

testing photon and dilepton rates in thermal QCD¹

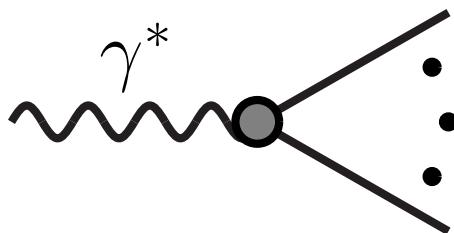
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¹ supported by the SNF under grant 200020-168988

motivation

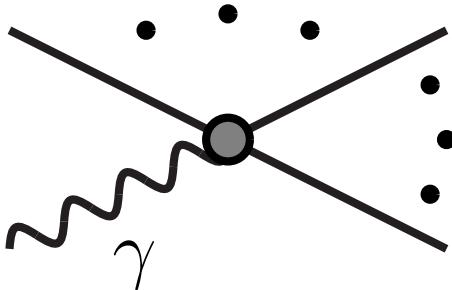
in vacuum, only virtual photons carry spectral weight



$$M^2 = \omega^2 - k^2 > 0$$

quantum-mechanical language is useful, e.g. $\gamma^* \rightarrow \pi^+ \pi^-$

in a thermal medium, spectral weight extends to $M^2 \leq 0$



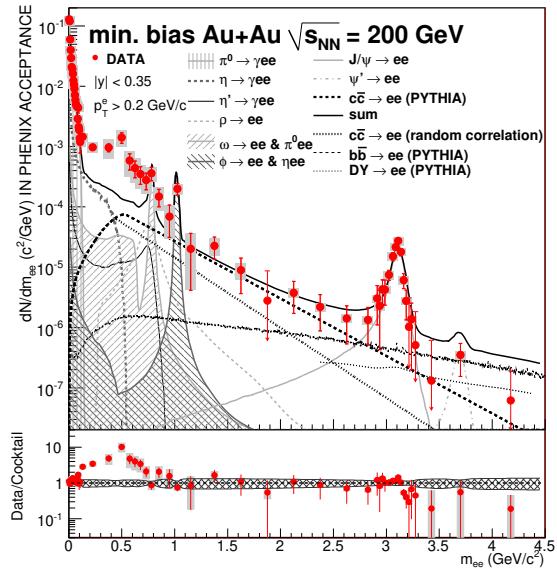
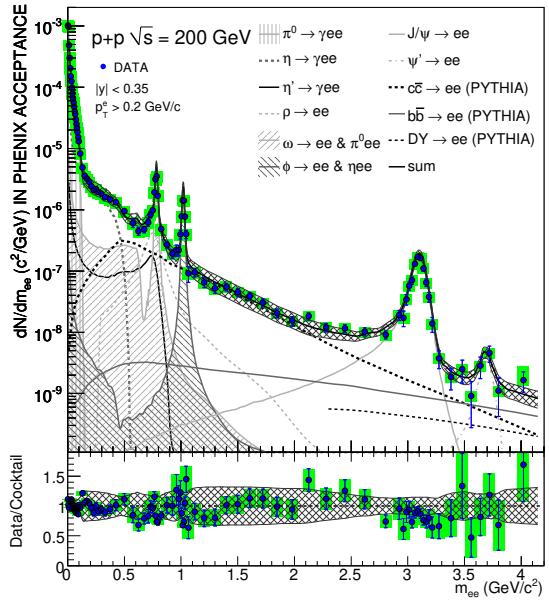
now a better language is that of classical plasma physics

the spectral function should be smooth across the light-cone²

\Rightarrow basic prediction: $T > 0$ adds spectral weight at small M^2

² S. Caron-Huot, *O(g) plasma effects in jet quenching*, 0811.1603

a medium can be searched for in dilepton (γ^*) data³



³ A. Adare *et al.* [PHENIX Collaboration], *Detailed measurement of the $e^+ e^-$ pair continuum in $p + p$ and $Au+Au$ collisions at $\sqrt{s_{NN}} = 200 \text{ GeV}$ and implications for direct photon production*, 0912.0244

