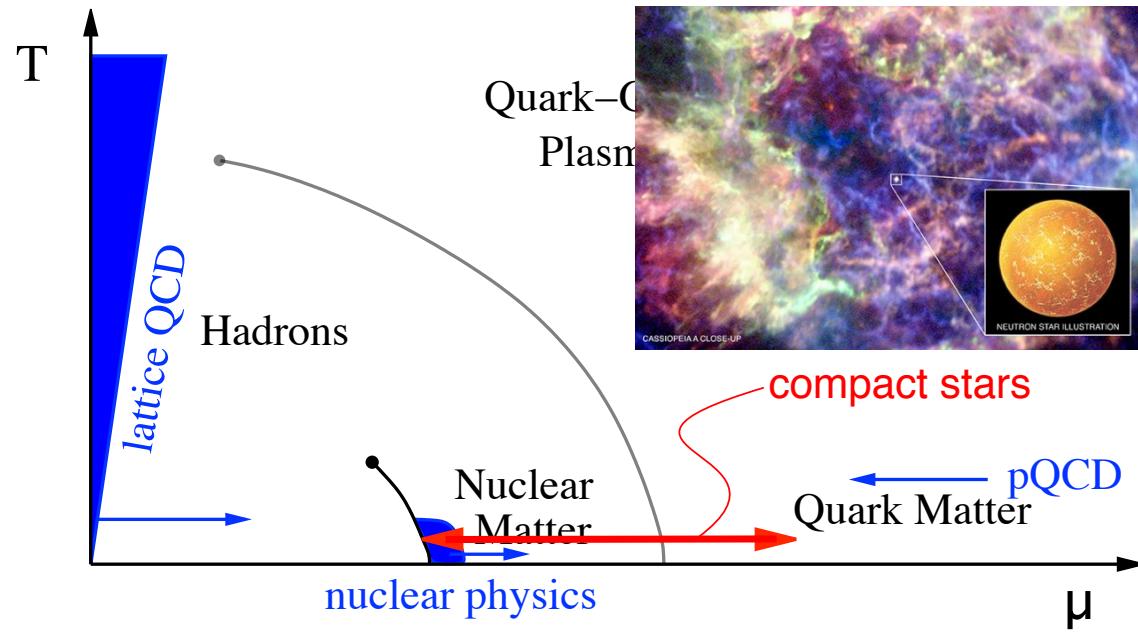
N. Kovensky, A. Schmitt, in preparation

Dense QCD: theoretical approaches (page 1/2)



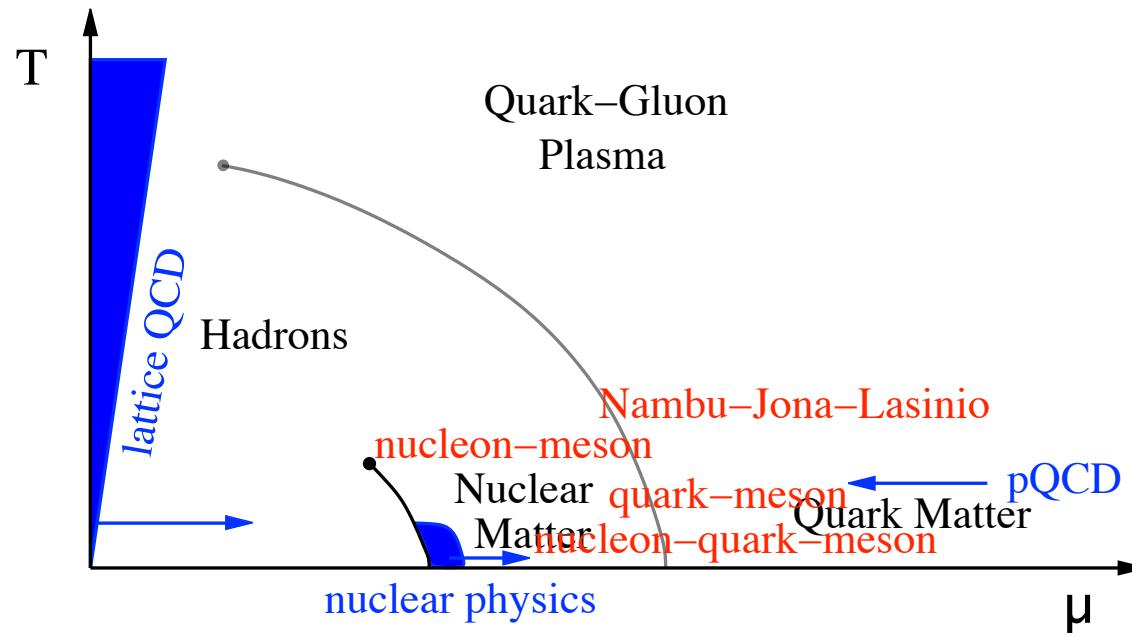
- lattice QCD: sign problem at nonzero μ (some recent progress)
- perturbative QCD: restricted to ultra-high densities
- “standard” nuclear physics:
restricted to densities \lesssim nuclear saturation density n_0

Dense QCD: theoretical approaches (page 1/2)



- lattice QCD: sign problem at nonzero μ (some recent progress)
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Dense QCD: theoretical approaches (page 2/2)

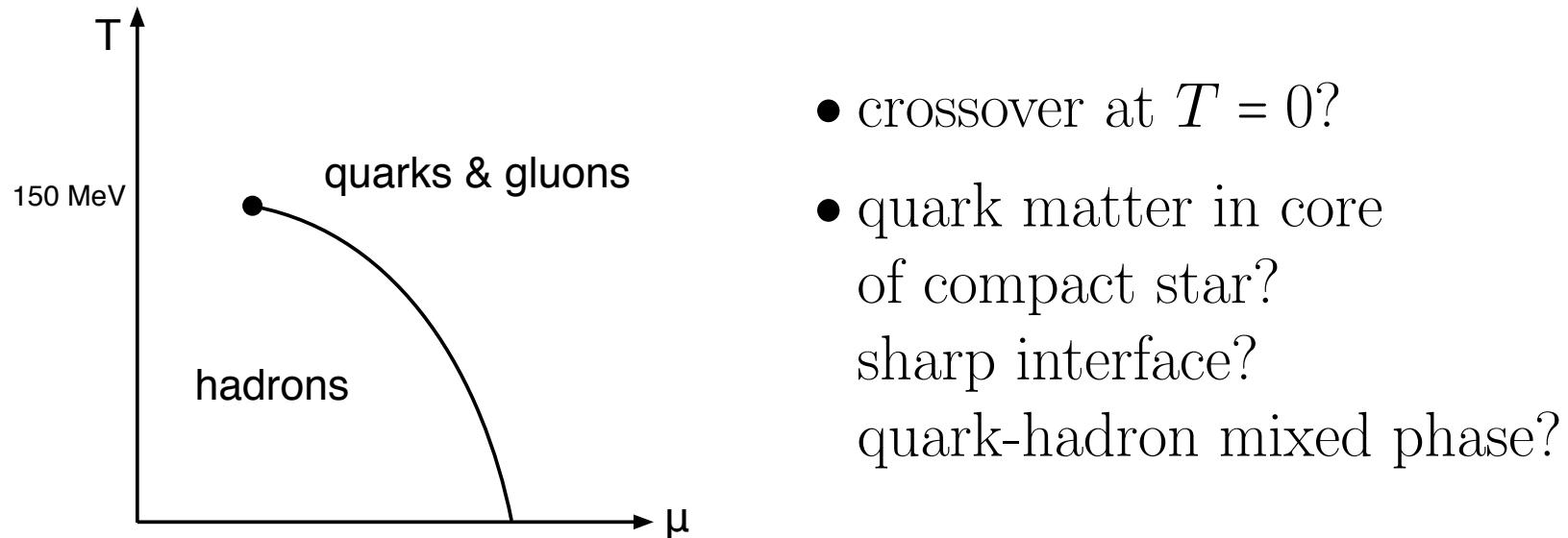


- Nambu–Jona-Lasinio (usually no nuclear matter)
 - quark-meson (no nucleons), nucleon-meson (no quarks)
 - nucleon-quark-meson (patched together, many parameters)
- even without rigor: models for compact stars hard to construct!

Quark-hadron continuity at high density?

order parameter	Polyakov loop (confinement)	chiral condensate
spontaneously breaks	\mathbb{Z}_{N_c}	$SU(N_f) \times SU(N_f)$
symmetry exact for	pure Yang-Mills ($m_q = \infty$)	chiral limit ($m_q = 0$)

→ no qualitative difference between hadronic and quark matter

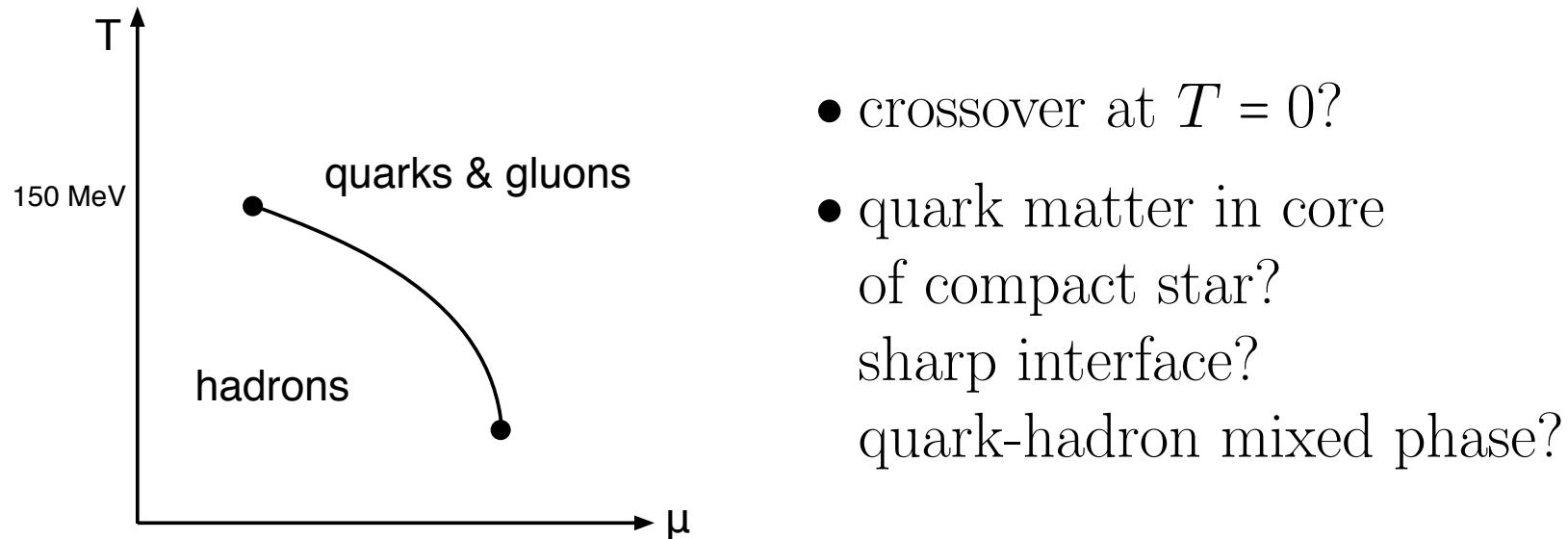


(for simplicity ignore Cooper pairing)