status on the pQCD side

basic relations (assuming thermal equilibrium)⁴

dilepton rate:

$$\frac{d\Gamma_{\mu^-\mu^+}(\omega, k)}{d\omega d^3k} \approx \frac{\alpha_{\text{em}}^2 n_B(\omega)}{3\pi^3 M^2} \sum_{i=1}^{N_f} Q_i^2 \rho_V(\omega, k) + \mathcal{O}(\alpha_{\text{em}}^3)$$

photon rate:

$$\frac{d\Gamma_\gamma(k)}{d^3k} = \frac{\alpha_{\text{em}} n_B(k)}{2\pi^2 k} \sum_{i=1}^{N_f} Q_i^2 \rho_V(k, k) + \mathcal{O}(\alpha_{\text{em}}^2)$$

here $n_B(\omega) \equiv 1/[\exp(\omega/T) - 1]$ is the bose distribution

⁴ L.D. McLerran and T. Toimela, *Photon and Dilepton Emission from the Quark-Gluon Plasma: Some General Considerations*, PRD31(85)545; H.A. Weldon, *Reformulation of Finite Temperature Dilepton Production*, PRD42(90)2384; C. Gale and J.I. Kapusta, *Vector dominance model at finite temperature*, NPB357(91)65

leading-order result

definition of spectral function ($\eta \equiv (- +++)$):

$$\begin{aligned}\rho_V(\omega, k) &= \text{Im } \Pi_R(\omega, k) \\ &\equiv \int_{\mathcal{X}} e^{i(\omega t - \mathbf{k} \cdot \mathbf{x})} \left\langle \frac{1}{2} [V^\mu(t, \mathbf{x}), V_\mu(0)] \right\rangle\end{aligned}$$

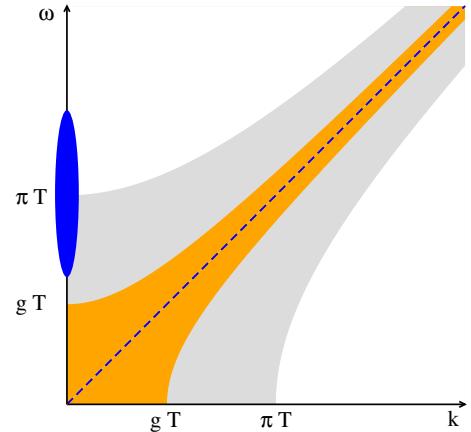
expression vanishes for $M^2 \rightarrow 0$:

$$\rho_V(\omega, k) = \frac{N_c T M^2}{2\pi k} \ln \left\{ \frac{\cosh(\frac{\omega+k}{4T})}{\cosh(\frac{\omega-k}{4T})} - \frac{\omega \theta(k-\omega)}{2T} \right\}$$

NLO result at vanishing momentum ($k = 0$)⁵



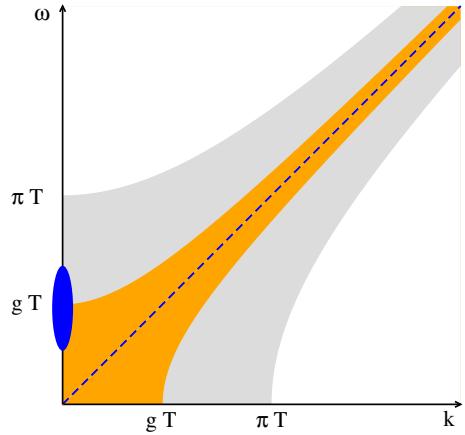
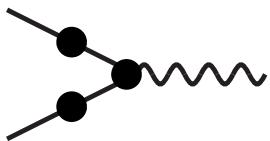
etc