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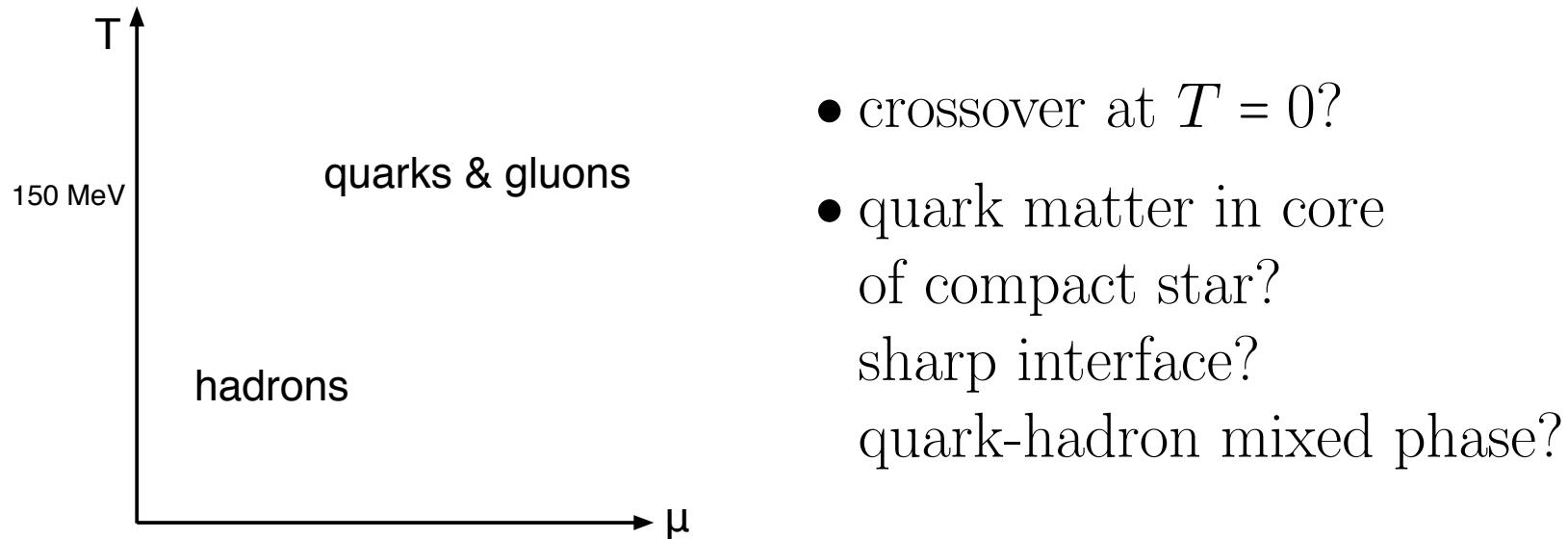


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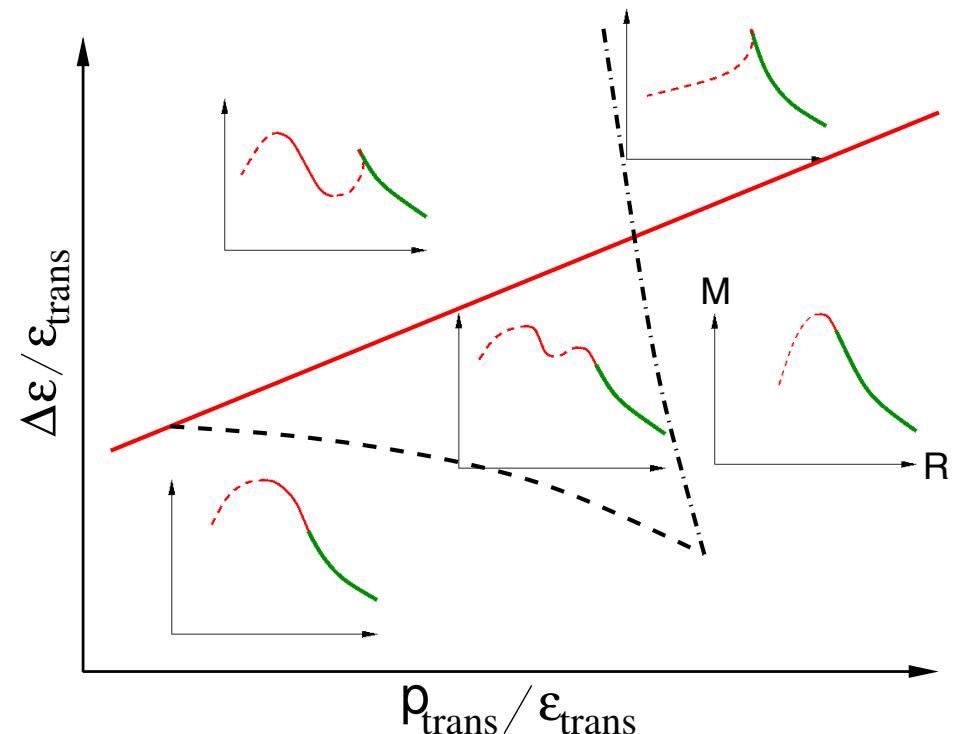
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(for simplicity ignore Cooper pairing)

Observable consequences of first-order transition?

- qualitative difference in mass/radius curve
M. G. Alford, S. Han and M. Prakash,
PRD 88, 083013 (2013)
- sequential 1st-order transitions?
M. G. Alford and A. Sedrakian, PRL
119, 161104 (2017)

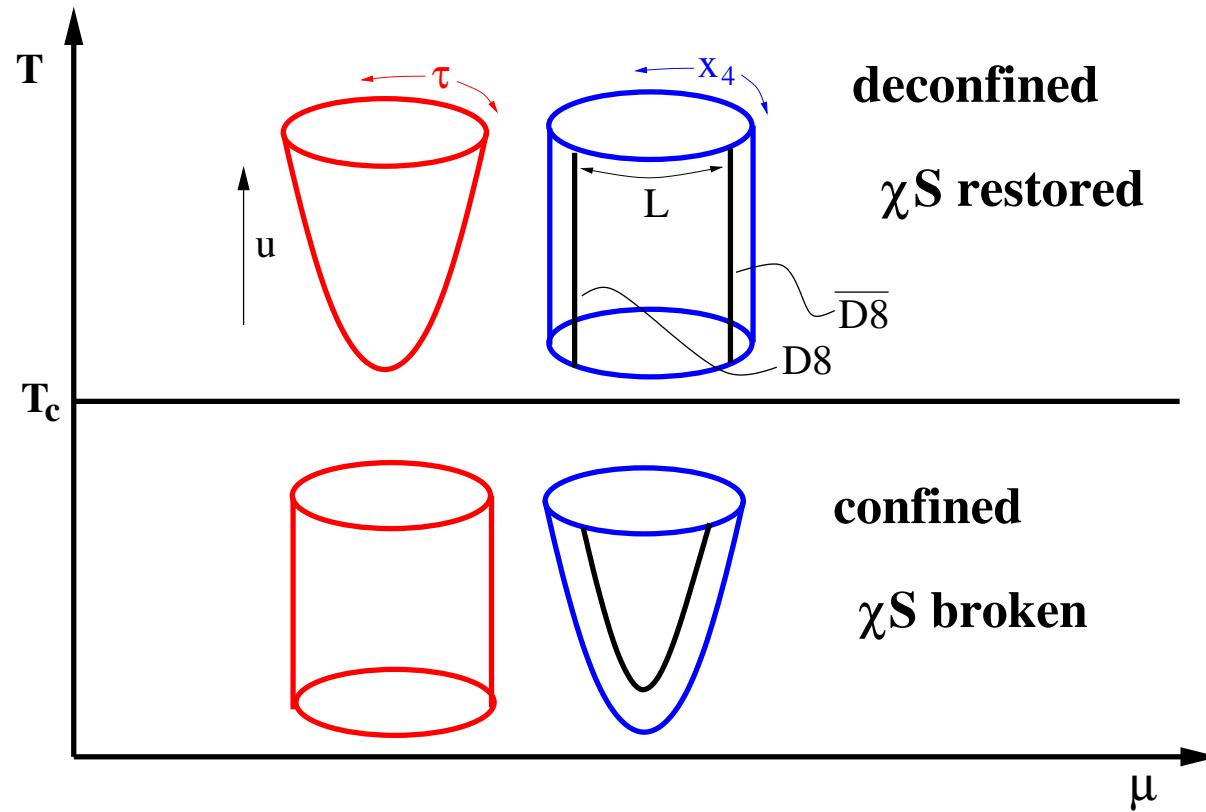


- different gravitational wave signal in neutron star mergers?
E. R. Most *et al.*, PRL 122, 061101 (2019)
- gravitational wave from bubble nucleation during supernova?
G. Cao and S. Lin, arXiv:1810.00528 [nucl-th]

Can holography help?

- dual of QCD: probably exists, but currently out of reach
- reliable strong-coupling calculation (usually infinite coupling)
- successful (qualitative) predictions for heavy-ion collisions
(supersymmetric YM plasma instead of quark-gluon plasma)
- Sakai-Sugimoto model
 - E. Witten, Adv. Theor. Math. Phys. 2, 505 (1998)
 - T. Sakai and S. Sugimoto, Prog. Theor. Phys. 113, 843 (2005)
 - top-down approach with only 3 parameters
 - dual to large- N_c QCD, however in inaccessible limit
 - successfully applied to meson, baryon, glueball spectra
- holographic approach to neutron stars
 - D3/D7: C. Hoyos *et al.*, PRL 117, 032501 (2016); E. Annala *et al.*, JHEP 1812, 078 (2018)
 - V-QCD: N. Jokela *et al.*, JHEP 1903, 041 (2019); C. Ecker *et al.*, 1908.03213 [astro-ph.HE]
 - Sakai-Sugimoto with pointlike baryons: T. Hirayama *et al.*, 1902.08477 [hep-ph]

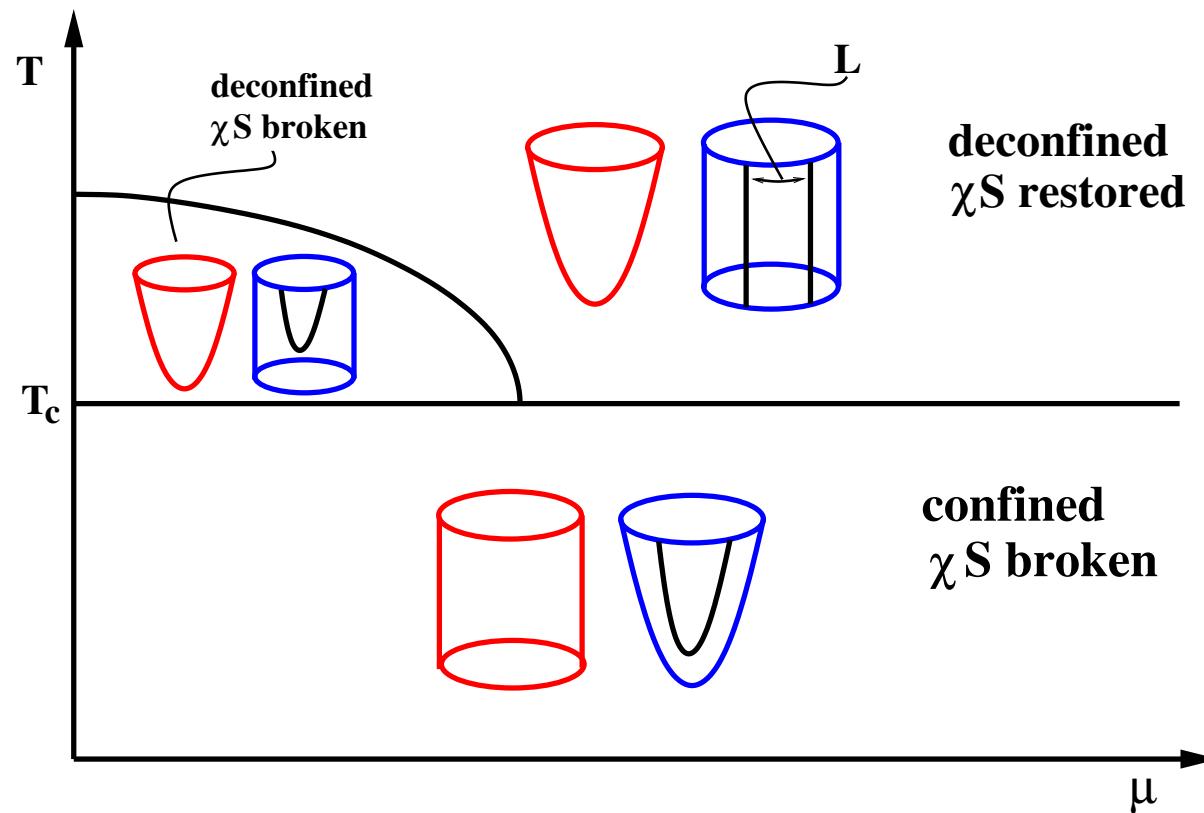
Phases in the Sakai-Sugimoto model (page 1/3)



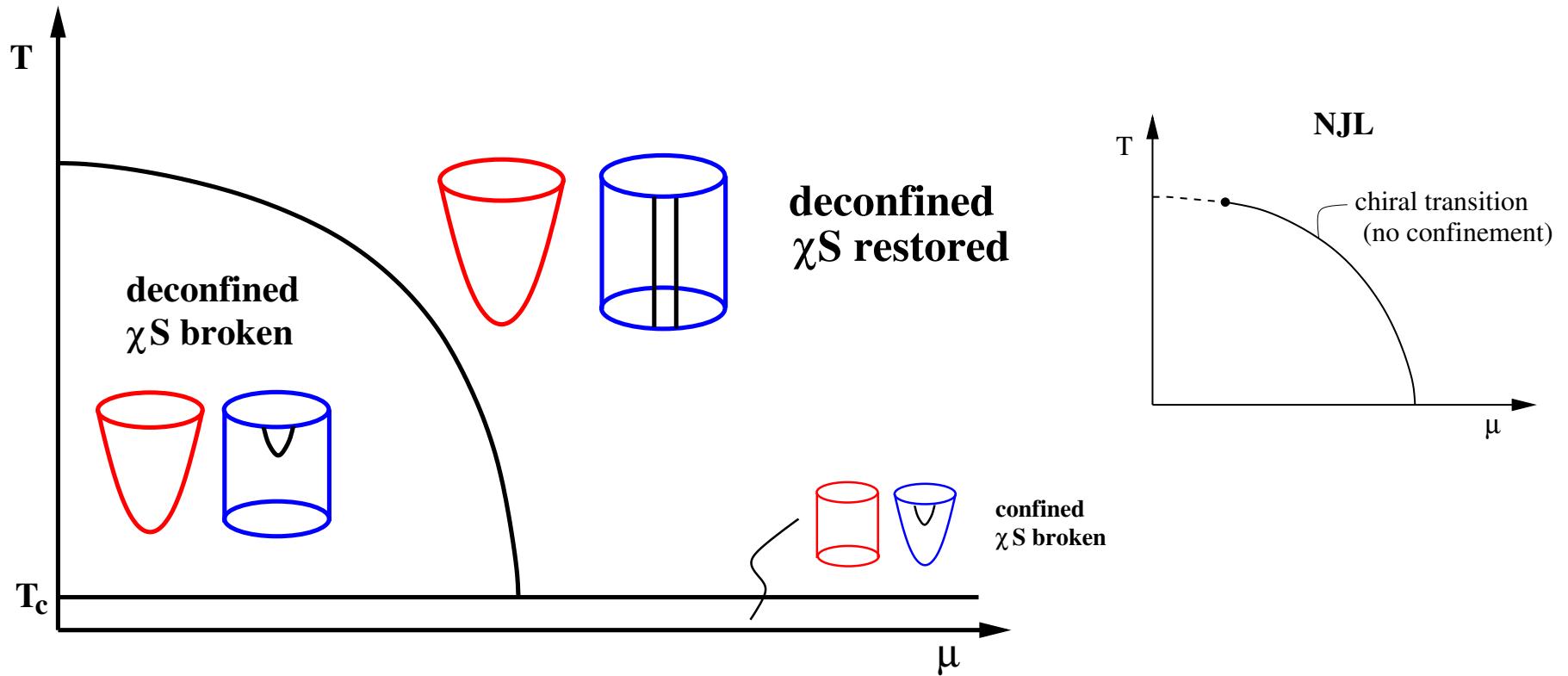
- in probe brane ("quenched") approximation: phase transition unaffected by quantities on flavor branes (μ, B, \dots)
- not unlike expectation from large- N_c QCD

Phases in the Sakai-Sugimoto model (page 2/3)

- less “rigid” behavior for smaller L
- deconfined, chirally broken phase for $L < 0.3\pi/M_{KK}$
 O. Aharony, J. Sonnenschein, S. Yankielowicz, Annals Phys. 322, 1420 (2007)
 N. Horigome, Y. Tanii, JHEP 0701, 072 (2007)



Phases in the Sakai-Sugimoto model (page 3/3)



- “decompactified” limit \rightarrow gluon dynamics decouple
- “NJL-like” dual field theory

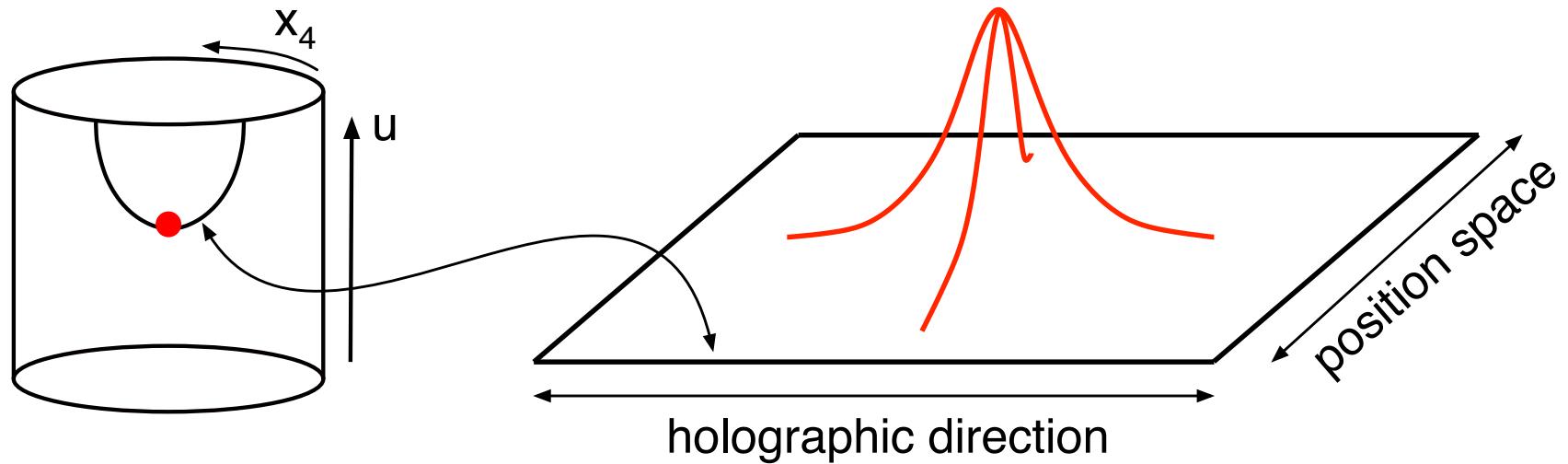
E. Antonyan, J. A. Harvey, S. Jensen, D. Kutasov, hep-th/0604017