⇒ only a small correction

⁵ R. Baier, B. Pire and D. Schiff, *Dilepton production at finite temperature: Perturbative treatment at order α_s* , PRD38(88)2814; Y. Gabellini, T. Grandou and D. Poizat, *Electron-positron annihilation in thermal QCD*, AP202(90)436; T. Altherr and P. Aurenche, *Finite temperature QCD corrections to lepton-pair formation in a quark-gluon plasma*, ZPC45(89)99

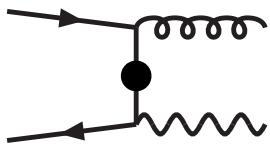
hard thermal loop (HTL) resummation in soft regime⁶



⇒ a large enhancement

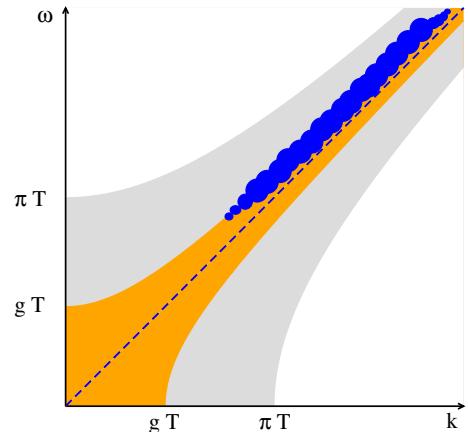
⁶ E. Braaten, R.D. Pisarski and T.-C. Yuan, *Production of soft dileptons in the quark-gluon plasma*, PRL64(90)2242

HTL resummation for hard momenta ($k \sim \pi T$)⁷



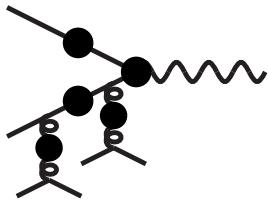
if $M \rightarrow 0$ there is a singularity
from soft t -channel exchange, which
is regulated by landau damping

$$\rho_V = \dots + \frac{\alpha_s N_c C_F T^2}{4} \ln \left(\frac{T^2}{[M^2 \rightarrow \alpha_s T^2]} \right)$$

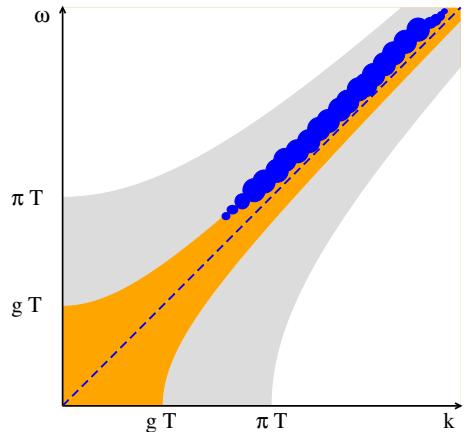


⁷ J.I. Kapusta, P. Lichard and D. Seibert, *High-energy photons from quark-gluon plasma versus hot hadronic gas*, PRD44(91)2774 [Erratum-ibid. D47(93)4171]; R. Baier, H. Nakagawa, A. Niégawa and K. Redlich, *Production rate of hard thermal photons and screening of quark mass singularity*, ZPC53(92)433; T. Altherr and P.V. Ruuskanen, *Low mass dileptons at high momenta in ultrarelativistic heavy ion collisions*, NPB380(92)377

LPM resummation for hard momenta ($k \sim \pi T$)^{8,9}



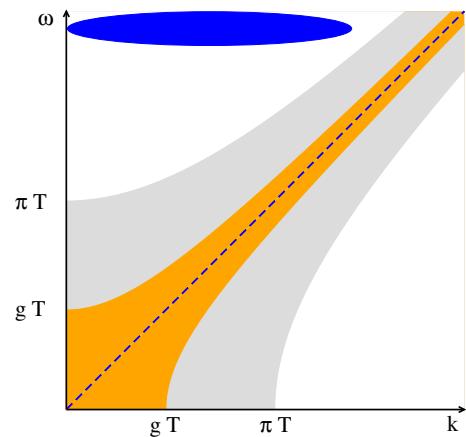
removing the divergence is not enough: there are terms of similar magnitude from multiple scatterings with collinear enhancement