J. L. Davis, M. Gutperle, P. Kraus, I. Sachs, JHEP 0710, 049 (2007)

F. Preis, A. Rebhan and A. Schmitt, Lect. Notes Phys. 871, 51 (2013)

Adding baryons

- baryons in AdS/CFT: wrapped D-branes with N_c string endpoints
E. Witten, JHEP 9807, 006 (1998); D. J. Gross, H. Ooguri, PRD 58, 106002 (1998)
- baryons in Sakai-Sugimoto:
 - D4-branes wrapped on S^4
 - equivalently: instantons on D8-branes (\rightarrow skyrmions)
T. Sakai, S. Sugimoto, Prog. Theor. Phys. 113, 843-882 (2005)
H. Hata, T. Sakai, S. Sugimoto, S. Yamato, Prog. Theor. Phys. 117, 1157 (2007)



Approximations for holographic nuclear matter

- Pointlike approximation
O. Bergman, G. Lifschytz, M. Lippert, JHEP 0711, 056 (2007)
- Finite-width, but non-interacting, instantons
K. Ghoroku, K. Kubo, M. Tachibana, T. Taminato and F. Toyoda, PRD 87, 066006 (2013)
S.-w. Li, A. Schmitt, Q. Wang, PRD 92, 026006 (2015)
F. Preis, A. Schmitt, JHEP 1607, 001 (2016); EPJ Web Conf. 137, 09009 (2017)
- Interacting instantons from exact two-instanton solution
(this talk) K. Bitaghsir Fadafan, F. Kazemian, A. Schmitt, JHEP 1903, 183 (2019)
beyond chiral limit: N. Kovensky, A. Schmitt (in preparation)
- see also: “Homogeneous ansatz”
M. Rozali, H. H. Shieh, M. Van Raamsdonk and J. Wu, JHEP 0801, 053 (2008)
S.-w. Li, A. Schmitt, Q. Wang, PRD 92, 026006 (2015)
M. Elliot-Ripley, P. Sutcliffe and M. Zamaklar, JHEP 1610, 088 (2016)

Setup

- D8-brane action

$$S = \underbrace{T_8 V_4 \int_{x^\mu} \int_z e^{-\Phi} \sqrt{\det(g + 2\pi\alpha' F)}}_{\text{Dirac-Born-Infeld (DBI)}} + \underbrace{\frac{N_c}{8\pi^2} \int_{x^\mu} \int_z \hat{A}_0 \text{Tr}[F_{ij} F_{kz}] \epsilon_{ijk}}_{\text{Chern-Simons (CS)}}$$

- gauge fields in the bulk (\rightarrow global symmetry at the boundary)

$$N_f = 2 : \quad F_{\mu\nu} = \hat{F}_{\mu\nu} + F_{\mu\nu}^a \sigma_a$$

- abelian part $U(1)$: chemical potential $\mu = \hat{A}_0(z = \pm\infty)$

- non-abelian part $SU(2)$: baryon number (instantons)

$$N_B = -\frac{1}{8\pi^2} \int_{\vec{x}} \int_z \text{Tr}[F_{ij} F_{kz}] \epsilon_{ijk}$$

Many-instanton approximation (page 1/2)

- ansatz for interacting many-instanton system from flat-space instantons

$$F^2 \simeq \sum_{n=1}^{N_I} F_0^2(n) + \frac{1}{2} \sum_{n=1}^{N_I} \sum_{n \neq m}^{N_I} \mathcal{I}(m, n)$$

- single instanton with width $\rho = \rho_0 u_c^{3/4}$ and deformation $\gamma = \gamma_0 u_c^{3/2}$

$$F_0^2(n) \sim \frac{\rho^4}{[(\vec{x} - \vec{x}_n)^2 + \frac{z^2}{\gamma^2} + \frac{\rho^2}{\gamma^2}]^4}$$

- 2-body interaction from exact 2-instanton solution $F(m, n)$

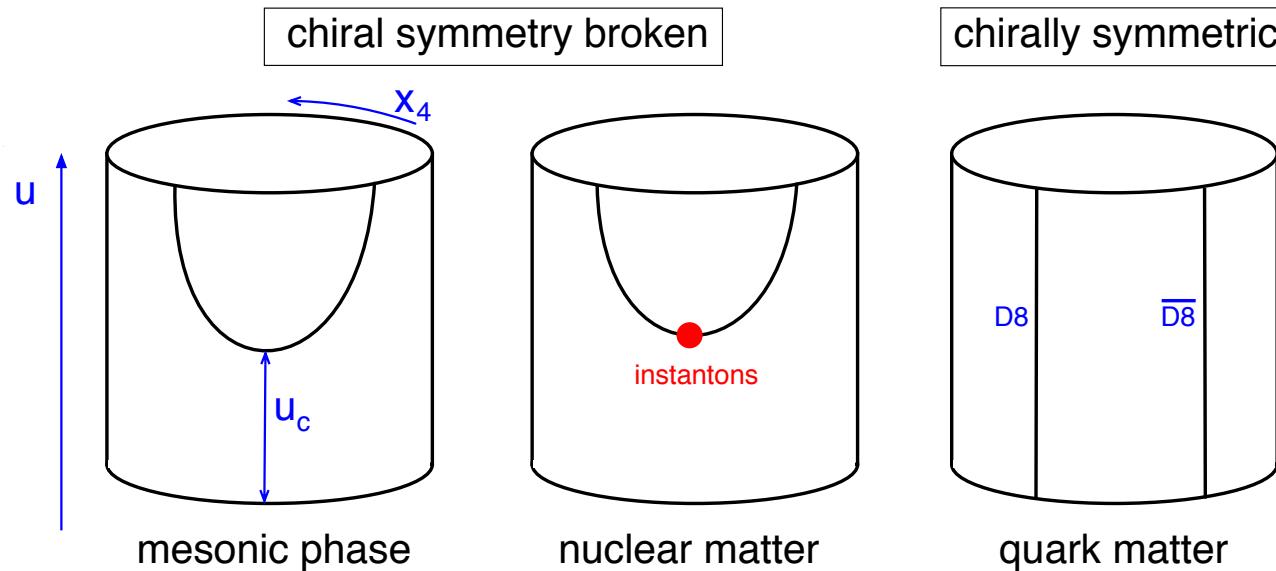
ADHM: M. F. Atiyah, N. J. Hitchin, V. G. Drinfeld and Y. I. Manin, PLA 65, 185 (1978)
 nucleon interaction in Sakai-Sugimoto: K. Y. Kim and I. Zahed, JHEP 0903, 131 (2009)
 K. Hashimoto, T. Sakai and S. Sugimoto, Prog. Theor. Phys. 122, 427 (2009)

$$\mathcal{I}(m, n) = F^2(m, n) - F_0^2(m) - F_0^2(n)$$

- choose lattice structure in \mathbb{R}^3 , apply nearest-neighbor approximation, neglect layers in holographic direction
- assume $SU(2)$ orientation of instantons to be the same
- take spatial average $F^2 \rightarrow \langle F^2 \rangle$

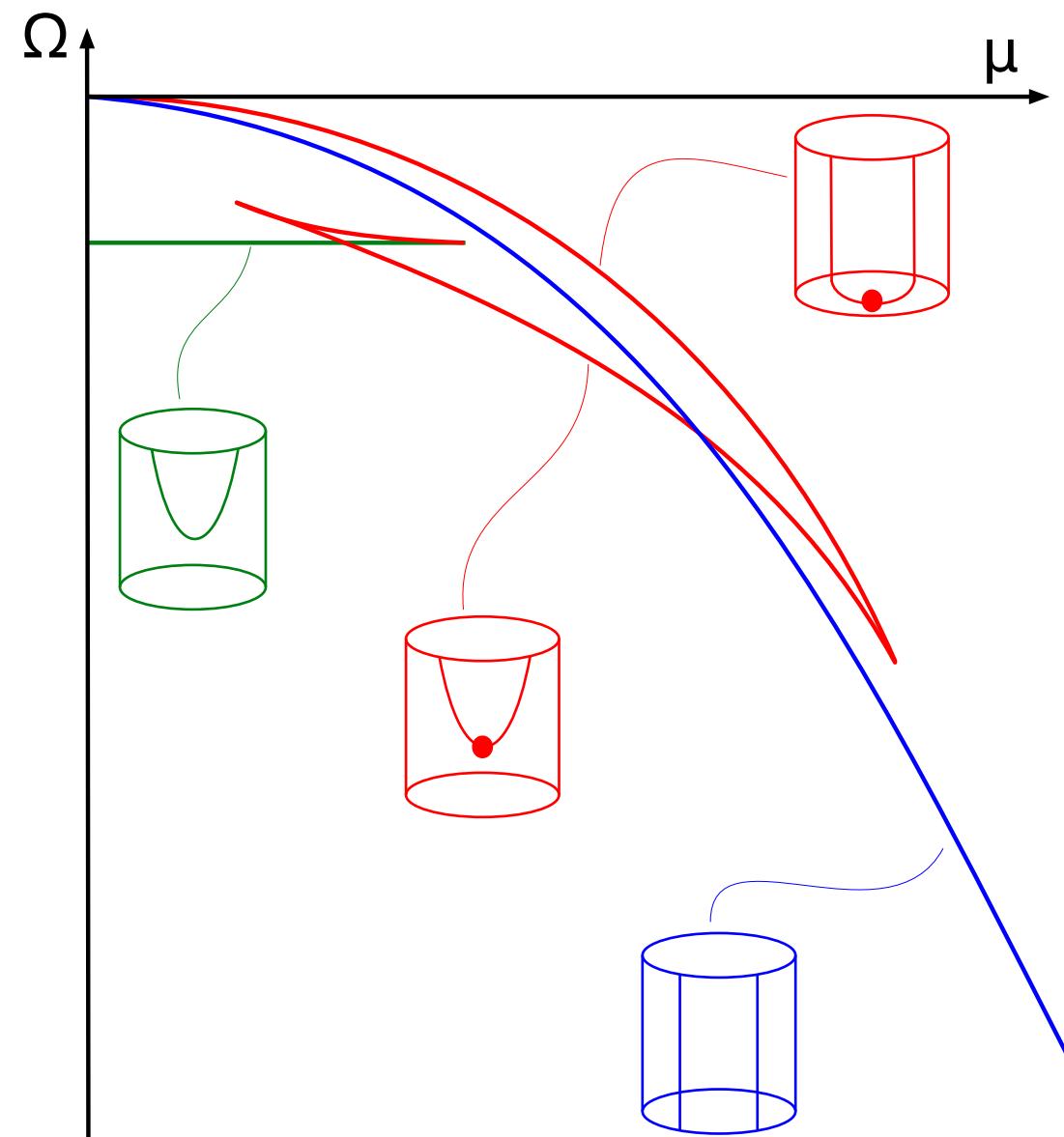
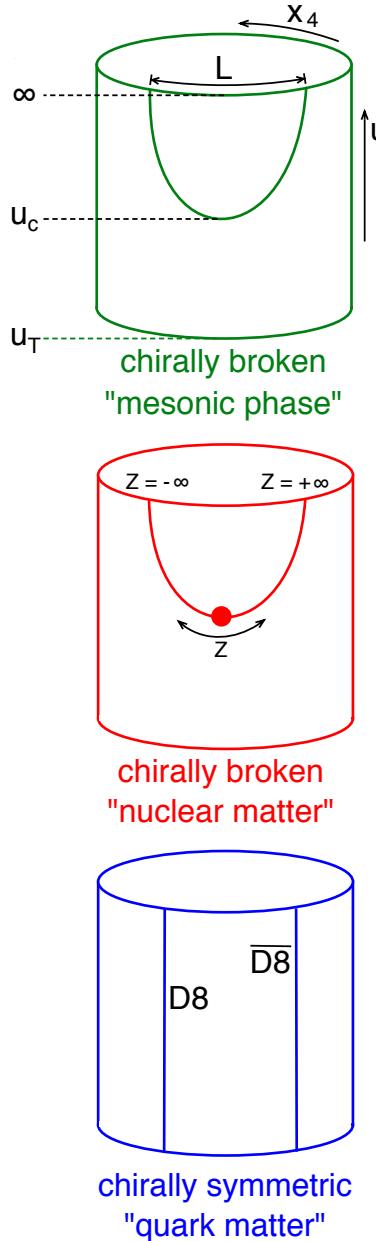
Many-instanton approximation (page 2/2)

- solve EOMs for $U(1)$ gauge field $\hat{A}_0(u)$ and embedding $x_4(u)$
- minimize free energy wrt u_c , N_I for given μ (and T)

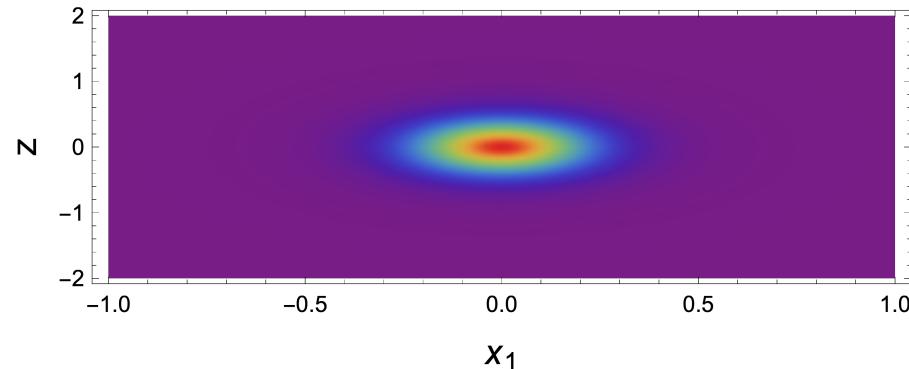
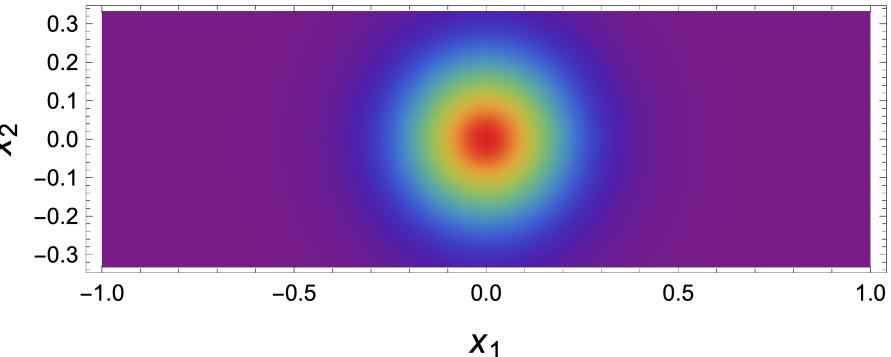


- compare free energy of all three phases
- 5 parameters (M_{KK} , L , λ , ρ_0 , γ_0):
scan parameter space F. Preis, A. Schmitt, JHEP 1607, 001 (2016)
or fit parameters to low-density nuclear matter properties
K. Bitaghsir Fadafan, F. Kazemian, A. Schmitt, JHEP 1903, 183 (2019)

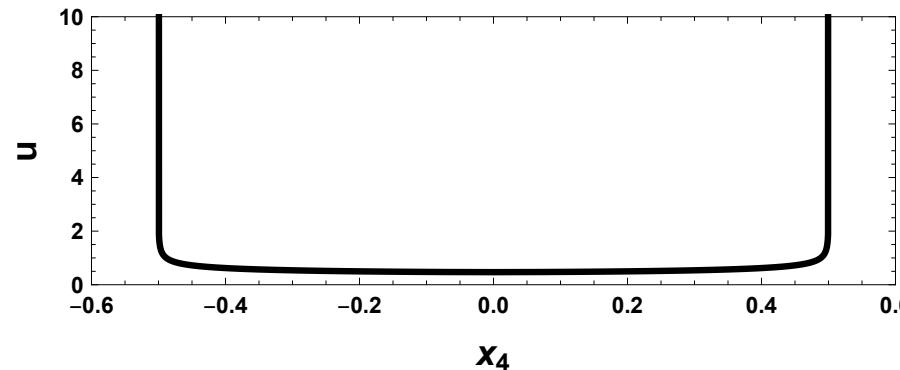
Main result (schematically)



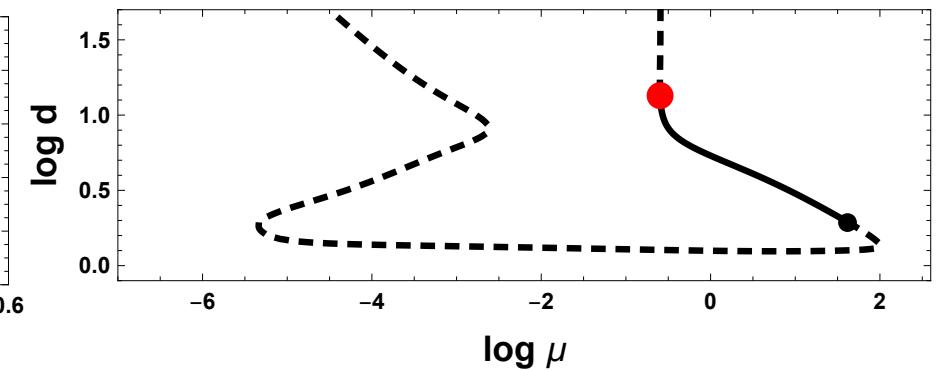
Instanton profiles

deformation from $\text{SO}(4)$ 

instanton lattice

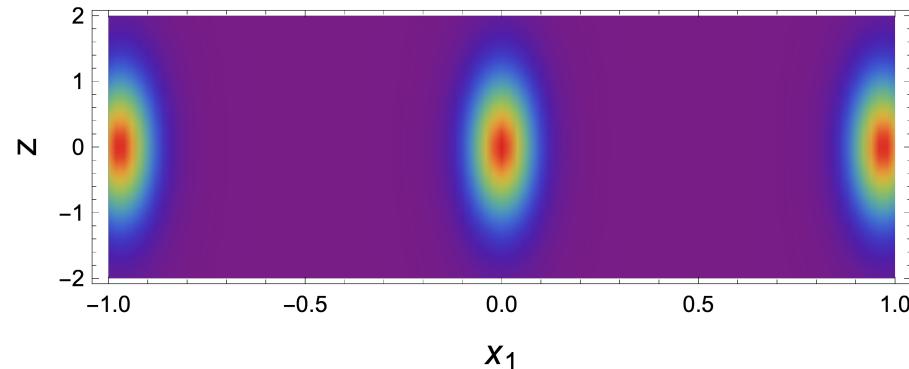
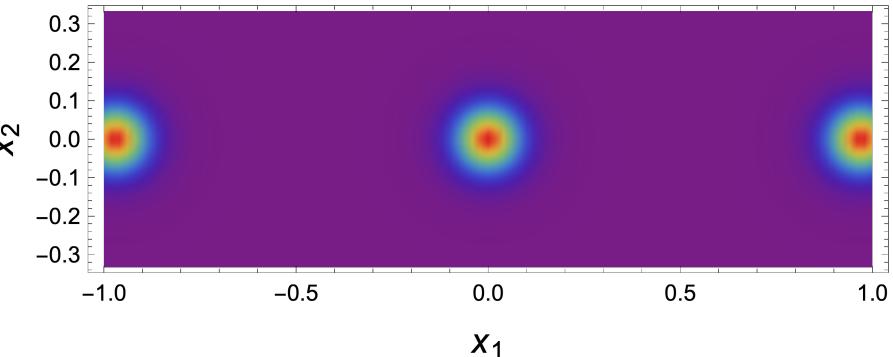