⁸ P.B. Arnold, G.D. Moore and L.G. Yaffe, *Photon emission from ultrarelativistic plasmas*, hep-ph/0109064; hep-ph/0111107; P. Aurenche, F. Gelis, G.D. Moore and H. Zaraket, *Landau-Pomeranchuk-Migdal resummation for dilepton production*, hep-ph/0211036; M.E. Carrington, A. Gynther and P. Aurenche, *Energetic di-leptons from the Quark Gluon Plasma*, 0711.3943

⁹ NLO: J. Ghiglieri *et al*, *Next-to-leading order thermal photon production in a weakly coupled quark-gluon plasma*, 1302.5970; J. Ghiglieri and G.D. Moore, *Low Mass Thermal Dilepton Production at NLO in a Weakly Coupled Quark-Gluon Plasma*, 1410.4203

for $M \gg \pi T$ the vacuum term dominates^{10,11}



$$\rho_V = \frac{N_c M^2}{4\pi} \left(1 + \frac{3\alpha_s C_F}{4\pi} \right) + \frac{4\alpha_s N_c C_F}{9} \left(1 + \frac{4k^2}{3M^2} \right) \frac{\pi^2 T^4}{M^2} + \mathcal{O}\left(\frac{\alpha_s T^6}{M^4}\right)$$

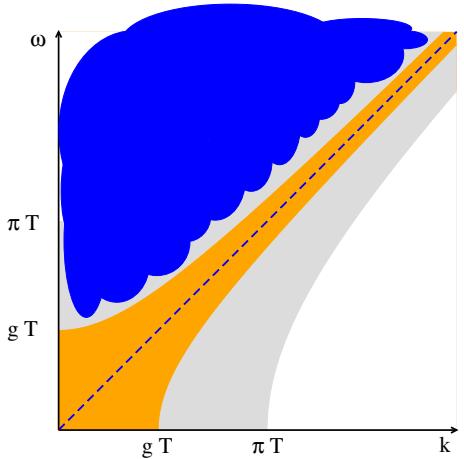
¹⁰ S. Caron-Huot, *Asymptotics of thermal spectral functions*, 0903.3958

¹¹ NNNLO: Y. Burnier and ML, *Towards flavour diffusion coefficient and electrical conductivity without UV contamination*, 1201.1994; P.A. Baikov, K.G. Chetyrkin and J.H. Kühn, *Order α_s^4 QCD Corrections to Z and τ Decays*, 0801.1821

away from light-cone no resummation is needed¹²



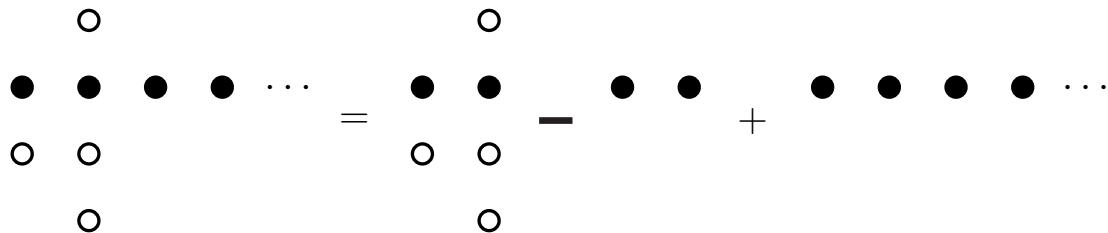
compute the NLO imaginary-time
correlator and take the cut:



$$\begin{aligned} \Pi_E(K) &\equiv \int_0^{1/T} d\tau \int_{\mathbf{x}} e^{iK \cdot X} \langle (\bar{\psi} \gamma^\mu \psi)(\tau, \mathbf{x}) (\bar{\psi} \gamma_\mu \psi)(0, \mathbf{0}) \rangle_T \\ \Pi_R(\mathcal{K}) &= \Pi_E|_{k_n \rightarrow -i[\omega + i0^+]} , \quad \rho_V = \text{Im } \Pi_R \end{aligned}$$