flavor brane embedding

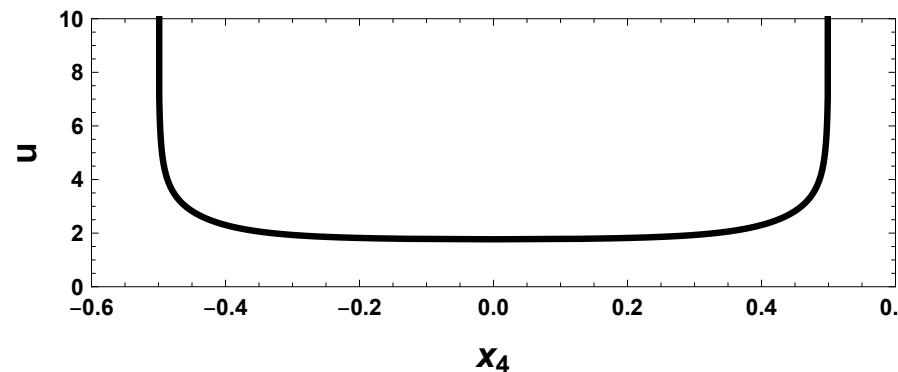


instanton overlap

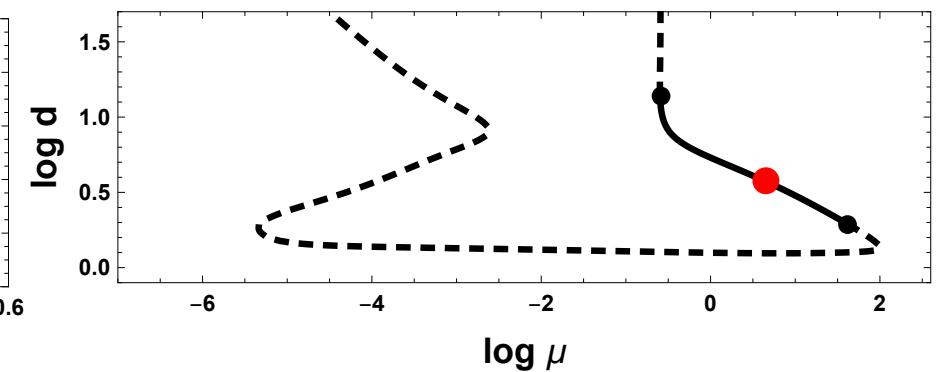
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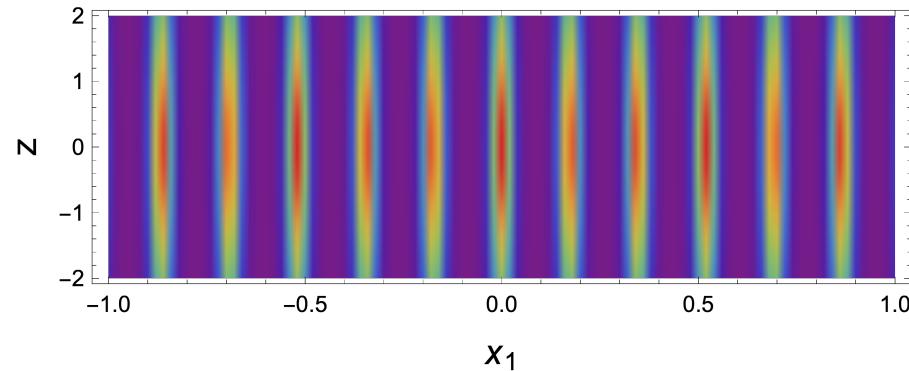
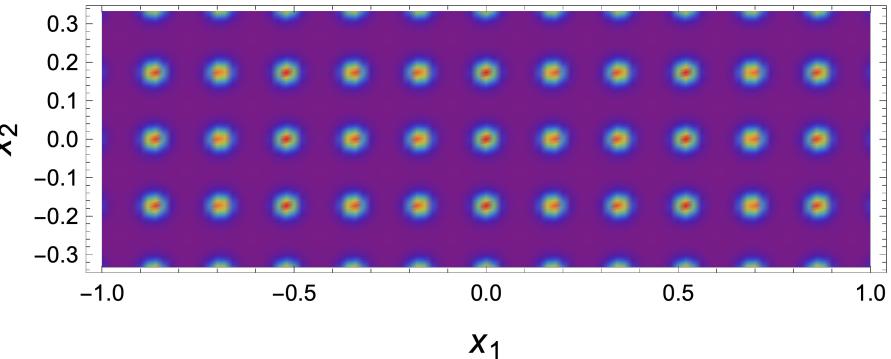


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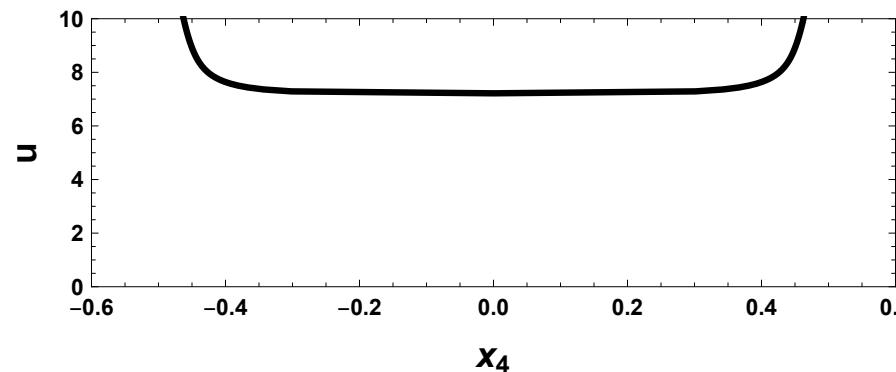


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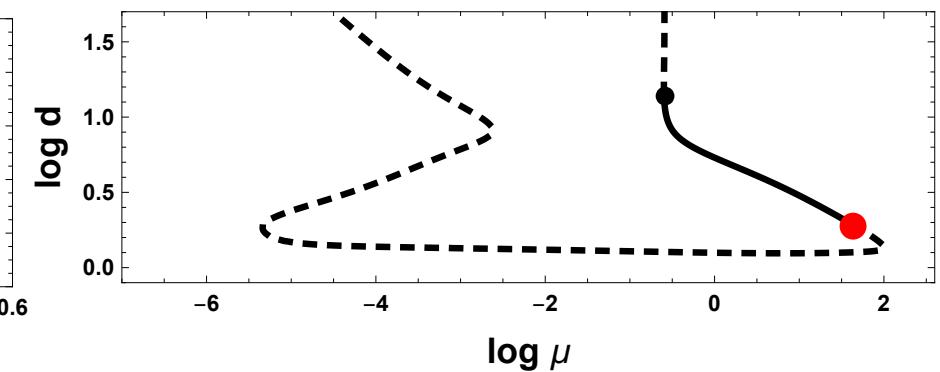
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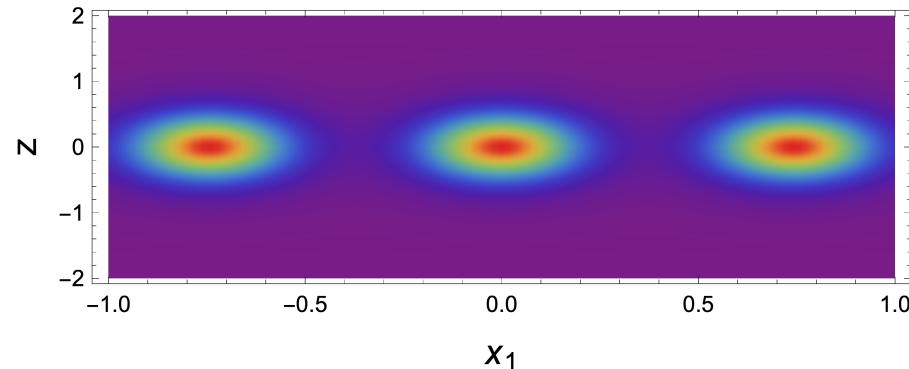
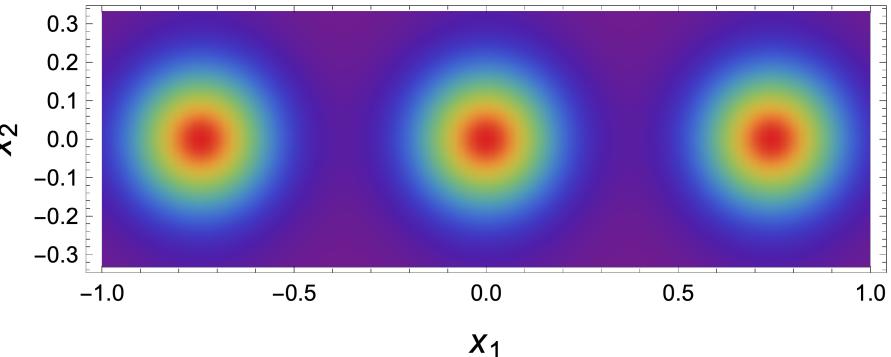