¹² ML, Thermal 2-loop master spectral function at finite momentum, 1304.0202; NLO thermal dilepton rate at non-zero momentum, 1310.0164

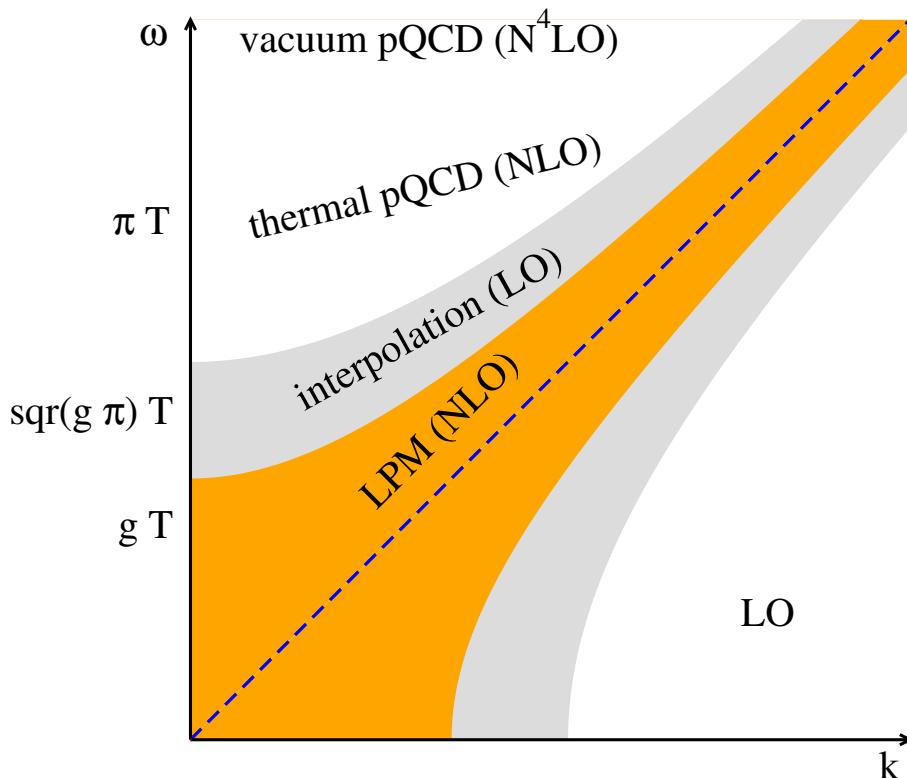
NLO and LPM results can be interpolated into each other¹³



$$\rho_V|_{\text{resummed NLO}} \equiv \rho_V|_{\text{NLO}} - \rho_V|_{\text{LPM}}^{\text{expanded}} + \rho_V|_{\text{LPM}}^{\text{full}}$$

¹³ I. Ghisoiu and ML, *Interpolation of hard and soft dilepton rates*, 1407.7955

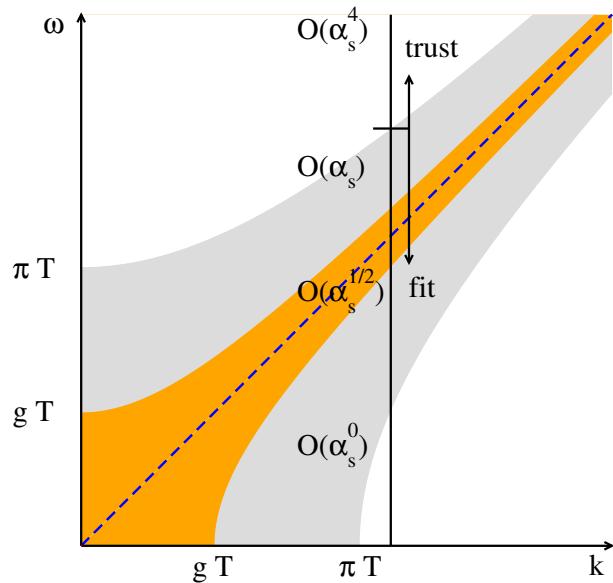
summary: different techniques needed in different regimes