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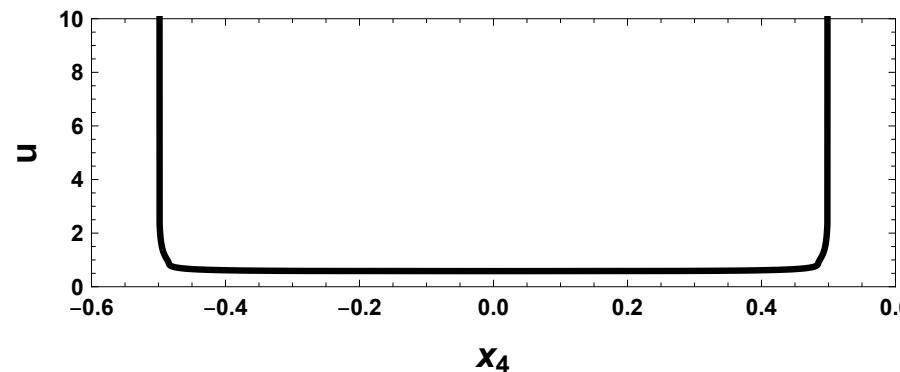


instanton overlap

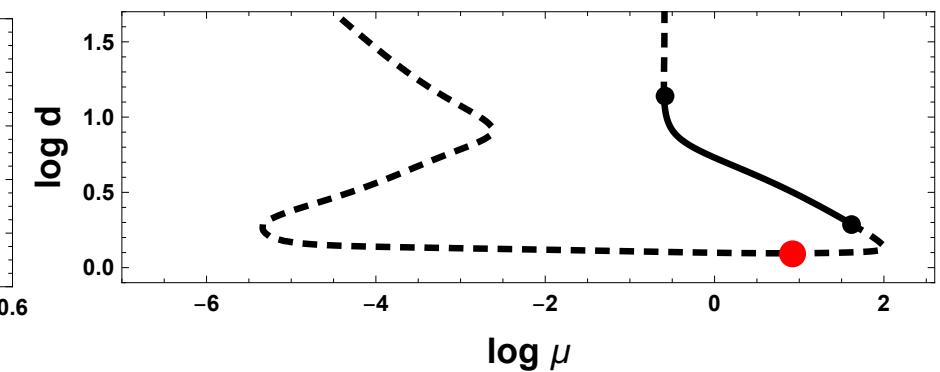
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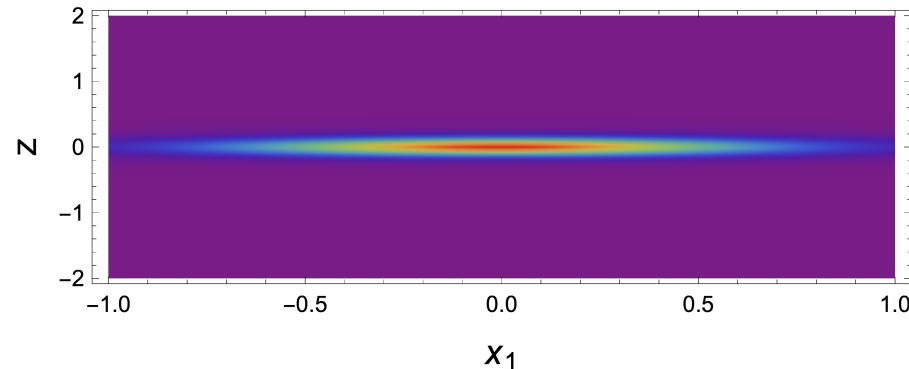
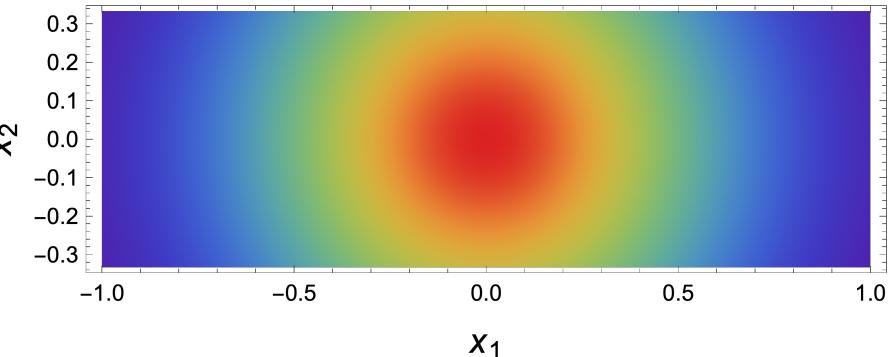


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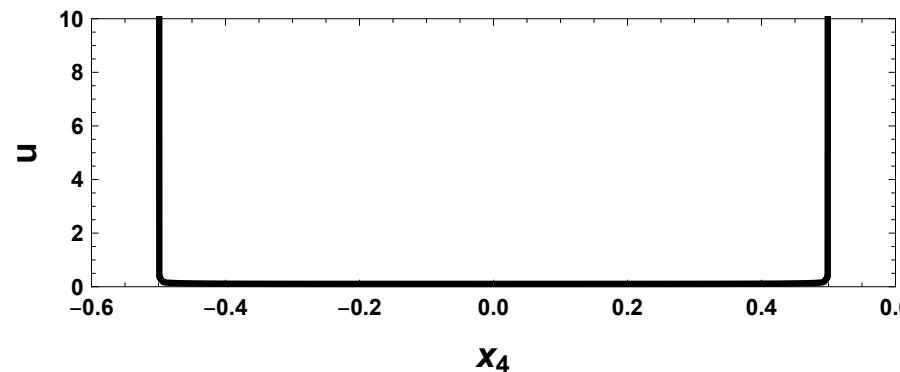


instanton overlap

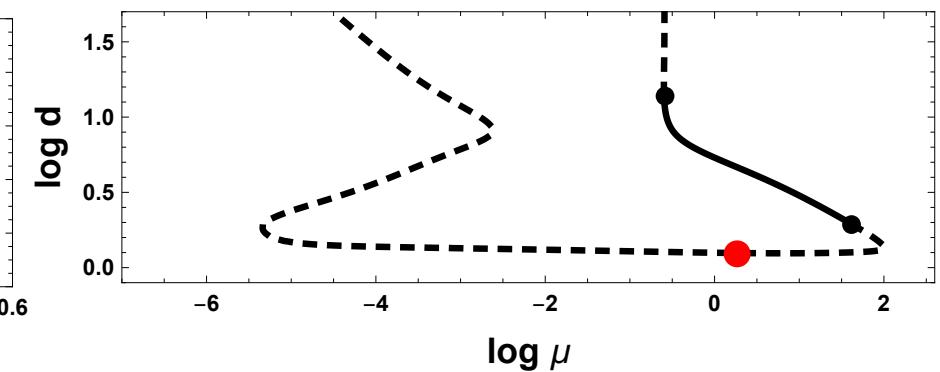
Instanton profiles

deformation from $\text{SO}(4)$ 

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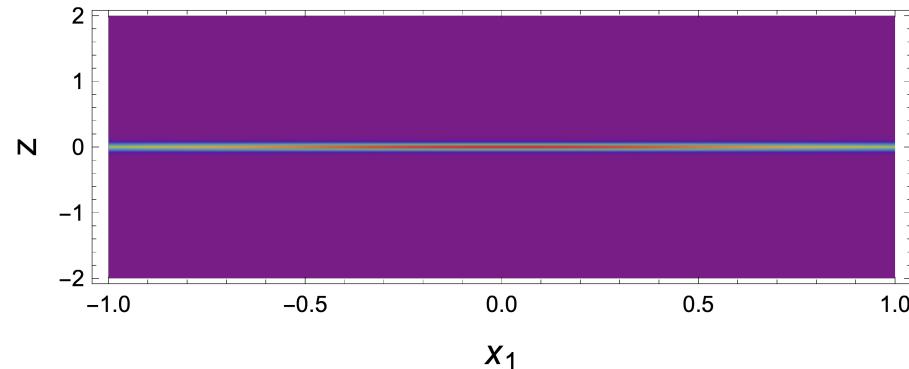
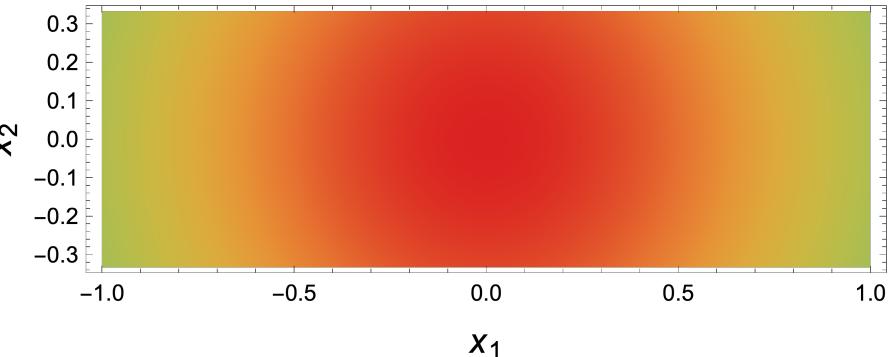


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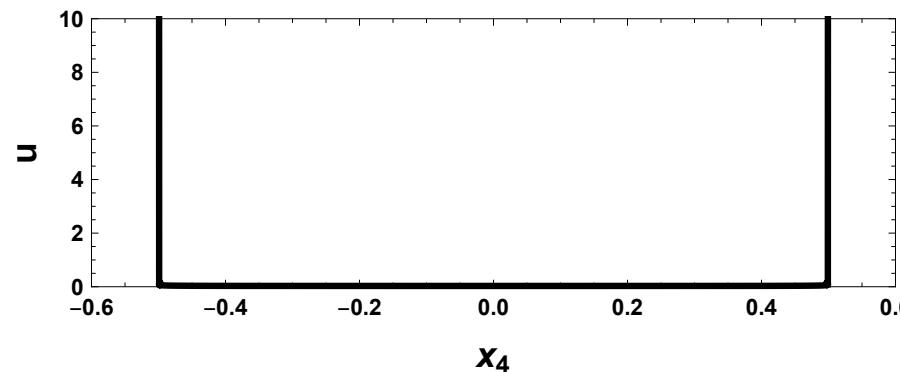


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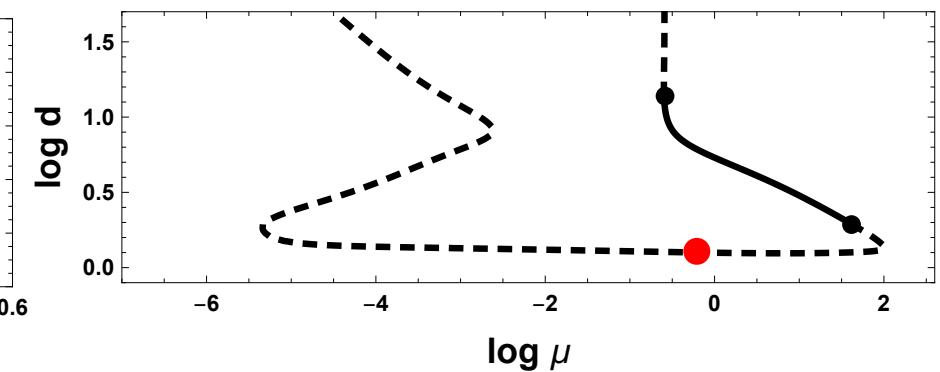
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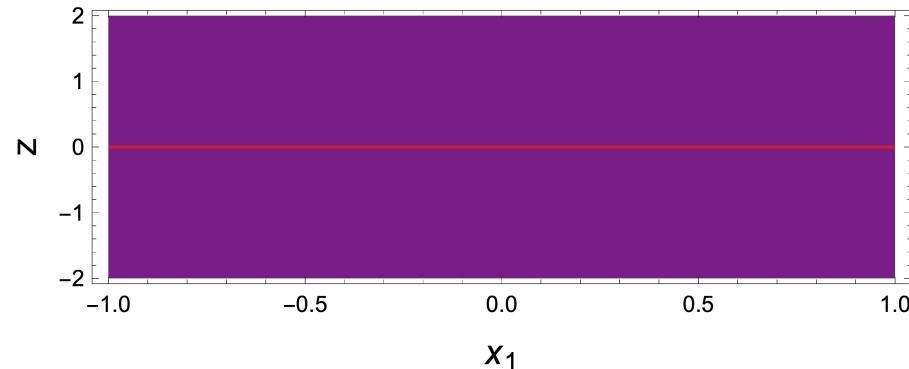
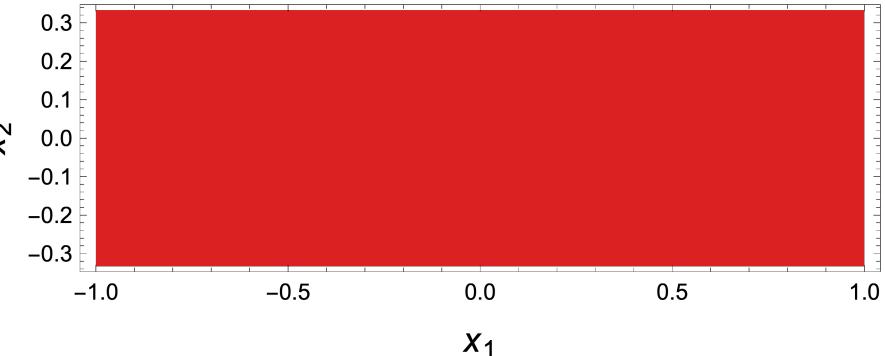


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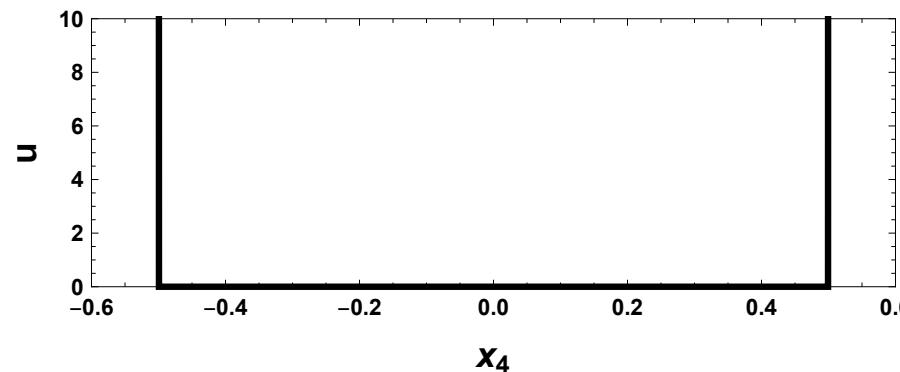


instanton overlap

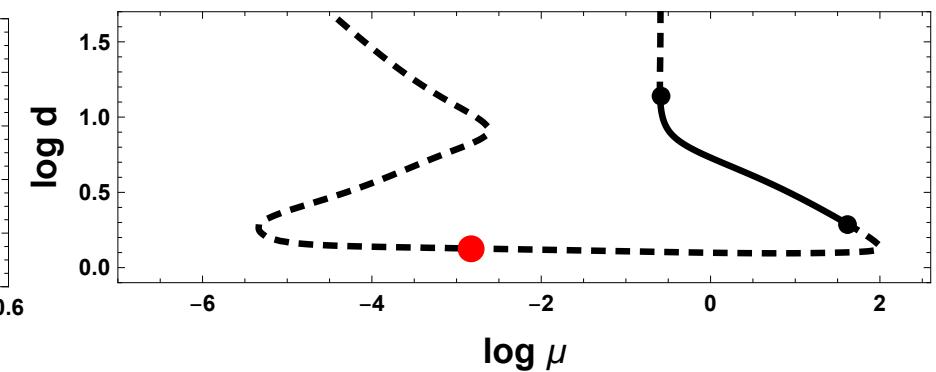
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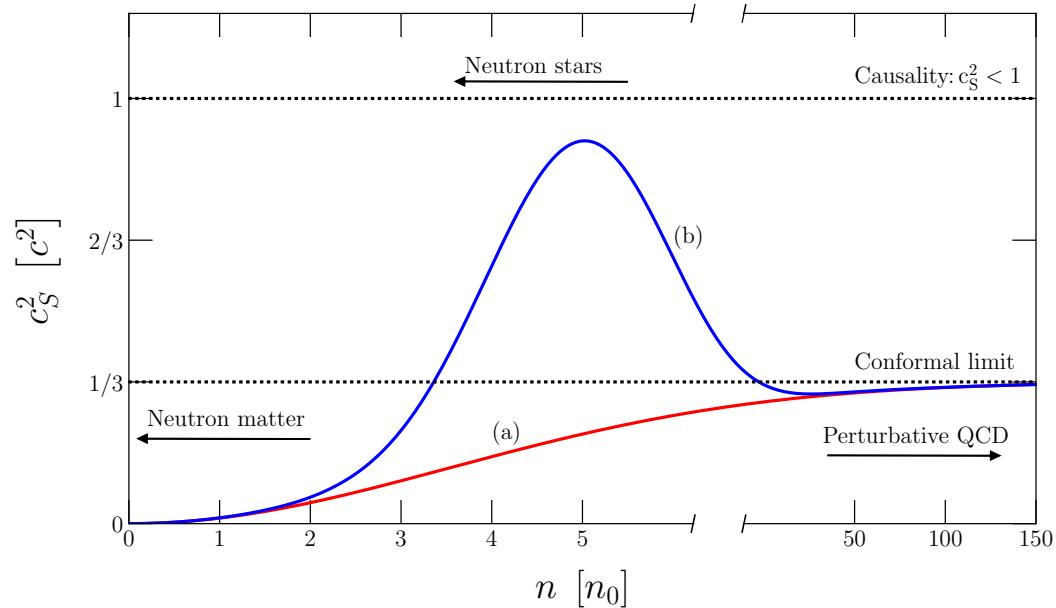


instanton overlap

Observations

- hadron and quark phases connect continuously
(instanton interactions crucial!)
 - geometrically: continuous transformation between connected and disconnected flavor branes
 - instantons smear out in spatial direction and become infinitesimally thin in holographic direction
 - continuity at zero chemical potential due to masslessness of quarks
(like a second-order phase transition)
- instantons avoid overlapping by becoming smaller at high density
- actual quark-hadron transition is of first order and at extremely large $\mu \simeq 30 \text{ GeV}$ (compact stars: $\mu \lesssim 0.5 \text{ GeV}$)

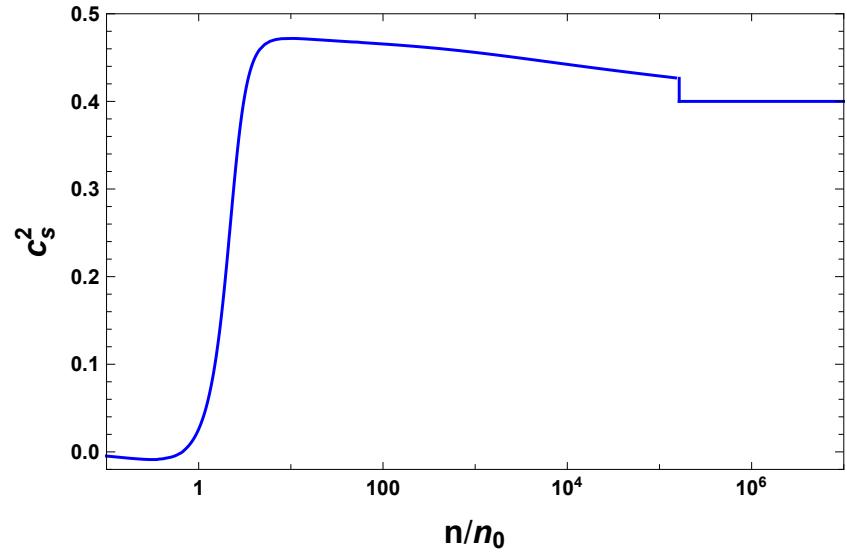
Speed of sound



sound speed
 \leftrightarrow stiffness of matter
 \leftrightarrow neutron star masses
 schematic plot from I. Tews *et al.*,
 Astrophys. J. 860, 149 (2018)

- Sakai-Sugimoto: non-monotonic speed of sound
 K. Bitaghsir Fadafan, F. Kazemian,
 A. Schmitt, JHEP 1903, 183 (2019)

- see also: quarkyonic speed of sound
 L. McLerran, S. Reddy,
 PRL 122, 122701 (2019)



Summary

- location and nature of the quark-hadron transition at large baryon densities is unknown (sign problem)
- a potential first-order quark-hadron transition has observable consequences for neutron star physics
- Sakai-Sugimoto model allows for consistent treatment of nuclear and quark matter
- if instanton interactions are included, nuclear and quark matter phases are continuously connected

Outlook

- include nonzero quark masses
N. Kovensky, A. Schmitt, in preparation
- isospin asymmetry → from symmetric nuclear matter to neutron star matter
- nonzero temperature and/or magnetic field → phase diagrams
- equation of state → neutron star mass/radius, deformability