what can lattice do?

a first attempt in quenched QCD¹⁴

$$G_V(\tau, k) = \int_0^\infty \frac{d\omega}{\pi} \rho_V(\omega, k) \frac{\cosh[\omega(\frac{\beta}{2} - \tau)]}{\sinh[\frac{\omega\beta}{2}]} , \quad \beta \equiv \frac{1}{T}$$



¹⁴ J. Ghiglieri, O. Kaczmarek, ML, F. Meyer, *Lattice constraints on the thermal photon rate*, 1604.07544

polynomial interpolation (assuming smoothness, $V \rightarrow \infty$)

we pick a point above which pQCD is reasonable, for instance

$$\omega_0 \simeq \sqrt{k^2 + (\pi T)^2} ,$$

and use that to fix two coefficients from pQCD:

$$\beta \equiv \rho_V(\omega_0, k) \Big|_{\text{resummed NLO}} , \quad \gamma \equiv \partial_\omega \rho_V(\omega_0, k) \Big|_{\text{resummed NLO}}$$

then the most general polynomial odd in ω takes the form¹⁵

$$\rho_{\text{fit}} \equiv \frac{\beta \omega^3}{2\omega_0^3} \left(5 - \frac{3\omega^2}{\omega_0^2} \right) - \frac{\gamma \omega^3}{2\omega_0^2} \left(1 - \frac{\omega^2}{\omega_0^2} \right) + \sum_{n \geq 0}^{n_{\max}} \frac{\delta_n \omega^{1+2n}}{\omega_0^{1+2n}} \left(1 - \frac{\omega^2}{\omega_0^2} \right)^2$$

¹⁵ replace polynomial by padé: B.B. Brandt, A. Francis, T. Harris, H.B. Meyer and A. Steinberg, *An estimate for the thermal photon rate from lattice QCD*, 1710.07050

lattice details

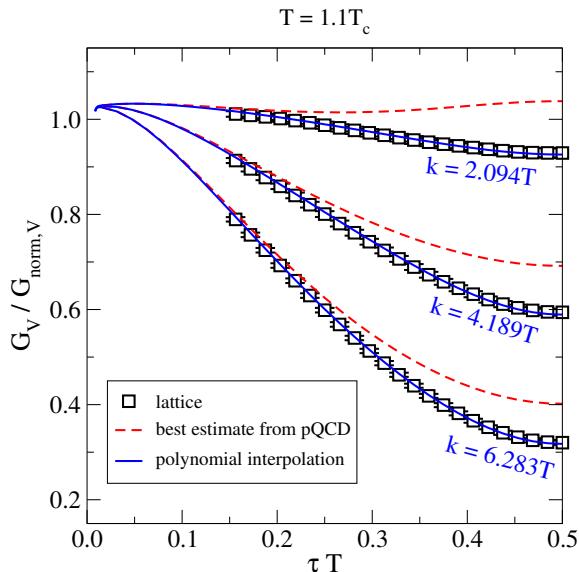
imaginary-time observable:

$$G_V(\tau, k) \equiv \int_{\mathbf{x}} e^{-i\mathbf{k} \cdot \mathbf{x}} \langle V^i(\tau, \mathbf{x}) V^i(0) - V^0(\tau, \mathbf{x}) V^0(0) \rangle_c$$

ensemble:

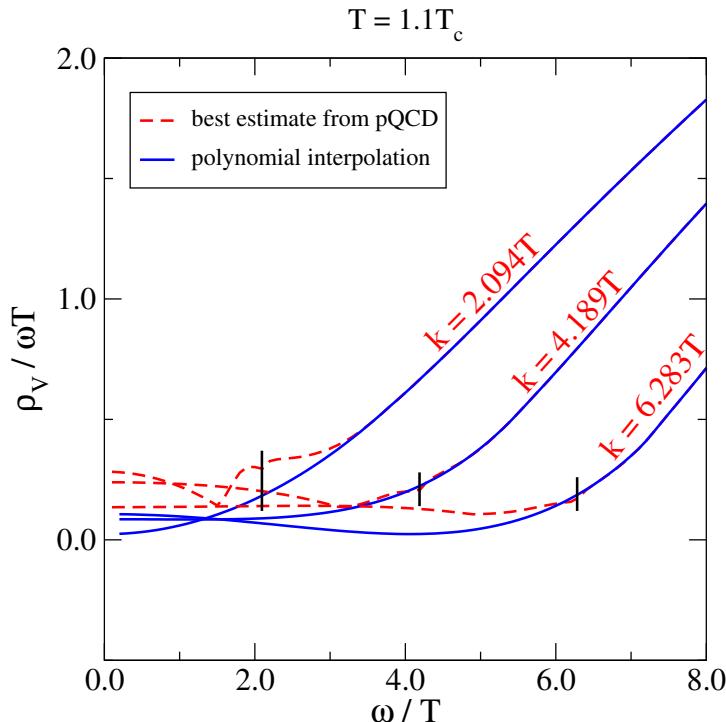
β_0	$N_s^3 \times N_\tau$	confs	$T/T_c _{t_0}$	k/T
7.192	$96^3 \times 32$	314	1.12	2.094, 4.189, 6.283
7.544	$144^3 \times 48$	358	1.14	
7.793	$192^3 \times 64$	242	1.15	
7.192	$96^3 \times 28$	232	1.28	1.833, 3.665, 5.498
7.544	$144^3 \times 42$	417	1.31	
7.793	$192^3 \times 56$	273	1.31	

imaginary-time correlators after continuum extrapolation



$$\frac{G_{\text{norm},V}}{6T^3} \equiv \pi(1 - 2\tau T) \frac{1 + \cos^2(2\pi\tau T)}{\sin^3(2\pi\tau T)} + \frac{2\cos(2\pi\tau T)}{\sin^2(2\pi\tau T)}$$

one-parameter best fits for $\rho_V/(\omega T)$ ($\delta_0 \sim$ intercept)

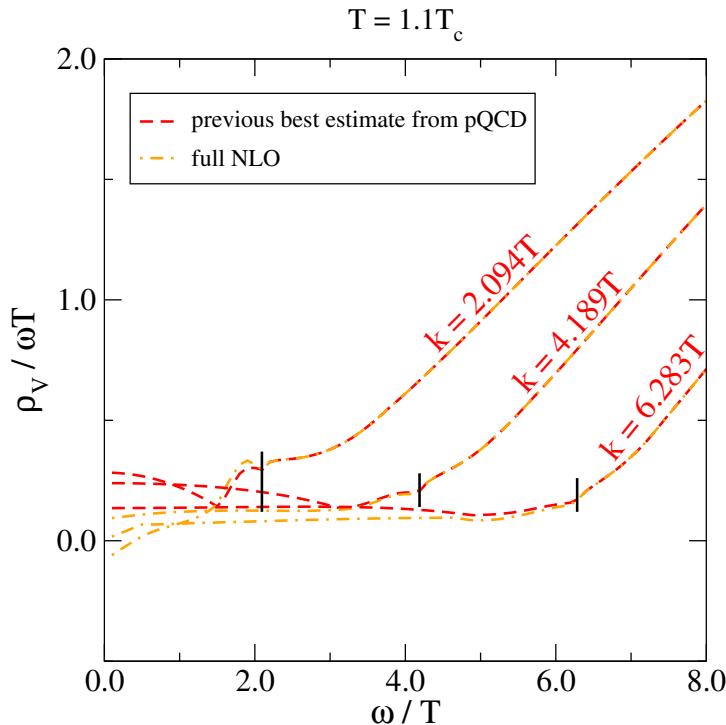


there is a clear reduction in the spacelike domain $M^2 < 0$

towards a better understanding of the spacelike domain¹⁶

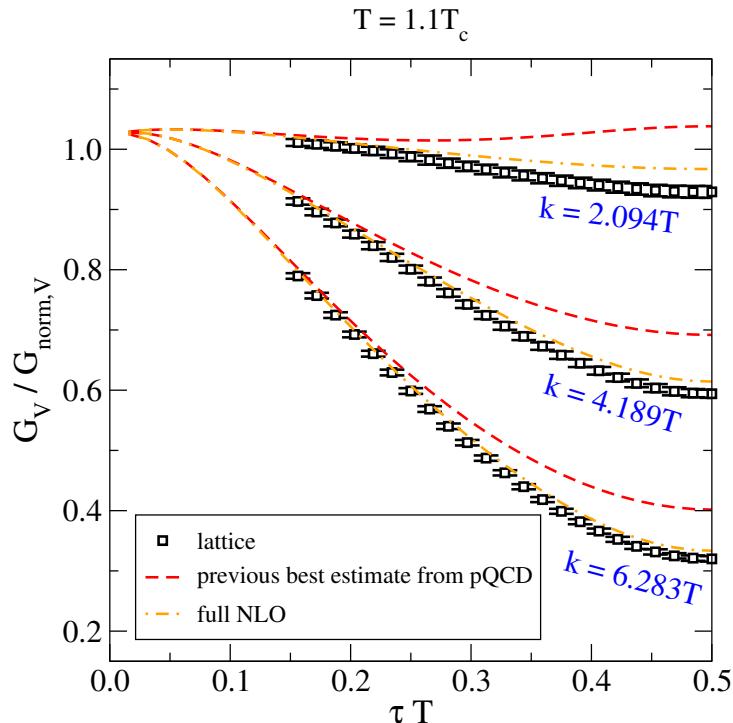
¹⁶ thanks to Harvey Meyer for inspiration and encouragement

what if replace fit by an NLO computation for $M^2 < 0$?¹⁷



¹⁷ G. Jackson, 50+ page draft, work in progress; G. Jackson and ML, work in progress

the corresponding imaginary-time correlators



agreement is even better at $T = 1.3T_c$

summary and outlook

lattice and AdS folks might prefer pQCD to fail...

here nice results exist and they seem to work reasonably well

⇒ sanity check for spectral reconstruction / input for hydro?

backup: ρ_V should indeed become negative in the very IR

for $\omega, k \ll T$ the general theory of statistical fluctuations applies,¹⁸ and permits for a “hydrodynamic” prediction:¹⁹

$$\frac{\rho_V(\omega, k)}{\omega} = \left(\frac{\omega^2 - k^2}{\omega^2 + D^2 k^4} + 2 \right) \chi_q D$$

here D is the diffusion coefficient, and χ_q parametrizes the constant correlator $\langle V^0(\tau, \mathbf{0})V^0(0, \mathbf{0}) \rangle = \chi_q T$

therefore $\lim_{\omega \rightarrow 0} \rho_V(\omega, k)/\omega$ crosses zero at $k = 1/(\sqrt{2}D)$

¹⁸ e.g. E.M. Lifshitz and L.P. Pitaevskii, *Statistical Physics, Part 2*, §88-89

¹⁹ e.g. J. Hong and D. Teaney, *Spectral densities for hot QCD plasmas in a leading log approximation*, 1003.0699